Alpha(s), Heavy Quark Mass and Sum Rules

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Outline

- Introduction
- Heavy quark masses
- Strong coupling
- Summary

Introduction

- Masses
  Quark masses still suffer from bigger uncertainties $\gtrsim 1\%$ PDG, no free particles
  $\hookrightarrow$ improvement desirable
- Couplings

  $\delta\alpha \sim 0.3\text{ppb}, \quad \delta G_F \sim 0.5\text{ppm}, \quad \delta\alpha_s(M_Z) \sim 0.5\%$ PDG

  improvement desirable $\hookleftarrow$
- Quark masses and strong coupling are fundamental parameters of the Standard Model
  $\rightsquigarrow$ enter in many physical observables

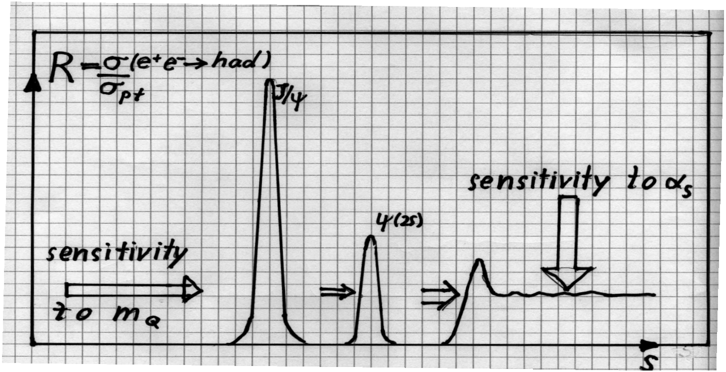
Heavy quarks

Experiment: *R*-ratio for charm- & bottom-quarks

Extraction of mass from cross section

$$R(s) = \frac{\sigma(e^+e^-\to\text{Hadrons})}{\sigma(e^+e^-\to\mu^+\mu^-)}$$

experiment ↩ ↪ theory



Heavy quarks

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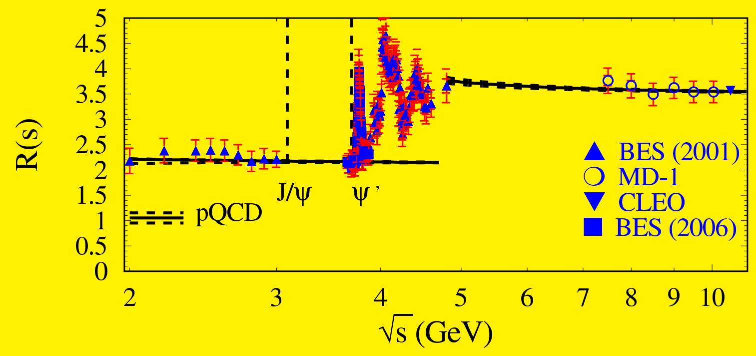


Bottom-quark case analog

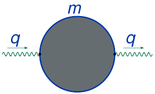
Relation: Experiment $\Longleftrightarrow$ Theory

- Heavy quark correlator

$$\Pi^{\mu\nu}(q,j) = i \int dx\, e^{iqx} \langle 0 | T j^{\mu}(x) j^{\nu}(0) | 0 \rangle$$

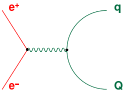
<u>Here:</u> $j^{\mu}(x)$ electromagnetic heavy quark <u>vector current</u>

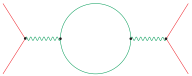
$$\Pi^{\mu\nu}(q) = (-g^{\mu\nu} + q^{\mu} q^{\nu}/q^2)\Pi(q^2) \sim$$



- $$R(s) = \frac{\sigma(e^+e^- \to \text{Hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 12\pi \,\text{Im}\left[\Pi(q^2 = s + i\epsilon)\right]$$

$$\int d\Pi \left| \quad \right|^2 = 2\ \text{Im}\left(\quad \right)$$





- With the help of dispersion-relations:

$$\boxed{\Pi(q^2) = \Pi(q^2 = 0) + \frac{q^2}{12\pi^2} \int ds \frac{R(s)}{s(s - q^2)}}$$

- Exp. moments are related to derivatives of $\Pi(q^2)$ at $q^2 = 0$

Heavy Quarks

Relation: Theory $\Longleftrightarrow$ Experiment

- Exp. moments are related to derivatives of $\Pi(q^2)$ at $q^2 = 0$:

$$\frac{12\pi}{n!}\left(\frac{d}{dq^2}\right)^n \Pi(q^2)\bigg|_{q^2=0} = \mathcal{M}_n^{\mathrm{exp}} = \int \frac{ds}{s^{n+1}} R^{\mathrm{exp}}(s)$$

- In terms of expansion coefficients:

$$\Pi(q^2) = \frac{3Q_f^2}{16\pi^2}\sum_n \overline{C}_n^v \left(\frac{q^2}{4m^2}\right)^n ,$$

Q_f: charge of quark

$m = m(\mu)$: $\overline{\text{MS}}$ mass

$\overline{C}_n^v$ can be calculated perturbatively SVZ

- First and higher derivatives of $\Pi(q^2)$ allow direct determination of the $\overline{\text{MS}}$ charm- and bottom-quark mass:

$$\overline{m}(\mu) = \frac{1}{2}\left(Q_f^2 \frac{9}{4}\frac{\overline{C}_n^v}{\mathcal{M}_n^{\mathrm{exp}}}\right)^{1/(2n)}$$

$\longleftarrow$ Theory

$\longleftarrow$ Experiment

c-quarks: Novikov et al. '78; b-quarks: Reinders et al. '85

$\overline{C}_n^v$ depend on the quark mass through $\log(m(\mu)^2/\mu^2)$

Calculation of C_n

$\Pi(q^2)$ in low energy limit

- 3-loop(order α_s^2) coefficients $\overline{C}_n$
  up to n=8 Chetyrkin,Kühn,Steinhauser 96
  up to higher moments $n \sim 30$ Czakon et al. 06; Maierhöfer, Maier, Marquard 07
  for correlators VV, AA, PP, SS
- 4-loop(order α_s^3) coefficients $\overline{C}_n$
  - Vector case: (R-ratio method)
    - first moments $\overline{C}_0$, $\overline{C}_1$
      K. G. Chetyrkin, J. H. Kühn, C.S.'06; R. Boughezal, M. Czakon, T. Schutzmeier'06
    - second moment $\overline{C}_2$ A. Maier, P. Maierhöfer, P. Marquard'08
    - third moment $\overline{C}_3$ A. Maier, P. Maierhöfer, P. Marquard, A.V. Smirnov '09
    - fourth moment $\overline{C}_{4,\ldots,10}$ Y. Kiyo, A. Maier, P. Maierhöfer, P. Marquard '09
  - Pseudoscalar case: (Lattice method)
    - first moments $\overline{C}_0$, $\overline{C}_1$, $\overline{C}_2$ I. Allison, E. Dalgic, C.T.H. Davies, E. Follana, R.R. Horgan, K. Hornbostel, G.P. Lepage, C. McNeile, J. Shigemitsu, H. Trottier, R.M. Woloshyn, K.G. Chetyrkin, J.H. Kühn, M. Steinhauser, C.S. 08
    - third moment $\overline{C}_3$ A. Maier, P. Maierhöfer, P. Marquard'08
    - fourth moment $\overline{C}_4$ A. Maier, P. Maierhöfer, P. Marquard, A.V. Smirnov '09
    - fifth moment $\overline{C}_{5,\ldots,10}$ Y. Kiyo, A. Maier, P. Maierhöfer, P. Marquard '09
  - Axial-vector and scalar case:
    - first moments $\overline{C}_0$, $\overline{C}_1$ C. S.'08
    - third moment $\overline{C}_3$ A. Maier, P. Maierhöfer, P. Marquard, A.V. Smirnov '09
    - fourth moment $\overline{C}_{4,\ldots,10}$ Y. Kiyo, A. Maier, P. Maierhöfer, P. Marquard '09

Charm quark mass

Contributions to $\mathcal{M}_n^{\text{exp}}$

- narrow resonances $R = \frac{9\,\Pi\, M_R\, \Gamma_e}{\alpha^2(s)}\,\delta(s - M_R^2)$ PDG
- threshold region ($2\,m_D - 4.8\,\text{GeV}$) (BES)
- perturbative continuum ($E \geq 4.8\,\text{GeV}$) (Theory)

n	$m_c(3\,\text{GeV})$	exp	α_s	μ	np	total
1	986	9	9	2	1	13
2	976	6	14	5	0	16
3	978	5	15	7	2	17
4	1004	3	9	31	7	33

$$m_c(3\,\text{GeV}) = 986 \pm 13\,\text{MeV}$$

$$m_c(m_c) = 1279 \pm 13\,\text{MeV}$$

Chetyrkin, Kühn, Maierhöfer, Marquard, Steinhauser, C.S.

Bottom quark case leads in analogy to:

$$m_b(10\,\text{GeV}) = 3610 \pm 16\,\text{MeV}$$

$$m_b(m_b) = 4163 \pm 16\,\text{MeV}$$

Chetyrkin, Kühn, Maierhöfer, Marquard, Steinhauser, C.S.

Potential improvements

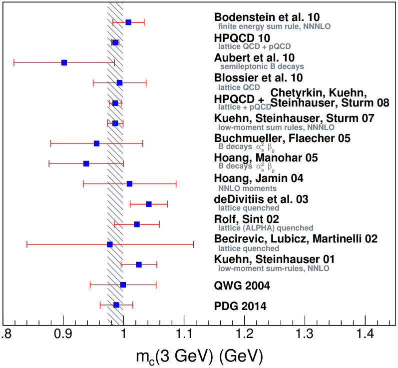
- More “aggressive” choice for $\delta\alpha_s$:
  $\alpha_s = 0.1189 \pm 0.002 \quad \Rightarrow \quad \alpha_s = 0.1185 \pm 0.0006$ (PDG)
- Theory error from perturbation series (2 – 3 permille) instead of μ-variation

Experimental improvements

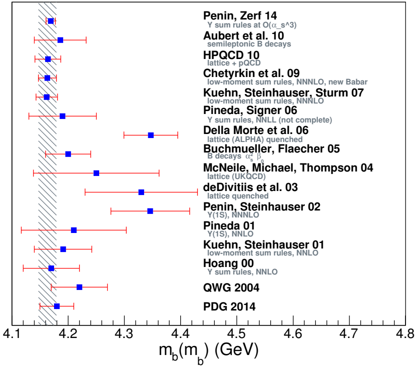
- Charm: narrow resonances ($J/\Psi, \Psi'$) dominate
  $\Gamma_e(1S) = 5.55 \pm 0.14 \pm 0.02$ keV; $\Gamma_e(2S) = 2.36 \pm 0.04$ keV PDG
  improvement?
- Bottom: improved data

Further determinations

charm-quarks



bottom-quarks



Heavy quarks and α_s

lattice

Idea: Replace moments $\mathcal{M}$ obtained from *R*-ratio by computation of correlator through lattice simulations $\mathcal{R}$

- Allows to substitute electromagnetic current by pseudoscalar operator $r_{2n+2} = \left(\overline{C}^p_n/\overline{C}^{p,(0)}_n\right)^{\frac{1}{2n-2}}$, $n = 2, 3, \ldots$
  $\overline{C}^p_n$: expansion coeff. of pseudoscalar correlator
- Quark mass:

$$\overline{m}_c(\mu) = \frac{m^{exp}_{\eta_c}}{2} \frac{r^{pQCD}_{2n+2}}{\mathcal{R}^{LQCD}_{2n+2}}$$

← Pert. theory

← Lattice Sim.

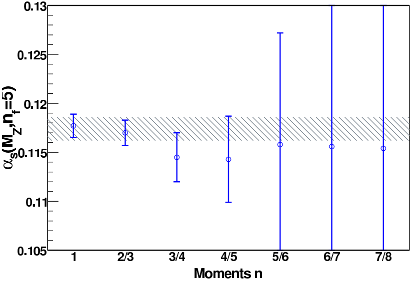
I. Allison, et al.

- $m_{\eta_c} = 2.980$ GeV, meson mass difference Υ'-Υ
  $m_c(3\text{ GeV})=0.986(10)$ GeV, $m_b(m_b) = 4.164(23)$ GeV
  HPQCD + Chetyrkin, Kühn, Steinhauser, C.S. $\hookrightarrow$ 6 MeV smaller lattice spacing $\leftarrow$HPQCD ⬏
- Strong coupling:
  Lowest moment dimensionless
  $r_4(\alpha_s, \mu/m_c) = \mathcal{R}^{LQCD}_4$ + ratios of moments, weak dependence on quark mass

Strong coupling

lattice

- Lowest moment & ratios of moments $\rightsquigarrow$ weak dependence on quark mass $\rightsquigarrow$ extract α_s
- First step extract $\alpha_s(3\text{ GeV}, n_f = 4)$, then run to $\alpha_s(M_z, n_f = 5)$



Result:

$\alpha_s(M_z) = 0.1174(12)$

lattice + pQCD HPQCD + Chetyrkin, Kühn, Steinhauser, C.S.

Updated by HPQCD:

$\alpha_s(M_z) = 0.1183(7)$

smaller lattice spacing

[PDG: $\alpha_s(M_z) = 0.1185(6)$]

R, high energy limit / massless limit

$$R^{NS} = 3 \sum_i Q_i^2 \left(1 + \frac{\alpha_s}{\pi} + \# \left(\frac{\alpha_s}{\pi}\right)^2 + \# \left(\frac{\alpha_s}{\pi}\right)^3 + \# \left(\frac{\alpha_s}{\pi}\right)^4 + \cdots \right)$$

parton model

QED Källen+ Sabry 1955

Chetyrkin, Kataev, Tkachov; Dine, Sapirstein; Celmaster 1979

Gorishny, Kataev, Larin; Surguladze, Samuel 1991 Chetyrkin /gen. gauge/ 1996

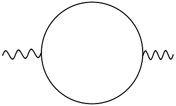
Baikov, Chetyrkin, Kühn 2008; Baikov, Chetyrkin, Kühn 2010 (Feynman Gauge only)

$$R^{SI} = \left(\sum_i Q_i\right)^2 \left(\qquad \# \left(\frac{\alpha_s}{\pi}\right)^3 + \boxed{\# \left(\frac{\alpha_s}{\pi}\right)^4} + \cdots \right)$$

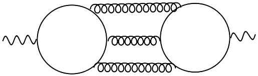
Baikov, Chetyrkin, Kühn, Rittinger 2012

D

D^{NS}



D^{SI}



Strong coupling

- from analysis of $\sigma(e^+e^- \to \text{hadrons})$ data in $\lesssim$10 GeV region

CLEO, BES, MD-1

$$\alpha_s(9\text{ GeV}) = 0.182^{+0.022}_{-0.025} \to \alpha_s(M_Z) = 0.119^{+0.009}_{-0.011}$$

Kühn, Steinhauser, Teubner

$\delta\alpha_s$ dominated by experimental error

- from τ decays

$$R_{\tau,V+A} = \frac{\Gamma(\tau \to \text{hadrons} + \nu_\tau)}{\Gamma(\tau \to \ell + \bar{\nu}_\ell + \nu_\tau)} = 3|V_{ud}|^2 S_{EW}(1+\delta_{EW}+\delta_2+\delta_{NP}+\delta_0)$$

$$1 + \delta_0 = 2\int_0^{M_\tau^2} \frac{ds}{M_\tau^2}\left(1 - \frac{s}{M_\tau^2}\right)^2\left(1 + \frac{2s}{M_\tau^2}\right) R(s)$$

$$R_{\tau,V+A} = 3.471 \pm 0.011$$

Davier, Höcker, Zhang; ALEPH, OPAL, CLEO,...

$$\alpha_s(M_\tau) = 0.332 \pm 0.005_{exp} \pm 0.015_{theo}$$

Baikov, Chetyrkin, Kühn

$$\alpha_s(M_Z) = 0.1202 \pm 0.0006_{exp} \pm 0.0018_{theo} \pm 0.0003_{evol}$$

$\delta\alpha_s$ from τ dominated by theory

Summary

- Heavy quark current correlators can be used to perform a precise quark mass determination in combination with experimentally measured *R*-ratio and with lattice simulations
- Discussed extraction of charm- and bottom-quark masses from *R*-ratio and lattice simulations including NNNLO results in pQCD
- Discussed determinations of strong coupling from lattice simulations, $\sigma(e^+e^- \to \text{hadrons})$ and τ-decays including NNNLO results in pQCD

Backup slides

Charm moments

n	$\mathcal{M}_n^{\rm res}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\rm thresh}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\rm cont}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\rm exp}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\rm np}$ $\times 10^{(n-1)}$
1	0.1201(25)	0.0318(15)	0.0646(11)	0.2166(31)	−0.0001(2)
2	0.1176(25)	0.0178(8)	0.0144(3)	0.1497(27)	0.0000(0)
3	0.1169(26)	0.0101(5)	0.0042(1)	0.1312(27)	0.0007(14)
4	0.1177(27)	0.0058(3)	0.0014(0)	0.1249(27)	0.0027(54)

J.H. Kühn, M. Steinhauser, C.S.

Bottom moments

n	$\mathcal{M}_n^{\text{res},(1S-4S)}$ $\times 10^{(2n+1)}$	$\mathcal{M}_n^{\text{thresh}}$ $\times 10^{(2n+1)}$	$\mathcal{M}_n^{\text{cont}}$ $\times 10^{(2n+1)}$	$\mathcal{M}_n^{\text{exp}}$ $\times 10^{(2n+1)}$
1	1.394(23)	0.287(12)	2.911(18)	4.592(31)
2	1.459(23)	0.240(10)	1.173(11)	2.872(28)
3	1.538(24)	0.200(8)	0.624(7)	2.362(26)
4	1.630(25)	0.168(7)	0.372(5)	2.170(26)

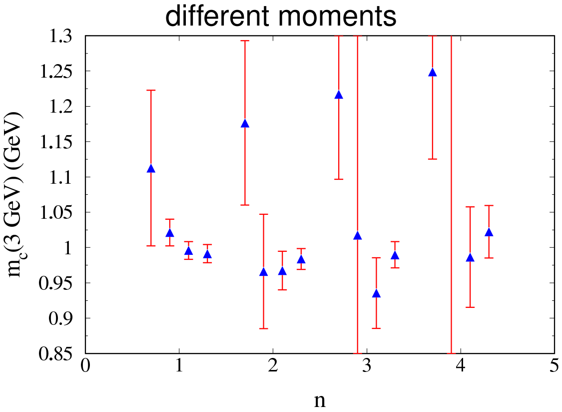
Chetyrkin, Kühn, Maierhöfer, Marquard, Steinhauser, C.S.

Perturbative expansion

$$
\begin{aligned}
m_c &= \frac{1}{2}\left(\frac{9Q_c^2}{4}\frac{\bar{C}_n^{\mathrm{Born}}}{\mathcal{M}_n^{\mathrm{exp}}}\right)^{\frac{1}{2n}}\left(1+r_n^{(1)}\alpha_s+r_n^{(2)}\alpha_s^2+r_n^{(3)}\alpha_s^3\right) \\
&\propto 1-\begin{pmatrix}0.328\\0.524\\0.618\\0.662\end{pmatrix}\alpha_s-\begin{pmatrix}0.306\\0.409\\0.510\\0.575\end{pmatrix}\alpha_s^2-\begin{pmatrix}0.262\\0.230\\0.299\\0.396\end{pmatrix}\alpha_s^3,
\end{aligned}
$$

error from next order $\leq r_n^{max}\alpha_s^4 <$ 2 to 3 permille
(smaller than μ-variation)

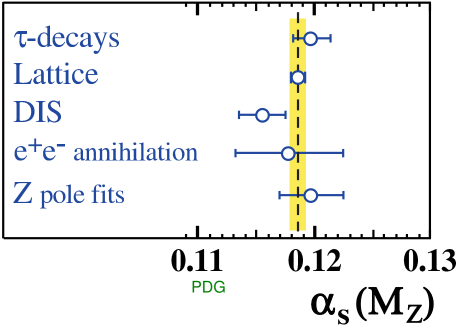
Charm quark masses



Chetyrkin, Kühn, Maierhöfer, Marquard, Steinhauser, C.S.

Strong coupling

PDG



PDG

$$\alpha_s(M_z) = 0.1185(6)$$