ISOLDE Nuclear Reaction and Nuclear Structure Course

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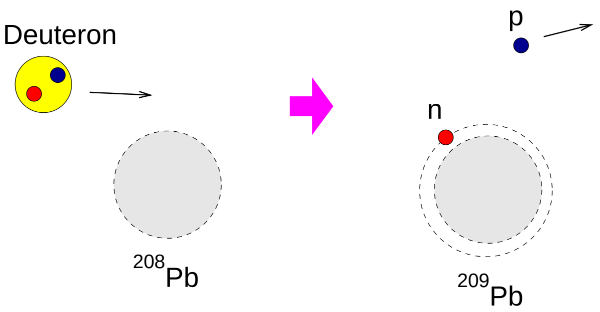
Transfer reactions

Example: d+^{208}Pb → p + ^{209}Pb



Transfer reactions: Q-value considerations

Consider: $a + A \rightarrow b + B$

- Energy balance (in CM frame):

$$E^{i}_{\rm cm} + M_a c^2 + M_A c^2 = E^{f}_{\rm cm} + M_b c^2 + M_B c^2$$

- Q_0 value:

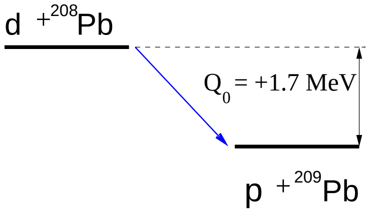
$$Q_0 = M_a c^2 + M_A c^2 - M_b c^2 - M_B c^2$$

$$E^{f}_{\rm cm} = E^{i}_{\rm cm} + Q_0$$

  - $Q_0 > 0$: the system gains kinetic energy (exothermic reaction)
  - $Q_0 < 0$: the system loses kinetic energy (endothermic reaction)

Transfer reactions: Q-value considerations

Example: d+^{208}Pb → p + ^{209}Pb



$$Q_0 = M_d c^2 + M(^{208}\text{Pb})c^2 - M_p c^2 - M(^{209}\text{Pb})c^2 = +1.7\,\text{MeV}$$

Transfer reactions: Q-value considerations

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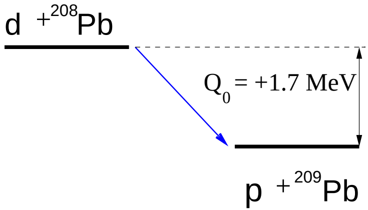
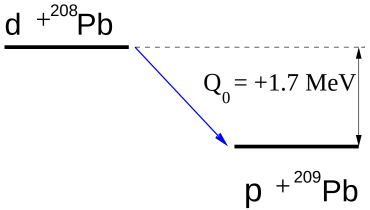


$$Q_0 = M_d c^2 + M(^{208}\text{Pb})c^2 - M_p c^2 - M(^{209}\text{Pb})c^2 = +1.7\,\text{MeV}$$

☞ $Q_0 > 0$: the outgoing proton will gain energy with respect to the incident deuteron.

Transfer reactions: Q-value considerations

Example: d+^{208}Pb $\rightarrow$ p + ^{209}Pb



$$Q_0 = M_d c^2 + M(^{208}\text{Pb})c^2 - M_p c^2 - M(^{209}\text{Pb})c^2 = +1.7\,\text{MeV}$$

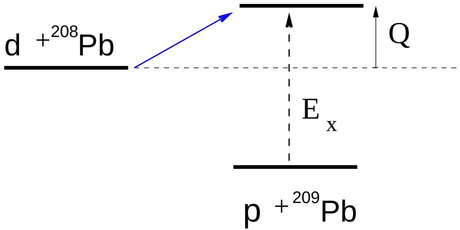
☞ $Q_0 > 0$: the outgoing proton will gain energy with respect to the incident deuteron.

For a transfer reaction, the Q value is just the difference in binding energies of the transferred particle/cluster in the initial and final nuclei:

$$Q_0 = \varepsilon_b(f) - \varepsilon_b(i) = 3.936 - 2.224 = +1.7\,\text{MeV}$$

Transfer reactions: Q-value considerations

If the transfer leads to an excited state, the Q-value will change, and hence the kinetic energy of the outgoing nuclei.



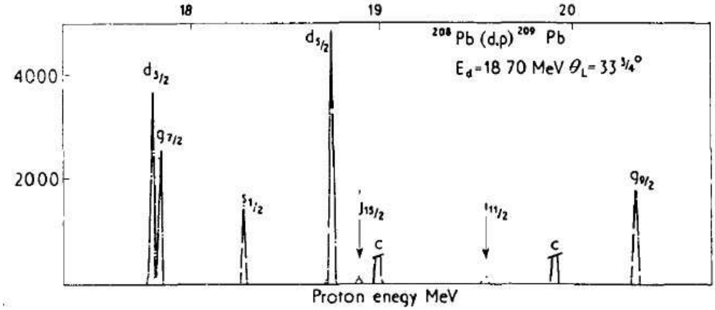
Energy balance:

$$E^f_{\rm cm} = E^i_{\rm cm} + Q = E^i_{\rm cm} + Q_0 - E_x$$

☞*If we know Q_0 we can infer the excitation energies (E_x) measuring the final kinetic energy of outgoing fragments.*

What we do observe in a transfer experiment?

Example: $d + {}^{208}Pb \rightarrow p + {}^{209}Pb$



- The proton energy spectrum shows some peaks which reflect the excitation energy spectrum of the residual nucleus (^{209}Pb).
- The population probability will depend on the reaction dynamics and on the structure properties of these states.

What we do observe in a transfer experiment?

We would like to infer other properties besides excitation energies:

- Angular momentum / parity of populated states.
- Information on the internal structure of these states.

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$$\Downarrow$$

SPECTROSCOPY

What we do observe in a transfer experiment?

We would like to infer other properties besides excitation energies:

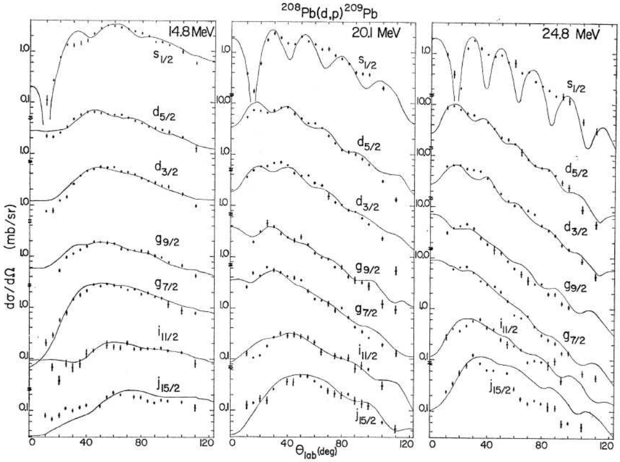
- Angular momentum / parity of populated states.
- Information on the internal structure of these states.

$$\Downarrow$$

SPECTROSCOPY

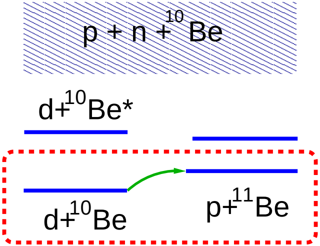
The excitation function spectrum does not provide in general enough information to extract these properties. What additional information can we use....?

What we do observe in a transfer experiment?



☞*Angular distributions of transfer cross sections are very sensitive to the internal structure of the populated states*

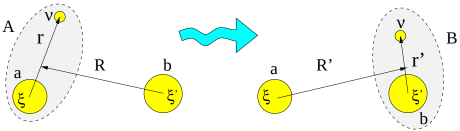
DWBA modelspace



☞*In a transfer calculation, the modelspace will contain states belonging to different mass partitions, and hence to different internal Hamiltonians.*

DWBA method for transfer reactions

- Transfer process: $\underbrace{(a+v)}_{A} + b \rightarrow a + \underbrace{(b+v)}_{B}$



- Full Hamiltonian:
  - Prior form: $\xi_\alpha = \{\xi, \xi', \mathbf{r}\}$

$$H = \hat{T}_{\mathbf{R}} + H_\alpha(\xi_\alpha) + V_\alpha(\mathbf{R},\mathbf{r}) = \hat{T}_{\mathbf{R}} + \underbrace{H_A(\xi,\mathbf{r}) + H_b(\xi')}_{H_\alpha(\xi_\alpha)} + \underbrace{V_{vb} + U_{ab}}_{V_\alpha(\mathbf{R},\mathbf{r})}$$

  - Post form: $\xi_\beta = \{\xi, \xi', \mathbf{r}'\}$

$$H = \hat{T}_{\mathbf{R}'} + H_\beta(\xi_\beta) + V_\beta(\mathbf{R}',\mathbf{r}') = \hat{T}_{\mathbf{R}'} + \underbrace{H_B(\xi',\mathbf{r}') + H_a(\xi)}_{H_\beta(\xi_\beta)} + \underbrace{V_{av} + U_{ab}}_{V_\beta(\mathbf{R}',\mathbf{r}')}$$

Reminder of the exact transition amplitude

Exact scattering amplitude: $f_{\beta\alpha}(\theta) = -(\mu_\beta/2\pi\hbar^2)\mathcal{T}_{\beta,\alpha}$, with

$$\mathcal{T}^{\text{post}}_{\beta,\alpha} = \int\int \chi^{(-)*}_\beta(\mathbf{K}_\beta, \mathbf{R}_\beta)\Phi^*_\beta(\xi_\beta)(V_\beta - U_\beta)\Psi^{(+)}_{\mathbf{K}_\alpha}(\mathbf{R}_\alpha, \xi_\alpha)d\xi_\beta d\mathbf{R}_\beta$$

- $\Psi^{(+)}_{\mathbf{K}_\alpha}(\mathbf{R}_\alpha, \xi_\alpha)$: exact WF
- $\Phi_\beta(\xi_\beta)$: internal (proj + target) WF for final state
- $\chi^{(+)}_\beta(\mathbf{K}_\beta, \mathbf{R}_\beta)$: distorted wave generated by auxiliary potential U_β

$$\Psi^{(+)}_{\mathbf{K}_\alpha}(\mathbf{R}_\alpha, \xi_\alpha) \xrightarrow{R\gg} \underbrace{e^{i\mathbf{K}_\alpha\cdot\mathbf{R}_\alpha}\Phi_\alpha(\xi_\alpha)}_{\text{incident}} + \underbrace{f_{\alpha,\alpha}(\theta)\frac{e^{iK_\alpha R_\alpha}}{R_\alpha}\Phi_\alpha(\xi_\alpha)}_{\text{elastic}} + \underbrace{\sum_{n>0} f_{\beta,\alpha}(\theta)\frac{e^{iK_\beta R_\beta}}{R_\beta}\Phi_\beta(\xi_\beta)}_{\text{transfer}}$$

Evaluation of scattering amplitude in Born approximation: general case

- Define auxiliary potentials in entrance and exit channels: $U_\alpha(\mathbf{R}_\alpha)$, $U_\beta(\mathbf{R}_\beta)$

$$\left[E - \varepsilon_\alpha - \hat{T}_{\mathbf{R}_\alpha} - U_\alpha(\mathbf{R}_\alpha)\right]\chi_\alpha^{(+)}(\mathbf{K}_\alpha, \mathbf{R}_\alpha) = 0$$
$$\left[E - \varepsilon_\beta - \hat{T}_{\mathbf{R}_\beta} - U_\beta(\mathbf{R}_\beta)\right]\chi_\beta^{(+)}(\mathbf{K}_\beta, \mathbf{R}_\beta) = 0$$

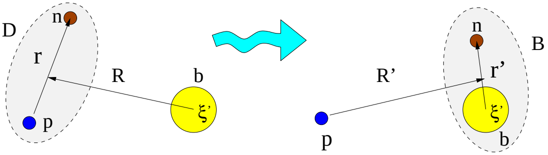
- Retain only elastic component of $\Psi^{(+)}_{\mathbf{K}_\alpha}$ (Born approximation):

$$\Psi^{(+)}_{\mathbf{K}_\alpha}(\mathbf{R}_\alpha, \xi_\alpha) \approx \chi_\alpha^{(+)}(\mathbf{K}_\alpha, \mathbf{R}_\alpha)\Phi_\alpha(\xi_\alpha)$$

- DWBA scattering amplitude:

$$\mathcal{T}^{\rm DWBA}_{\beta,\alpha} = \int\int \chi_\beta^{(-)*}(\mathbf{K}_\beta, \mathbf{R}_\beta)\Phi^*_\beta(\xi_\beta)(V_\beta - U_\beta)\chi_\alpha^{(+)}(\mathbf{K}_\alpha, \mathbf{R}_\alpha)\Phi_\alpha(\xi_\alpha)d\xi_\beta d\mathbf{R}_\beta$$

The important (d,p) case



Post-form DWBA transition amplitude: $V_\beta = V_{pn} + U_{pb}$

$$\mathcal{T}_{d,p}^{\mathrm{DWBA}} = \int\int \chi_p^{(-)*}(\mathbf{K}_p, \mathbf{R}')\Phi_\beta^*(\xi_\beta)(V_{pn} + U_{pb} - U_{pB})\chi_d^{(+)}(\mathbf{K}_d, \mathbf{R})\Phi_\alpha(\xi_\alpha)d\xi_\beta d\mathbf{R}'$$

- For medium-mass/heavy targets: $U_{pb} \approx U_{pB} \Rightarrow V_{pn} + U_{pb} - U_{pB} \approx V_{pn}(\mathbf{r})$
- Internal states and internal coordinates:

$$\Phi_\alpha(\xi_\alpha) = \varphi_d(\mathbf{r})\phi_b(\xi') \qquad \xi_\alpha = \{\xi', \mathbf{r}\}$$

$$\Phi_\beta(\xi_\beta) = \Phi_B(\xi', \mathbf{r}') \qquad \xi_\beta = \{\xi', \mathbf{r}'\}$$

(d,p) case: parentage decomposition of target nucleus

➩ We need to evaluate the overlap integral

$$\int d\xi' \; \phi_B^*(\xi', \mathbf{r}')\phi_b(\xi')$$

➩ Use the parentage decomposition of $B \to b + n$

$$\Phi_B(\xi', \mathbf{r}') = C_{bn}^B \phi_b(\xi')\varphi_{bn}(\mathbf{r}') \; + \; \left\{ \text{other components of b} \right\}$$

$$\Rightarrow \int d\xi' \; \phi_B^*(\xi', \mathbf{r}')\phi_b(\xi') = C_{bn}^B \varphi_{bn}(\mathbf{r}')$$

☞ C_{bn}^B = spectroscopic amplitude
☞ $|C_{bn}^B|^2 = S_{bn}^B$ = spectroscopic factor
☞ $\varphi_{bn}(\mathbf{r}')$ usually approximated by a bound state wavefuncion of n relative to the core b.

Examples of parentage decompositions

1. Double-magic nucleus plus a single nucleon:

$$|^{209}\mathrm{Bi(g.s.)}\rangle_{9/2^-} \approx \left[|^{208}\mathrm{Pb}(0^+)\rangle \otimes |\nu 1h_{9/2}\rangle\right]_{9/2^-}$$

☞ *almost* single-particle configuration ($S^{\ell sj}_{IJ} \approx 1$).

2. Deformed core plus an extra nucleon:

$$|^{11}\mathrm{Be(gs)}\rangle_{1/2^+} = \alpha \left[|^{10}\mathrm{Be}(0^+)\rangle \otimes |\nu 2s_{1/2}\rangle\right]_{1/2^+} + \beta \left[|^{10}\mathrm{Be}(2^+)\rangle \otimes |\nu 1d_{5/2}\rangle\right]_{1/2^+} + \ldots$$

with $|\alpha|^2 + |\beta|^2 + \ldots = 1$

3. The spectroscopic factor reflects the occupation number of a single-particle level so it can be even larger than 1!

Scattering amplitude and cross sections

➩ In post form:

$$\mathcal{T}^{\rm DWBA}_{d,p} = C^B_{bn} \int\int \chi^{(-)*}_p(\mathbf{K}_p, \mathbf{R}')\varphi^*_{bn}(\mathbf{r}')\, V_{pn}(\mathbf{r})\,\chi^{(+)}_d(\mathbf{K}_d, \mathbf{R})\varphi_{pn}(\mathbf{r})d\mathbf{r}'d\mathbf{R}'$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\beta,\alpha} = \frac{\mu_\alpha\mu_\beta}{(2\pi\hbar^2)^2}|C^B_{bn}|^2\left|\int\int \chi^{(-)*}_p(\mathbf{K}_p, \mathbf{R}')\varphi^*_{bn}(\mathbf{r}')\, V_{pn}(\mathbf{r})\,\chi^{(+)}_d(\mathbf{K}_d, \mathbf{R})\varphi_{pn}(\mathbf{r})d\mathbf{r}'d\mathbf{R}'\right|^2$$

$$|C^B_{bn}|^2 = S^B_{bn} = \text{spectroscopic factor}$$

☞*In DWBA, the transfer cross section is proportional to the product of the projectile and target spectroscopic factors. Comparing the data with DWBA calculations, one can extract the values of* S^B_{bn}

^{1}H(^{11}Be,^{10}Be)^{2}H example

$$|^{11}\text{Be}\rangle = \alpha\, |^{10}\,\text{Be}(0^{+}) \otimes \nu 2s_{1/2}\rangle + \beta\, |^{10}\,\text{Be}(2^{+}) \otimes \nu 1d_{5/2}\rangle + \ldots$$

➩ In DWBA:

$$\sigma(0^{+}) \propto |\alpha|^{2}; \quad \sigma(2^{+}) \propto |\beta|^{2}$$

$^{1}H(^{11}Be,^{10}Be)^{2}H$ example

$$|^{11}Be\rangle = \alpha\,|^{10}\,Be(0^{+})\otimes\nu 2s_{1/2}\rangle + \beta\,|^{10}\,Be(2^{+})\otimes\nu 1d_{5/2}\rangle + \ldots$$

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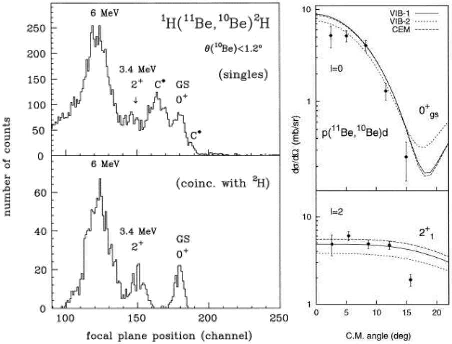
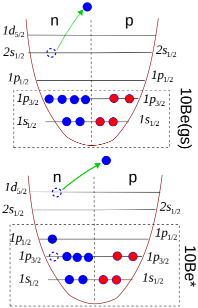
Fortier et al, PLB461, 22 (1999)





Prior form and post/prior equivalence

- Exact transition amplitude in prior form:

$$f^{\text{prior}}_{\beta,\alpha} = -\frac{\mu_\beta}{2\pi\hbar^2} \int\int \Psi^{(-)*}_{\mathbf{K}_\beta}(\mathbf{R}_\beta, \mathbf{r}_\beta)(V_\alpha - U_\alpha)\chi^{(+)}_\alpha(\mathbf{K}_\alpha, \mathbf{R}_\alpha)\Phi_\alpha(\xi_\alpha)d\xi_\alpha d\mathbf{R}_\alpha \quad ,$$

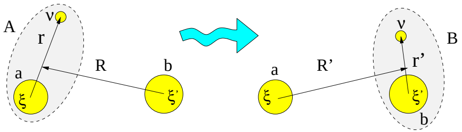
- DWBA approximation: $\Psi^{(-)}_{\mathbf{K}_\beta}(\mathbf{R}_\beta, \mathbf{r}_\beta) \simeq \chi^{(-)}_\beta(\mathbf{K}', \mathbf{R}')\varphi_{bv}(\mathbf{r}')$:
- For a (d,p) reaction:

$$f^{\text{prior}}_{(d,p)}(\theta) = -\frac{\mu_\beta}{2\pi\hbar^2} C^B_{bn} \int \chi^{(-)*}_p(\mathbf{K}', \mathbf{R}')\varphi^*_{bn}(\mathbf{r}')(V_{bn} + U_{pb} - U_{db})\varphi_{pn}(\mathbf{r})\chi^{(+)}_d(\mathbf{K}, \mathbf{R})d\mathbf{R}d\mathbf{r}$$

- In either the exact case, or under DWBA:

$$f^{\text{prior}}_{\beta,\alpha}(\theta) = f^{\text{post}}_{\beta,\alpha}(\theta) \qquad \text{(post/prior equivalence)}$$

General case: parentage decomposition of projectile and target states



Integral in internal coordinates $\xi_\beta = \{\xi, \xi', \mathbf{r}'\}$

$$\int \underbrace{\Phi^*_\beta(\xi_\beta)}\overbrace{\Phi_\alpha(\xi_\alpha)}\, d\xi_\beta = \int \underbrace{\Phi_B(\xi', \mathbf{r}')\phi_a(\xi)}\overbrace{\Phi_A(\xi, \mathbf{r})\phi_b(\xi')}\, \underbrace{d\xi d\xi' d\mathbf{r}'}_{d\xi_\beta}$$

So, we need to evaluate the overlap integrals

$$\int \Phi^*_B(\xi', \mathbf{r}')\phi_b(\xi')d\xi' \quad \text{and} \quad \int \Phi_A(\xi, \mathbf{r})\phi_a(\xi)d\xi$$

Parentage decompositions (C^A_{va}, C^B_{vb} = spectroscopic amplitudes):

$$\Phi_A(\xi, \mathbf{r}) = C^A_{va}\phi_a(\xi)\phi_v\varphi_{av}(\mathbf{r}) + \Phi^C_A$$
$$\Phi_B(\xi', \mathbf{r}') = C^B_{vb}\phi_b(\xi')\phi_v\varphi_{bv}(\mathbf{r}') + \Phi^C_B$$

Parentage decompositions (continued)

Using the parentage decompositions of $A \to a + v$ and $B \to b + v$

$$\int \Phi_A(\xi, \mathbf{r}) \phi_a(\xi) d\xi = C^B_{av} \varphi_{av}(\mathbf{r})$$

$$\int \Phi^*_B(\xi', \mathbf{r}') \phi_b(\xi') d\xi' = C^A_{bv} \varphi^*_{bv}(\mathbf{r}')$$

Three-body DWBA transition amplitude

$$\mathcal{T}^{\mathrm{DWBA}}_{\beta,\alpha} = C^B_{vb} C^A_{va} \int \int \chi^{(-)*}_{\beta}(\mathbf{K}', \mathbf{R}') \varphi_{bv}(\mathbf{r}') \left(V_\beta - U_\beta\right) \chi^{(+)}_{\alpha}(\mathbf{K}, \mathbf{R}) \varphi_{av}(\mathbf{r}) d\mathbf{R}' d\mathbf{r}'.$$

☞ So, $d\sigma/d\Omega \propto |C^B_{vb}|^2 |C^A_{va}|^2$ (spectroscopic factors)

Spectroscopic factors: angular momentum considerations

- We need to evaluate:

$$\int d\xi \; \phi_a(\xi)\phi_A(\xi, \mathbf{r}) \quad \text{and} \quad \int d\xi' \; \phi_b(\xi')\phi_B(\xi', \mathbf{r}')$$

- Since $A = a + v$ we can use the parentage decomposition:

$$\boxed{\phi_A^{JM}(\xi, \mathbf{r}) = \sum_{I\ell j} C_{IJ}^{\ell sj} \left[\phi_a^I(\xi) \otimes \varphi_{bv}^{\ell sj}(\mathbf{r})\right]_{JM}}$$

  - $\varphi_{bv}^{\ell sj}(\mathbf{r})$: wavefunction of the valence particle (v) relative to the core a.
  - $C_{IJ}^{\ell sj}$ = spectroscopic amplitudes
  - $S_{IJ}^{\ell sj} = |C_{IJ}^{\ell sj}|^2$ = spectroscopic factors

Spectroscopic factors: angular momentum considerations

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  - $C_{IJ}^{\ell sj}$ = spectroscopic amplitudes
  - $S_{IJ}^{\ell sj} = |C_{IJ}^{\ell sj}|^2$ = spectroscopic factors

☞ *The spectroscopic factor gives the probability of finding the nucleon v in the configuration ℓsj bound to the core in the state with spin I.*

DWBA transition amplitude

- Using the parentage decompositions for *A* and *B*:

$$\int \Phi^*_\beta(\xi_\beta)\Phi_\alpha(\xi_\alpha)\, d\xi d\xi' = C^{\ell sj}_{IJ} C^{\ell' sj'}_{I'J'} \varphi^{\ell' sj'}_{bv}(\mathbf{r}')^* \varphi^{\ell sj}_{av}(\mathbf{r})$$

- Three-body DWBA transition amplitude

$$\mathcal{T}^{\text{DWBA}}_{\beta,\alpha} = C^{\ell sj}_{IJ} C^{\ell' sj'}_{I'J'} \int\int \chi^{(-)*}_\beta(\mathbf{K}',\mathbf{R}')\varphi^{\ell' sj'}_{bv}(\mathbf{r}')^*(V_\beta - U_\beta)\chi^{(+)}_\alpha(\mathbf{K},\mathbf{R})\varphi^{\ell sj}_{av}(\mathbf{r})d\mathbf{R}'d\mathbf{r}'$$

- Differential cross section:

$$\frac{d\sigma_{\alpha,\beta}}{d\Omega} = \frac{\mu_\alpha\mu_\beta}{(2\pi\hbar^2)^2} S^{\ell sj}_{IJ} S^{\ell' sj'}_{I'J'} \frac{K_f}{K_i} \left| \int\int \chi^{(-)*}_\beta(\mathbf{K}',\mathbf{R}')\varphi^{\ell' sj'}_{bv}(\mathbf{r}')^*(V_\beta - U_\beta)\chi^{(+)}_\alpha(\mathbf{K},\mathbf{R})\varphi^{\ell sj}_{av}(\mathbf{r})d\mathbf{R}'d\mathbf{r}' \right|^2$$

☞ *In DWBA, the transfer cross section is proportional to the product* $S^{\ell sj}_{IJ} S^{\ell' sj'}_{I'J'}$

Prior form of the transition amplitude

Exact transition amplitude in prior form:

$$f^{\rm prior}_{\beta,\alpha} = -\frac{\mu_\beta}{2\pi\hbar^2}\int\int \Psi^{(-)*}_{\mathbf{K}_\beta}(\mathbf{R}_\beta,\mathbf{r}_\beta)(V_\alpha - U_\alpha)\chi^{(+)}_\alpha(\mathbf{K}_\alpha,\mathbf{R}_\alpha)\Phi_\alpha(\xi_\alpha)d\xi_\alpha d\mathbf{R}_\alpha \quad ,$$

DWBA approximation: $\Psi^{(-)}_{\mathbf{K}_\beta}(\mathbf{R}_\beta,\mathbf{r}_\beta) \simeq \chi^{(-)}_\beta(\mathbf{K}',\mathbf{R}')\varphi_{bv}(\mathbf{r}')$:

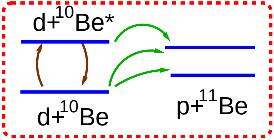
$$f^{\rm prior}_{\beta,\alpha}(\theta) = -\frac{\mu_\beta}{2\pi\hbar^2}C^{B*}_{bv}C^{A}_{av}\int\int \chi^{(-)*}_\beta(\mathbf{K}',\mathbf{R}')\varphi^*_{bv}(\mathbf{r}')(V_\alpha - U_\alpha)\varphi_{av}(\mathbf{r})\chi^{(+)}_\alpha(\mathbf{K},\mathbf{R})d\mathbf{R}d\mathbf{r}$$

In DWBA:

$$f^{\rm prior}_{\beta,\alpha}(\theta) = f^{\rm post}_{\beta,\alpha}(\theta)$$

Beyond DWBA: CCBA formalism

In presence of strongly coupled excited states in the initial or final partition, the CC and DWBA formalisms can be combined → CCBA



Exact scattering amplitude (*post form*): ($\beta \neq \alpha$)

$$\mathcal{T}^{\rm post}_{\beta,\alpha} = \int\int \chi^{(-)*}_{\beta}(\mathbf{K}_\beta, \mathbf{R}_\beta)\Phi^{*}_{\beta}(\xi_\beta)(V_\beta - U_\beta)\underbrace{\Psi^{(+)}_{\mathbf{K}_\alpha}(\mathbf{R}_\alpha, \xi_\alpha)}d\xi_\beta d\mathbf{R}_\beta$$

$$\Psi^{(+)}_{\mathbf{K}_\alpha}(\mathbf{R}_\alpha, \xi_\alpha) \approx \Psi^{\rm CC}_{\mathbf{K}_\alpha}(\mathbf{R}, \xi) = \phi_0(\xi)\chi_0(\mathbf{R}) + \sum_{n>0}\phi_n(\xi)\chi_n(\mathbf{R})$$

Beyond DWBA: CCBA formalism

When there are strongly coupled excited states in the initial or final partition, the CC and DWBA formalisms can be combined → CCBA

Ej: ^{172}Yb(p,d) Ascuitto et al, Nucl Phys. A226 (1974) 454

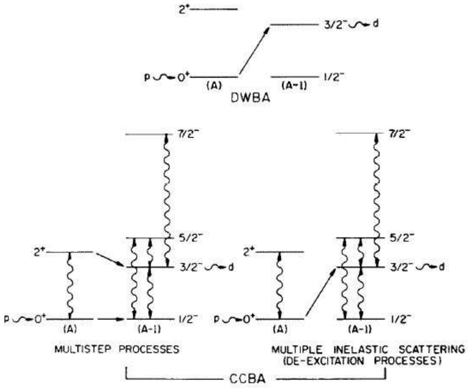


Fig. 1. Schematic representation of processes *explicitly* included in a DWBA analysis and those in a CCBA analysis. The multistep processes (for simplicity we show a single route) are predominantly determined by parentage conditions, while the de-excitation processes depend on the strength of the inelastic coupling.

Beyond DWBA: CCBA formalism

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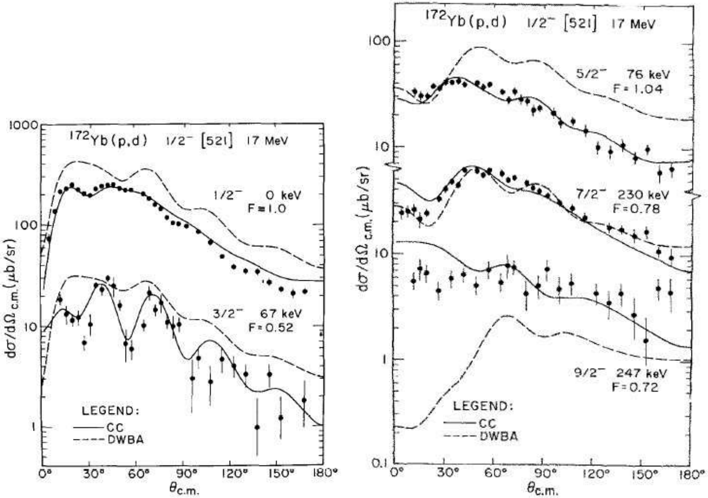
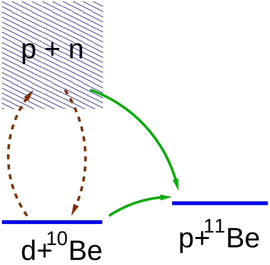


Transfer reactions with exotic nuclei

Transfer reactions with weakly bound nuclei

- DWBA approximates the total WF by the elastic channel and assumes that transfer occurs in one step (Born approximation).
- For weakly bound projectiles (eg. deuterons), breakup is an important channel and can influence the transfer process.



- $\Psi^{(+)}_{\mathbf{K}_d}(\mathbf{R},\mathbf{r})$ includes breakup components, but these are lost when we make the DWBA approximation ($\Psi^{(+)} \approx \chi^{(+)}_d(\mathbf{K}_d,\mathbf{R})\varphi_d(\mathbf{r})$) $\Rightarrow$ need to go beyond DWBA

Adiabatic approximation

- For a (d,p) reaction, $\Psi^{(+)}_{\mathbf{K}_d}(\mathbf{R},\mathbf{r})$ is a solution of

$$[\hat{T}_{\mathbf{R}} + H_d(\mathbf{r}) + U_{pb} + U_{nb} - E]\Psi^{(+)}_{\mathbf{K}_\alpha} = 0$$

- At sufficiently high energies ($E \gg \varepsilon$) we can make the adiabatic approximation:

$$H_d(\mathbf{r}) = \hat{T}_{\mathbf{r}} + V_{pn}(\mathbf{r}) \simeq \varepsilon_d$$

- $\Psi^{(+)}_{\mathbf{K}_d}(\mathbf{R},\mathbf{r}) \simeq \Psi^{(\mathrm{ad})}_{\mathbf{K}_d}(\mathbf{R},\mathbf{r}) = \chi^{ad}_d(\mathbf{R},\mathbf{r})\varphi_d(\mathbf{r})$

$$[\hat{T}_{\mathbf{R}} + \varepsilon_d + U_{pb} + U_{nb} - E]\chi^{ad}_d(\mathbf{R},\mathbf{r}) = 0$$

☞ (still complicated; depends parametrically on **r**!)

Zero-range adiabatic approximation (ADWA)

- For the transfer matrix element, we need only $\Psi^{(+)}_{\mathbf{K}_d}(\mathbf{R},\mathbf{r})$ for small $|\mathbf{r}|$
- Zero-range approximation : $\chi^{ad}_d(\mathbf{R},\mathbf{r}) \simeq \chi^{ad}_d(\mathbf{R},0) \equiv \chi^{JS}_d(\mathbf{R})$

$$[\hat{T}_{\mathbf{R}} + \varepsilon_d + U^{JS}(R) - E]\chi^{JS}_d(\mathbf{R}) = 0 \qquad \text{(Johnson-Soper)}$$

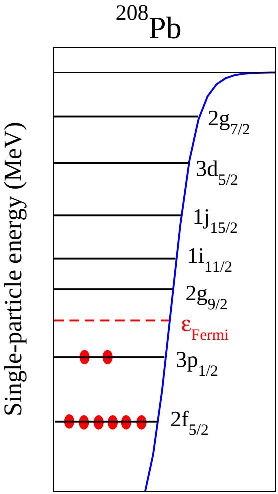
$$U^{JS}(R) = U_{pb}(R) + U_{nb}(R)$$

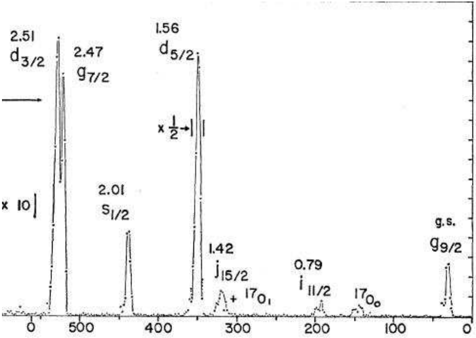
- Or, with finite-range corrections :

$$U^{JT}(R) = \frac{\langle \varphi_{pn}(\mathbf{r})|V_{pn}(U_{nb} + U_{pb})|\varphi_{pn}(\mathbf{r})\rangle}{\langle \phi_{pn}(\mathbf{r})|V_{pn}|\phi_{pn}(\mathbf{r})\rangle} \qquad \text{(Johnson–Tandy)}$$

Extracting structure information from transfer reactions

Example: $d + {}^{208}Pb \rightarrow p + {}^{209}Pb$





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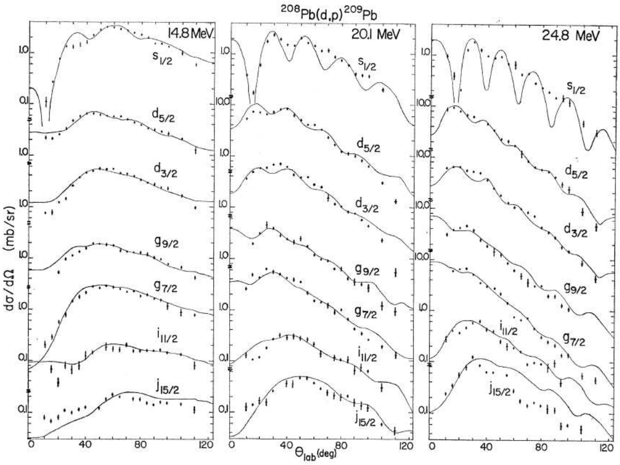
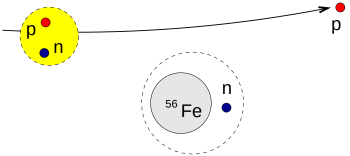
Extracting structure information from transfer reactions

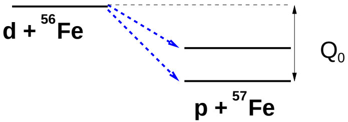


☞*Angular distributions of transfer cross sections are very sensitive to the single-particle configuration of the transferred nucleon/s.* $\Rightarrow \varphi_{nlj}(\mathbf{r})$

Transfer example

Physical example: ^{56}Fe(d,p)^{57}Fe at E_d = 12 MeV





Transfer example: ^{56}Fe(d,p)^{57}Fe

DWBA scattering amplitude:

$$f^{\rm DWBA}(\theta)_{i\to f} = -\frac{\mu_\beta}{2\pi\hbar^2} C_i C_f \langle \chi^{(-)}_{\rm p-^{57}Fe} \phi_{\rm ^{57}Fe} | V_{\rm prior/post} | \chi^{(+)}_{\rm d-^{56}Fe} \phi_d \rangle$$

Transfer example: ^{56}Fe(d,p)^{57}Fe

DWBA scattering amplitude:

$$f^{\rm DWBA}(\theta)_{i\to f} = -\frac{\mu_\beta}{2\pi\hbar^2} C_i C_f \langle \chi^{(-)}_{\rm p-^{57}Fe} \phi_{\rm ^{57}Fe} | V_{\rm prior/post} | \chi^{(+)}_{\rm d-^{56}Fe} \phi_d \rangle$$

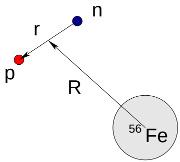
- $\chi_{\rm d-^{56}Fe}, \chi_{\rm p-^{57}Fe}$: initial and final distorted waves
- ϕ_d: projectile bound wavefunction($p-n$)
- $\phi_{\rm ^{57}Fe}$: final (residual) wavefunction (n+^{56}Fe)
- C_i, C_f: initial / final spectroscopic amplitudes.
- $V_{\rm prior/post}$: transition potential in PRIOR or POST form

Transfer example: ^{56}Fe(d,p)^{57}Fe

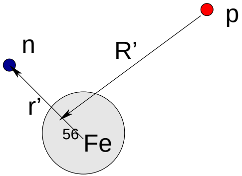
DWBA scattering amplitude:

$$f^{\rm DWBA}(\theta)_{i\to f} = -\frac{\mu_\beta}{2\pi\hbar^2} C_i C_f \langle \chi^{(-)}_{\rm p-^{57}Fe}\phi_{\rm ^{57}Fe}|V_{\rm prior/post}|\chi^{(+)}_{\rm d-^{56}Fe}\phi_d\rangle$$

PRIOR



POST



$$V_{\rm prior} = V_{\rm n-56Fe} + \underbrace{U_{\rm p-56Fe} - U_{\rm d-56Fe}}_{\tt remnant}$$

$$V_{\rm post} = V_{\rm p-n} + \underbrace{U_{\rm p-56Fe} - U_{\rm p-57Fe}}_{\tt remnant}$$

Transfer example: ^{56}Fe(d,p)^{57}Fe

Essential physical ingredients in a DWBA calculation:

- **Potentials (5):**
  - Distorted potential for entrance channel (complex): d+^{56}Fe
  - Distorted potential for exit channel (complex): p+^{57}Fe
  - Core-core interaction (complex): p+^{56}Fe
  - Binding potential for projectile (real): p+n
  - Binding potential for target (real): n+^{56}Fe
- **Spectroscopic amplitudes:** C_i, C_f

Transfer example: ^{56}Fe(d,p)^{57}Fe

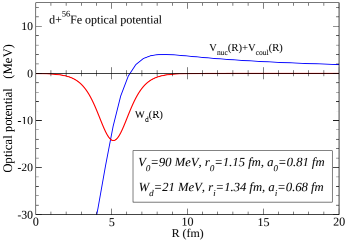
Physical ingredients: Optical and binding potentials (NPA(1971) 529)

System	$\mathbf{V}_0$ (MeV)	$\mathbf{r}_0$ (fm)	$\mathbf{a}_0$ (fm)	$\mathbf{W}_d$ (MeV)	$\mathbf{r}_i$ (fm)	$\mathbf{a}_i$ (fm)	$\mathbf{r}_C$ (fm)
d+^{56}Fe	90	1.15	0.81	21.0	1.34	0.68	1.15
p+^{56,57}Fe	47.9	1.25	0.65	11.5	1.25	0.47	1.15
$p+n$ [1]	72.15	0.00	1.484	-	-	-	
n+^{56}Fe	B.E.	1.25	0.65	-	-	-	

⇒

$$U(R) = -V_0 f_{WS}(R) + 4\, i\, a\, W_d \frac{df_{WS}(R)}{dR}$$

$$f_{WS}(R) = \frac{1}{1+\exp\left(\frac{R-R_0}{a}\right)}$$



[1] Gaussian geometry: $V(r) = -V_0 \exp[-(r/a_0)^2]$.

Transfer example: ^{56}Fe(d,p)^{57}Fe

Spectroscopic factors:

$$\phi_B^{JM}(\xi, \mathbf{r}) = \sum_{I\ell j} A_{\ell s j}^{IJ} \left[\phi_b^I(\xi) \otimes \varphi_{\ell s j}(\mathbf{r})\right]_{JM}$$

In our example:

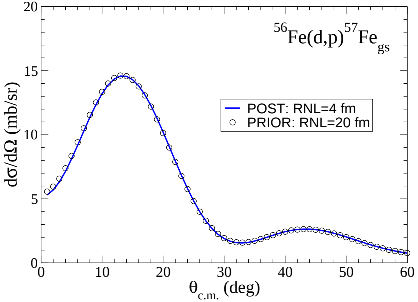
- d=p+n: Mostly 1*s* configuration with spectroscopic factor 1.
- ^{57}Fe=^{56}Fe+n

$$|^{57}\mathrm{Fe; gs}\rangle_{1/2^-} = \alpha \left[|^{56}\mathrm{Fe; gs}\rangle \otimes |n;\ 2p_{1/2}\rangle\right]_{1/2^-} + \beta \left[|^{56}\mathrm{Fe}; 2^+\rangle \otimes |n;\ 2p_{3/2}\rangle\right]_{1/2^-} + \ldots$$

  - $\alpha, \beta, \ldots$: spectroscopic amplitudes
  - $|\alpha|^2, |\beta|^2, \ldots$: spectroscopic factors

Transfer example: ^{56}Fe(d,p)^{57}Fe

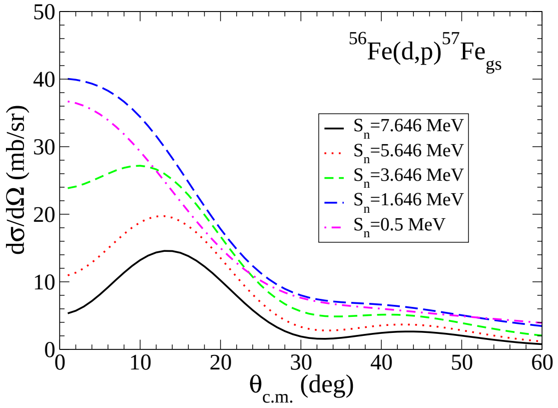
Post and prior equivalence:



☞ *Post and prior give identical results, provide that the parameters are adequate for convergence.*

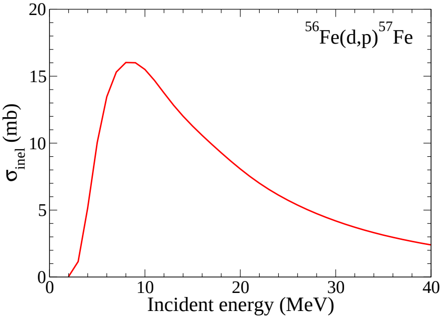
Transfer example: ^{56}Fe(d,p)^{57}Fe

Dependence with binding energy:



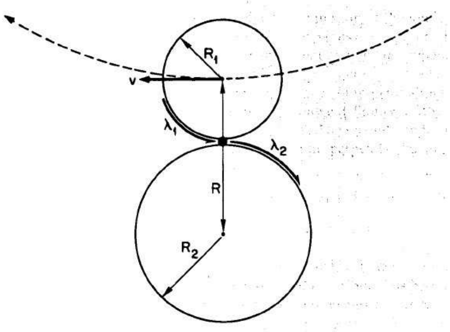
Transfer example: ^{56}Fe(d,p)^{57}Fe

Dependence with beam energy:



Transfer example: ^{56}Fe(d,p)^{57}Fe

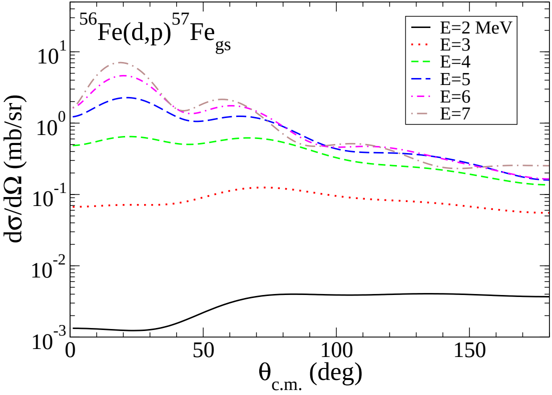
☞ Due to orbital and momentum matching conditions there is an optimal Q–value



Transfer example: ^{56}Fe(d,p)^{57}Fe

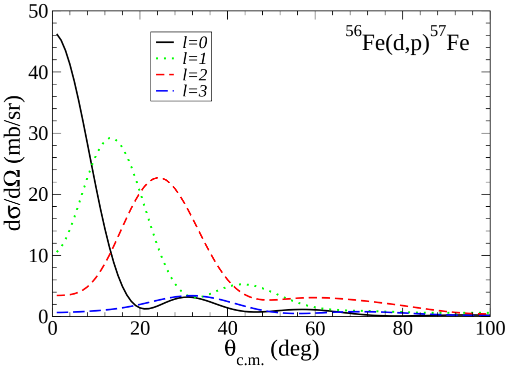
Dependence with beam energy

- $E \gg V_b$: diffractive structure, forward peaked.
- $E \ll V_b$: smooth dependence with θ, backward peaked.



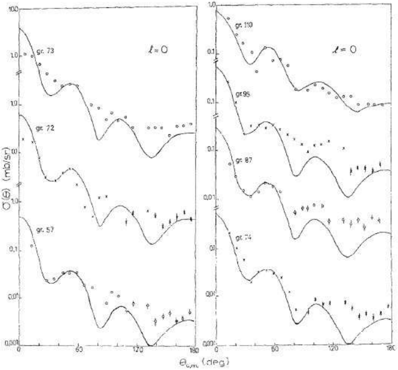
Transfer example: ^{56}Fe(d,p)^{57}Fe

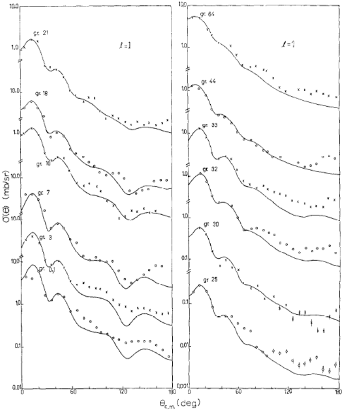
Selectivity of ℓ:



Transfer example: ^{56}Fe(d,p)^{57}Fe

Selectivity of ℓ:





H.M. Sen Gupta et al, Nucl. Phys. A160, 529 (1971)

Advanced topic: Coupled-Reaction Channels method

Coupled Reaction Channels

- Model wavefunction:

$$\Psi = \phi_A(\xi, \mathbf{r})\phi_b(\xi')\chi_\alpha(\mathbf{R}_\alpha) + \phi_a(\xi)\phi_B(\xi', \mathbf{r}')\chi_\beta(\mathbf{R}_\beta)$$

- Coupled-reaction channels (CRC) equations: $[H - E]\Psi = 0$

$$\begin{aligned}
[E - \varepsilon_\alpha - T_R - U_\alpha(\mathbf{R}_\alpha)]\chi_\alpha(\mathbf{R}_\alpha) &= \int d\mathbf{R}_\beta K_{\alpha,\beta}(\mathbf{R}_\alpha, \mathbf{R}_\beta)\chi_\beta(\mathbf{R}_\beta) \\
\left[E - \varepsilon_\beta - T_R - U_\beta(\mathbf{R}_\beta)\right]\chi_\beta(\mathbf{R}_\beta) &= \int d\mathbf{R}_\alpha K_{\alpha,\beta}(\mathbf{R}_\alpha, \mathbf{R}_\beta)\chi_\alpha(\mathbf{R}_\alpha)
\end{aligned}$$

- Non-local kernels:

$$K_{\alpha,\beta}(\mathbf{R}_\beta, \mathbf{R}_\alpha) = \int d\xi d\xi' d\mathbf{r}\; \phi_a(\xi)\phi_B(\xi', \mathbf{r}')(H - E)\phi_A(\xi, \mathbf{r})\phi_b(\xi')$$

☞ *CRC equations hav e to be solved iteratively due to NL kernels.*

DWBA approximation from CRC

- Iterative solution of the CRC equations:

$$\begin{aligned}[E-\varepsilon_\alpha-T_R-U_\alpha(\mathbf{R}_\alpha)]\chi_\alpha(\mathbf{R}_\alpha) &\approx 0 \\ \left[E-\varepsilon_\beta-T_R-U_\beta(\mathbf{R}_\beta)\right]\chi_\beta(\mathbf{R}_\beta) &\approx \int d\mathbf{R}_\alpha K_{\alpha,\beta}(\mathbf{R}_\alpha,\mathbf{R}_\beta)\chi_\alpha(\mathbf{R}_\alpha)\end{aligned}$$

- DWBA scattering amplitude (prior):

$$f^{\rm DWBA}_{\beta,\alpha} = -\frac{\mu_\beta}{2\pi\hbar^2}\int\int \widetilde{\chi}^{(-)}_\beta(\mathbf{R}_\beta)(\phi_a\phi_B|V_{\rm prior}|\phi_A\phi_b)\widetilde{\chi}^{(+)}_\alpha(\mathbf{R}_\alpha)d\mathbf{R}_\alpha d\mathbf{r}$$

- Distorted waves:

$$\begin{aligned}[E-\varepsilon_\alpha-T_R-U_\alpha(\mathbf{R}_\alpha)]\widetilde{\chi}_\alpha(\mathbf{R}_\alpha) &= 0 \\ \left[E-\varepsilon_\beta-T_R-U_\beta(\mathbf{R}_\beta)\right]\widetilde{\chi}_\beta(\mathbf{R}_\beta) &= 0\end{aligned}$$

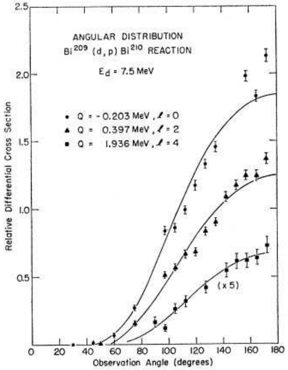
- Structure form-factor:

$$(\phi_a\phi_B|V_{\rm prior}|\phi_A\phi_b) \equiv \int d\xi d\xi' \;\phi_a(\xi)\phi_B(\xi',\mathbf{r}')V_{\rm prior}\phi_A(\xi,\mathbf{r})\phi_b(\xi')$$

Sub-coulomb transfer

Below the Coulomb barrier:

- The transfer angular distribution is less sensitive to the orbital angular momentum ℓ.
- Less ambiguities due to nuclear potentials (useful to determine spectroscopic factors).



Brief summary on transfer reactions

- Inclusion of transfer couplings in the Schrodinger equation gives rise to a set of coupled equations with non-local kernels (Coupled Reactions Channels)
- If transfer couplings are weak, the CRC equations can be solved in Born approximation $\Rightarrow$ DWBA approximation
- The DWBA amplitude is proportional to the product of the projectile and target spectroscopic factors.
- The analysis of transfer reactions provide information on:
  - Spectroscopic factors.
  - Quantum number for single-particle configurations (n, ℓ, j).
  - Binding interactions.
  - Reactions mechanisms.