Status and future prospects for Higgs production computations

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Quy Nhon, 15 August 2014
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- Gluon fusion
- TTH
- Higgs strahlung
- VBF
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Current status for the total cross section: \[ \frac{\sigma}{\sigma_{SM}} = 1.00 \pm 0.13 \left[ \pm 0.09 \text{(stat.)}^{+0.08}_{-0.07} \text{(theo.)} \pm 0.07 \text{(syst.)} \right] \]
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- Theo. and exp. uncertainties are of the same order.
- Need to improve our theory predictions!
Outline

- There is a need to improve our theoretical predictions!
  - Requires higher order computations!

- Outline:
  - The gluon fusion cross section: Status
  - NNLO corrections to H + jet and Higgs pairs.
  - Towards N3LO corrections to inclusive Higgs production.
The gluon fusion cross section: Status
The dominant Higgs production mechanism at the LHC is gluon fusion.

Loop induced process!

→ Leads to technical complications!
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- Complication 1:
  Everything is shifted by one loop order.
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  Loops with massive virtual particles are generically beyond the state-of-the-art starting at two loops.
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Loop induced process!

- Leads to technical complications!

Complication 1:
Everything is shifted by one loop order.

Complication 2:
Loops with massive virtual particles are generically beyond the state-of-the-art starting at two loops.

Conclusion:
Higher-order computations for gluon fusion are extremely difficult!
Gluon fusion

- For a light Higgs boson, the dimension five operator describing a tree-level coupling of the gluons to the Higgs boson

\[ \mathcal{L} = \mathcal{L}_{QCD,5} - \frac{1}{4v} C_1 H G^a_{\mu \nu} G_a^{\mu \nu} \]

- Benefit: Removes both complications in one go!
Gluon fusion

- For a light Higgs boson, the dimension five operator describing a tree-level coupling of the gluons to the Higgs boson

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- Benefit: Removes both complications in one go!
- If we aim for precision, how good is this ‘crude’ approximation..?
  - Corrections in the top mass can be systematically computed.
  - Experience from NNLO shows that this approximation works amazingly well!
- Caveat! This is not true if other scales are involved that can be higher than the top threshold!
Gluon fusion: Status

- Status of the inclusive cross section:
  - NLO corrections including full top-mass effects.
  - NNLO corrections in effective theory.
  - Top mass corrections at NNLO.
  - Leading electroweak corrections.
  - Resummation up to NNLL.

- Fully differential cross sections are available up to NNLO!

- Next goal: Inclusive cross section at N3LO in the effective theory
  - More on this later!
NNLO corrections to 
H + jet and Higgs pairs
Higgs + jet

- The two-loop corrections to H+jet in the effective theory have been computed
  [Gehrmann, Glover, Jaquier, Koukoutsakis]

- Last year first steps were taken towards the computing the full NNLO corrections.
  [Boughezal, Caola, Melnikov, Petriello, Schulze]

- First process computed at NNLO where a jet function is required already at LO.

- Infrared singularities were subtracted using (a variant of) Stripper. [Czakon]
Higgs + jet

- Inclusive NNLO cross section computed last year (gluons only.):

\[
\sigma_{\text{LO}}(pp \to Hj) = 2713^{+1216}_{-776} \text{ fb}, \\
\sigma_{\text{NLO}}(pp \to Hj) = 4377^{+760}_{-738} \text{ fb}, \\
\sigma_{\text{NNLO}}(pp \to Hj) = 6177^{+204}_{-242} \text{ fb}.
\]

[Boughezal, Caola, Melnikov, Petriello, Schulze]

- At LoopFest preliminary differential distributions were announced.

  ➡ Using (a variant of) Stripper.

  ➡ Using antenna subtraction.

  [Boughezal, Caola, Melnikov, Petriello, Schulze]

  [Chen, Gehrmann, Glover, Jaquier]
**Higgs pair production**

- Last year NNLO correction to Higgs pair production in the large top-mass limit production became available.

\[
\begin{align*}
\sigma_{\text{LO}} &= 17.8^{+5.3}_{-3.8} \text{ fb} \\
\sigma_{\text{NLO}} &= 33.2^{+5.9}_{-4.9} \text{ fb} \\
\sigma_{\text{NNLO}} &= 40.2^{+3.2}_{-3.5} \text{ fb}
\end{align*}
\]

[de Florian, Mazzitelli]

- NNLO computations in EFT are normalised to exact LO matrix element
  - At NLO: \(~10\%\) agreement.

[Grigo, Hoff, Melnikov, Steinhauser]
Going beyond NNLO: towards N3LO
The need for N3LO

\[ \frac{\sigma}{\sigma_{SM}} = 1.00 \pm 0.13 \left[ \pm 0.09 \text{(stat.)}^{+0.08}_{-0.07} \text{(theo.)} \pm 0.07 \text{(syst.)} \right] \]

⇒ We need to update our theory prediction!

• Next contribution is the N3LO contribution in the effective theory.
  ⇒ Huge challenge!
  ⇒ Never has an N3LO computation been done for a hadron collider!

• Recently: Several approximate N3LO results have been presented.
  ⇒ Only full N3LO result will be final judge!
Contributions at N3LO

At N3LO, there are 5 contributions:

- Triple virtual
- Real-virtual squared
- Double virtual real
- Double real virtual
- Triple real
## Reverse-unitarity @ N3LO

### Growth in complexity for real emission

<table>
<thead>
<tr>
<th>Order</th>
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The threshold expansion

- There are 1000’s of integrals to compute!
  - Tough nut to crack!
  - Concentrate on some approximation first.

- The gluon fusion cross section depends on one single parameter:
  \[ z = \frac{m^2}{s} \]

- Close to threshold \((z \sim 1)\), we can approximate the triple real cross section by a power series:
  \[ \hat{\sigma}(z) = \sigma_{-1} + \sigma_0 + (1 - z) \sigma_1 + O(1 - z)^2 \]

- Goal:
  - First term captures complete 3-loop correction + emission of soft gluons.
The soft-virtual approximation

- The computation of the first term has been completed!
  [Anastasiou, CD, Dulat, Furlan, Gehrmann, Herzog, Mistlberger]

- Many different contributions are needed:
  - 22 three-loop.
    [Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus]
  - 3 double-virtual-real.
    [CD Gehrmann, Li, Zhu]
  - 7 real-virtual-squared.
    [Anastasiou, CD, Dulat, Herzog, Mistlberger; Kilgore]
  - 10 double-real-virtual.
    [Anastasiou, CD, Dulat, Furlan, Herzog, Mistlberger; Li, von Manteuffel, Schabinger, Zhu]
  - 8 triple real.
    [Anastasiou, CD, Dulat, Mistlberger]
  - three-loop splitting functions.
    [Moch, Vermaseren, Vogt]
  - three-loop beta function.
    [Tarasov, Vladimirov, Zharkov; Larin, Vermaseren]
  - three-loop Wilson coefficient.
    [Chetyrkin, Kniehl, Steinhauser; Schroeder, Steinhauser; Chetyrkin, Kuhn, Sturm]
Caveat!

Source of ambiguity:

\[
\int dx_1 \, dx_2 \, [f_i(x_1) \, f_j(x_2) \, z \, g(z)] \left[ \frac{\hat{\sigma}_{ij}(s, z)}{z \, g(z)} \right]_{\text{threshold}} \quad \lim_{z \to 1} g(z) = 1
\]
Generalizations

- Soft-virtual corrections are universal, and the result can be extended to other processes.

- Can be used to predict the rapidity distribution of the Higgs boson at N3LO at threshold. [Ahmed, Mandal, Rana, Ravindran]

- Recently the 3-loop form factor for $bb \rightarrow H$ was computed. [Gehrmann, Kara]

- The result was immediately extended to N3LO corrections to Higgs production in bottom fusion at threshold. [Ahmed, Rana, Ravindran]

  ➡ N.B.: Bottom-fusion cross section available fully differentially at NNLO! [Buehler, Herzog, Lazopoulos, Mueller]

- Caveat for threshold approximation still applies!
Looking into the future…

- The soft-virtual term is only the beginning!
- Real-virtual-squared contribution already fully known.
  [Anastasiou, CD, Dulat, Herzog, Mistlberger; Kilgore]
- Next-to-soft term known for triple real contribution.
  [Anastasiou, CD, Dulat, Mistlberger]
- Two-loop matrix element for H+j known
  [Gehrmann, Glover, Jaquier, Koukoutsakis]
  ➡ Phase space integration requires contribution from collinear regions!
- Once the N3LO result for the Higgs is available, more will follow!
  ➡ Drell-Yan, bb -> H, …
Conclusion

- LHC Run II will require very precise QCD computations for Higgs production.
  - Theory uncertainties are same size as experimental ones.
- A lot of progress was made regarding (differential) predictions at NNLO.
  - $H + \text{jet, Higgs pairs, } bb \rightarrow H$.
- N3LO result for inclusive cross section is in the making.
  - Threshold term already available!
  - More terms in threshold expansion and the full results are in the making.
  - Requires a lot of new and advanced technologies from the theory side!