

Top-quark mass theory

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PHYSICS AT LHC AND BEYOND

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Outline :

- Introduction : recent interests, current status
- Top-quark mass : theoretical definition
- New methods (leptonic observables)

Introduction

- Top-quark is still the heaviest elementary particle

$$m_t = y_t \langle \Phi \rangle$$

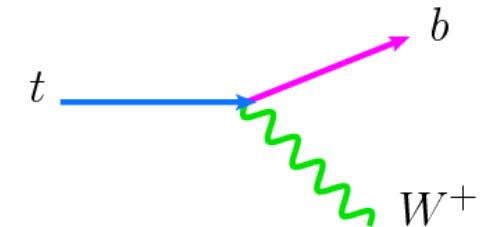
- $y_t \sim 1 \Rightarrow$ strongly coupled to Higgs field
 - Only the Fermion with natural mass scale
 - Related with EWSB and BSM?



- Top-quark decays before hadronization

$$\Gamma_t \simeq \frac{G_F m_t^3}{8\sqrt{2}\pi} |V_{tb}|^2 \sim 1.5 \text{ [GeV]} \gg \Lambda_{\text{QCD}}$$

- No time to form color-singlet hadron with light quarks
 - Decay as a (free) quark, preserving spin information



Ranking up interests to the top mass value

after the discovery of Higgs boson
with $m_h = 125.5 \text{ GeV}$

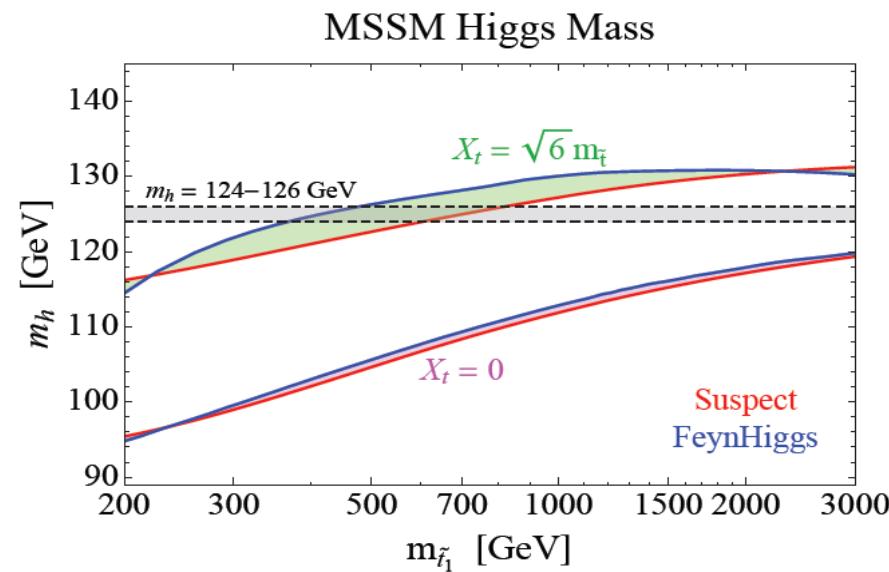
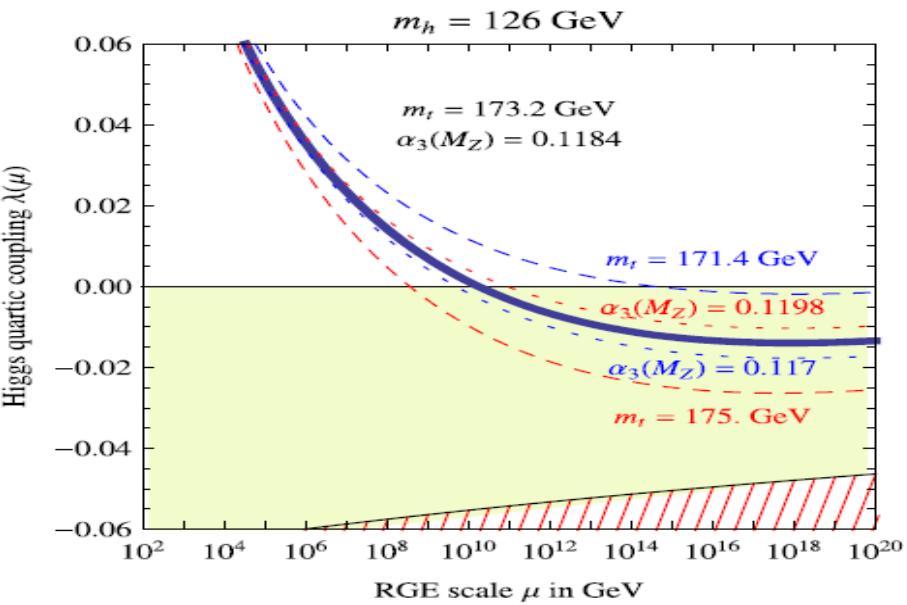
- SM vacuum at the Plank scale

$$16\pi^2 \mu \frac{d\lambda}{d\mu} = 24\lambda^2 - 6y_t^4 + \dots$$

Precision input of m_t is crucial to the scale at which the SM breaks down.

- Higgs mass in the MSSM

$$\delta m_h^2 = \frac{3y_t^4 v^2}{16\pi^2} \left[\ln \frac{m_{\tilde{t}}^2}{m_t^2} + X_t^2 - \frac{X_t^4}{12} \right]$$

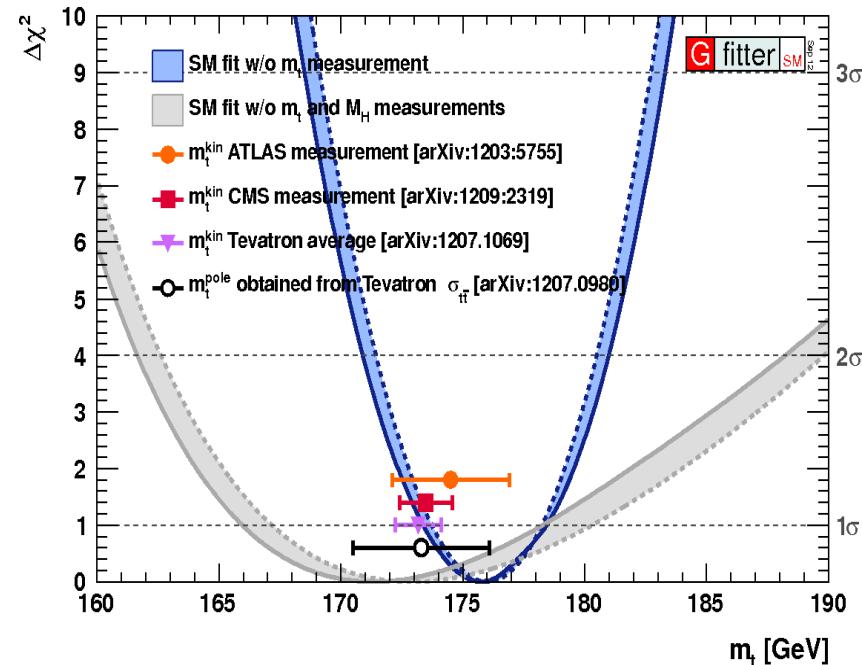


Top-quark mass in the EW fit

- Top-quark mass in the EW fit

$$\delta\rho \sim \frac{3y_t^2}{32\pi^2} - \frac{3g'^2}{32\pi^2} \ln \frac{m_h}{m_Z}$$

Theoretically clear interpretation:
pole mass or MS mass depending on
the scheme you use

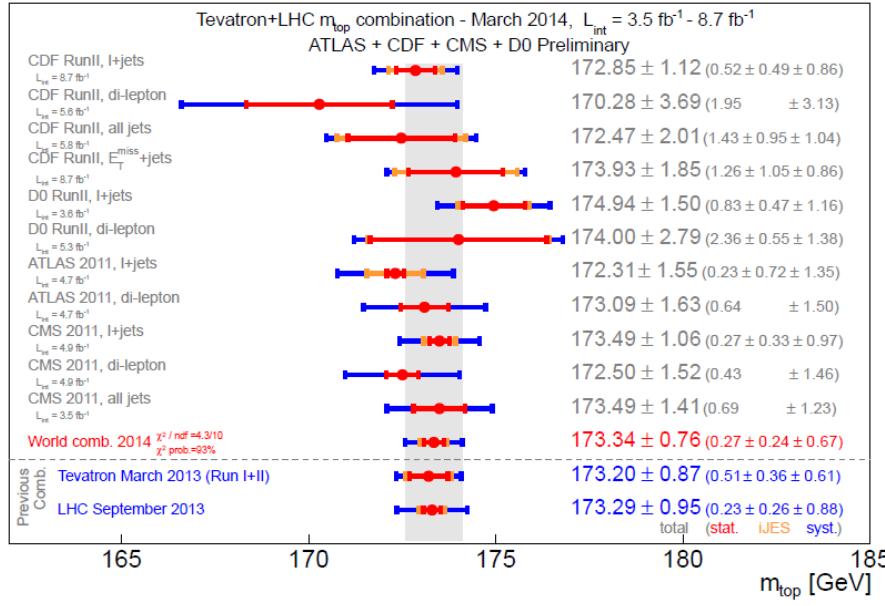


1. Inputs of m_h and m_t give an indication for new physics.
2. Assuming no new physics and input of $m_h=125.5$ GeV
 \rightarrow indirect observation of the top-quark mass

First Tevatron & LHC combination result

$$m_t = 173.34 \pm 0.27(\text{stat}) \pm 0.71(\text{syst}) \text{ GeV}$$

arXiv:1403.4427



- Very good accuracy based on mainly Template method.
- However, the obtained mass is not well-defined theoretically.
- There must be a difference btw the measured mass and the pole mass.

- The problem is how much the difference is. $m_{\text{MC}} - m_{\text{pole}} = ?$

Top-quark mass definitions

- Short Distance Masses

$$m_{\text{pole}} - m_{\text{SD}} \simeq + \frac{1}{16\pi^2} C_F \int_{|q| < \mu_{\text{SD}}} d^3 \vec{q} \frac{\alpha_s(q^2)}{q^2} \simeq \mu_{\text{SD}} \frac{\alpha_s(\mu_{\text{SD}})}{\pi} c_{\text{SD}}$$

- Because of color confinement, pole of the quark propagator can not be seen.
- SD mass removes IR gluon effects from the definition of pole mass.
- Perturbative calculation in the pole mass scheme has Renormalon ambiguity, but those in the short distance mass scheme do not.
- MC mass should be close to the Jet mass scheme where cut-off scale is near Γ_t , but we don't know the exact coefficient. [if the MC describes QCD correctly.]

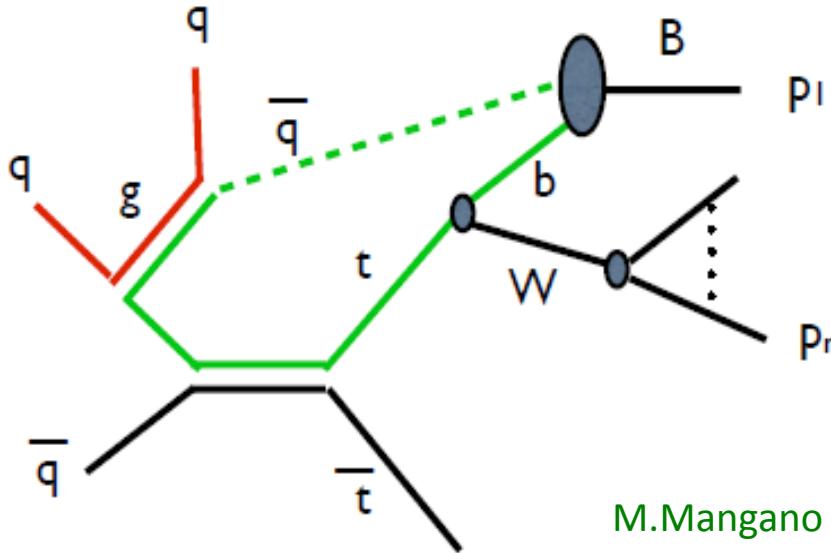
MSbar mass: $\mu = m_t$

1S mass, PS mass,,,,: $\mu = m_t \alpha_s$

Jet mass, (MC mass): $\mu = \Gamma_t$

Color Reconnection

Skands,Wicke (07); Argyropoulos,Sjostrand(14)



- Unavoidable since top has color charge.
- Hard to calculate by perturbative QCD.
→ modeling to fit the data.
- Dijet from W decay form color singlet
→ less CR effects to dijet.

- Uncertainties of CR modeling within exp. constraints give an error in top mass determination; $\delta m_t = \pm \mathcal{O}(0.5 - 1 \text{ GeV})$
- Expected to be more constrained by using LHC data, however, theory development must be required as well.

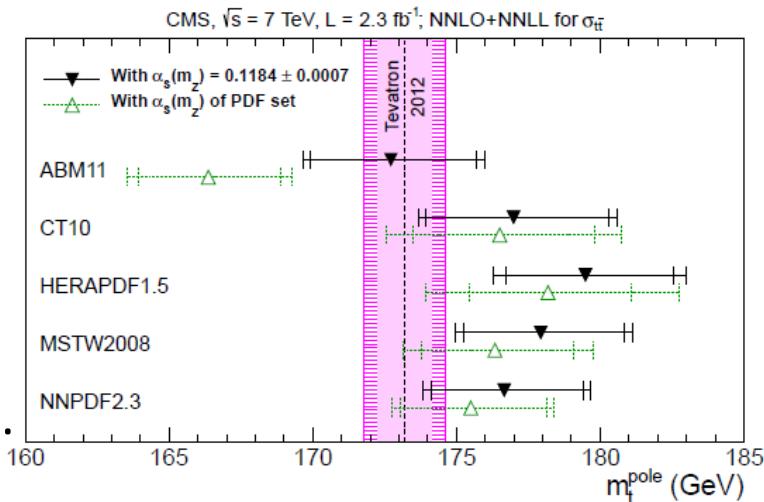
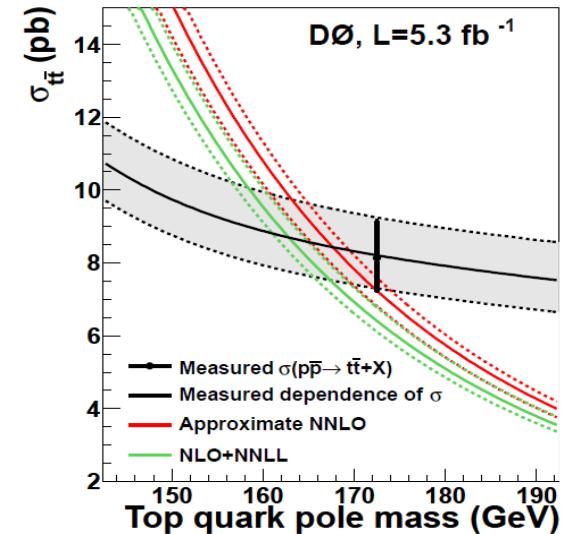
Mass from the Total Cross-Section

Langenfeld,Moch,Uwer (07), Alekhin,Djouadi,Moch(12)

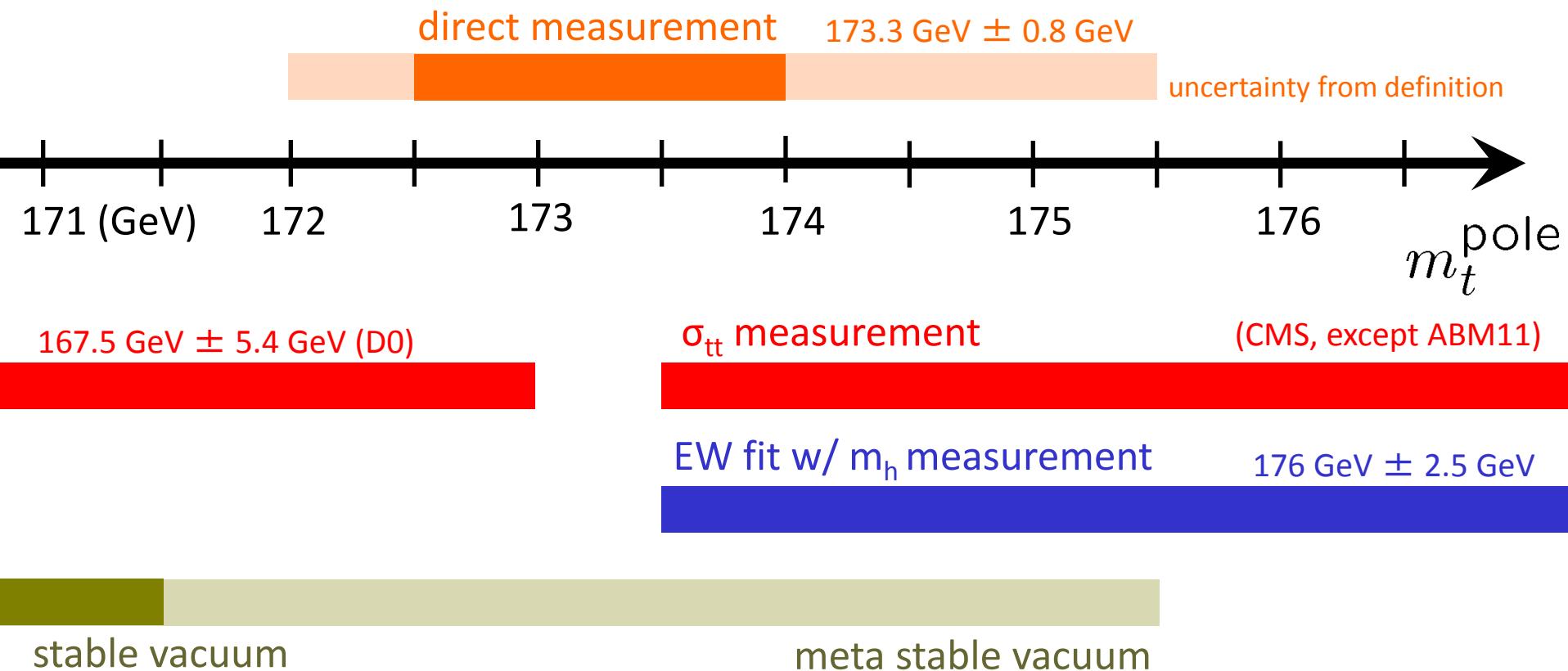
$\sigma_{t\bar{t}}(m_{\text{pole}} \text{ or } \bar{m})$ known up to NNLO

Barnreuther,Czakon,Mitov (12),
Czakon,Fiedler,Mitov (13)

- Inclusive observables closely related to the operator matrix elements.
- Can be described independently from kinematical parameters. (cf. R-ratio)
- Direct comparison to the QCD prediction in the MSbar mass scheme, etc.
- Involve PDF, α_s uncertainties, and estimation of total CS from fiducial CS needs kinematical mass.



Top-Quark Pole Mass



- Direct measurement : possible shift by theoretical definition
and errors from color reconnection.
- σ_{tt} method, EW fit have clear connection with theoretically well-defined mass.

New Methods for the Top Mass Observation

- Avoid to use jet momenta to determine the well-defined top-quark mass.
- Good for cross check by various methods which has different syst. errors.

- Kinematical Endpoint (M_{T2})

$$m_t = 173.9 \pm 0.9 \text{ (stat)} \pm 1.2 \text{ (syst)} \text{ GeV}$$

Tevatron,
CMS, EPJC73,2494

- B-hadron lifetime (L_{xy}) Hill,Incandela,Lamb (05)

$$m_t = 173.5 \pm 1.5 \text{ (stat)} \pm 1.3 \text{ (syst)} \pm 2.6 \text{ (p_T^{top})} \text{ GeV}$$

CMS-PAS-TOP-12-030

- J/ψ method Kharchilava (00)

- $t\bar{t} + 1\text{-jet}$ invariant-mass distribution Alioli et. al (13),

- Boost invariant energy peak Agashe,Franceschini,Kim (13)

- Leptonic observable methods Kawabata,Shimizu,Sumino,HY (14); Frixione,Mitov (14)

Leptonic Observable Methods

- Lepton momentum can be measured very accurately.
- Lepton momentum is not affected by hadronization effects, and less affected by color reconnection etc.
- (Note:) Missing momentum is affected by hadronic activities such as JES, pile up, etc.



- J/ ψ methods : $M_{\mu\mu\ell}$, J/ ψ frag fnc from b-quark
Kharchilava (00)
- Mellin moments of E_l , p_{Tl} , ... :
Frixione,Mitov (14), Biswas,Melnikov,Shulze (10),
- Weight function methods :
Kawabata,Shimizu,Sumino,HY (14)

Weight Function Method

S.Kawabata, Y.Shimizu, Y.Sumino, H.Y. (11,13,14)

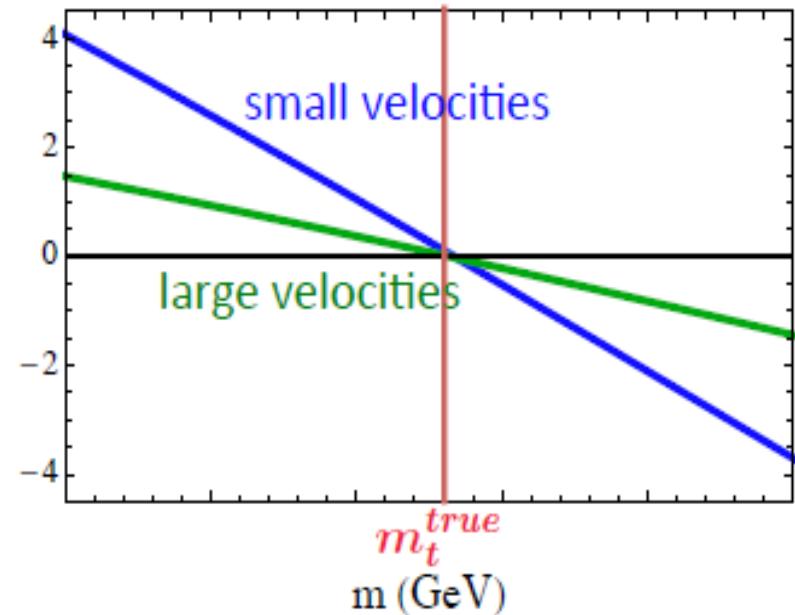
$$I(m) = \int dE_\ell \mathcal{D}(E_\ell) W(E_\ell, m)$$

$\mathcal{D}(E_\ell)$: energy distribution of lepton.

We can construct a weight function $W(E, m)$ which satisfy

$$I(m = m_{\text{true}}) = 0$$

independently of velocity distribution of top-quark.



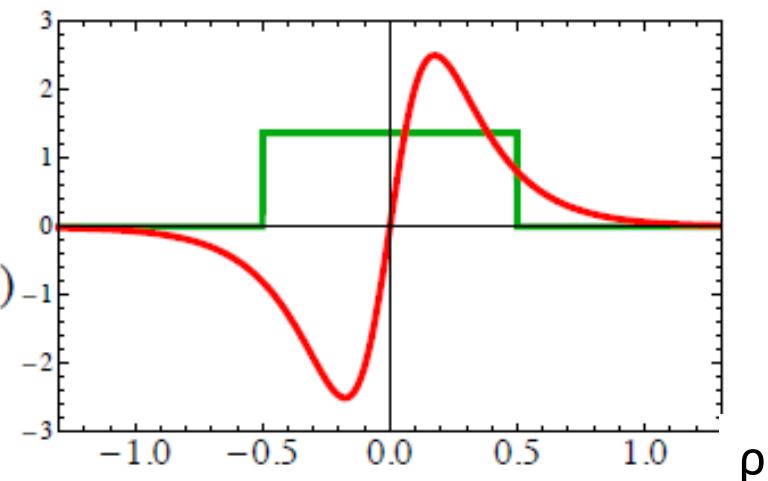
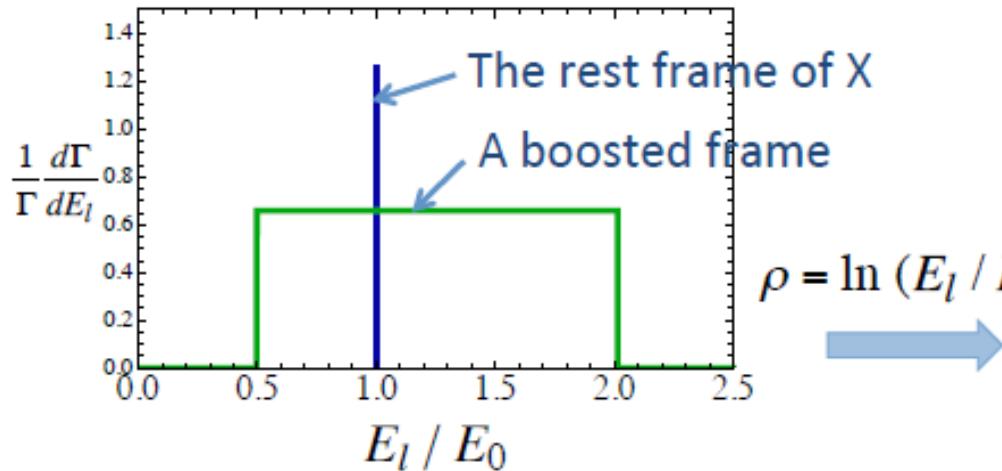
- Free from production mechanism (top has to be unpolarized)
- Free from the PDF uncertainty, initial-state radiation
- Free from hadronization modeling (because it uses only lepton)

Weight Function Method

$$W(E_\ell; m) = \int \frac{dE}{EE_\ell} \mathcal{D}_0(E, m) [\text{odd fnc. of } \rho] \Big|_{e^\rho = E_\ell/E}$$

- Any odd fnc. of $\rho \rightarrow$ we could define many kinds of weight fnc.
- $\mathcal{D}_0(E_\ell)$: Lepton energy distribution in the top-quark rest-frame
→ Predictable in pQCD (incl. HO corr. with pole mass/MS scheme)

For a two-body decay : $X \rightarrow \ell + Y$ (X is scalar or unpolarized)

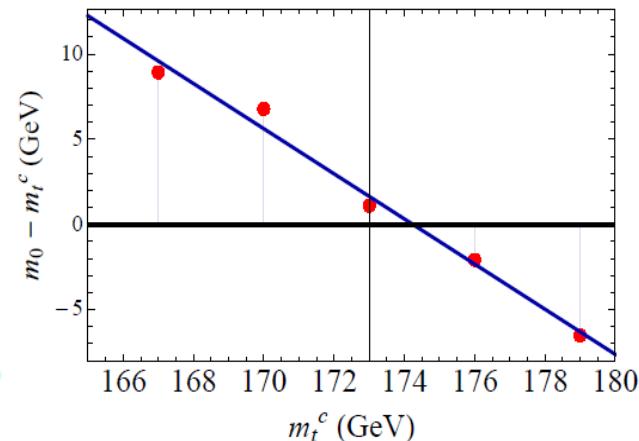
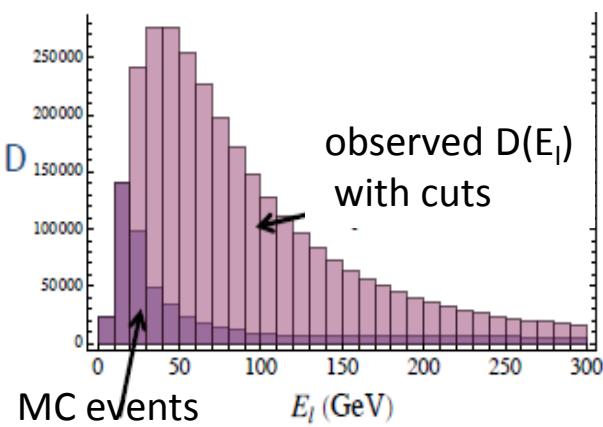


Simulation analysis in LO

- In real experiments, pure entire lepton energy distribution can not be available, due to background, acceptance cuts, selection cuts etc.
- Estimation on these effects often involves the production mechanism, PDF uncertainty, hadronization model dependence, etc.
- Simulation study in lepton+jets channel:
For the events lost by lepton cuts, we compensate with MC events.
→ Mass choice in the MC can be solved by self-consistency.

Event selection cuts

- 1 muon with $p_T > 20\text{GeV}$, $|\eta| < 2.4$
- At least 4 jets
- At least 1 b-tag
- $p_T(j_1) > 55$, $p_T(j_2) > 25$,
 $p_T(j_3) > 15$, $p_T(j_4) > 8 \text{ GeV}$



Simulation analysis in LO

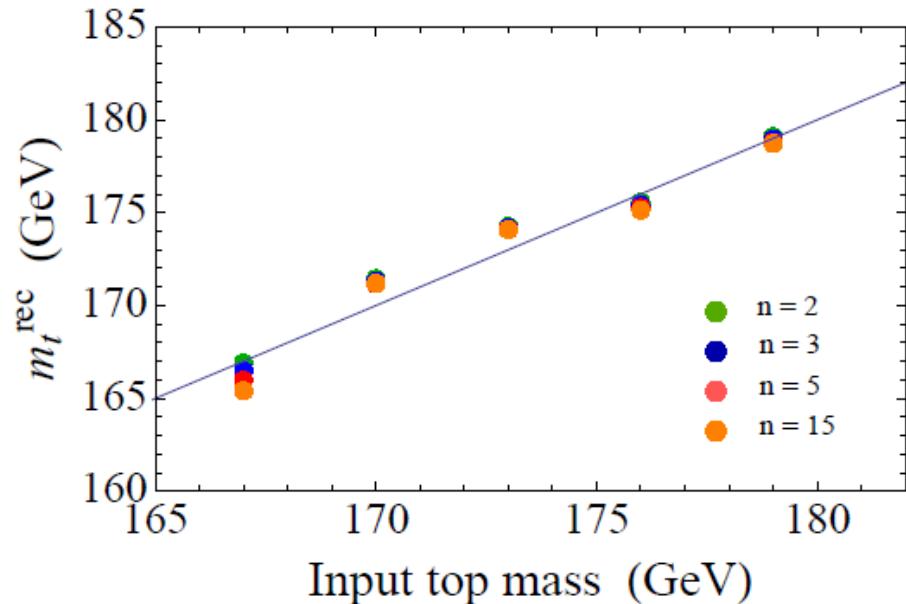
- Stat. error ~ 0.4 GeV for 100 fb^{-1}
- Dominant syst. error comes from compensating events [scale choice,,]
- Hadronic ambiguity remains small. [JES, b-tag,,]
- Successful mass reconstruction within those errors.

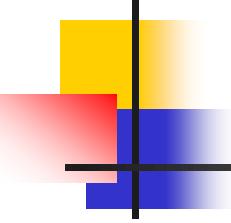
Prospects:

- NLO analysis should reduce errors.
- NLO correction to the weight function gives direct measurement of pole mass /MSbar mass at NLO.
- Finite width effects can be incorporated as a correction to the NWA.

Uncertainties [GeV] ($100 \text{ fb}^{-1}, e + \mu$)

Signal stat. error	0.4	←
μ_F scale	+1.5/-1.6	←
Jet energy scale	+0.0/-0.1	
BG stat. error	0.4	

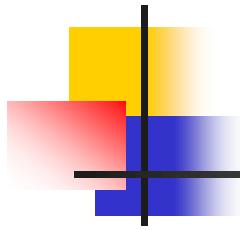




Summary

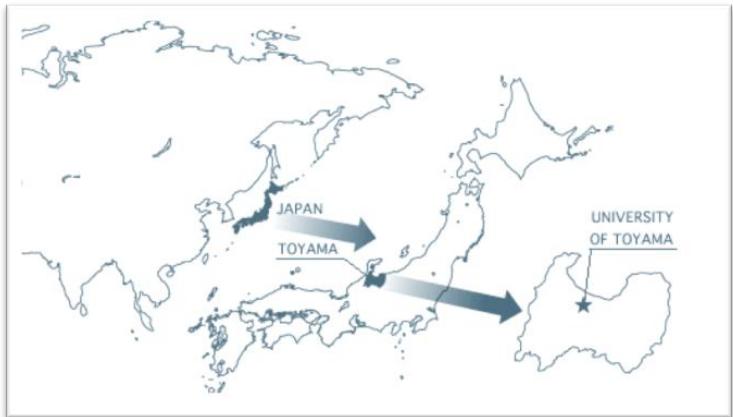
- Top quark mass has been determined in 0.5% accuracy, but there should be still $O(1 \text{ GeV})$ uncertainties due to the definition of the measured mass.
- Various new methods have been proposed which have different systematic errors.
- Some of them can access to the pole mass/MS mass directly.
- Leptonic observable methods seem attractive, since lepton mom. can be measured accurately, and is not affected by hadronization effects (less affected by CR.)
- Goal at the LHC may be the **$O(1 \text{ GeV})$** determination of the well-defined top-quark mass (pole, MS,,,).

[At ILC, 1S mass can be determined in 50 MeV, MS mass in 100MeV]



(Adv.) HPNP2015

2nd Toyama International Workshop on “Higgs as a Probe of New Physics”



- Feb. 11-15, 2015 at Toyama, JAPAN
- Higgs and new physics at TeV scale
- Registration will start soon
- Enjoy seafood at Toyama
in the best season!

HPNP2015

February 11-15, 2015, University of Toyama, Japan

Invited Speakers

Abdesslam Arhrib*	(Abdelmalek Essaadi U.)	Edmond L. Berger	(ANL)
Fawzi Boudjema*	(Annecy, LAPTH)	Chengwei Chiang	(NCU)
Eung-Jin Chun	(KIAS)	Abdelhak Djouadi	(LPT, Orsay)
Keisuke Fujii	(KEK)	Christophe Grojean*	(CERN)
Howard E. Haber	(UC Santa Cruz)	Kaoru Hagiwara	(KEK)
Sven Heinemeyer	(IFCA)	Pyungwon Ko	(KIAS)
Maria Krawczyk	(U. of Warsaw)	Ernest Ma	(UC. Riverside)
Stefano Moretti	(U. of Southampton)	Salah Nasri	(UAE U. & Oran U.)
Michael Peskin	(SLAC)	Rui Santos	(ISEL, U. of Lisbon)
Reisaburo Tanaka	(LAL, Orsay)	C.-P. Yuan*	(Michigan State U.)

Public Lecture (15th February, Afternoon)

Hitoshi Murayama (Kavli IPMU / UC Berkeley)

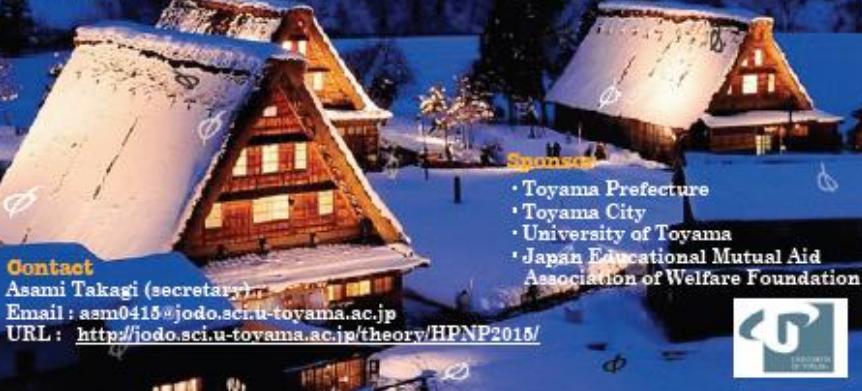
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* To be confirmed

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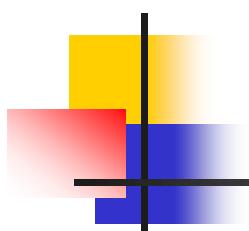
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URL : <http://jodo.sci.u-toyama.ac.jp/theory/HPNP2015/>

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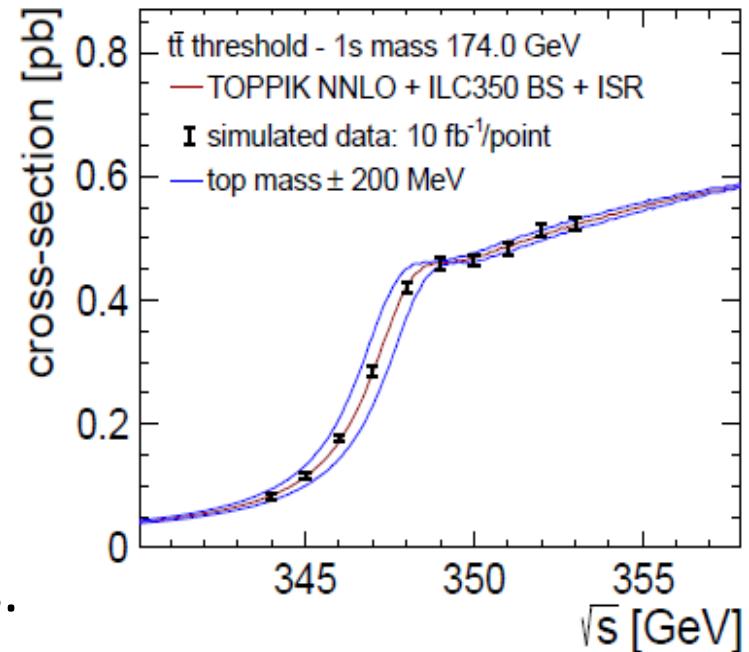


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Threshold Scan at the ILC

Fujii,Matsui,Sumino 94, Martinez,
Miquel 03, Seidel,Simon 12

- ✓ QCD calc. up to NN(N)LO
- ✓ Beam spectrum, ISR effects
- ✓ Beam polarization reduces BG events.
- ✓ Higgs-exchange effects → y_t measurement Horiguchi et al. 13



$$\delta m_t = 16 \text{ MeV}$$

$$\delta \Gamma_t = 21 \text{ MeV}$$

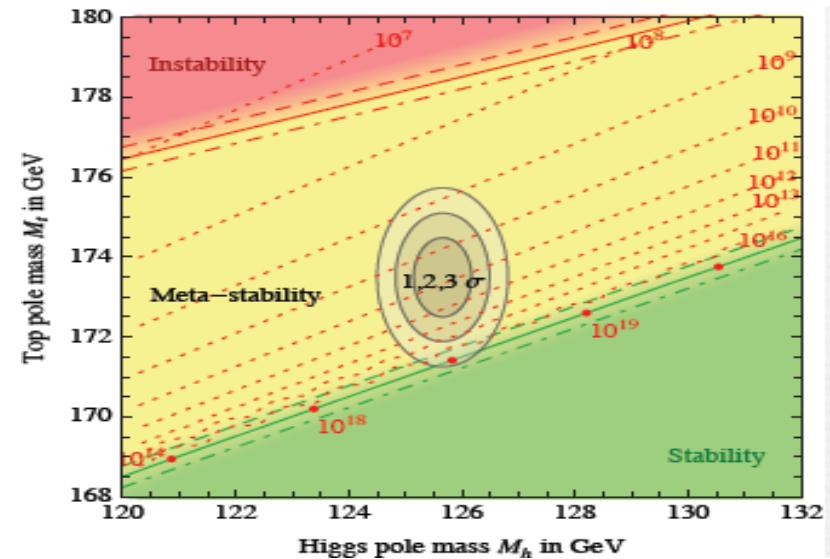
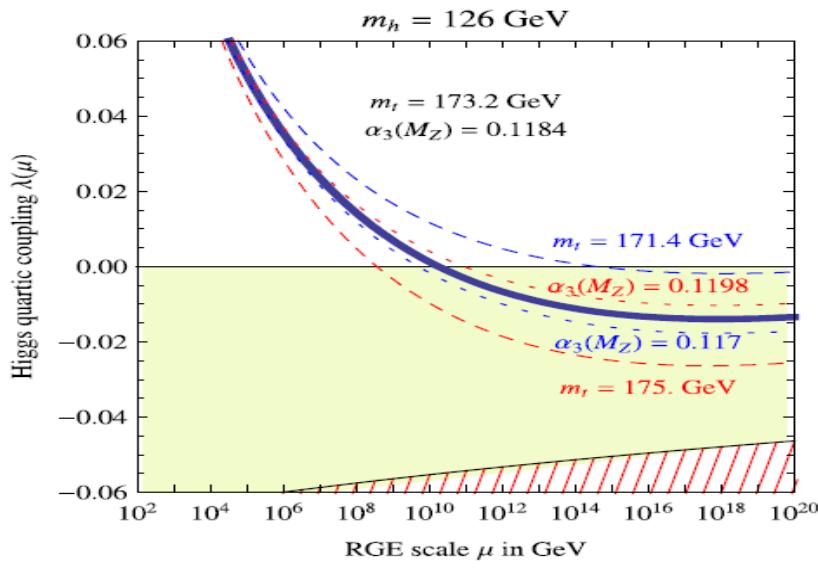
$$\delta y_t = 4.2\% \quad \alpha_s \text{ fixed}$$

← PS-mass (can be converted to MS mass)
another $\delta m \sim 40 \text{ MeV}$ by conversion
← overall normalization rather uncertain

SM vacuum stability

- RGE of Higgs quartic coupling $16\pi^2 \mu \frac{d\lambda}{d\mu} = 24\lambda^2 - 6y_t^4 + \dots$

Top-quark mass is crucial for higher-scale behavior of the SM vacuum
 Is it accidental or not? We need more accurate input of the top-quark mass



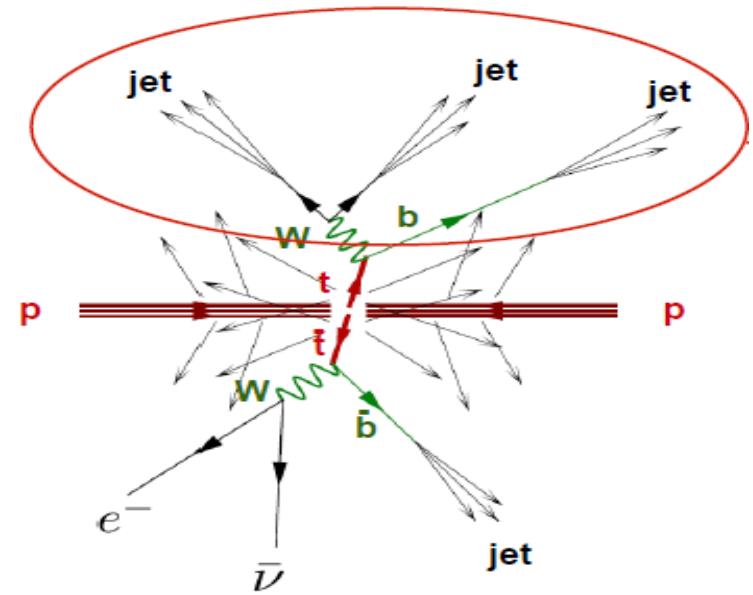
Top-quark mass

- Kinematical measurement

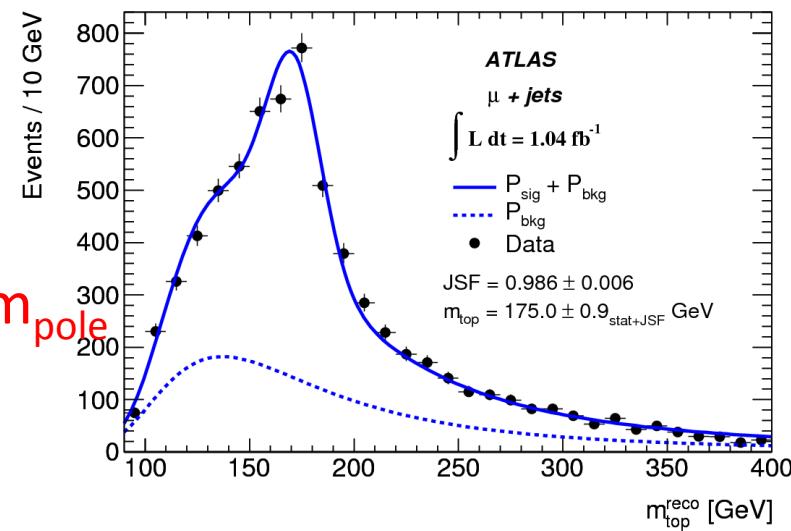
Tevatron : $173.2 \pm 0.5 \pm 0.7$ GeV

LHC : $173.3 \pm 0.3 \pm 0.9$ GeV

LHC September 2013



- ✓ Fit the observed distribution (m_{jjj}) with Templates.
- ✓ In principal, it could not be the true “pole mass” but “Pythia mass” $m_{jjj} \neq m_{\text{pole}}$
- ✓ Uncontrollable non-pert. QCD effects $\sim O(1 \text{ GeV})$

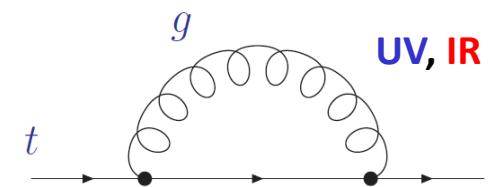


Pole mass vs. $\overline{\text{MS}}$ mass

Pole mass: pole position of the propagator

but, quark can never be free, on-shell pole is not well-defined,
bad convergence (IR sensitive)

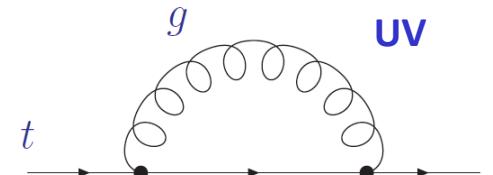
$$S_F(p)^{-1} = p - m - \Sigma(p) \simeq p - m_{pole}$$



Short-distance mass ($\overline{\text{MS}}, \text{PS}, \text{1S}$):

just a parameter of the Lagrangian, better convergence

$$\delta m_{\overline{\text{MS}}} = \Sigma^{(loop)}(p)|_{\frac{1}{\epsilon} - \gamma_E + \ln 4\pi}$$



Conversion between them is known perturbatively

$$M_{pole} = m(\mu) \left(1 + \alpha_s(\mu) d_1 + \alpha_s^2(\mu) d_2 + \dots \right)$$

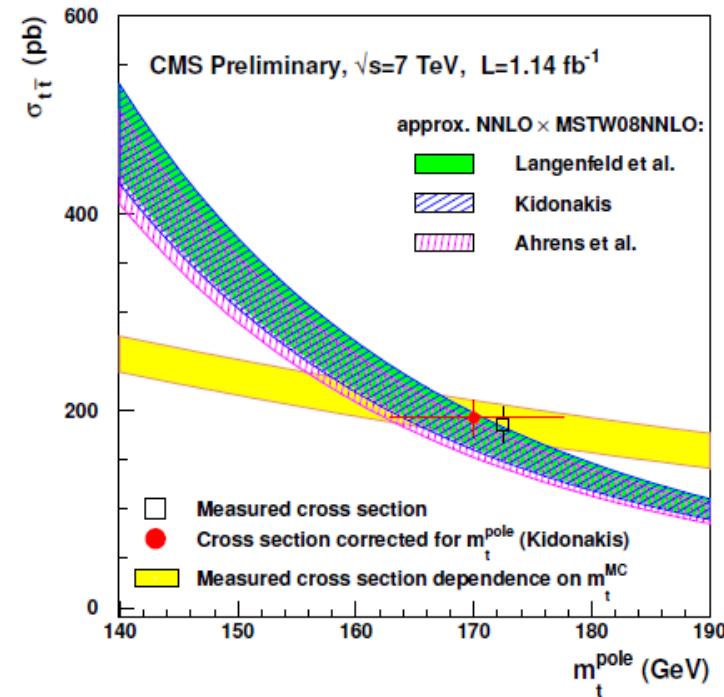
Hoang,Teubner 99
Kiyo,Sumino 02,,,

Mass from cross-section:

Compare theoretically calculated cross-section in the MS/pole and the observed cross-section at experiments.

$$\frac{\delta m_t}{m_t} \simeq 0.2 \frac{\delta \sigma_{t\bar{t}}}{\sigma_{t\bar{t}}}$$

5% measurement
 \Rightarrow 1% accuracy in mass



$\overline{\text{MS}}$ scheme is better convergence than pole mass scheme

Tevatron : $M_{\text{MS}} = 163 \pm 3$ [GeV]

LHC : $M_{\text{MS}} = 166 \pm 3$ [GeV] (MSTW PDF)

Langenfeld,Moch,Uwer 09

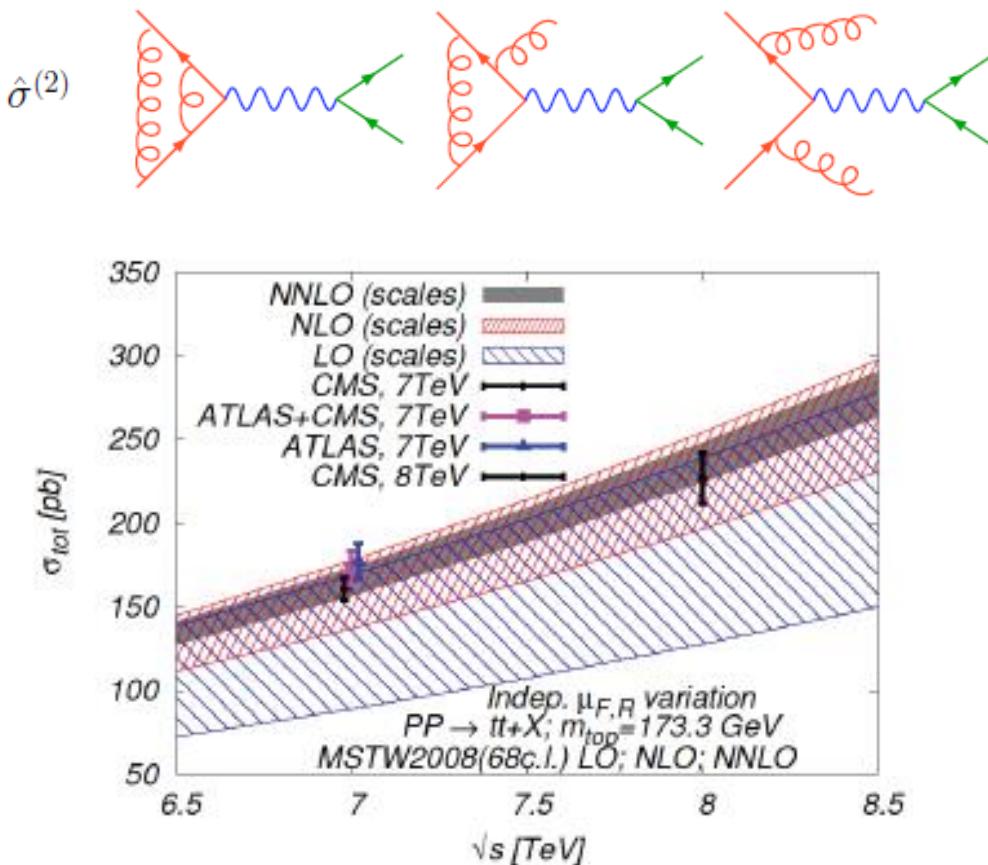
Alekhin,Djouadi,Moch 13

S.Alioli et al. 13, Dawling,Moch 13

- Similar analysis may be studied by using distributions , ttj process, etc

NNLO cross-section completed at Hadron Colliders

..., Bernreuther, Czakon,
Fiedler, Mitov 12, 13



Collider	σ_{tot} [pb]	scales [pb]	pdf [pb]
Tevatron	7.009	+0.259(3.7%) -0.374(5.3%)	+0.169(2.4%) -0.121(1.7%)
LHC 7 TeV	167.0	+6.7(4.0%) -10.7(6.4%)	+4.6(2.8%) -4.7(2.8%)
LHC 8 TeV	239.1	+9.2(3.9%) -14.8(6.2%)	+6.1(2.5%) -6.2(2.6%)
LHC 14 TeV	933.0	+31.8(3.4%) -51.0(5.5%)	+16.1(1.7%) -17.6(1.9%)

Concurrent uncertainties:

- | | |
|-------------------------------|--------------|
| Scales | $\sim 3\%$ |
| pdf (at 68%cl) | $\sim 2-3\%$ |
| α_s (parametric) | $\sim 1.5\%$ |
| m_{top} (parametric) | $\sim 3\%$ |

Alternative methods at Hadron Colliders

- ttj system invariant-mass,
- trilepton inv. mass using J/ ψ ,
- Endpoint distribution (M_{T2}),
- B-meson decay length



Aiming ~ 1 GeV accuracy,
but the problems are always
systematics (JES, ISR, PDF,,)

- Weight function method using lepton energy distribution

Kawabata,Shimizu,Sumino,HY in progress

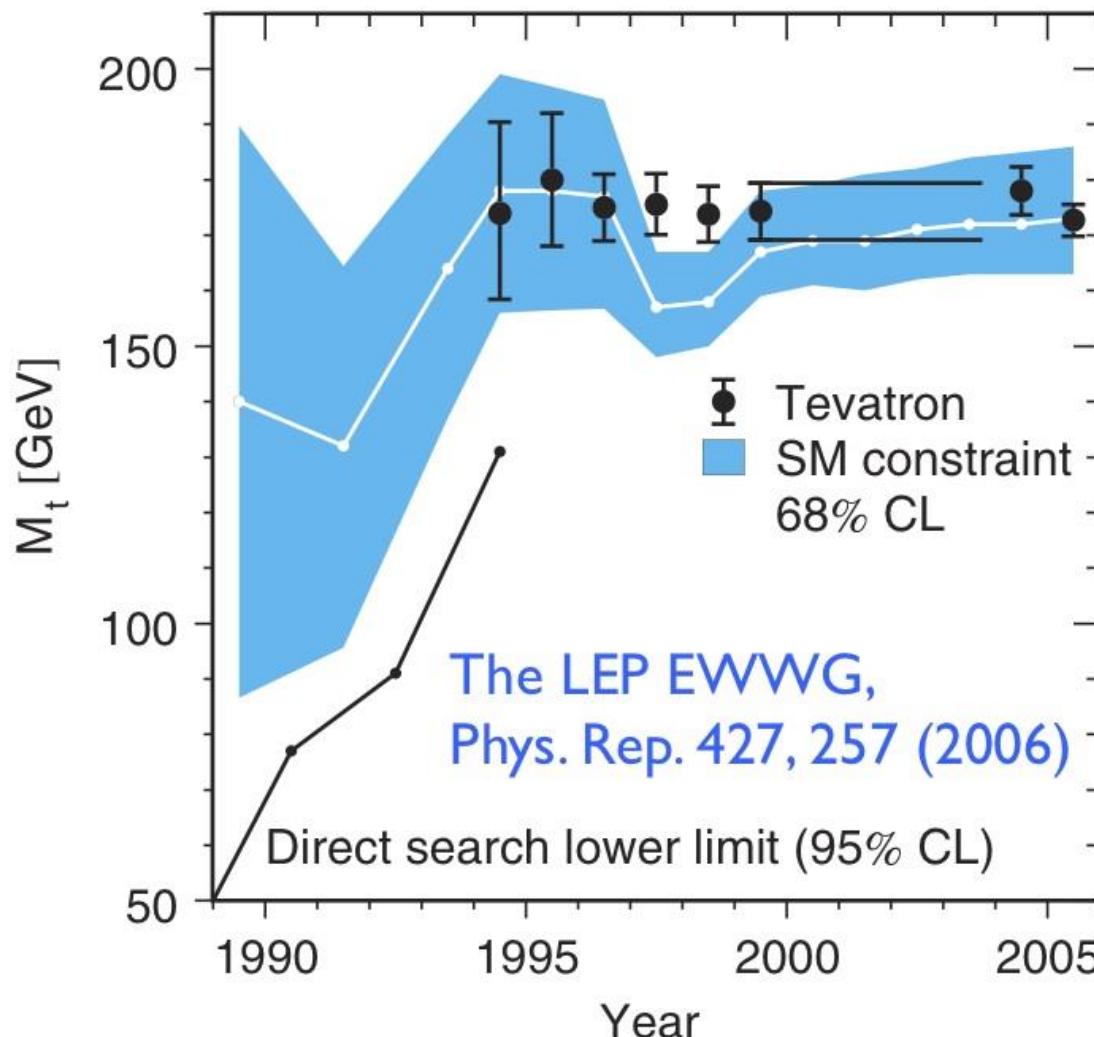
Define a convolution integral of **lepton energy distribution** and **a weight function**

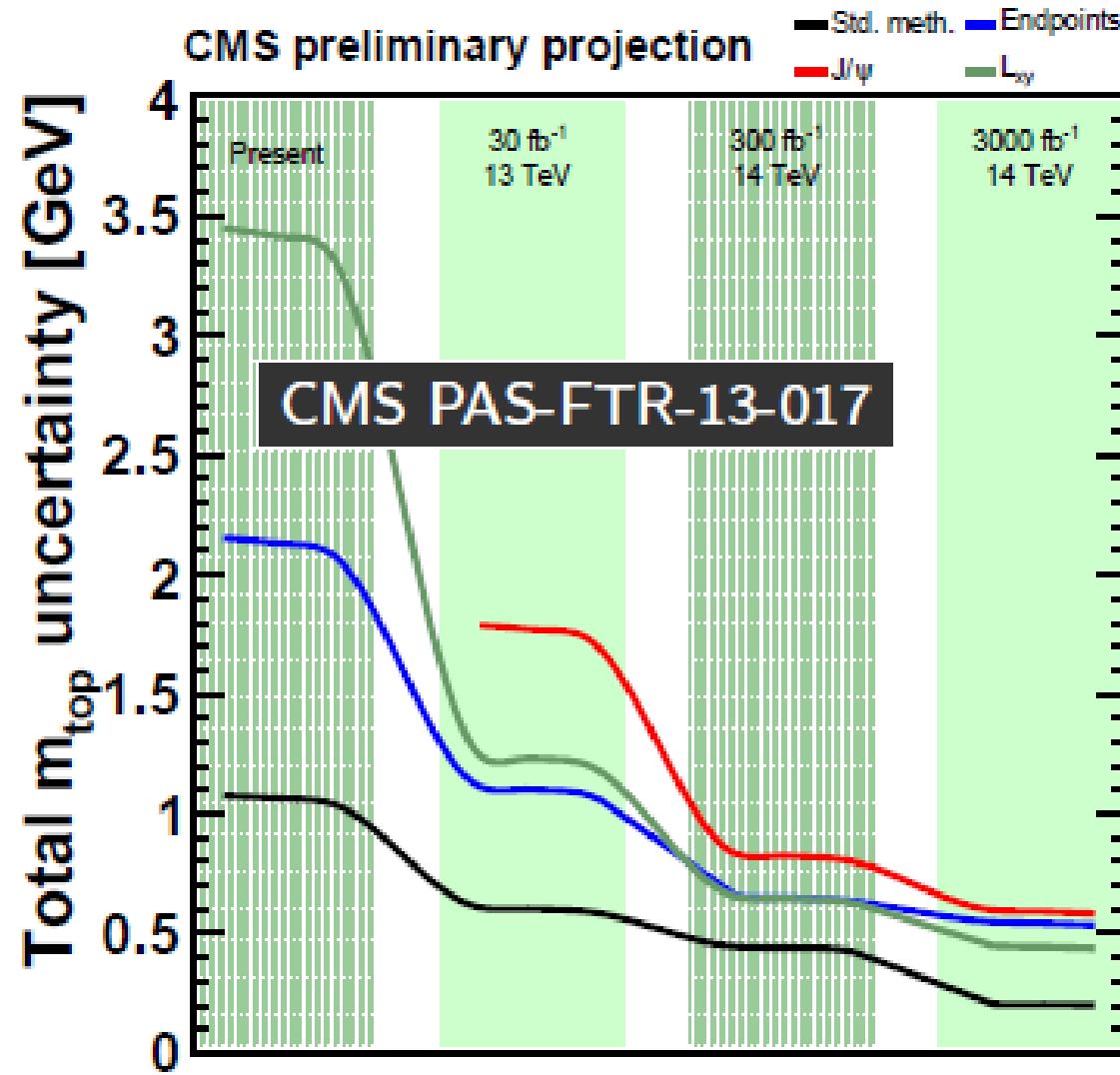
$$I(M) = \int dE_\ell D(E_\ell) W(E_\ell, M)$$

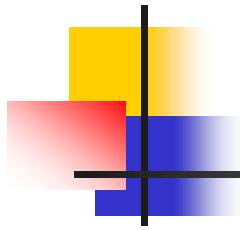
which satisfy $I(M = m_t) = 0$

- Lepton energy is accurately measured
- Suppress systematic uncertainty
- Effects of cuts are studied by MC

トップクォーク質量測定







トップクオーケ

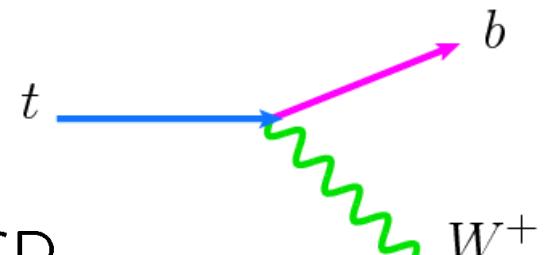
Top-quark :

- 1995年にTevatron実験で発見
 - 質量が他の夸克に比べて非常に重い

$$m_t \simeq 173 \text{ GeV} \gg m_b, m_c$$

- ・ハドロン化する前に崩壊する ($t \rightarrow bW^+$)

$$\Gamma_t \simeq \frac{G_F m_t^3}{8\sqrt{2}\pi} |V_{tb}|^2 \sim 1.5 \text{ [GeV]} \gg \Lambda_{\text{QCD}}$$



- ・ ヒッグス場との大きな湯川結合 ($y_t \sim 1$) → EWSBに関係がある?

Definitions of quark mass

Pole mass : $S_F(p) = p - m - \Sigma(p) \simeq p - m_{\text{pole}}$

プロパゲーターの極の位置として定義される、物理的な質量の定義。
 しかし、クオークは閉じ込められていて、フリーなクオークは現実的には存在しないので、pole質量にこだわる必要はない。
 (pole質量を用いた摂動計算は、収束性が悪いことが知られている。)

MSbar mass : $\delta m = \sum^{\text{(loop)}}(p)|_{\frac{1}{\epsilon} - \gamma_E + \ln 4\pi}$

発散項(+幾何学的因子)のみを引き算して定義する、unphysicalな質量の定義。
 単なる理論のパラメーター。繰り込みスケールに依存する。
 高次補正の計算には適している。

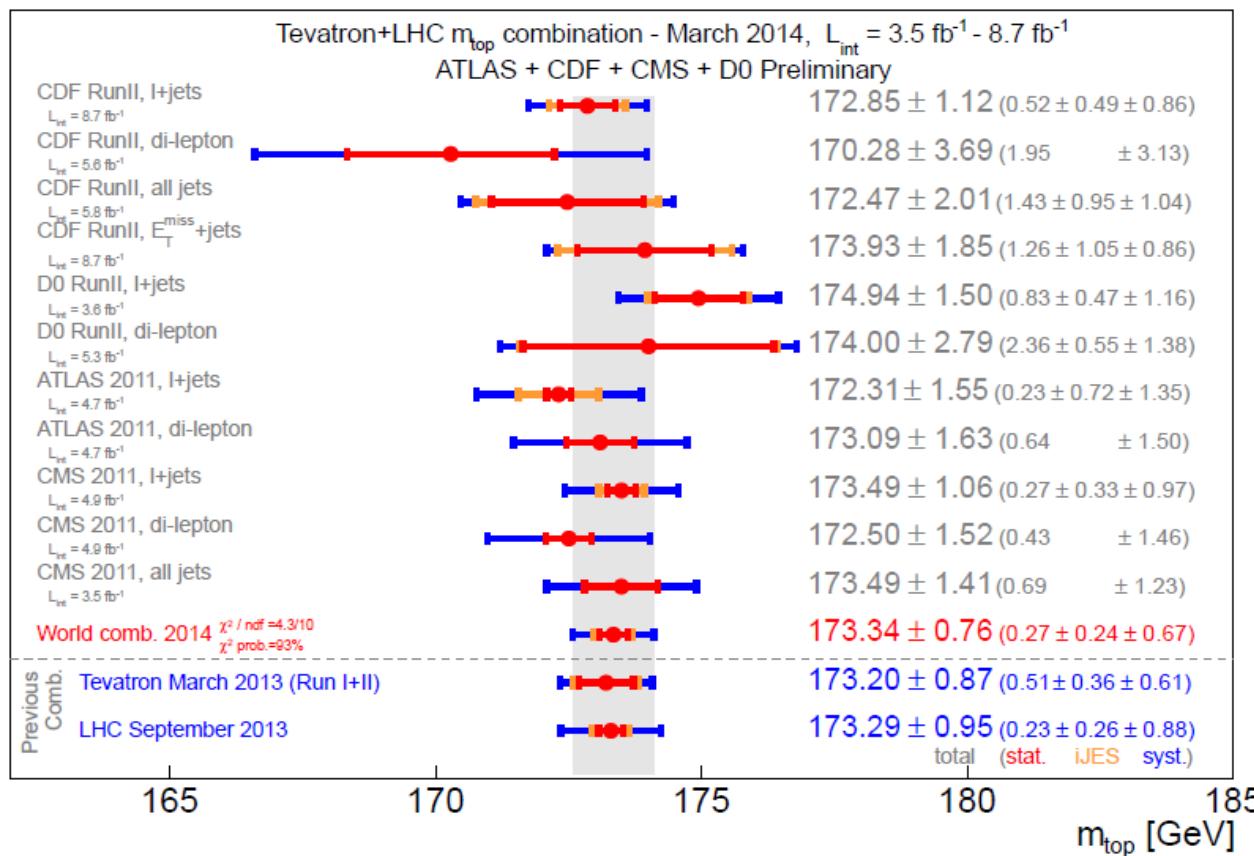
Threshold masses : クオーク対の束縛状態を記述するのに適した定義。

- 有限繰り込みによって、摂動の各次数での対応関係は知られている。

トップクォーク質量測定

First Tevatron + LHC combination result (arXiv:1403.4427)

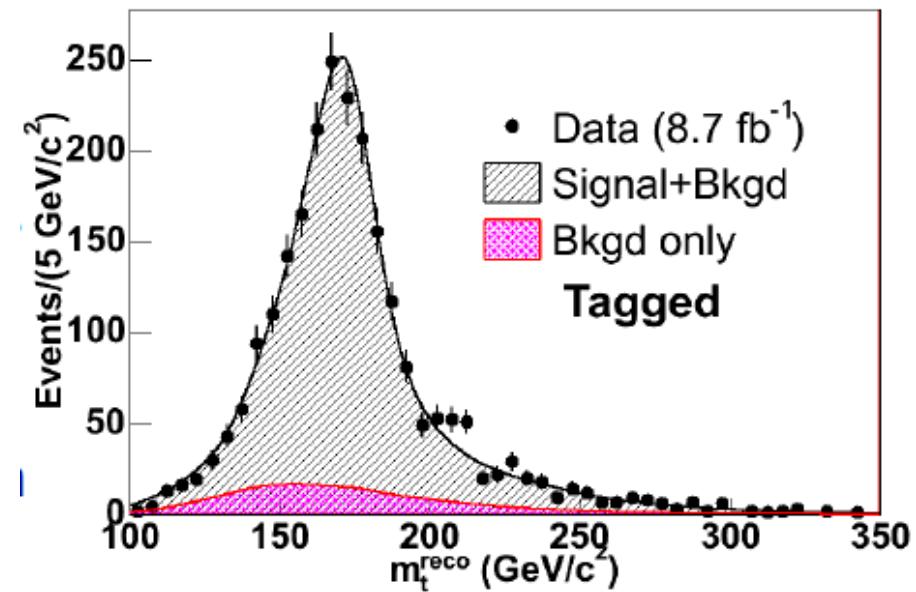
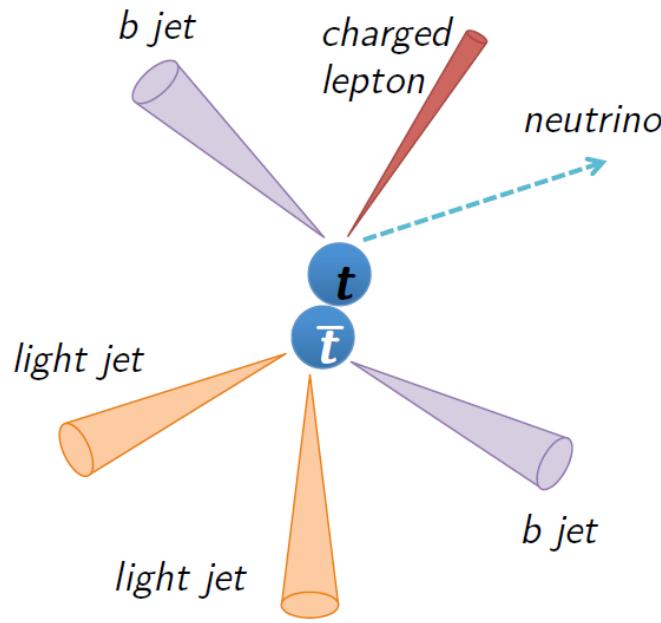
$$m_t = 173.34 \pm 0.27(\text{stat}) \pm 0.71(\text{syst}) \text{ GeV}$$



トップクォーク質量測定

ハドロンコライダーでの質量測定で、最も精度良いのはbjj不変質量分布

$$t\bar{t} \rightarrow b\ell\nu\bar{b}q\bar{q}' : \text{lepton} + \cancel{E}_T + 4\text{-jets (2 b-tagged)}$$



(実際の解析には、Template method, ME method などが用いられている)

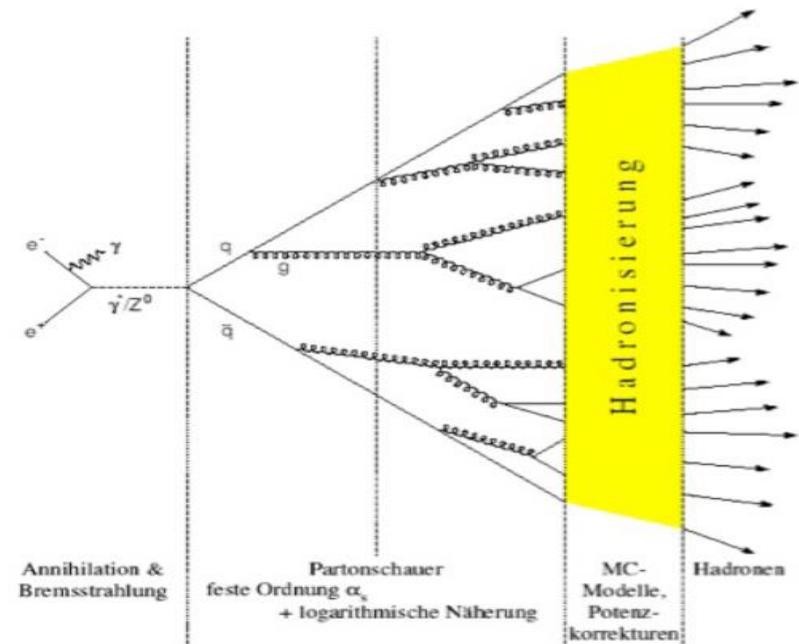
トップクォーク質量測定

- b_{jj} の不变質量のピークは、
トップクォークのポール質量の値か? **No !!**

$$M_{bjj}^2 = (p_{j1} + p_{j2} + p_{j3})^2 \not\simeq (p_b + p_q + p_{q'})^2 = p_t^2 = m_{pole}^2$$

- カラーを持たないハドロンの運動量をどれだけ正確に測れたとしても、カラーを持ったクォークの質量になることは決してない。
- ハドロンコライダーでは、更に、Initial-State Radiation, Underlying Eventなどの寄与が加わる。

$$M_{bjj} - m_{pole} \simeq \mathcal{O}(\Lambda_{\text{QCD}} - 1 \text{ GeV})$$



クオーケ質量の定義について

それぞれ定義の間の対応関係は、摂動の各次数で計算可能(有限繰り込み)

$$m_t = \overline{m}(\mu) \left[1 + \frac{\alpha_s}{\pi} d_1(\mu) + \dots \right] \quad d_1 = \frac{4}{3} + \ln \frac{\mu}{m}$$

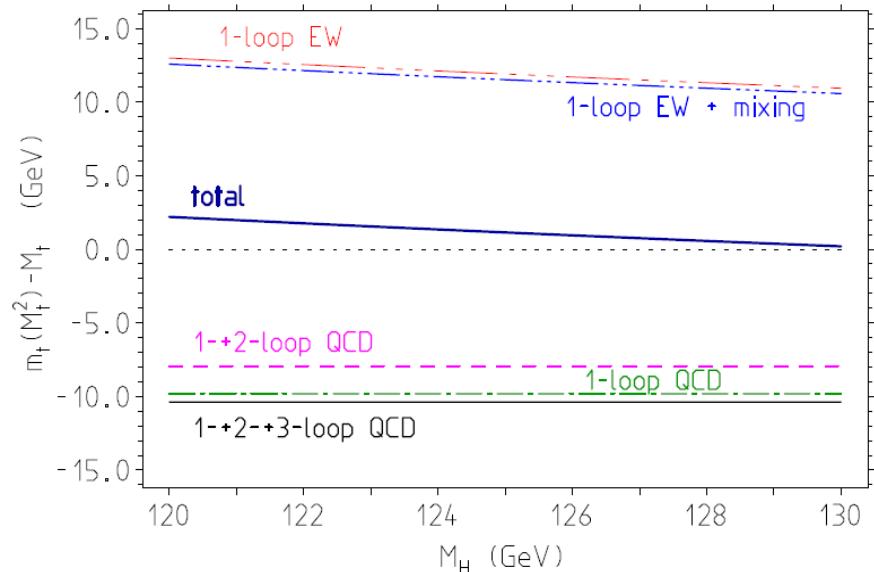
例えば、 $\overline{m}(\overline{m}) = 173 \left(1 - \frac{0.1}{\pi} \cdot \frac{4}{3} \right) \simeq 166 \text{ GeV}$ one-loop level

- Jagerlehner,Kalmykov,Kniehl (12)

EW補正もMSbarスキームで計算すると、
topの”MSbar質量”は、pole質量よりも重い。

$$m_f(\mu^2) = 2^{-3/4} (G_F^{\overline{\text{MS}}}\!(\mu^2))^{-1/2} y_f(\mu^2)$$

ただし、湯川結合の値は変わらない



トップクォーク質量測定

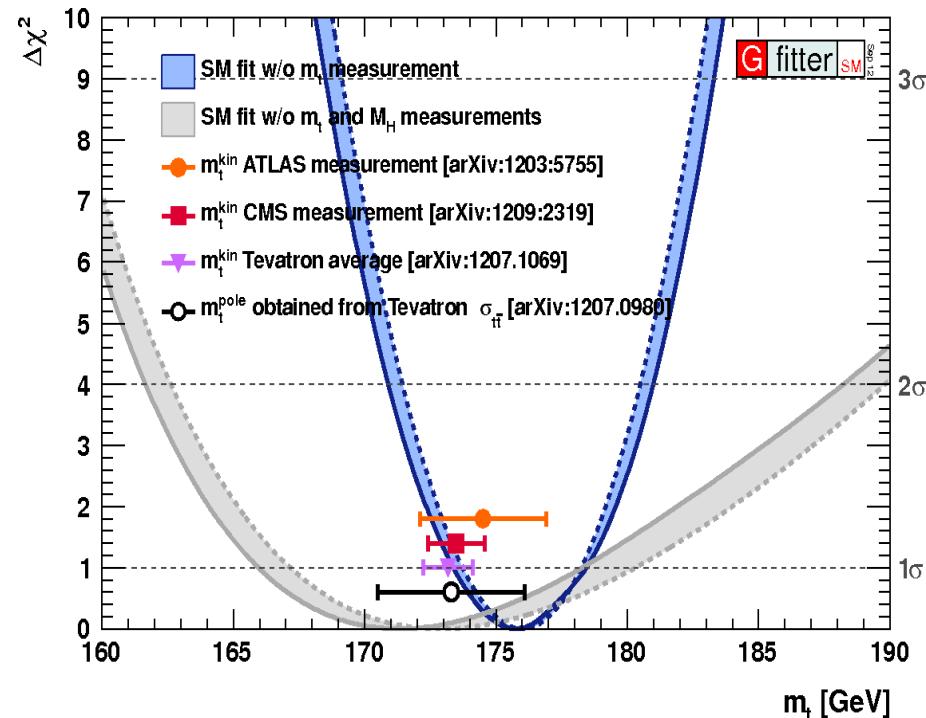
今後重要な課題：理論的に明確に定義された質量の実験測定

1. EW Precision Measurementsで質量を決める。

ρ パラメータへは、top質量は2次で、Higgs質量はlogで効く。

$$\delta\rho \sim \frac{3y_t^2}{32\pi^2} - \frac{3g'^2}{32\pi^2} \ln \frac{m_h}{m_Z}$$

- 得られる質量の定義が明確（理論計算に使ったスキーム）。
- Pole massよりもMSbar massの方が、摂動の収束性が良い。
- 他のBSM寄与の存在を無視。



トップクォーク質量測定

3. その他の新しい手法 (質量の定義が明確で無いものも含む)

- Kinematical Endpoint (M_{T2})

$$m_t = 173.9 \pm 0.9 \text{ (stat)}^{+1.2}_{-1.8} \text{ (syst) GeV} \quad \text{CMS, EPJC73,2494}$$

- B-hadron lifetime (L_{xy}) Hill,Incandela,Lamb (05)

$$m_t = 173.5 \pm 1.5 \text{ (stat)} \pm 1.3 \text{ (syst)} \pm 2.6 \text{ (p_T^{top}) GeV}$$

CMS-PAS-TOP-12-030

- J/ψ method Kharchilava (00)

- $t\bar{t} + 1\text{-jet}$ invariant-mass distribution Alioli et. al (13)

- Leptonic observable methods Agashe,Franceschini,Kim (13), Kawabata,Shimizu,Sumino,HY (14), Frixione,Mitov (14)

Threshold scan at the ILC

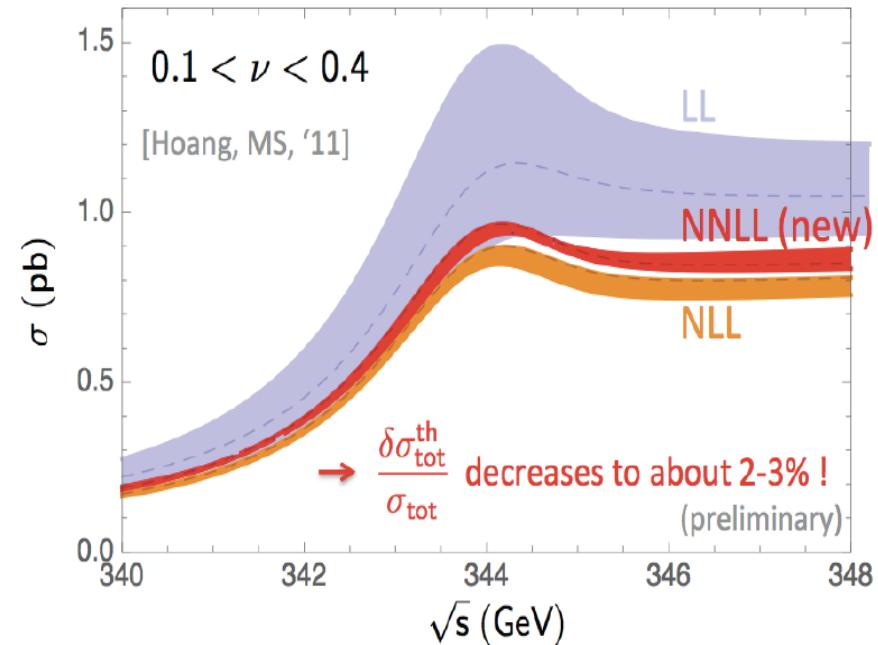
ILC実験での対生成断面積の測定

Threshold Scan : しきい値付近での生成断面積の変化を見る

- トップクォーク対の束縛状態の形成による、共鳴ピークの存在。
- ピークの位置、形、大きさなどの測定によって、 m_t , Γ_t , α_s の値が精密に決定できる。
(更に、トップ湯川結合の効果も)
- Multi-parameter fit Martinez,Miquel (03)

$$\delta m_t \simeq 30 \text{ MeV}, \delta \Gamma_t \simeq 0.3 \text{ GeV}, \delta \alpha_s \simeq 0.001$$

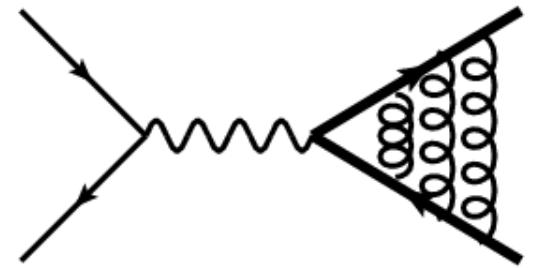
$$\delta m_t(\overline{\text{MS}}) \simeq 100 \text{ MeV} \text{ (有限繰り込み@3-loop)}$$



→ revised by Horiguchi et al (13)

トップクォークの束縛状態

- トップクォーク対の束縛状態を記述する
シュレディンガー方程式：



$$\left[(E + i\Gamma_t) - \left\{ -\frac{\nabla^2}{m_t} + V_{QCD}^{(c)}(r) \right\} \right] G^{(c)}(\vec{x}, E + i\Gamma_t) = \delta^3(\vec{x})$$

- QCDポテンシャル： $V_{QCD}^{(c)}(r) = C^{(c)} \frac{\alpha_s(\mu_B)}{r} \times \left[1 + \frac{\alpha_s}{\pi} v_1^{(c)}(r) + \dots \right]$

(湯川ポテンシャル = Higgs粒子交換の効果も結構ある)

- 束縛エネルギー： $\Delta E = \frac{C_F}{2} m_t \alpha_s^2 \simeq 2 \text{ GeV} > \Gamma_t$

トップクォークが崩壊するまでに、束縛状態を形成する時間が少しだけある。

- NRQCDに基づいたシステムティックな定式化 [NNLO(~2000), NNNLO(~2013?)]

25%

10% (?)

Synopsis of NNLO results (NRQCD)

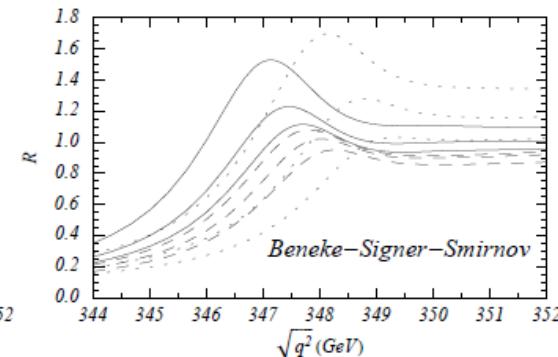
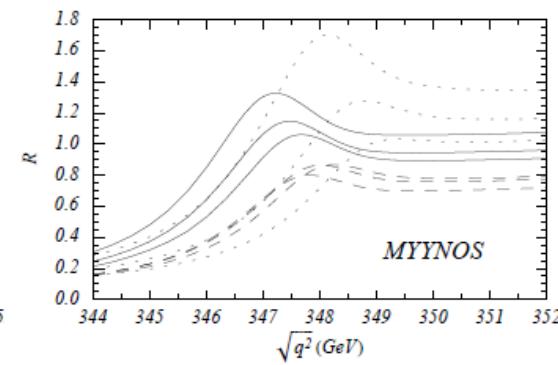
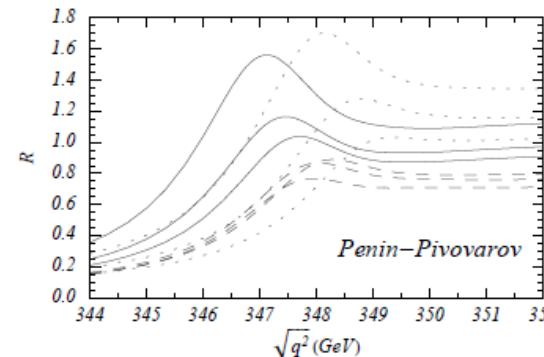
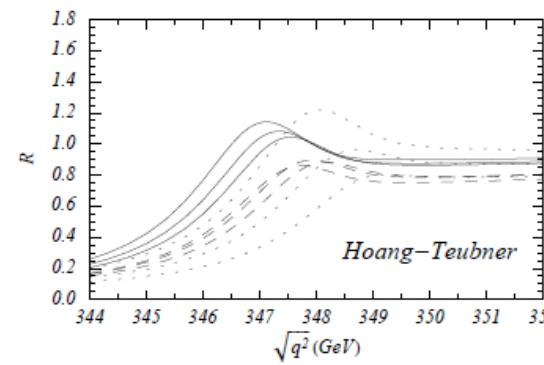
Hoang et.al.

$$\sigma_{\text{NNLO}} = \sigma_0 \sum_i \left(\frac{\alpha_s}{v} \right)^i \times \{\alpha_s^2, \alpha_s v, v^2\}$$

α_s とv(速度)を
展開パラメータにとる

- LO → NLO → NNLO
(点線) (破線) (実線)

- Peak positionの予言は、
数10 MeV程度の不定性
→質量測定の精度
- Normalizationの不定性は、
~25%程度残っている。



さらなる発展

NNLL Resummation :

Hoang, Stahlhofen (11)

resummation of ultrasoft logarithm

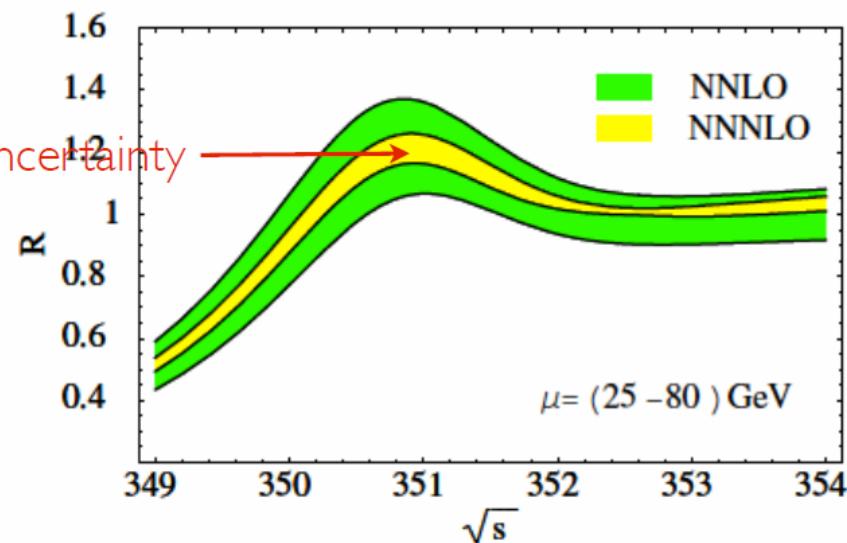
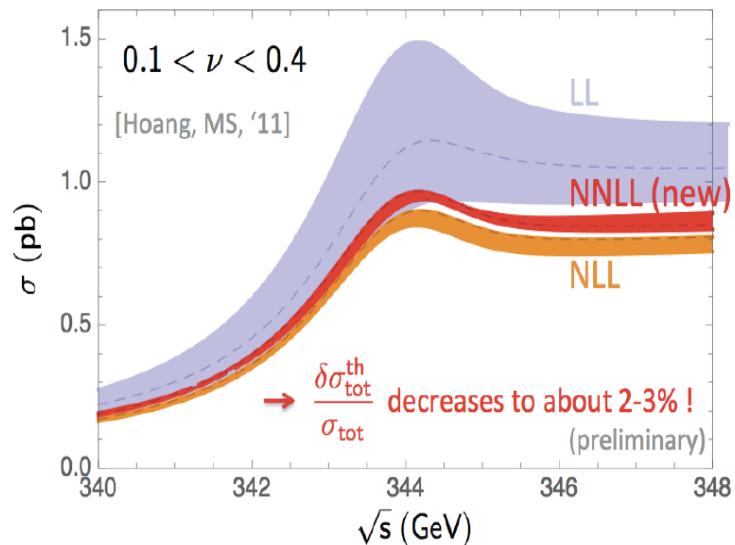
$$R = \frac{\sigma_{t\bar{t}}}{\sigma_{\mu^+\mu^-}} = v \sum_k \left(\frac{\alpha_s}{v} \right)^k \sum_i (\alpha_s \ln v)^i \times \left\{ 1 \text{ (LL)}; \alpha_s, v \text{ (NLL)}; \alpha_s^2, \alpha_s v, v^2 \text{ (NNLL)}; \dots \right\}$$

NNNLO calculation :

Beneke, Kiyo, Schuller (08, 13)

Complete NNNLO calculation almost done

Can be improved by NNNLL resummation?



まとめ：

- 加速器における高エネルギー散乱現象は、レプトンコライダーであっても、多数のハドロンの生成を伴い、その現象はQCDによって理解される。
- QCDを摂動的に用いるには、摂動論の適応出来る物理量を定義し、それに対して、クオークグルーオンを用いた計算を行う。
- 摂動論で取り扱える物理量を定義する際に重要な概念となるのが、赤外発散とその相殺である。
- 赤外有限な物理量として、包括過程、ジェットを用いた物理を説明した。
- トップクオークについて、特に質量の定義、測定法について外観した。

