# $B \rightarrow D^{(*)} \tau \bar{\nu}$ in various new physics models 

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# (1) Introduction and motivation 

(2) Test of some New Physics models

- 2HDM of type III
- Leptoquark models
- MSSM with $R$-parity violation
(3) Conclusions

- Tree-level (TL) process. Large $\mathcal{B}^{(\mathrm{SM})} \sim(1-2) \%$.

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- $B$-decays with $\tau$ in the final state offer possibilities to study NP effects not present in processes with light leptons.
- Hadronic uncertainties better controlled (or can be!).
- Popular NP test via

$$
R(D)=\frac{\mathcal{B}\left(B \rightarrow D \tau \bar{\nu}_{\tau}\right)}{\mathcal{B}\left(B \rightarrow D \ell \bar{\nu}_{\ell}\right)}, \quad R\left(D^{*}\right)=\frac{\mathcal{B}\left(B \rightarrow D^{*} \tau \bar{\nu}_{\tau}\right)}{\mathcal{B}\left(B \rightarrow D^{*} \ell \bar{\nu}_{\ell}\right)} \quad(\ell=e, \mu)
$$

in order to cancel/reduce theoretical uncertainties in $V_{c b} / \mathrm{FFs}$.

The BABAR results [arxiv: :1205.5442],

$$
\begin{aligned}
& R(D)^{\exp }=0.440 \pm 0.058 \pm 0.042, \quad R(D)^{\mathrm{SM}}=0.297 \pm 0.017 \\
& R\left(D^{*}\right)^{\exp }=0.332 \pm 0.024 \pm 0.018, \quad R\left(D^{*}\right)^{\mathrm{SM}}=0.252 \pm 0.003
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disagree with the SM at the $3.4 \sigma$ level (combining with Belle result, we obtain $3.5 \sigma$ ).

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## Motivation

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[BABAR('13), arXiv:1303.0571]


## Leptoquarks ?

1 h MSSM ?
smth else?

- We assume that there is NO right-handed neutrino.


## $\mathcal{H}_{\text {eff }}$ describing the $b \rightarrow c \tau \bar{\nu}$ process

$$
\begin{gathered}
\mathcal{H}_{\mathrm{eff}}=\frac{4 G_{F}}{\sqrt{2}} V_{c b}[(\underbrace{(1)}_{\mathrm{SM}}+\underbrace{\left.C_{V_{1}}\right) \mathcal{O}_{V_{1}}+C_{V_{2}} \mathcal{O}_{V_{2}}+C_{S_{1}} \mathcal{O}_{S_{1}}+C_{S_{2}} \mathcal{O}_{S_{2}}+C_{T} \mathcal{O}_{T}}_{\mathrm{NP}}] \\
\mathcal{O}_{V_{1}}=\left(\bar{c}_{L} \gamma^{\mu} b_{L}\right)\left(\bar{\tau}_{L} \gamma_{\mu} \nu_{L}\right), \quad \mathcal{O}_{V_{2}}=\left(\bar{c}_{R} \gamma^{\mu} b_{R}\right)\left(\bar{\tau}_{L} \gamma_{\mu} \nu_{L}\right), \\
\mathcal{O}_{S_{1}}=\left(\bar{c}_{L} b_{R}\right)\left(\bar{\tau}_{R} \nu_{L}\right), \quad \mathcal{O}_{S_{2}}=\left(\bar{c}_{R} b_{L}\right)\left(\bar{\tau}_{R} \nu_{L}\right), \\
\mathcal{O}_{T}=\left(\bar{c}_{R} \sigma^{\mu \nu} b_{L}\right)\left(\bar{\tau}_{R} \sigma_{\mu \nu} \nu_{L}\right) .
\end{gathered}
$$

- E.g. in the 2HDM-II

$$
C_{S_{1}}=-\frac{m_{b} m_{\tau}}{m_{H^{ \pm}}^{2}} \tan ^{2} \beta
$$

which is excluded by $B A B A R \quad \forall \tan \beta / m_{H^{ \pm}} \Rightarrow$ the $S_{1}$ scenario is discarded!

NB: the pseudotensor operator is not independent of $\mathcal{O}_{T}$ due to the relation $\bar{c} \sigma_{\mu \nu} \gamma_{5} b=-\frac{i}{2} \epsilon_{\mu \nu \alpha \beta} \bar{c} \sigma^{\alpha \beta} b$.

## Constraints on NP from $R(D) \& R\left(D^{*}\right)$

Assuming the presence of only one NP type, we do $\chi^{2}$ fit of the combined $B A B A R+$ Belle result on $R(D) \& R\left(D^{*}\right)$ and obtain the constraints on the NP Wilson coefficients:





## 2HDM of type III with non-minimal flavour violation

- 2HDM III with MFV cannot explain $R(D)$ and $R\left(D^{*}\right)$ simultaneously.
- Both Higgs doublets couple simultaneously to up and down quarks (in the 2HDM-II one doublet couples to down quarks and charged leptons, while the other one gives masses to the up quarks).

$$
\mathcal{L}_{\text {Yukawa }}=\bar{Q}_{f L}^{a}\left[Y_{f i}^{d} \epsilon_{a b} H_{d}^{b *}-\varepsilon_{f i}^{d} H_{u}^{a}\right] d_{i R}-\bar{Q}_{f L}^{a}\left[Y_{f i}^{u} \epsilon_{a b} H_{u}^{b *}+\varepsilon_{f i}^{u} H_{u}^{a}\right] u_{i R}+\text { h.c. }
$$




$$
m_{i j}^{d}=v_{d} Y_{i j}^{d}+v_{u} \varepsilon_{i j}^{d}
$$

$$
m_{i j}^{u}=v_{u} Y_{i j}^{u}+v_{d} \varepsilon_{i j}^{u}
$$

- $\varepsilon_{13,31}^{d}, \varepsilon_{23,32}^{d}, \varepsilon_{12,21}^{d}$ are stringently constrained from FCNC processes $B_{(s)} \rightarrow \mu^{+} \mu^{-}, K_{L} \rightarrow \mu^{+} \mu^{-}\left(|\varepsilon| \lesssim \# 10^{-5}\right)$.
- $\varepsilon_{12,21}^{u}$ and $\varepsilon_{23,13}^{u}$ are constrained from $D \rightarrow \mu^{+} \mu^{-}$and $b \rightarrow(s, d) \gamma$ $\left(|\varepsilon| \lesssim \# 10^{-2}\right)$. BUT $\varepsilon_{31,32}^{u}$ are still unconstrained!
- Lepton-Higgs couplings are assumed to be like in the 2HDM-II.
[Crivellin at al.('12), arXiv:1206.2634], [Crivellin @Moriond 2014, arXiv:1405.3701]


## Constraining 2HDM-III

$$
C_{S_{1}} \simeq \frac{1}{2 \sqrt{2} G_{F}} \frac{m_{\tau}}{v} \varepsilon_{33}^{d} \frac{\sin \beta \tan ^{2} \beta}{M_{H^{ \pm}}^{2}}
$$

-disfavoured by BABAR!

$$
C_{S_{2}} \simeq \frac{1}{2 \sqrt{2} G_{F} V_{c b}} \frac{m_{\tau}}{v} \varepsilon_{32}^{u *} \frac{\sin \beta \tan \beta}{M_{H^{ \pm}}^{2}}
$$



Allowed $1 \sigma$ regions from $R(D)$ (blue) and $R\left(D^{*}\right)$ (yellow) for $\tan \beta=50$ and
$M_{H}=500 \mathrm{GeV}$
[Crivellin at al.('12), arXiv:1206.2634]

- 2HDM-III can explain $R(D)$ and $R\left(D^{*}\right)$ simultaneously using a single free parameter $\varepsilon_{32}^{u}$.
- Search for $A^{0}, H^{0} \rightarrow \bar{t} c$ and $t \rightarrow h^{0} c$ at the LHC to test the model.


## Leptoquark models

$\mathcal{L}_{\text {eff }}$ with generic dimensionless $S U(3) \times S U(2) \times U(1)$ invariant non-diagonal couplings of scalar and vector LQs ( 6 models)

$$
\begin{aligned}
\mathcal{L}_{F=0}^{\mathrm{LQ}}= & \left(h_{1 L}^{i j} \bar{Q}_{i L} \gamma^{\mu} L_{j L}+h_{1 R}^{i j} \bar{d}_{i R} \gamma^{\mu} \ell_{j R}\right) U_{1 \mu}+h_{3 L}^{i j} \bar{Q}_{i L} \boldsymbol{\sigma} \gamma^{\mu} L_{j L} \boldsymbol{U}_{3 \mu} \\
& +\left(h_{2 L}^{i j} \bar{u}_{i R} L_{j L}+h_{2 R}^{i j} \bar{Q}_{i L} i \sigma_{2} \ell_{j R}\right) R_{2} \\
\mathcal{L}_{F=-2}^{\mathrm{LQ}}= & \left(g_{1 L}^{i j} \bar{Q}_{i L}^{c} i \sigma_{2} L_{j L}+g_{1 R}^{i j} \bar{u}_{i R}^{c} \ell_{j R}\right) S_{1}+g_{3 L}^{i j} \bar{Q}_{i L}^{c} i \sigma_{2} \boldsymbol{\sigma} L_{j L} S_{3} \\
& +\left(g_{2 L}^{i j} \bar{d}_{i R}^{c} \gamma^{\mu} L_{j L}+g_{2 R}^{i j} \bar{Q}_{i L}^{c} \gamma^{\mu} \ell_{j R}\right) V_{2 \mu}
\end{aligned}
$$

[Buchmüller et al.('87), Phys.Lett.B191]

Quantum numbers

|  | $\boldsymbol{S}_{\mathbf{1}}$ | $\boldsymbol{S}_{\mathbf{3}}$ | $\boldsymbol{V}_{\mathbf{2}}$ | $\boldsymbol{R}_{\mathbf{2}}$ | $\boldsymbol{U}_{\mathbf{1}}$ | $\boldsymbol{U}_{\mathbf{3}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| spin | 0 | 0 | 1 | 0 | 1 | 1 |
| $F=3 B+L$ | -2 | -2 | -2 | 0 | 0 | 0 |
| $S U(3)_{c}$ | $3^{*}$ | $3^{*}$ | $3^{*}$ | 3 | 3 | 3 |
| $S U(2)$ | 1 | 3 | 2 | 2 | 1 | 3 |
| $U(1)_{Y=Q-T_{3}}$ | $1 / 3$ | $1 / 3$ | $5 / 6$ | $7 / 6$ | $2 / 3$ | $2 / 3$ |



$$
\begin{aligned}
\mathrm{LQ}= & R_{2}^{2 / 3}, U_{1 \mu}^{2 / 3}, U_{3 \mu}^{2 / 3} \\
& \left(S_{1}^{1 / 3}, S_{3}^{1 / 3}, V_{2 \mu}^{1 / 3}\right)
\end{aligned}
$$

## Leptoquark contribution to $B \rightarrow D^{(*)} \tau \bar{\nu}$

General Wilson coefficients (at $M_{\mathrm{LQ}}$ scale) for all possible types of $L Q s$ contributing to the $b \rightarrow c \tau \bar{\nu}$ process :

$$
\begin{aligned}
C_{V_{1}} & \simeq \frac{1}{2 \sqrt{2} G_{F} V_{c b}}\left[\frac{g_{1 L}^{33} g_{1 L}^{23 *}}{2 M_{S_{1}}^{2}}-\frac{g_{3 L}^{33} g_{3 L}^{23 *}}{2 M_{S_{3}}^{2}}+\frac{h_{1 L}^{23} h_{1 L}^{33 *}}{M_{U_{1}}^{2}}-\frac{h_{3 L}^{23} h_{3 L}^{33 *}}{M_{U_{3}}^{2}}\right] \\
C_{V_{2}} & =0 \\
C_{S_{1}} & \simeq \frac{1}{2 \sqrt{2} G_{F} V_{c b}}\left[-\frac{2 g_{2 L}^{33} g_{2 R}^{23 *}}{M_{V_{2}}^{2}}-\frac{2 h_{1 L}^{23} h_{1 R}^{33 *}}{M_{U_{1}}^{2}}\right] \\
C_{S_{2}} & \simeq \frac{1}{2 \sqrt{2} G_{F} V_{c b}}\left[-\frac{g_{1 L}^{33} g_{1 R}^{23 *}}{2 M_{S_{1}}^{2}}-\frac{h_{2 L}^{23} h_{2 R}^{33 *}}{2 M_{R_{2}}^{2}}\right] \\
C_{T} & \simeq \frac{1}{2 \sqrt{2} G_{F} V_{c b}}\left[\frac{g_{1 L}^{33} g_{1 R}^{23 *}}{8 M_{S_{1}}^{2}}-\frac{h_{2 L}^{23} h_{2 R}^{33 *}}{8 M_{R_{2}}^{2}}\right]
\end{aligned}
$$

- We neglect $\mathcal{O}\left(\lambda^{2}\right)$ terms and keep only the leading terms proportional to $V_{t b}$.
- In the simplified scenario with only $R_{2}^{2 / 3}$ or $S_{1}^{1 / 3} \mathrm{LQ}$ contribution,

$$
C_{S_{2}}\left(M_{\mathrm{LQ}}\right)= \pm 4 C_{T}\left(M_{\mathrm{LQ}}\right) \Rightarrow \text { for } M_{\mathrm{LQ}} \sim 1 \mathrm{TeV}, C_{S_{2}}\left(\mu_{b}\right) \simeq \pm 7.8 C_{T}\left(\mu_{b}\right)
$$

[Sakaki,Tanaka, AT, Watanabe('13), arXiv:1309.0301]
$S_{1}$ with only $L$ couplings

$S_{1}$ with $L, R$ couplings

$R_{2}$ with $L, R$ couplings


- The constraints on $g_{1(3) L}^{33} g_{1(3) L}^{23 *}\left(S_{1,3}\right)$ from $R(D) \& R\left(D^{*}\right)$ and $\mathcal{B}\left(B \rightarrow X_{s} \nu \bar{\nu}\right)$ are consistent only at $3 \sigma$ level and force the couplings to be rather small.
- The $U_{3} \mathrm{LQ}$ scenario with $h_{3 L}^{23} h_{3 L}^{33 *}$ is excluded by $R(D) \& R\left(D^{*}\right)$ and $\mathcal{B}\left(B \rightarrow X_{s} \nu \bar{\nu}\right)$.
- $h_{1 L}^{23} h_{1 L}^{33 *}\left(U_{1}\right)$ remain unconstrained from $\mathcal{B}\left(B \rightarrow X_{s} \nu \bar{\nu}\right)$ and the magnitude of $\mathcal{O}(1)$ can be sufficient to explain the data.
- For $M_{S_{1}, R_{2}} \sim 1 \mathrm{TeV}$, one can have $g_{1 L}^{33} g_{1 R}^{23 *}, h_{2 L}^{23} h_{2 R}^{33 *} \sim \mathcal{O}(1)$.
- The other $V_{2}$ and $U_{1}$ LQ scenarios with $L, R$ couplings are disfavoured as in the 2HDM-II.

$$
\begin{gathered}
W_{R \mathrm{PV}}=\frac{1}{2} \lambda_{i j k} L_{i} L_{j} E_{k}^{c}+\lambda_{i j k}^{\prime} L_{i} Q_{j} D_{k}^{c} \\
\mathcal{L}_{\mathrm{eff}}^{R \mathrm{PV}}=-\sum_{j, k=1}^{3} V_{2 k}\left[\frac{\lambda_{3 j 3} \lambda_{j k 3}^{\prime *}}{m_{\tilde{e}_{L}^{j}}^{2}}\left(\bar{c}_{L} b_{R}\right)\left(\bar{\tau}_{R} \nu_{L}\right)+\frac{\lambda_{33 j}^{\prime} \lambda_{3 k j}^{\prime *}}{m_{\tilde{d}_{R}^{j}}^{2}}\left(\bar{c}_{L} \tau_{R}^{c}\right)\left(\bar{\nu}_{R}^{c} b_{L}\right)\right] \\
=- \\
\sum_{j, k=1}^{3} V_{2 k}[\frac{\lambda_{3 j 3} \lambda_{j k 3}^{\prime *}}{m_{\tilde{e}_{L}^{j}}^{2}} \underbrace{\left(\bar{c}_{L} b_{R}\right)\left(\bar{\tau}_{R} \nu_{L}\right)}_{\mathcal{O}_{S_{1}}}+\frac{\lambda_{33 j}^{\prime} \lambda_{3 k j}^{\prime *}}{2 m_{\tilde{d}_{R}^{j}}^{2}} \underbrace{\left(\bar{c}_{L} \gamma^{\mu} b_{L}\right)\left(\bar{\tau}_{L} \gamma_{\mu} \nu_{L}\right)}_{\mathcal{O}_{V_{1}}}]
\end{gathered}
$$



The corresponding Wilson coefficients are

$$
C_{S_{1}}=\frac{1}{2 \sqrt{2} G_{F} V_{c b}} \sum_{j, k=1}^{3} V_{2 k} \frac{\lambda_{3 j 3} \lambda_{j k 3}^{\prime *}}{2 m_{\tilde{e}_{L}^{j}}^{2}}, \quad C_{V_{1}}=\frac{1}{2 \sqrt{2} G_{F} V_{c b}} \sum_{j, k=1}^{3} V_{2 k} \frac{\lambda_{33 j}^{\prime} \lambda_{3 k j}^{\prime *}}{2 m_{\tilde{d}_{R}^{j}}^{2}}
$$

[Tanaka, Watanabe('12), arXiv:1212.1878]

The largest effect on $R\left(D^{(*)}\right)$ is obtained for
$R \mathrm{PV}$ couplings for $k=2$ :

$$
\mathcal{B}^{\exp }\left(B \rightarrow X_{s} \nu \bar{\nu}\right)<6.4 \times 10^{-4}
$$

$$
C_{V_{1}}^{\mathrm{kMSSM}} \simeq \frac{1}{2 \sqrt{2} G_{F} V_{c b}} \sum_{j=1}^{3} \frac{\lambda_{33 j}^{\prime} \lambda_{32 j}^{\prime *}}{2 m_{\tilde{d}_{R}^{j}}^{2}}
$$

The same $R \mathrm{PV}$ couplings appear also in the NP contribution to $b \rightarrow s \nu \bar{\nu}$ :

$$
\begin{aligned}
\mathcal{H}_{\mathrm{eff}} & =\frac{4 G_{F}}{\sqrt{2}} V_{t b} V_{t s}^{*}\left[C_{L}^{(\mathrm{SM})}+C_{L}\right] \mathcal{O}_{L} \\
\mathcal{O}_{L} & =\left(\bar{s}_{L} \gamma^{\mu} b_{L}\right)\left(\bar{\nu}_{L} \gamma_{\mu} \nu_{L}\right) \\
C_{L}^{\not \mathrm{MSSM}} & \simeq \frac{1}{2 \sqrt{2} G_{F} V_{t b} V_{t s}^{*}} \sum_{j=1}^{3} \frac{\lambda_{33 j}^{\prime} \lambda_{32 j}^{\prime *}}{m_{\tilde{d}_{R}^{j}}^{2}}
\end{aligned}
$$

[ALEPH('00), arXiv:0010022]

[Tanaka, Watanabe('12), arXiv:1212.1878]

- MSSM with $R \mathrm{PV}$ is inconsistent with both $B \rightarrow D^{(*)} \tau \bar{\nu}$ and $B \rightarrow X_{s} \nu \bar{\nu}$ at the same time.
- $\mathcal{L}_{\mathrm{eff}}^{R \mathrm{PV}}$ involves the interaction which induces $\mathrm{LFV} \Rightarrow$ one can have $\nu$ with flavour different from $\tau$. In this case the conclusion remains same.
(1) Not only FCNC loop processes can provide a window to NP search. Tree-level decays are as good and often even more interesting, especially when the hadronic uncertainties are well controlled.
(2) Excess in $\bar{B} \rightarrow D \tau \bar{\nu}$ and $\bar{B} \rightarrow D^{*} \tau \bar{\nu}$, observed by BABAR and Belle, helped discarding 2HDM-II.
(3) 2HDM of type III with MSSM-like Higgs potential and flavour-violation in the up sector can explain these deviations from the SM.
(1) Some of the leptoquark models can also explain the observed discrepancy in $R(D)$ and $R\left(D^{*}\right)$ and can provide quite good constraints on leptoquark couplings which are allowed to be $\sim \mathcal{O}(1)$.
(0) MSSM with $R$-parity violation is not likely to be consistent with both $B \rightarrow D^{(*)} \tau \bar{\nu}$ and $B \rightarrow X_{s} \nu \bar{\nu}$ at the same time and therefore is disfavoured.
(- More precise data that will be given in a future super $B$ factory experiment will allow us to identify the relevant NP operator(s) and test some particular NP models if the deviation from the SM persists.

BACKUP SLIDES

- $R$ ratios (to be improved at Belle II)

$$
R\left(D^{(*)}\right)=\frac{\mathcal{B}\left(B \rightarrow D^{(*)} \tau \bar{\nu}\right)}{\mathcal{B}\left(B \rightarrow D^{(*)} \ell \bar{\nu}\right)}
$$

- $\tau$ forward-backward asymmetry,

$$
\begin{gathered}
\mathcal{A}_{\mathrm{FB}}=\frac{\int_{0}^{1} \frac{d \Gamma}{d \cos \theta} d \cos \theta-\int_{-1}^{0} \frac{d \Gamma}{d \cos \theta} d \cos \theta}{\int_{-1}^{1} \frac{d \Gamma}{d \cos \theta} d \cos \theta}=\frac{\int b_{\theta}\left(q^{2}\right) d q^{2}}{\Gamma} \\
\frac{d^{2} \Gamma}{d q^{2} d \cos \theta}=a_{\theta}\left(q^{2}\right)+b_{\theta}\left(q^{2}\right) \cos \theta+c_{\theta}\left(q^{2}\right) \cos ^{2} \theta
\end{gathered}
$$

- $\tau$ polarization parameter by studying further $\tau$ decays,

$$
P_{\tau}=\frac{\Gamma\left(\lambda_{\tau}=1 / 2\right)-\Gamma\left(\lambda_{\tau}=-1 / 2\right)}{\Gamma\left(\lambda_{\tau}=1 / 2\right)+\Gamma\left(\lambda_{\tau}=-1 / 2\right)}
$$

- $D^{*}$ longitudinal polarization using the $D^{*} \rightarrow D \pi$ decay,

$$
P_{D^{*}}=\frac{\Gamma\left(\lambda_{D^{*}}=0\right)}{\Gamma\left(\lambda_{D^{*}}=0\right)+\Gamma\left(\lambda_{D^{*}}=1\right)+\Gamma\left(\lambda_{D^{*}}=-1\right)}
$$

## How to distinguish between NP scenarios : correlations (illustration)

Applying the constraints on $C_{S_{2}}$ or $C_{T}$ from the $\chi^{2}$ fit of $R(D) \& R\left(D^{*}\right)$ at $3 \sigma$ level,


arXiv: 1309.0301]

Measurements of these observables in addition to more precise determination of $R\left(D^{(*)}\right)$ are the key issue in order to identify the origin of the present excess of $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}$.

BUT this is NOT an easy experimental task $)^{(\cdot)}$


- To reduce the FF uncertainties, one can explore the $q^{2}$-dependent ratio

$$
R_{D^{(*)}}\left(q^{2}\right) \equiv \frac{d \mathcal{B}\left(\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}\right) / d q^{2}}{d \mathcal{B}\left(\bar{B} \rightarrow D^{(*)} \ell \bar{\nu}\right) / d q^{2}}
$$

- For our convenience, to remove the divergence of $R_{D}$ at $q^{2}=\left(m_{B}-m_{D}\right)^{2} \dagger$ and the phase space suppression of $R_{D^{(*)}}$ at $q^{2} \sim m_{\tau}^{2}$, we introduce

$$
\begin{aligned}
R_{D}^{\prime}\left(q^{2}\right) & \equiv R_{D}\left(q^{2}\right) \times \frac{\lambda_{D}\left(q^{2}\right)}{\left(m_{B}^{2}-m_{D}^{2}\right)^{2}} \times\left(1-\frac{m_{\tau}^{2}}{q^{2}}\right)^{-2} \\
R_{D^{*}}^{\prime}\left(q^{2}\right) & \equiv R_{D^{*}}\left(q^{2}\right) \times\left(1-\frac{m_{\tau}^{2}}{q^{2}}\right)^{-2}
\end{aligned}
$$

$\dagger$ Since the $\mu$-mode is supposed to be SM-like, $\mathcal{B}_{\mu}^{-1} \propto\left(H_{V}^{s}\right)^{-2} \propto \lambda_{D}^{-1}\left(q^{2}\right)$.



Belle II, $\mathcal{L}=10 \mathrm{ab}^{-1}$


Belle II, $\mathcal{L}=10 \mathrm{ab}^{-1}$

[Sakaki, Tanaka, AT, Watanabe, in preparation]
$\mathcal{H}_{\text {eff }}$ describing the $b \rightarrow s \nu_{j} \bar{\nu}_{i}$ process

$$
\begin{gathered}
\mathcal{H}_{\mathrm{eff}}=\frac{4 G_{F}}{\sqrt{2}} V_{t b} V_{t s}^{*}\left[\left(\delta_{i j} C_{L}^{(\mathrm{SM})}+C_{L}^{i j}\right) \mathcal{O}_{L}^{i j}+C_{R}^{i j} \mathcal{O}_{R}^{i j}\right] \\
\mathcal{O}_{L}^{i j}=\left(\bar{s}_{L} \gamma^{\mu} b_{L}\right)\left(\bar{\nu}_{j L} \gamma_{\mu} \nu_{i L}\right), \quad \mathcal{O}_{R}^{i j}=\left(\bar{s}_{R} \gamma^{\mu} b_{R}\right)\left(\bar{\nu}_{j L} \gamma_{\mu} \nu_{i L}\right)
\end{gathered}
$$

- In the SM,

$$
C_{L}^{(\mathrm{SM})}=\frac{\alpha}{2 \pi \sin ^{2} \theta_{W}} X\left(m_{t}^{2} / M_{W}^{2}\right), \quad C_{R}^{(\mathrm{SM})}=0
$$

- $S_{1,3}$ and $V_{2 \mu}$ LQs give the following contribution to $b \rightarrow s \nu_{j} \bar{\nu}_{i}$ :

$$
\begin{aligned}
C_{R}^{i j} & =-\frac{V_{c s}}{2 \sqrt{2} G_{F} V_{t s}^{*}} \frac{g_{2 L}^{3 i} g_{2 L}^{2 j *}}{M_{V_{2}}^{2}} \\
C_{L}^{i j} & =-\frac{V_{c s}}{2 \sqrt{2} G_{F} V_{t s}^{*}}\left[\frac{g_{1 L}^{3 i} g_{1 L}^{2 j *}}{2 M_{S_{1}}^{2}}+\frac{g_{3 L}^{3 i} g_{3 L}^{2 j *}}{2 M_{S_{3}}^{2}}-2 \frac{h_{3 L}^{2 i} h_{3 L}^{3 j *}}{M_{U_{3}}^{2}}\right]
\end{aligned}
$$

- For simplicity, after the rotation of the down-type quarks into the mass eigenstate basis, we neglect the subleading $\mathcal{O}(\lambda)$ terms and keep only the $V_{t b} V_{c s}^{*} \simeq 1$ terms.

The studied distributions are given by

$$
\begin{aligned}
\frac{d \Gamma(\bar{B} \rightarrow D \tau \bar{\nu})}{d q^{2}}= & \frac{G_{F}^{2}\left|V_{c b}\right|^{2}}{192 \pi^{3} m_{B}^{3}} q^{2} \sqrt{\lambda_{D}\left(q^{2}\right)}\left(1-\frac{m_{\tau}^{2}}{q^{2}}\right)^{2} \times\{ \\
& \left|1+C_{V_{1}}+C_{V_{2}}\right|^{2}\left[\left(1+\frac{m_{\tau}^{2}}{2 q^{2}}\right) H_{V, 0}^{s 2}+\frac{3}{2} \frac{m_{\tau}^{2}}{q^{2}} H_{V, t}^{s 2}\right] \\
& +\frac{3}{2}\left|C_{S_{1}}+C_{S_{2}}\right|^{2} H_{S}^{s 2}+8\left|C_{T}\right|^{2}\left(1+\frac{2 m_{\tau}^{2}}{q^{2}}\right) H_{T}^{s 2} \\
& +3 \mathcal{R} e\left[\left(1+C_{V_{1}}+C_{V_{2}}\right)\left(C_{S_{1}}^{*}+C_{S_{2}}^{*}\right)\right] \frac{m_{\tau}}{\sqrt{q^{2}}} H_{S}^{s} H_{V, t}^{s} \\
& \left.-12 \mathcal{R} e\left[\left(1+C_{V_{1}}+C_{V_{2}}\right) C_{T}^{*}\right] \frac{m_{\tau}}{\sqrt{q^{2}}} H_{T}^{s} H_{V, 0}^{s}\right\}
\end{aligned}
$$

where $H_{i}$ are the helicity amplitudes,

$$
H_{i}\left(q^{2}\right) \propto\left\langle D^{(*)}\right| \mathcal{O}_{i}|\bar{B}\rangle
$$

The studied distributions are given by

$$
\begin{aligned}
& \frac{d \Gamma\left(\bar{B} \rightarrow D^{*} \tau \bar{\nu}\right)}{d q^{2}}=\frac{G_{F}^{2}\left|V_{c b}\right|^{2}}{192 \pi^{3} m_{B}^{3}} q^{2} \sqrt{\lambda_{D^{*}}\left(q^{2}\right)}\left(1-\frac{m_{\tau}^{2}}{q^{2}}\right)^{2} \times\{ \\
& \quad\left(\left|1+C_{V_{1}}\right|^{2}+\left|C_{V_{2}}\right|^{2}\right)\left[\left(1+\frac{m_{\tau}^{2}}{2 q^{2}}\right)\left(H_{V,+}^{2}+H_{V,-}^{2}+H_{V, 0}^{2}\right)+\frac{3}{2} \frac{m_{\tau}^{2}}{q^{2}} H_{V, t}^{2}\right] \\
& \quad-2 \operatorname{Re} e\left[\left(1+C_{V_{1}}\right) C_{V_{2}}^{*}\right]\left[\left(1+\frac{m_{\tau}^{2}}{2 q^{2}}\right)\left(H_{V, 0}^{2}+2 H_{V,+} H_{V,-}\right)+\frac{3}{2} \frac{m_{\tau}^{2}}{q^{2}} H_{V, t}^{2}\right] \\
& \quad+\frac{3}{2}\left|C_{S_{1}}-C_{S_{2}}\right|^{2} H_{S}^{2}+8\left|C_{T}\right|^{2}\left(1+\frac{2 m_{\tau}^{2}}{q^{2}}\right)\left(H_{T,+}^{2}+H_{T,-}^{2}+H_{T, 0}^{2}\right) \\
& \quad+3 \mathcal{R e}\left[\left(1+C_{V_{1}}-C_{V_{2}}\right)\left(C_{S_{1}}^{*}-C_{S_{2}}^{*}\right)\right] \frac{m_{\tau}}{\sqrt{q^{2}}} H_{S} H_{V, t} \\
& \quad-12 \mathcal{R e}\left[\left(1+C_{V_{1}}\right) C_{T}^{*}\right] \frac{m_{\tau}}{\sqrt{q^{2}}}\left(H_{T, 0} H_{V, 0}+H_{T,+} H_{V,+}-H_{T,-} H_{V,-}\right) \\
& \left.\quad+12 \mathcal{R e}\left[C_{V_{2}} C_{T}^{*}\right] \frac{m_{\tau}}{\sqrt{q^{2}}}\left(H_{T, 0} H_{V, 0}+H_{T,+} H_{V,-}-H_{T,-} H_{V,+}\right)\right\}
\end{aligned}
$$

$\bar{B} \rightarrow \boldsymbol{D} \tau \bar{\nu}(3 \mathrm{FFs}):$

$$
\begin{aligned}
& H_{V, 0}^{s}\left(q^{2}\right) \equiv H_{V_{1}, 0}^{s}\left(q^{2}\right)=H_{V_{2}, 0}^{s}\left(q^{2}\right)=\sqrt{\frac{\lambda_{D}\left(q^{2}\right)}{q^{2}}} F_{1}\left(q^{2}\right) \\
& H_{V, t}^{s}\left(q^{2}\right) \equiv H_{V_{1}, t}^{s}\left(q^{2}\right)=H_{V_{2}, t}^{s}\left(q^{2}\right)=\frac{m_{B}^{2}-m_{D}^{2}}{\sqrt{q^{2}}} F_{0}\left(q^{2}\right) \\
& H_{S}^{s}\left(q^{2}\right) \equiv H_{S_{1}}^{s}\left(q^{2}\right)=H_{S_{2}}^{s}\left(q^{2}\right)=\frac{m_{B}^{2}-m_{D}^{2}}{m_{b}-m_{c}} F_{0}\left(q^{2}\right) \\
& H_{T}^{s}\left(q^{2}\right) \equiv H_{T,+-}^{s}\left(q^{2}\right)=H_{T, 0 t}^{s}\left(q^{2}\right)=-\frac{\sqrt{\lambda_{D}\left(q^{2}\right)}}{m_{B}+m_{D}} F_{T}\left(q^{2}\right)
\end{aligned}
$$

with hadronic amplitudes defined as,

$$
\begin{aligned}
H_{V_{1,2}, \lambda}^{\lambda_{M}}\left(q^{2}\right) & =\varepsilon_{\mu}^{*}(\lambda)\left\langle M\left(\lambda_{M}\right)\right| \bar{c} \gamma^{\mu}\left(1 \mp \gamma_{5}\right) b|\bar{B}\rangle, \\
H_{S_{1,2}, \lambda}^{\lambda_{M}}\left(q^{2}\right) & =\left\langle M\left(\lambda_{M}\right)\right| \bar{c}\left(1 \pm \gamma_{5}\right) b|\bar{B}\rangle, \\
H_{T, \lambda \lambda^{\prime}}^{\lambda_{M}}\left(q^{2}\right) & =\varepsilon_{\mu}^{*}(\lambda) \varepsilon_{\nu}^{*}\left(\lambda^{\prime}\right)\left\langle M\left(\lambda_{M}\right)\right| \bar{c} \sigma^{\mu \nu}\left(1-\gamma_{5}\right) b|\bar{B}\rangle
\end{aligned}
$$

where $\lambda_{M}$ and $\lambda$ denote the meson and virtual intermediate boson helicities in the $B$ rest frame respectively.

$$
\begin{aligned}
& \bar{B} \rightarrow D^{*} \tau \bar{\nu}(7 \mathrm{FFs}): \\
& H_{V, \pm}\left(q^{2}\right) \equiv H_{V_{1}, \pm}^{ \pm}\left(q^{2}\right)=-H_{V_{2}, \mp}^{\mp}\left(q^{2}\right)=\left(m_{B}+m_{D^{*}}\right) A_{1}\left(q^{2}\right) \mp \frac{\sqrt{\lambda_{D^{*}}\left(q^{2}\right)}}{m_{B}+m_{D^{*}}} V\left(q^{2}\right) \\
& H_{V, 0}\left(q^{2}\right) \equiv H_{V_{1}, 0}^{0}\left(q^{2}\right)=-H_{V_{2}, 0}^{0}\left(q^{2}\right)=\frac{m_{B}+m_{D^{*}}}{2 m_{D^{*}} \sqrt{q^{2}}}\left[-\left(m_{B}^{2}-m_{D^{*}}^{2}-q^{2}\right) A_{1}\left(q^{2}\right)\right. \\
& \left.+\frac{\lambda_{D^{*}}\left(q^{2}\right)}{\left(m_{B}+m_{D^{*}}\right)^{2}} A_{2}\left(q^{2}\right)\right] \\
& H_{V, t}\left(q^{2}\right) \equiv H_{V_{1}, t}^{0}\left(q^{2}\right)=-H_{V_{2}, t}^{0}\left(q^{2}\right)=-\sqrt{\frac{\lambda_{D^{*}}\left(q^{2}\right)}{q^{2}}} A_{0}\left(q^{2}\right) \\
& H_{S}\left(q^{2}\right) \equiv H_{S_{1}}^{0}\left(q^{2}\right)=-H_{S_{2}}^{0}\left(q^{2}\right)=-\frac{\sqrt{\lambda_{D^{*}}\left(q^{2}\right)}}{m_{b}+m_{c}} A_{0}\left(q^{2}\right) \\
& H_{T, \pm}\left(q^{2}\right) \equiv \pm H_{T, \pm t}^{ \pm}\left(q^{2}\right)=\frac{1}{\sqrt{q^{2}}}\left[ \pm\left(m_{B}^{2}-m_{D^{*}}^{2}\right) T_{2}\left(q^{2}\right)+\sqrt{\lambda_{D^{*}}\left(q^{2}\right)} T_{1}\left(q^{2}\right)\right] \\
& H_{T, 0}\left(q^{2}\right) \equiv H_{T,+-}^{0}\left(q^{2}\right)=H_{T, 0 t}^{0}\left(q^{2}\right)=\frac{1}{2 m_{D^{*}}}\left[-\left(m_{B}^{2}+3 m_{D^{*}}^{2}-q^{2}\right) T_{2}\left(q^{2}\right)\right. \\
& \left.+\frac{\lambda_{D^{*}}\left(q^{2}\right)}{m_{B}^{2}-m_{D^{*}}^{2}} T_{3}\left(q^{2}\right)\right]
\end{aligned}
$$

