Practice Exercises Using CATKIN

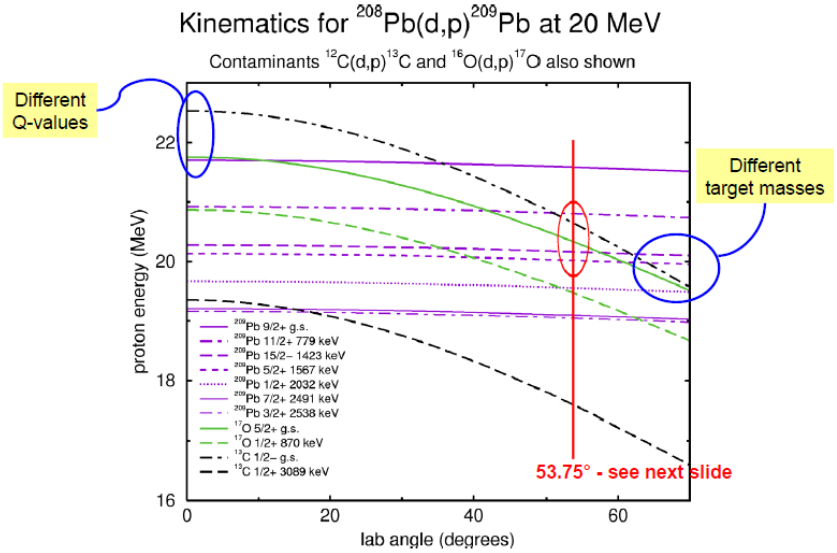
W.N. Catford

CERN Reactions School 2014

These exercises are designed to introduce *catkin* via some calculations that illustrate the differences between normal and inverse kinematics, with reference to the lecture notes for this school. In fact, the main motivation is to study and think about the kinematics calculations once you have done them, so *catkin* is in no way essential – any kinematics programme is suitable – although anybody who already has an alternative programme will probably have thought about all of the issues studied here, already! Another convenient way to perform kinematics calculations is included in LISE++.

QUESTION ONE

This question concerns "normal kinematics" calculations. That is, a light particle is incident on a heavier target particle. The figure below, taken from the lectures, shows calculations for deuterons incident on Pb at 10 MeV/nucleon (a total of 20 MeV for the deuteron, measured in the laboratory frame).



1 (a)

Make a similar plot for the hypothetical case of the (d,p) reaction on a target of ^{25}Na at 5.0 MeV/nucleon (10 MeV deuterons). (Hypothetical because ^{25}Na has a half-life of 59.1 seconds and realistically it can't be made as a target). Don't bother to do the calculations for the carbon and oxygen targets, we just need the ^{25}Na target for this question. Use whatever software you like, to do the plot; in the simplest case you could just use excel and copy/paste the columns from the lower part of the main kinematics page in *catkin*. Make the plot for (d,p) to the ground state of ^{26}Na and, also on the same plot, include the excited states at excitation energies in ^{26}Na of 0.407 MeV and 3.512 MeV.

1(b)

Now calculate the elastic scattering (d,d) at the same beam energy (5 MeV/nucleon), and add it to your plot. Also add a plot of (p,p) elastic scattering at 5 MeV/nucleon incident energy for the proton, showing what you would get for a proton beam on the ^{25}Na target. Finally, add the (d,t) reaction at 5 MeV/nucleon, to the ground state of ^{24}Na. Now, finally, add curves for (d,d) and (d,t) reactions to hypothetical states in the final nucleus at 0.5 MeV.

1(c)

How does the spacing in proton energy of the (d,p) kinematic lines, for final states of 0.0 and 0.407 MeV, compare with the spacing in excitation energy of the two states, i.e. 0.407 MeV? How does this depend upon laboratory angle?

1(d)

Considering just (d,p) for example, how different do the kinematical plots appear, if you use the centre of mass angle instead of the laboratory angle, for the proton?

QUESTION TWO

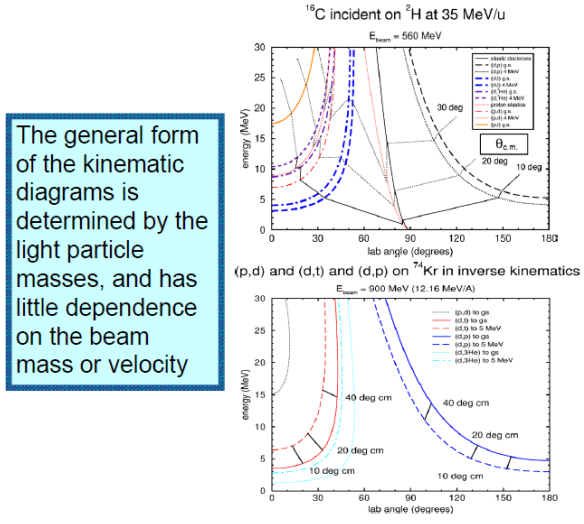
2(a)

Repeat the above kinematical calculations, in inverse kinematics: that is, with the deuteron (or proton in the case of proton elastic scattering) as the target. The beam should be ^{25}Na at 5.0 MeV/nucleon, i.e. 125 MeV, and you should make plots of energy versus laboratory angle.

[Note that classically (and here, $\beta = 10.3\%c$ so it is still just about acceptable to be guided by this) the velocity is given by $\beta(\%) = 4.6337\sqrt{E/A}$ where E is in MeV and A is the number of nucleons, so these calculations are the same relative velocity between the deuteron (or proton) and the ^{25}Na].

The relevant plot from the lectures that should look similar in many ways is shown below, for some rather different masses and energies for the beams. Note that the vertical axes (absolute energy) are not so different in these cases. In the plots shown below, the points corresponding to the same centre of mass angles (10, 20, 30… degrees) are connected.

The general form of the kinematic diagrams is determined by the light particle masses, and has little dependence on the beam mass or velocity



Note that the smaller values of centre of mass angles (which are the most important to measure experimentally, because these are the angles where the approximations in the theory can be expected to work the best) correspond to the lower energy branch of the double-valued solutions for the (d,t) reaction. For the (d,p) reaction, the lower values of centre of mass angle correspond to the backward laboratory angles (closer to 180 degrees in the laboratory). Also, you will find that catkin will give the centre of mass angle as $(180 - \theta_{cm})$; this is because catkin always measures angle relative to the beam direction, whereas the relevant reference direction for the physics is the direction pointing from the deuteron towards the ^{25}Na, which is opposite to the beam direction in the case of inverse kinematics.

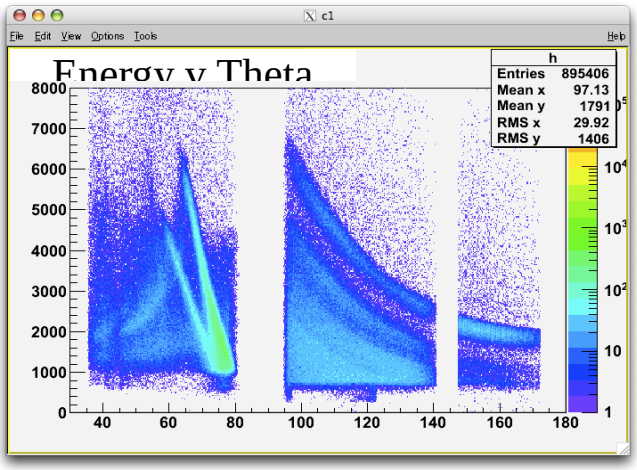
You will also notice that the inelastic plot for (d,d) to the hypothetical 0.5 MeV state will look quite a lot different to the plot for the elastic scattering. This will be obvious, following the discussion in Question 3.

2(b)

Calculate the difference in energy between the (d,p) curves for the population of the states at 0.0 and 0.407 MeV, and plot this as a function of the laboratory angle. Note that, in contrast to the normal kinematics case, there is a significant "compression" of the spacing between these lines at the laboratory angles of primary interest (which, for inverse kinematics, are those towards $180°$). This makes the experimental challenge more difficult, in terms of resolving closely spaced states in terms of energy, and is a factor that gives helical spectrometers such as HELIOS an advantage in certain experimental situations, as mentioned in a parallel series of experimental lectures.

2(c)

Compare your curves for the kinematics with the experimentally measured data shown below. These data are from an experiment performed at TRIUMF using SHARC and TIGRESS (thesis of G.L. Wilson, also of I.C. Celik). You should see evidence of the population, amongst other things, of states close to 0.0, 0.407 and 3.512 MeV in 26Na, via (d,p). Also, there is evidence of elastic scattering of the deuterons in the target, which are ejected from the target and are recorded in SHARC. Finally, identify which line(s) correspond to proton elastic scattering (this tells us that the targets apparently have ^{1}H in them, even though the experimenters asked for them to contain only ^{2}H). The plot is calibrated to give kinetic energy (in keV) versus laboratory angle in degrees.



2(d)

Use your results for (d,p) to the ground state of ^{26}Na to calculate the *Jacobian* quantity $d\theta_{lab}/d\theta_{cm}$ as a function of θ_{lab} and to plot it. Observe that the effect here is to "decompress" the angles in the change of frame from centre of mass to laboratory frame, so that a given span in laboratory angle covers a smaller range of angles in the centre of mass frame for the laboratory angles tending towards $180°$. This means that the most backward angles, whilst important because they correspond to small centre of mass angles, have their importance diminished in terms of the number of counts seen at these angles in an experiment.

2(e)

If you have LISE++ you can calculate* the angles at which the protons from the (d,p) reaction will punch completely through the silicon detector in SHARC. For this you need to know that the silicon detectors were aligned with their front faces parallel to the beam direction, and had thicknesses of 1000μm for laboratory angles $> 90°$ and 140μm for angles $< 90°$. (Actually, you will find that the 1000μm is sufficient to stop the protons from (d,p) for all angles $> 90°$).

Now, if you calculate the angles at which (d,d) and (p,p) punch through the 140μm silicon, you will find that they start to punch through for laboratory angles in the range 50-70°. You can see from the experimental data that the particles at more forward angles than the punch-through are depositing less and less energy in the silicon; this is because the rate of energy loss of the particles drops to smaller values as the energy of the particles increases. This explains the difference between the kinematics calculations and the data, for the elastic scattering. (Note that in the experiment, in SHARC, the 140μm silicon detector is backed by a second silicon detector of thickness 1000μm which can also be included in the analysis of the data, to recover the full energy experimentally. The data shown here do not include the second detector.)

*to use LISE++ in this way, you need to use the "Physical calculator" shown by the blue P icon. You can specify the type of ion, and the total kinetic energy (TKE), and it will calculate the energy deposited in a given thickness of silicon. You can also specify the angle of incidence, which in this case is equal to the difference between the laboratory angle and $90°$. It will also give you the range in silicon. With these calculations, you can calculate the punch-through angles for the elastics.

QUESTION THREE

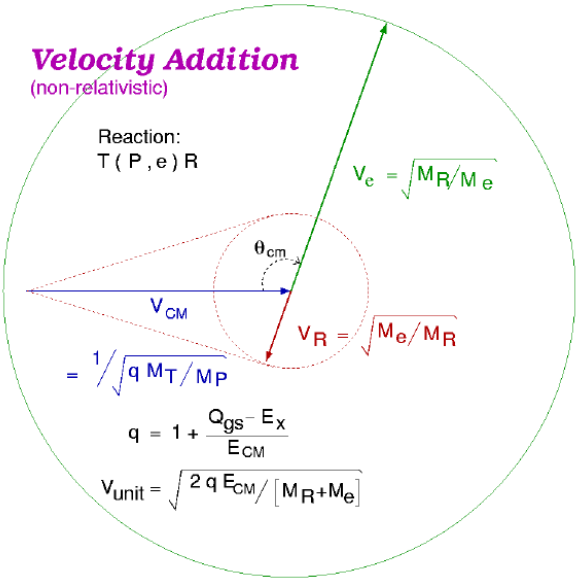
3(a)

The overall properties of the kinematics can be recognised intuitively using the velocity vector diagrams discussed in the lectures (even though it should be remembered that these obviously use the Galilean transformation of classical mechanics, to allow them to be drawn on a sheet of paper). The slide from the lectures that includes the formulae for the lengths of the vectors is in the following diagram.

The diagram shows the velocity ***of*** the centre of mass in the laboratory reference frame and also the velocity vectors, as measured ***in*** the centre of mass reference frame, of the ejectile and the heavier recoil. Momentum is conserved in the centre of mass frame, so the lengths of the two velocity

vectors in that frame simply reflect the different masses. Here, the symbols P, T, e and R are used to denote the projectile, target, (light) ejectile and (heavy) recoil respectively.

[A special case is elastic scattering, where $M_R = M_P$ and $M_e = M_T$ and also the Q-value and excitation energy are both zero so $q = 1$ exactly. Therefore the magnitudes of v_{CM} and v_e are equal. At a scattering angle in the centre of mass frame of zero (see diagram), the resultant velocity of the target nucleus after the collision is zero in the laboratory frame. As the angle of the deflection increases slightly, in the centre of mass frame, the resultant starts to trace around a circle and the elastically scattered target particles are seen to emerge just forward of $90°$ in the laboratory frame and with a kinetic energy that increases rapidly with angle. Initially, the length of the resultant vector is proportional to the centre of mass angle and hence the kinetic energy increases quadratically with centre of mass angle. Also, the vector triangle is isosceles so that $(90 - \theta_{lab}) = \theta_{cm}/2$ and hence the kinetic energy also increases approximately quadratically with laboratory angle, as the angle varies away from $90°$ before settling down to a slower rate of increase.]



Calculate the quantity q and hence the relative lengths of the vectors v_{CM}, v_R and v_e (here, E_{CM} is the incident kinetic energy as measured in the centre of mass frame: $E_{CM} = E_{lab} \times (\frac{M_T}{[M_T+M_P]})$ where E_{lab} is the total kinetic energy of the beam particles (projectiles) in MeV). Do this for $E_{lab} = 5$ MeV/nucleon, i.e. 125 MeV, and for the inverse kinematic reactions:

- d(^{25}Na,p)^{26}Na to the ground state
- d(^{25}Na,d)^{26}Na elastic scattering
- d(^{25}Na,t)^{24}Na to the ground state

The vector diagrams that you can construct in each case, and then imagining how they change as θ_{cm} changes, should make the overall form of the kinematical diagrams that you produced in Question 2 become obvious.

[Note that the ratio of lengths v_e/v_{CM} is given by the expression $\sqrt{qf(M_R/M_P)} \approx \sqrt{qf}$ where f is equal to 2 for (d,p), 2/3 for (d,t) and 1/2 for (p,d), being simply the ratio of masses and q is given by $q = 1 + (Q_{total}/E_{CM})$ where Q_{total} is the reaction Q-value taking into account the excitation energy. Now E_{CM} is related to the beam energy in the laboratory frame, as usual, by the simple relationship $E_{CM} = E_{lab} \times (M_T/[M_T + M_P])$ and $[M_T + M_P] \approx M_P$ for a light target, so we can write $E_{CM} \approx (E/A)_{beam} \times M_T$ where $(E/A)_{beam}$ means the energy per nucleon of the beam. Thus, $q \approx 1$ in the cases where the beam energy per nucleon $(E/A)_{beam}$ is large compared to the Q-value of the reaction, for example a Q-value of 1 or 2 MeV and a beam energy of order 10 MeV/nucleon. Then, the ratio of vector lengths is given approximately by the very simple expression $v_e/v_{CM} \approx \sqrt{f}$ which depends just on the type of reaction and not on any other details. This is why the kinematical diagrams all look so similar. If the Q-value becomes significantly negative and/or the beam energy is very low, etc., then the form of the vector diagrams will be different, but the approximations that give $q \approx 1$ are appropriate for a wide range of examples.]

[Note also that the scale of the vector diagram, given by v_{unit} in the figure, is proportional to $\sqrt{E_{CM}}$ when $q \approx 1$, which means to say that it is approximately proportional to $\sqrt{(E/A)_{beam}}$. If the velocities scale approximately in this fashion, then the kinetic energies of the observed particles (being the proportional to the square of the velocities, classically) will scale approximately as the energy per nucleon of the beam, $(E/A)_{beam}$.]

In summary, the classical vector addition diagrams can be a useful *aide memoire* to help you to visualize how the kinematical diagrams should look, for transfer reactions in inverse kinematics.

QUESTION FOUR

4(a)

This question concerns the production, in a two-body reaction, of a nucleus that then sequentially breaks up. Consider first the excitation energy at which ^{26}Na becomes unbound to neutron emission. Determine this from the *Calculator* page in *catkin*, using the section for separation energy.

4(b)

Now calculate the kinematics for the reaction d(^{25}Na,p)^{26}Na at 5.0 MeV/nucleon, to a hypothetical unbound state at 6.5 MeV excitation energy in ^{26}Na. For reactions corresponding to zero degrees in the centre of mass frame (as defined for example in Question 3), determine the energy of the ^{26}Na nucleus prior to sequential breakup via neutron decay (this recoil emerges from the reaction at zero degrees in the laboratory frame). Now use the Calculator page in catkin to determine the angular size of the cone into which the ^{25}Na decay products of the sequential neutron decay are focussed. Why will the mean kinetic energy of these ^{25}Na nuclei be given by (25/26) times the energy of the ^{26}Na that you calculated?

4(c)

The ^{25}Na nuclei originating from sequential decay of this 6.5 MeV state in ^{26}Na will all be focussed forward of some limiting angle, relative to the beam direction. Estimate this angle, taking into account the maximum angle of the ^{26}Na relative to the beam direction (from the two-body

kinematics for all scattering angles) and the cone size for sequential neutron emission that you already calculated. Compare this with the size of the cone (around the beam direction) that will contain all of the ^{25}Na nuclei arising from elastic scattering from the deuteron target. Also, how will the kinetic energy of the most intense of the elastically scattered ^{25}Na nuclei (i.e. small centre of mass scattering angles) compare to the mean energies of the ^{25}Na nuclei from sequential breakup, mentioned earlier?

Ultimately, a complex detection scenario such as this should be modelled in a proper simulation, but the type of simple calculations discussed above can help to give a physical insight into what to expect.

4(d)

For something rather different, suppose the target is replaced by ^{9}Be and the neutron transfer reaction is achieved via (^{9}Be,^{8}Be) in inverse kinematics, with the same beam energy for ^{25}Na as before. For reactions populating the ground state of ^{26}Na, what is the energy of ^{8}Be particles emerging at $90°$ in the laboratory frame ($\theta_{cm} = 29°$)?

4(e)

Since ^{8}Be is unbound to decay in two alpha particles, the ^{8}Be emitted at $90°$ in the laboratory immediately decays into two alpha particles contained within a cone with a size determined by the energy released in the breakup and the kinetic energy of the ^{8}Be. Use the *Calculator* page of *catkin* to calculate the size of that breakup cone.

NOTE ON LISE++

This programme is available from a download site at MSU (*google* LISE++ download). It runs under Microsoft Windows. On the web site you can see "latest updates" to determine the latest official version, and then go to "download" to get the executable programme. The programme was originally developed to calculate the expected secondary radioactive beams produced via fragmentation reactions in the mass-achromatic spectrometer LISE at GANIL. Conceptually similar spectrometers are in use at MSU (A1900), GSI (FRS) and RIKEN (BigRIPS) and LISE is now upgraded to LISE3. However, for present purposes, the really interesting part of LISE++ is the "PHYSICAL CALCULATOR" which is located in the task bar at the top, with a blue P (plus calculator) symbol. You can specify an ion by A and Z, and its energy via TKE (total kinetic energy), and then compute its energy loss in layers of any material and in particular silicon. The top left section defines the particle, the top right gives the energy loss in a thickness of silicon (you can change the angle of incidence with the "after/into" option), the bottom right will give the range in silicon, and the bottom left allows for very flexible calculations for particles with multiple layers of different materials.

Wilton Catford

CERN, April 2014