Models with Small CC at Finite T

also Nearly Braneless
Conformal Symmetry Breaking

Jay Hubisz
September 19, 2014
Syracuse University

Work in progress with Don Bunk and Bithika Jain

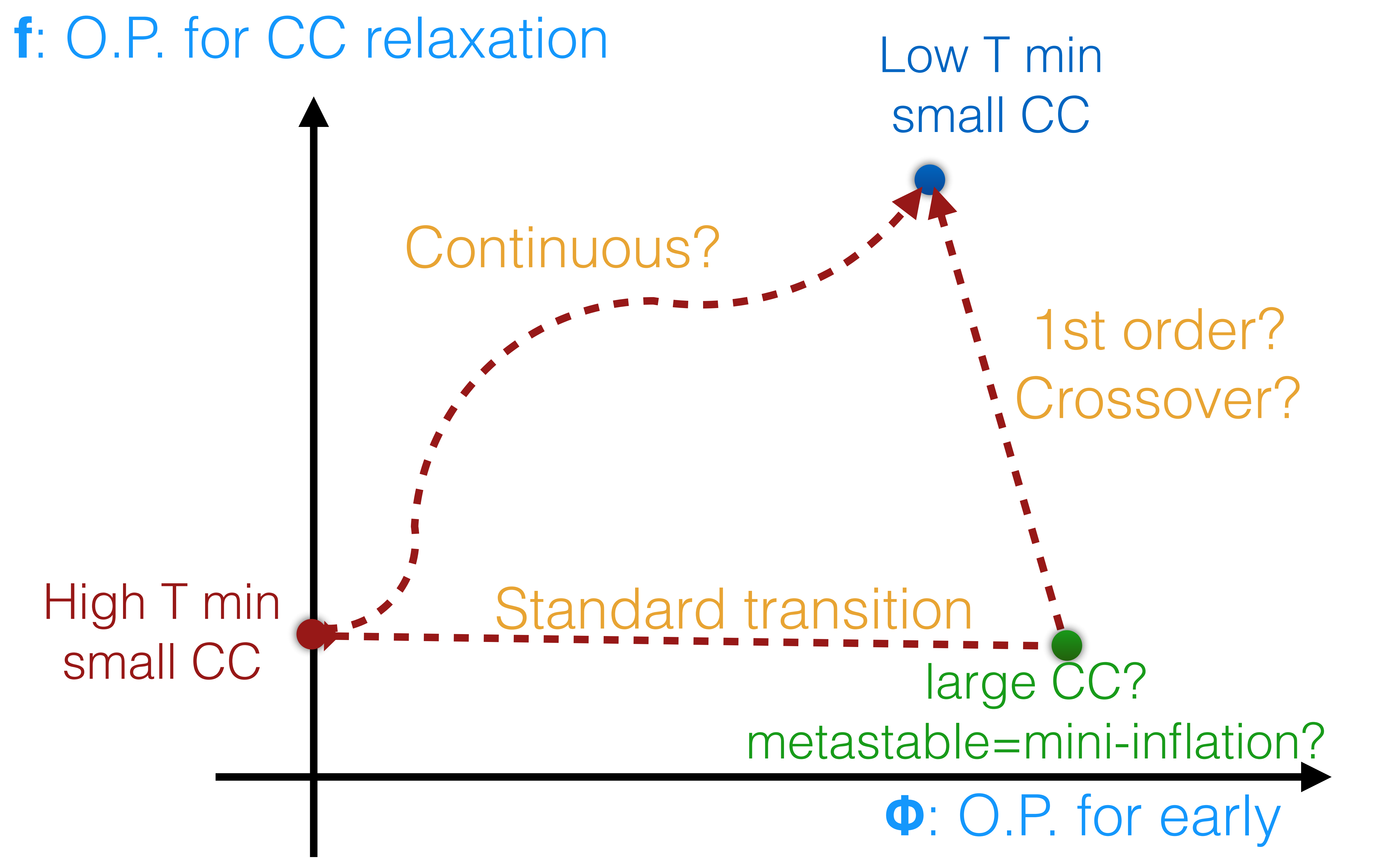
Pheno and exploration of: Bellazzini, Csáki, JH, Serra, Terning 1305.3919

2nd NPKI Workshop: Jeju Island

Motivation

- Cosmological history modified in models where the CC is protected dynamically, i.e. by
  non-linearly realized conformal invariance
    - Phase transitions generally contribute dynamically to vacuum energy - only end result is tiny
- How do such phase transitions proceed?
- Are results distinct from Creminelli, Nicolis, Rattazzi (hep-th/0107141)?
    - RS phase transition with Goldberger-Wise stablization
      (has hidden fine tuning)

Dueling order parameters



Scale Transformations

Dilatations:

$$x \to x' = e^{-\alpha} x$$

Operators transform:

$$\mathcal{O}(x) \to \mathcal{O}'(x) = e^{\alpha\Delta}\mathcal{O}(e^{\alpha}x)$$

Δ is non-perturbative quantum operator scaling dimension

very small bird, normal leaf?
kind of small bird, big leaf?

Linearized transformation of action with sourced O:

$$S \longrightarrow S + \sum_i \int d^4x\, \alpha g_i(\Delta_i - 4)\mathcal{O}_i(x)$$

Spontaneous breaking

CFT operator gets VEV:

$$\langle \mathcal{O}(x) \rangle = f^{\Delta}$$

Single corresponding goldstone boson:

Low, Manohar '01

$$\sigma(x) \to \sigma(e^{\alpha} x) + \alpha f$$

Non-linear realization in effective theory:

$$f \to f \, \chi \equiv f \, e^{\sigma/f}$$

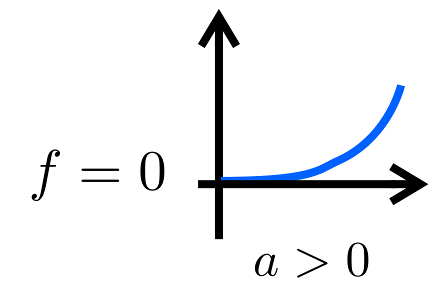
Restores symmetry to LEEFT

The Dilaton Quartic

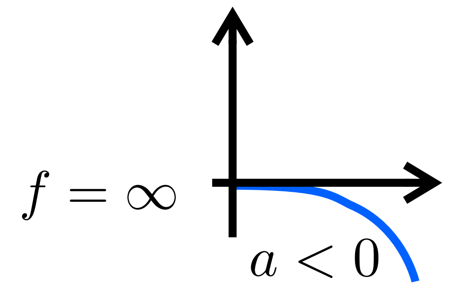
Most general terms invariant under dilatations:

$$\mathcal{L}_{\text{eff}} \approx -af^4\chi^4 + \frac{f^2}{2}(\partial_\mu \chi)^2 + \text{higher derivative terms}$$

↑
dilaton quartic

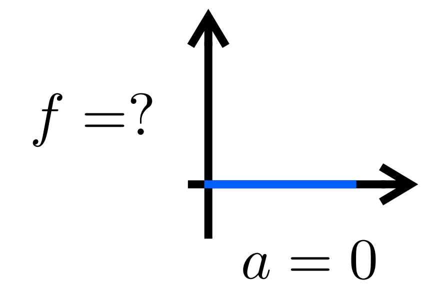


Obstruction to SBSI:



Fubini '76

- a > 0 → f = 0 (no breaking)
- a < 0 → f = ∞ (runaway)
- a = 0 → f = anything (flat direction)



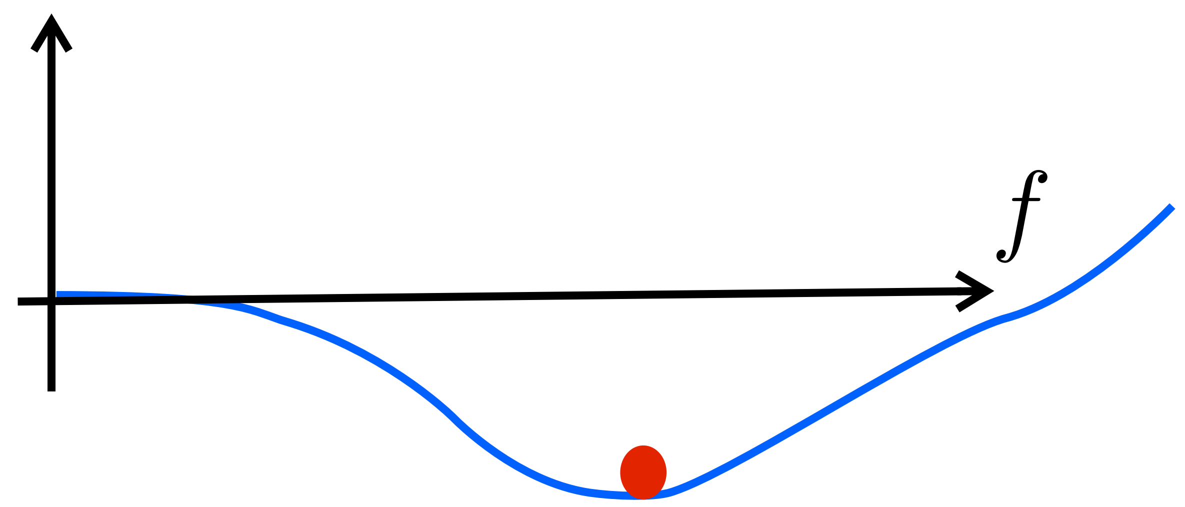
Near-Marginal Deformation

$$\delta S = \int d^4 x \lambda(\mu) \mathcal{O}$$

Quartic has dependence on near marginal coupling:

$$V(f) = a f^4 \rightarrow V(f) = f^4 F(\lambda(f))$$

slowly varying function of f



Deformation can stabilize f away from origin

$$V' = f^3 \left[4F(\lambda(f)) + \beta F'(\lambda(f)) \right] = 0$$

The Dilaton Mass

Expanding the potential:

$$m_{dil}^2 = f^2\beta\left[\beta F'' + 4F' + \beta' F'\right] \simeq 4f^2\beta F'(\lambda(f)) = -16f^2 F(\lambda(f))$$

small, so dilaton is light, right?

F is the cosmological constant in f units:

$$F_{NDA} \sim \frac{\Lambda^4}{16\pi^2 f^4} \sim 16\pi^2$$

Need *large* β to find minimum $V' = f^3\left[4F(\lambda(f)) + \beta F'(\lambda(f))\right] = 0$

Theory not conformal at scale f - **no light dilaton**

$$m_{dil}^2 \sim 256\pi^2 f^2 \sim \Lambda^2$$

3 TeV *not suppressed*

OR we can *tune* away the quartic to get a near flat-direction

CPR idea

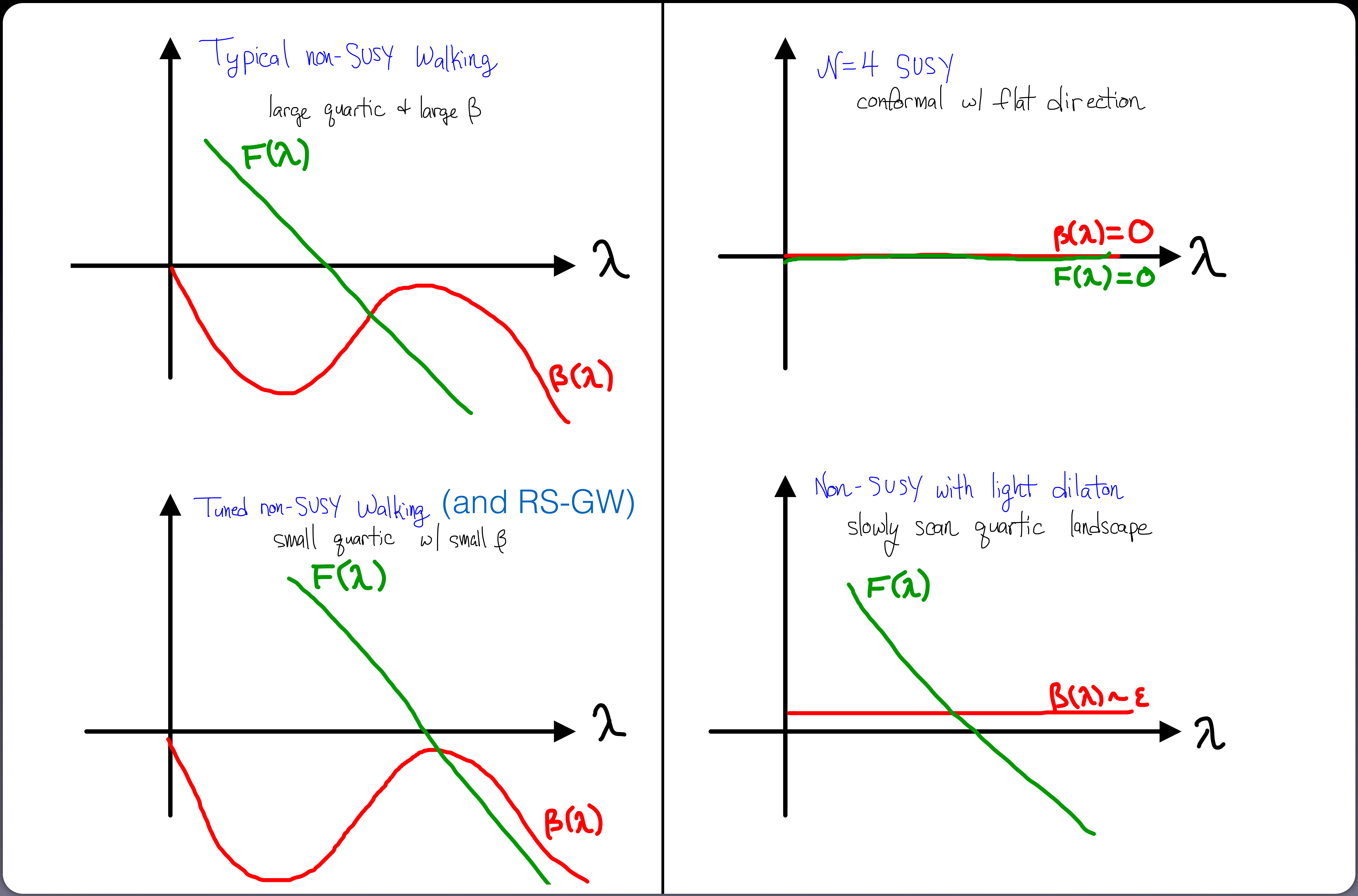
Contino, Pomarol, Rattazzi (Talk at Planck 2010)

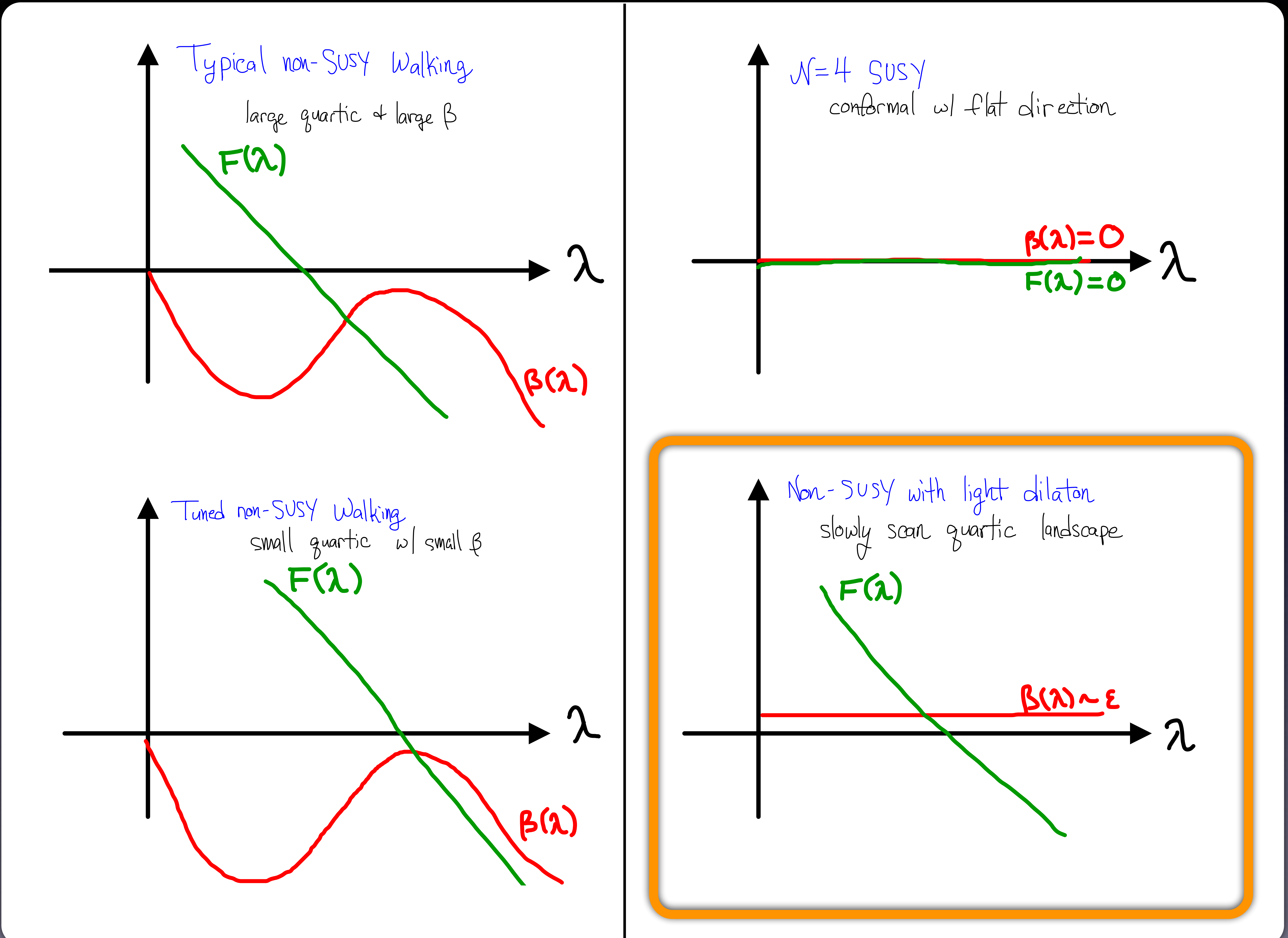
- F(λ) generically large, but if λ near marginal for **range** of λ, theory will scan over F with scale

$$\frac{d\lambda}{d\log\mu} = \beta(\mu) \equiv \epsilon \ll 1$$

- large F will not generate spontaneously broken scale invariance
  - minimum when F ~ 0
- dilaton mass proportional to ε

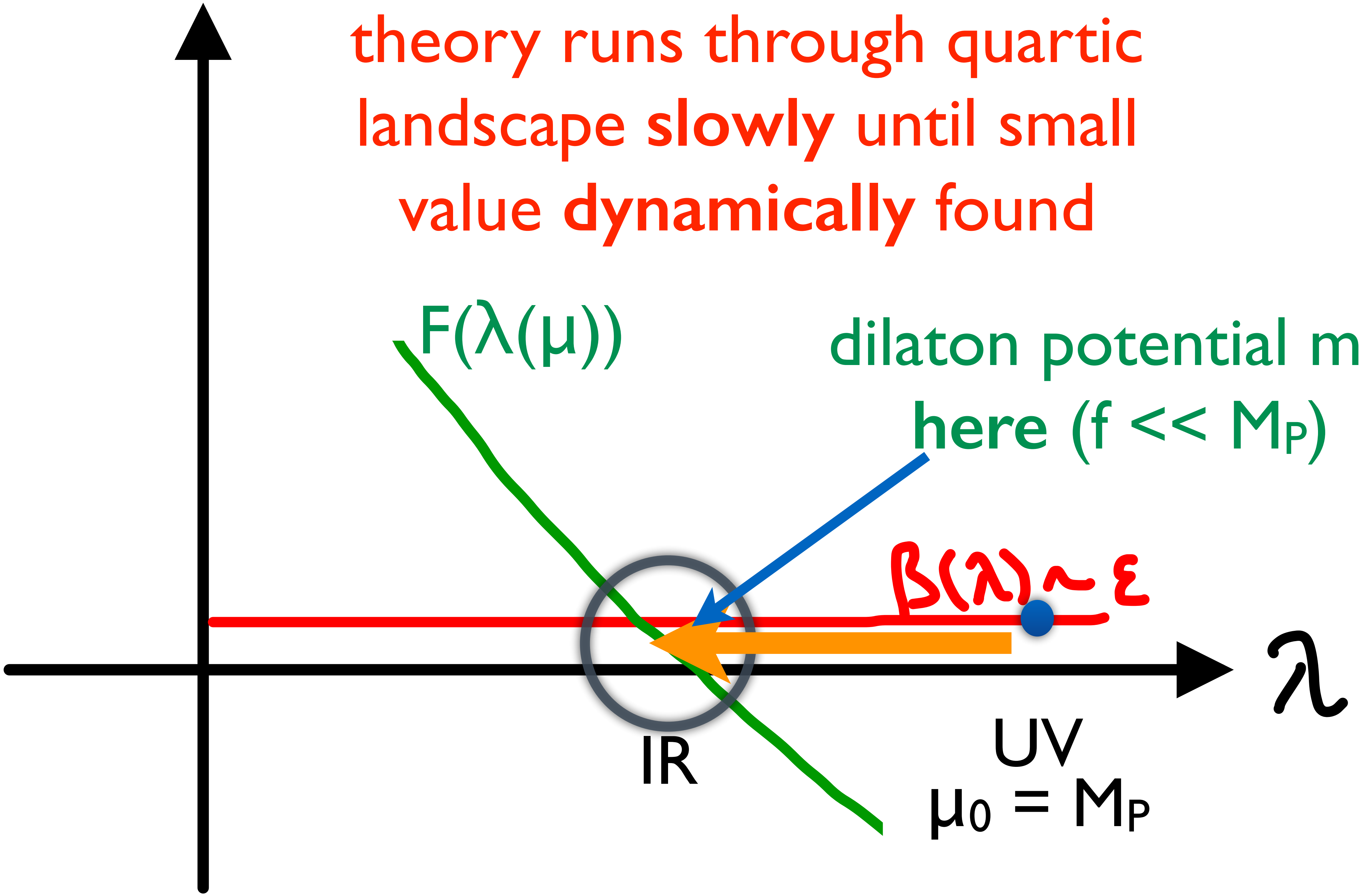
Various dynamical possibilities





Typical non-SUSY Walking
large quartic & large β
F(λ)
λ
β(λ)
N=4 SUSY
conformal w/ flat direction
β(λ)=0
λ
F(λ)=0
Tuned non-SUSY Walking
small quartic w/ small β
F(λ)
λ
β(λ)
Non-SUSY with light dilaton
slowly scan quartic landscape
F(λ)
β(λ)~ε
λ

Cartoon


theory runs through quartic landscape **slowly** until small value **dynamically** found
F(λ(μ))
dilaton potential min
here (f << MP)
β(λ) ~ ε
λ
IR
UV
μ0 = MP

Naturally Light Dilaton, Small CC

5D scalar min. coupled to gravity: $S = \int d^5x \sqrt{g}\left[\frac{1}{2}(\partial_M \phi)^2 - V(\phi) - \frac{1}{2\kappa^2}R\right]$

+ brane potentials at y_0 and y_1

AdS/CFT:

small β ⇔ slowly changing V(Φ): $V(\phi) = \Lambda_5 + \epsilon f(\phi)$

Csáki, Bellazzini, JH, Serra, Terning 1305.3919

see also Coradeschi, Lodone, Pappadopulo, Rattazzi, Vitale (1306.4601)

Metric Ansatz: flat 4D slices

$$ds^2 = e^{-2A(\tilde{y})}\eta_{\mu\nu}dx^\mu dx^\nu - d\tilde{y}^2$$

"True scale" coordinates: $\mu = ke^{-y}$

$$A(\tilde{y}) = y \qquad G(y) = A'(\tilde{y}(y))^2$$

$$ds^2 = e^{-2y}\eta_{\mu\nu}dx^\mu dx^\nu - \frac{dy^2}{G(y)}$$

AdS/CFT: EOM capture running even when far from AdS

Deviations from pure AdS encoded in G(y)

Scalar-Einstein Equations

Equations of motion:

$$G = \frac{\frac{-\kappa^2}{6} V(\phi)}{1 - \frac{\kappa^2}{12}\dot{\phi}^2}$$

$$\frac{\dot{G}}{G} = \frac{2\kappa^2}{3}\dot{\phi}^2$$

$$\ddot{\phi} = \left(4 - \frac{1}{2}\frac{\dot{G}}{G}\right)\dot{\phi} + \frac{1}{G}\frac{\partial V}{\partial \phi}$$

Can substitute G(y) in terms of scalar vev in last equation:

Master Evolution Equation:

$$\ddot{\phi} = 4\left(\dot{\phi} - \frac{3}{2\kappa^2}\frac{\partial \log V(\phi)}{\partial \phi}\right)\left(1 - \frac{\kappa^2}{12}\dot{\phi}^2\right)$$

Backreaction term

AdS/CFT:

$\phi \sim \log \lambda$

Captures running (and condensation) of sourced operators in ~CFT

Dilaton Effective Potential

Bulk action total derivative (Bellazzini et. al. 1305.3919) integrates to pure boundary term, along with brane localized potentials and jump contributions

$$V_{\text{eff}} = e^{-4y_0}\left[V_0(\phi(y_0)) - \frac{6}{\kappa^2}\sqrt{G(y_0)}\right] + e^{-4y_1}\left[V_1(\phi(y_1)) + \frac{6}{\kappa^2}\sqrt{G(y_1)}\right]$$

UV brane $\mu_0 = ke^{-y_0}$

IR brane $\mu_1 = ke^{-y_1}$

What is the behavior of the dilaton effective potential for various bulk scalar potentials?
(various deformations of CFT)

Task = work out UV and IR **asymptotics**

How is spontaneously broken scale invariance manifested?

Where's the IR brane?

B.C.'s from brane potential:

$$V_1 = \Lambda_1 + \gamma\,(\phi - \phi_1)^2$$

∞ in "stiff brane" limit

$$\dot{\phi}|_{y_1} = \left.\frac{\partial V_1(\phi)}{\partial \phi}\right|_{y_1}$$

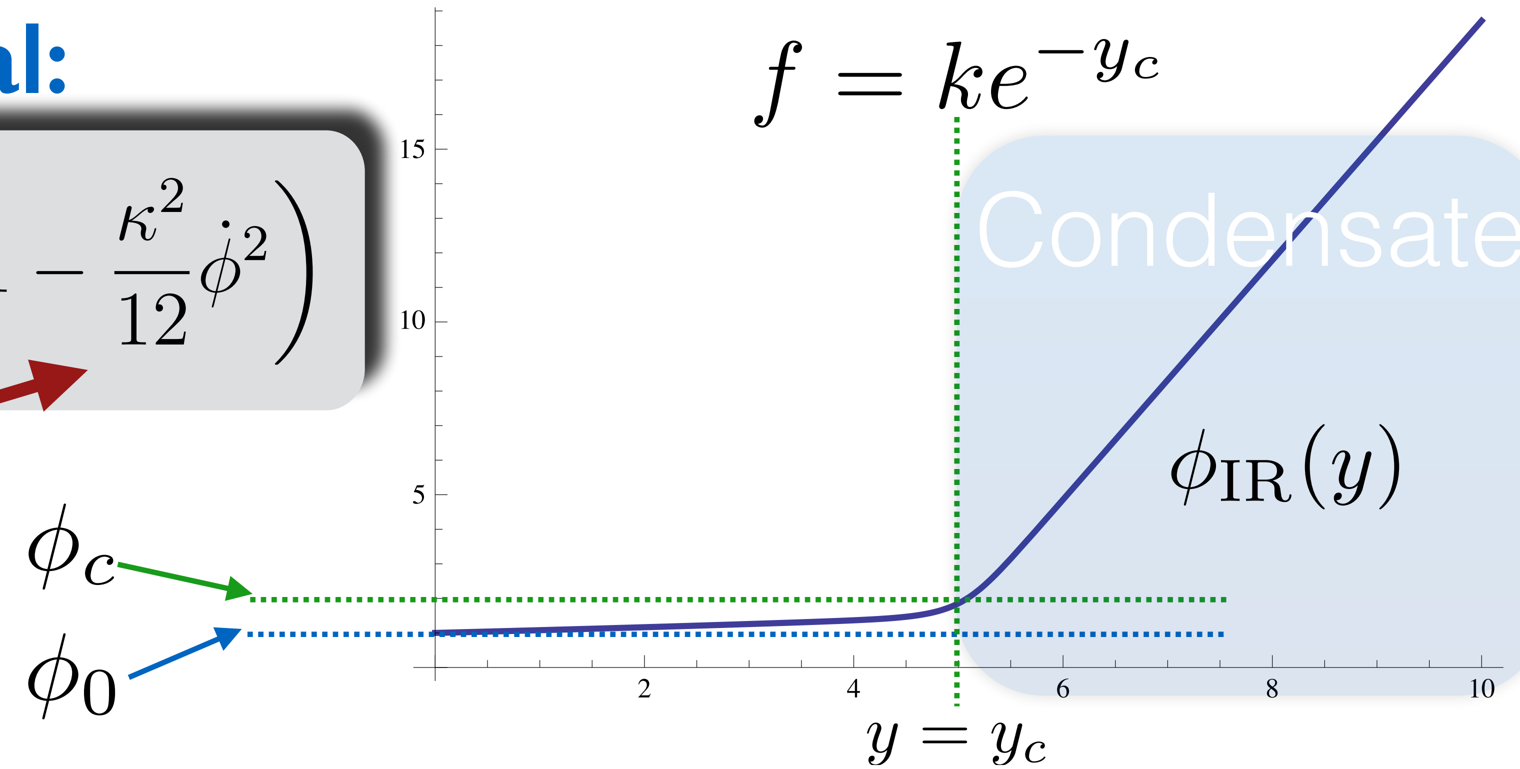
$$\phi = \phi_1$$

Generic Potential:

$$\ddot{\phi} = 4\left(\dot{\phi} - \frac{3}{2\kappa^2}\frac{\partial \log V(\phi)}{\partial \phi}\right)\left(1 - \frac{\kappa^2}{12}\dot{\phi}^2\right)$$

IR Universality

$$\phi_{\rm IR}(y) \approx \phi_c + \sqrt{\frac{12}{\kappa^2}}(y - y_c)$$



Imposing B.C.'s: $\phi_c + \sqrt{\frac{12}{\kappa^2}}(y_1 - y_c) = \phi_1$

$$\left(\frac{\mu_1}{f}\right) = e^{y_c - y_1} = \exp\left[\sqrt{\frac{\kappa^2}{12}}\,(\phi_1 - \phi_c)\right] \sim \mathcal{O}(.01 - 1)$$

NDA: $\phi_1 = \text{few} \cdot \sqrt{1/\kappa^2}$

Chacko, Mishra, Stolarski

Easily deep into non-AdS region

Constant V (exact CFT)

$$V(\phi) = -\frac{6k}{\kappa^2}$$

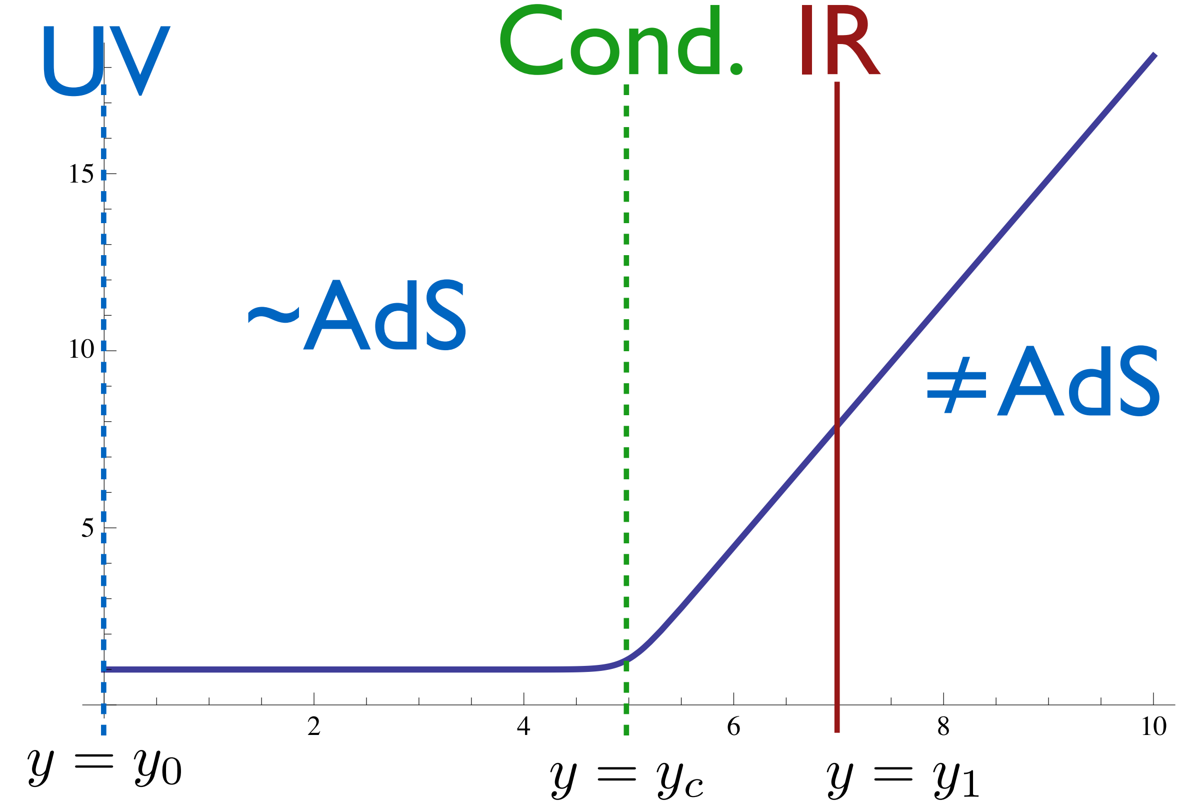
Master Evolution Equation:

$$\ddot{\phi} = 4\dot{\phi}\left(1 - \frac{\kappa^2}{12}\dot{\phi}^2\right)$$

Solution: Singularity at y→ ∞

$$\phi = \phi_0 \pm \frac{1}{4}\sqrt{\frac{12}{\kappa^2}} \log\left[e^{4(y-y_c)}\left(1+\sqrt{1+e^{8(y_c-y)}}\right)\right]$$

$$G = k^2\left[1 + \left(\frac{f}{\chi}\right)^8\right]$$



Asymptotics:

$$\phi \approx \begin{cases} \phi_0 \pm \frac{1}{4}\sqrt{\frac{12}{\kappa^2}}\left(\frac{f}{\mu_0}\right)^4 + \mathcal{O}\left(\left(\frac{f}{\mu_0}\right)^{12}\right) & \mu_0 \gg f \quad \text{UV} \\ \phi_0 \pm \frac{1}{4}\sqrt{\frac{12}{\kappa^2}}\left[\log\left(2\left(\frac{f}{\mu_1}\right)^4\right)\right] & \mu_1 \ll f \quad \text{IR} \end{cases}$$

$$G = k^2\begin{cases} 1 + \mathcal{O}\left(\left(\frac{f}{\mu_0}\right)^8\right) & \mu_0 \gg f \quad \text{UV} \\ \left(\frac{f}{\mu_1}\right)^4 + \mathcal{O}\left(\left(\frac{\mu_1}{f}\right)^4\right) & \mu_1 \ll f \quad \text{IR} \end{cases}$$

A'≠ constant!

Plug into Holographic Dilaton Potential

$$V_{\text{eff}} = e^{-4y_0}\left[V_0(\phi(y_0)) - \frac{6}{\kappa^2}\sqrt{G(y_0)}\right] + e^{-4y_1}\left[V_1(\phi(y_1)) + \frac{6}{\kappa^2}\sqrt{G(y_1)}\right]$$

Substitute and re-arrange:

$$V_{\text{dilaton}} = \left(\frac{\mu_0}{k}\right)^4\left[V_0(\phi_0) - \frac{6k}{\kappa^2}\right] \pm \frac{1}{4}\sqrt{\frac{12}{\kappa^2}}\left(\frac{f}{k}\right)^4\left[\frac{\partial V_0}{\partial\phi}(\phi_0) + 2k\sqrt{\frac{12}{\kappa^2}}\right] + \mathcal{O}\left[\left(\frac{f}{\mu_0}\right)^8\right]$$

Bare C.C. — Dilaton quartic

+ subleading terms of order $(\mu_1/f)^4$

2 Needed Tunings:

tune dilaton quartic to zero (*force* a flat direction)

fine-tune bare CC

Message:

soft wall *geometric* model of spontaneously broken scale invariance

IR brane plays **subdominant** role:

just cuts off growth of scalar field and curvature

General bulk potentials: UV

Small Bulk Mass Term

$$V = -\frac{6k^2}{\kappa^2}\left(1 + \frac{\kappa^2}{3}\epsilon\phi^2\right)$$

Approximate (small backreaction) solution:

$$\phi = \phi_0 e^{\epsilon y} + \widetilde{\lambda} e^{(4-\epsilon)(y-y_c)}$$

$$G(y) = k\left(1 + \mathcal{O}(e^{-8(y_c-y_0)})\right)$$

UV contribution to dilaton effective potential:

$$V_{\rm UV} = \left(\frac{\mu_0}{k}\right)^4 \left[V_0(\phi_0 e^{\epsilon y_0}) - \frac{6k}{\kappa^2}\right] + \widetilde{\lambda}_{\rm UV}\left(\frac{f}{k}\right)^{4-\epsilon}\left(\frac{\mu_0}{k}\right)^{-\epsilon}\frac{\partial V_0}{\partial\phi}(\phi_0 e^{\epsilon\phi})$$

Bare CC

marginal "almost quartic"

General bulk potentials: IR

$$\ddot{\phi} = 4\left(\dot{\phi} - \frac{3}{2\kappa^2}\frac{\partial \log V(\phi)}{\partial \phi}\right)\left(1 - \frac{\kappa^2}{12}\dot{\phi}^2\right)$$

IR Universality

Parametrize:

$$\dot{\phi} = \sqrt{\frac{12}{\kappa^2}}(1 - \delta v_{\rm IR})$$

Recall: $G(y) = \dfrac{\frac{-\kappa^2}{6}V(\phi)}{1 - \frac{\kappa^2}{12}\dot{\phi}^2} \approx -\dfrac{\kappa^2}{12}\dfrac{V(\phi)}{\delta v_{\rm IR}}$ In deep IR

Expand scalar EOM in small δv:

$$\dot{\delta v}_{\rm IR} = -8\left(1 - \frac{1}{8}\frac{d}{dy}\log V(\phi)\right)\delta v_{\rm IR}$$

Integrable in asymptotic limit!

$$\delta v_{\rm IR} = \tilde{\lambda}\frac{V(\phi(y))}{V(\phi(y_c))}e^{-8(y-y_c)}$$

$$G_{\rm IR}(y_1) = -\frac{\kappa^2}{12\tilde{\lambda}}V(\phi(y_c))e^{8(y_1-y_c)}$$

Generates **pure** quartic:

$$V_{\rm IR} = e^{-4y_1}\left[V_1(\phi(y_1)) + \frac{6}{\kappa^2}\sqrt{G(y_1)}\right] \to \sqrt{-\frac{\kappa^2}{12\tilde{\lambda}}V(\phi_c)}\left(\frac{f}{k}\right)^4$$

Resulting Dilaton Potential

$$V_{\text{dilaton}} = -\mu^{\epsilon} f^{4-\epsilon} + \lambda f^4$$

At min:

$$f \approx \mu \left(\frac{1}{\lambda}\right)^{1/\epsilon} \qquad V_{\text{dilaton}} = \frac{\epsilon}{4} \lambda f^4$$

CC is suppressed by ε!

Up to kinetic term normalization of dilaton:

$$m^2_{\text{dilaton}} \sim \epsilon f^2$$

Soft wall realization of small condensate contribution to CC in models with non-linearly realized SBSI

Finite Temperature

Black hole in AdS

Same action - euclidean signature, compactified time t⊂{0,β=1/T}

$$ds^2 = e^{-2y}\left[h(y)dt^2 + d\vec{x}^2\right] + \frac{1}{h(y)}\frac{dy^2}{G(y)}$$

Einstein Equations:

$$\frac{d}{dy}\log\frac{\dot{h}}{h} = 4\left(1 - \frac{1}{4}\frac{\dot{h}}{h} - \frac{\kappa^2}{12}\dot{\phi}^2\right)$$

$$\frac{\dot{G}}{G} = \frac{2\kappa^2}{3}\dot{\phi}^2$$

$$G = \frac{\frac{2\kappa^2}{3}\frac{V(\phi)}{h}}{\frac{d}{dy}\log\frac{\dot{h}}{h}}$$

Master Evolution Equation:

$$\ddot{\phi} = 4\left(\dot{\phi} - \frac{3}{2\kappa^2}\frac{\partial \log V}{\partial \phi}\right)\left(1 - \frac{1}{4}\frac{\dot{h}}{h} - \frac{\kappa^2}{12}\dot{\phi}^2\right)$$

Horizon function

Horizon B.C.'s:

$$\dot{\phi}\Big|_{y_h} = \frac{3}{2\kappa^2}\frac{\partial \log V}{\partial \phi}\Big|_{y_h}$$

Holographic Dilaton Potential:

$$V_{\text{dilaton}} = e^{-4y_0}h(y_0)\left[V_0(\phi(y_0)) - \frac{6}{\kappa^2}\sqrt{G(y_0)}\right] + e^{-4y_1}h(y_1)\left[V_1(\phi(y_1)) + \frac{6}{\kappa^2}\sqrt{G(y_1)}\right]$$

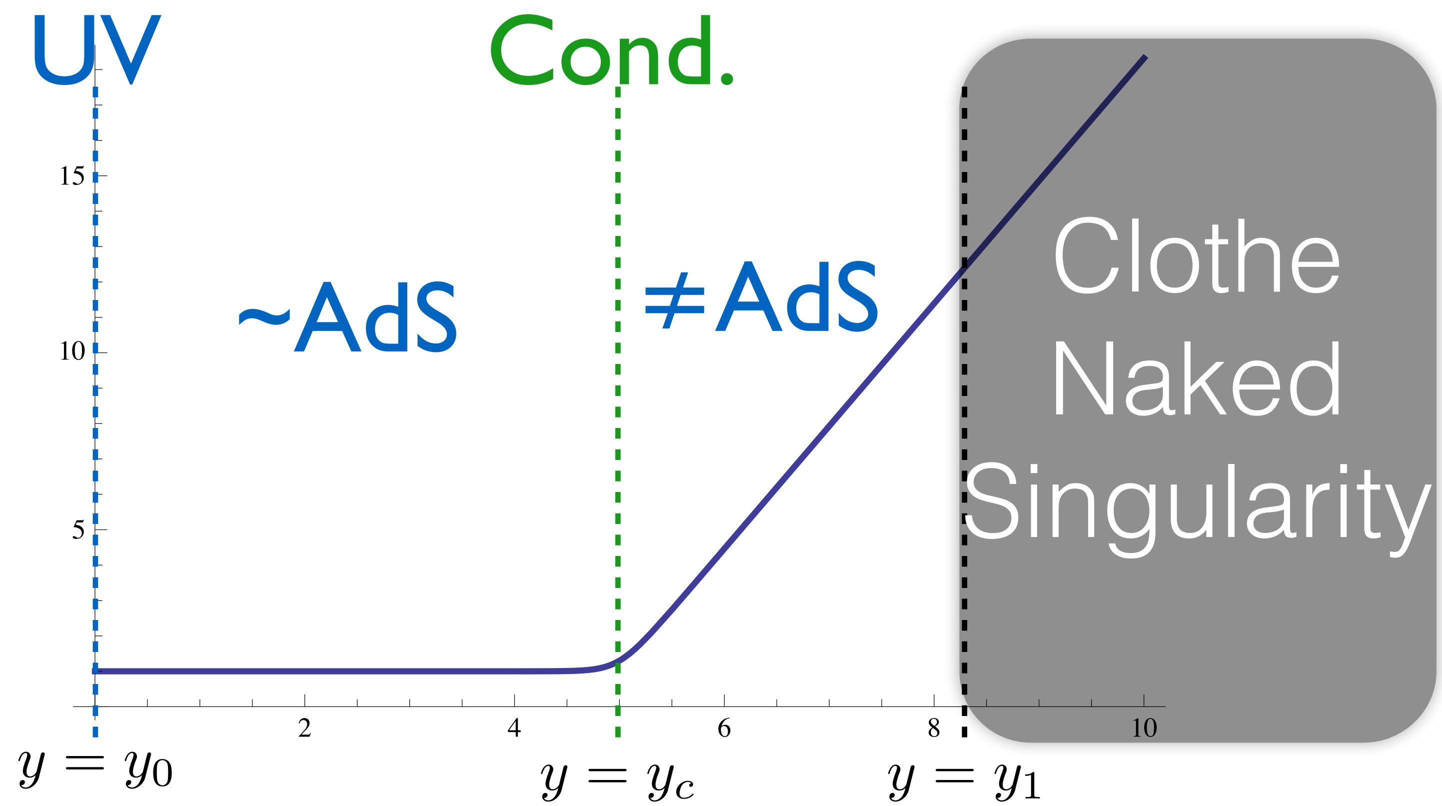
Black Brane

$$ds^2 = e^{-2y}\left[h(y)dt^2 + d\vec{x}^2\right] + \frac{1}{h(y)}\frac{dy^2}{G(y)}$$

Goal:

Obtain *continuous* V(f,T) moving between ~AdS and ~AdS$_c$

Distinct from:
Creminelli, Nicolis, Rattazzi
hep-th:0107141



$$h(y_1) = 0$$

Near horizon geometry typically divergent (hairy)
out-of-equilibrium

Equilibrium temp: adjust y$_1$ to remove conical singularity

$$T_{\rm eq} = \frac{e^{-y_1}}{2\pi}\sqrt{-\frac{\kappa^2}{6}\dot{h}(y_1)V(\phi_h)} \propto \frac{k}{2\pi}e^{-y_1}$$

Constant V (exact CFT)

Evolution equations again integrable:

$$\frac{d}{dy}\log\dot{\phi} = \frac{d}{dy}\log\frac{\dot{h}}{h}$$

Solution:

$$\phi = \phi_0 + C_l \log h$$

Incompatible with horizon B.C.'s and finite ϕ unless C_l=0

Finite temperature ruins the flat direction!

Constant ϕ→ **no condensate region** - UNBROKEN CFT

AdS-Schwarzchild

$$h = 1 - e^{4(y-y_h)} \qquad G = k^2$$

Plugging into holo. potential (free energy):

$$F_{\text{AdS-S}} \propto T^4$$

Usual result for unbroken CFT at finite T

General Potentials

$$V = -\frac{6k^2}{\kappa^2}\left(1 + \frac{\kappa^2}{3}\epsilon\phi^2\right)$$

Horizon B.C.'s: $\dot{\phi}\Big|_{y_h} = \frac{3}{2\kappa^2}\frac{\partial \log V}{\partial \phi}\Big|_{y_h} = \epsilon\phi\Big|_{y_h}$

In principle, there seems to be no great obstacle:
horizon doesn't force trivial ϕ = Constant

In practice, goal of numerical solutions with horizon at finite y_h and condensate region has proven **extraordinarily stubborn**
condensate forces horizon to $y=\infty$

Likely due to *fragility* of condensate under small perturbations (i.e. low temp)
appearing as numerical stability issues

Conclusions

- Models that make progress on the CC problem are few and far between
- We explore variants and pheno of holographic CPR:
  - Bellazzini, Csáki, JH, Serra, Terning 1305.3919
- Among this class of models are ones in which SBSI manifests as **continuous** geometries (**soft wall** SBSI) with IR brane playing lesser role as cutoff
- Should allow a continuous parametrizing of model at finite temperature
  - vital for understanding cosmological behavior of various models where CC problem has dynamical resolution
  - so far elusive, but …

Thank you!