ISOLDE Nuclear Reaction and Nuclear Structure Course

FRESCO examples

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Introduction

Input examples for inelastic scattering

Summary of physical ingredients for collective excitations

The main physical ingredient of DWBA and CC calculations are the coupling potentials:

$$V_{i,f}(\mathbf{R}) = \int d\xi \phi_f(\xi)^* V(\mathbf{R},\xi)\phi_i(\xi)$$

- Coulomb excitation:

$$V_{if}(\mathbf{R}) = \sum_{\lambda>0} \frac{4\pi}{2\lambda+1}\frac{Z_t e}{R^{\lambda+1}} \langle f; I_f M_f | \mathcal{M}(E\lambda,\mu) | i; I_i M_i \rangle Y_{\lambda\mu}(\hat{R})$$

- Nuclear excitation (collective model):

$$V_{if}(\mathbf{R}) \simeq -\frac{dV_0}{dR} \sum_{\lambda} \langle f; I_f M_f | \hat{\delta}_{\lambda\mu} | i; I_i M_i \rangle Y_{\lambda\mu}(\hat{R})$$

Reduced matrix elements

Wigner-Eckart theorem (Bohr&Mottelson convention):

$$\langle I_f M_f|\hat{O}_{\lambda\mu}|I_i M_i\rangle = (2I_f+1)^{-1/2}\langle I_f M_f|\lambda\mu I_i M_i\rangle\langle I_f||\hat{O}_\lambda||I_i\rangle_{\rm BM}$$

1. Coulomb case:

$$\langle f; I_f M_f|\mathcal{M}(E\lambda,\mu)|i; I_i M_i\rangle = (2I_f+1)^{-1/2}\langle I_f M_f|\lambda\mu I_i M_i\rangle\langle f; I_f||\mathcal{M}(E\lambda)||i; I_i\rangle$$

2. Nuclear case:

$$\langle f; I_f M_f|\hat{\delta}_{\lambda\mu}|i; I_i M_i\rangle = (2I_f+1)^{-1/2}\langle f; I_f M_f|I_i M_i\lambda\mu\rangle\langle f; I_f||\hat{\delta}_\lambda||i; I_i\rangle$$

So, in general:

$$\Delta V_{if}(\mathbf{R}) \propto \mathcal{F}_\lambda(R)\langle I_f||\mathcal{T}_\lambda(\xi)||I_i\rangle$$

Relation with physical quantities

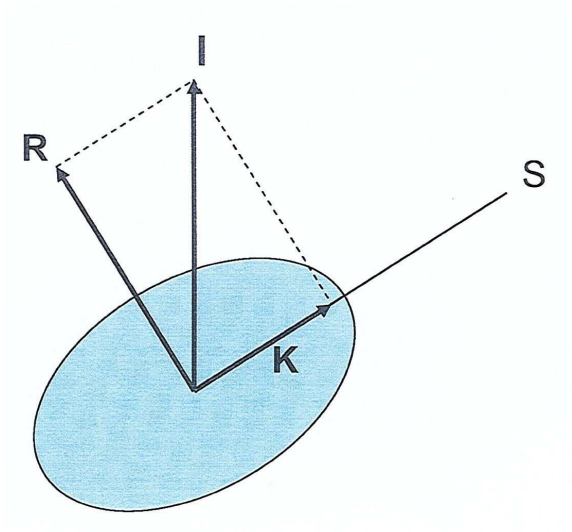
- For non-diagonal transitions ($I_i \neq I_f$)

$$B(E\lambda; i \rightarrow f) = \frac{1}{2I_i + 1} |\langle f\ I_f || \mathcal{M}(E\lambda) || i\ I_i \rangle_{\rm BM}|^2$$

- For diagonal transitions ($I_i = I_f \equiv I$) and $\lambda = 2$:

$$Q_2 = \sqrt{16\pi/5}\ (2I+1)^{-1/2}\ \langle II20|II\rangle\ \langle I||M(E2||I\rangle$$

Rotor model



- I=total spin (angular momentum) of the nucleus
- K=projection of I along symmetry axis

- The nucleus is described by a permanent deformation of matter and charge.
- The charge deformation for a multipole λ is characterized by the Coulomb intrinsic deformation: $M_n(E\lambda)$
- The matter deformation for a multipole λ is characterized by the deformation parameter (β_2) or the deformation length parameter (δ_λ)
- Transitions occur among states with the same value of K.

Reduced matrix elements in the strict rotor model

Coulomb excitation:

$$\langle K\ I_f||\mathcal{M}(E\lambda)||K\ I_i\rangle_{\rm BM} = \sqrt{2I_i+1}\langle I_i K\lambda 0|I_f K\rangle M_n(E\lambda)$$

⇨ $M_n(E\lambda)$=reduced matrix element of the charge deformation in intrinsic frame.

For nuclear excitation:

$$\langle f; K\ I_f||\hat{\delta}_\lambda||i; K\ I_i\rangle_{\rm BM} = \sqrt{2I_i+1}\langle I_i K\lambda 0|I_f K\rangle \beta_\lambda R_0$$

- β_λ= deformation parameter
- $\delta_\lambda = \beta_\lambda R_0$= deformation length parameter

⇨ $M_n(E\lambda)$ and β_λ represent the charge and matter deformation in the intrinsic frame

$$Mn(E\lambda) = \frac{3\ Z\ \beta_\lambda\ R^\lambda}{4\pi}$$

Reduced matrix elements for inelastic couplings

Coupling	TYPE	P(k)	STR
Coulomb (rotational)	10,11	$M_n(k)$	
Coulomb (general)	12,13	$\neq 0$ (arbitrary)	$M(Ek) = (-1)^{\frac{I-I'+\|I-I'\|}{2}} \langle I' \| M(Ek) \| I \rangle)$
Nuclear (rotational)	10,11	$DEF(k) = R\beta_k$	-
Nuclear (general)	12,13	$DEF(k) = R\beta_k$	$RDEF(k) = (-1)^{\frac{I-I'+\|I-I'\|}{2}} \langle I' \| \delta_k \| I \rangle$

When running inelastic (DWBA, CC) calculations with FRESCO...

- Do not confuse δ_λ (deformation length parameter in intrinsic frase) with $\hat{\delta}_{\lambda\mu}$ (deformation length operator in LAB frame):
  - δ_λ is a c-number and is the same for all transitions within the same rotational band
  - $\hat{\delta}_{\lambda\mu}$ is characterized by its matrix elements, which are different for each transition.
- When using $\langle I_f||\hat{\delta}_\lambda||I_i\rangle$ from literature, make sure what convention is used for the Wigner-Eckart theorem.
- FRESCO uses Bohr-Mottelson convention so

$$\langle I_f||\hat{\delta}_\lambda||I_i\rangle_{\rm BM} = \langle I_i||\hat{\delta}_\lambda||I_f\rangle_{\rm BM}$$

Inelastic scattering with general reduced matrix elements ^{10}Be(p,p')

^{10}Be(p,p') with general reduced matrix elements

```
0Be+p @ 63.7 MeV/u DWBA
AMELIST
FRESCO hcm=0.05 rmatch=40.0
    jtmin=0.0 jtmax=50 absend=-1
    thmin=0.1 thmax=90.00
    iter=1 nnu=24
    smats=2
      xstabl=1
      elab=637.7 /

PARTITION namep='10Be' massp=10.0113 zp=4
        namet='p' masst=1.0078 zt=1 nex=2 /
STATES jp=0.0 bandp=1 ep=0.000 cpot=1 ep=0.0 jt=0.0 /
STATES jp=2.0 bandp=1 ep=3.368 cpot=1 copyt=1 /
partition /

POT kp=1 ap=10 at=0.0 rc=1.0 /
POT kp=1 type=12 shape=10 p2=1 /
STEP ib=2 ia=1 k=2 str=7.8 /
STEP/

POT kp=1 type=1 p1=31.64 p2=1.145 p3=0.69
                  p4=8.78 p5=1.134 p6=0.69 /
POT kp=1 type=12 shape=10 p2=1 /
STEP ib=2 ia=1 k=2 str=1.97 /
STEP /
pot /

overlap /

coupling /
```

- `iter`=1: Solve CC to 1st order (DWBA)
- For the excited pair: `copyt=1`
- Coulomb deformation:
  - `type=12`: read reduced matrix elements
  - `shape=10`: radial dependence usual deformed charge distribution
  - `STR=7.8`: Coulomb r.m.e. (e fm^2)
- Nuclear deformation:
  - `type=12`: read reduced matrix elements
  - `shape=10`: derivative of undeformed potential
  - `STR=1.97`: nuclear reduced matrix element.

^{10}Be(p,p') with general reduced matrix elements

FRESCO namelist:

```
&FRESCO ITER=1 hcm=0.05 rmatch=40.0 elab=637.7
     jtmin=0.0 jtmax=50 absend=-1 thmin=0.1 thmax=90.00 smats=2 xstabl=1 /
```

ITER=1: Inelastic couplings to first order (DWBA)

PARTITIONS/STATES namelists:

```
&PARTITION namep='10Be' massp=10.0113 zp=4
          namet='p' masst=1.0078 zt=1 nex=2 /
&STATES jp=0.0 bandp=1 ep=0.000 cpot=1 ep=0.0 jt=0.0 /
&STATES jp=2.0 bandp=1 ep=3.368 cpot=1 copyt=1 /
```

NEX=2: Define two pairs of states

COPYT=1: The second target state is just a copy of the first one

ep,jp,bandp: Energy, spin, parity.

^{10}Be(p,p') with general reduced matrix elements

Coulomb potential:

```
&POT kp=1 ap=10 at=0.0 rc=1.0 /
&POT kp=1 type=12 shape=10 p2=1 /
```

- `type=12`: couple projectile states by deforming previous potential and read r.m.e
- `p1,...,p6`: consider couplings for multipoles k with `pk`$\neq 0$
- `shape=10`: usual deformed charge sphere: $V^k_{nm}(R) \propto M(Ek)/R^{k+1}$

```
&STEP ib=2 ia=1 k=2 str=7.8 /
&STEP/
```

Couple from state `IA=1` to state `IB=2` with multipole `K=2` and r.m.e. 7.8 e fm^2

`IB=2`: final state (2nd state within partition)

`IB=1`: initial state (1st state within partition)

`k=2` : multipole

`str=7.8`: reduced matrix element $\langle \text{IB}||\text{M(Ek}||\text{IA}\rangle = \sqrt{(2I_A+1)B(E\lambda; IA \rightarrow IB)}$

^{10}Be(p,p') with general reduced matrix elements

NUCLEAR potential:

```
&POT kp=1 type=1 p1=31.64 p2=1.145 p3=0.69 p4=8.78 p5=1.134 p6=0.69 /
&POT kp=1 type=12 shape=10 p2=1 /
```

- `type=12`: couple projectile states by deforming previous potential and read r.m.e.
- `shape=10`: standard derivative radial formfactor
- `p2=1`: quadrupole deformation

```
&STEP ib=2 ia=1 k=2 str=1.664 /
&STEP /
```

- `k=2`: quadrupole coupling
- `ia, ib`: indexes for states coupled.
- `STR=1.97`=$\langle IB||\delta_k||IA\rangle$ (fm)

Inelastic scattering within the rotor model: ^{10}Be(p,p')

^{10}Be(p,p') within rotor model

```
0Be+p @ 63.7 MeV/u DWBA (rotor model)
AMELIST
FRESCO hcm=0.05 rmatch=40.0
    jtmin=0.0 jtmax=50 absend=-1
    thmin=0.1 thmax=90.00
    iter=1 nnu=24
    smats=2
      xstabl=1
      elab=637.7 /

PARTITION namep='10Be' massp=10.0113 zp=4
        namet='p' masst=1.0078 zt=1 nex=2 /
STATES jp=0.0 bandp=1 ep=0.000 cpot=1 ep=0.0 jt=0.0 /
STATES jp=2.0 bandp=1 ep=3.368 cpot=1 copyt=1 /
partition /

POT kp=1 ap=10 at=0.0 rc=1.0 /
POT kp=1 type=10 shape=10 p2=7.8 /
POT kp=1 type=1 p1=31.64 p2=1.145 p3=0.69
                  p4=8.78 p5=1.134 p6=0.69 /
POT kp=1 type=10 shape=10 p2=1.97 /

pot /

overlap /

coupling /
```

- `iter`=1: Solve CC to 1st order (DWBA)
- For the excited pair: `copyt=1`
- Coulomb deformation:

  `type`=10 (rotor)
  `shape`=10 (usual deformed charge distribution)
  `Mn`=7.8 e fm^2
- Nuclear deformation:

  `type`=10 (rotor)
  `shape`=10 (derivative of undeformed potential)
  δ_2=1.97 fm.

^{10}Be(p,p') within rotor model

FRESCO namelist:

```
&FRESCO ITER=1 hcm=0.05 rmatch=40.0 elab=637.7
     jtmin=0.0 jtmax=50 absend=-1 thmin=0.1 thmax=90.00 smats=2 xstabl=1 /
```

ITER=1: Inelastic couplings to first order (DWBA)

PARTITIONS/STATES namelists:

```
&PARTITION namep='10Be' massp=10.0113 zp=4
          namet='p' masst=1.0078 zt=1 nex=2 /
&STATES jp=0.0 bandp=1 ep=0.000 cpot=1 ep=0.0 jt=0.0 /
&STATES jp=2.0 bandp=1 ep=3.368 cpot=1 copyt=1 /
```

- NEX=2: number of pairs of states
- COPYT=1: The second target state is just a copy of the first one
- ep,jp,bandp: Energy, spin, parity.

^{10}Be(p,p') within rotor model

Coulomb potential:

```
&POT kp=1 ap=10 at=0.0 rc=1.0 /
&POT kp=1 type=10 shape=10 p2=7.8 /
```

- `type=10`: rotor model
- `shape=10`: usual deformed charge distribution
- `P2=7.8`: intrinsic Mn value (e fm^2)

NUCLEAR potential:

```
&POT kp=1 type=1 p1=31.64 p2=1.145 p3=0.69 p4=8.78 p5=1.134 p6=0.69 /
&POT kp=1 type=10 shape=10 p2=1.97 /
```

- `type=10`: rotor model
- `shape=10`: derivative formfactor
- `p2=1.97`: deformation length