Measurement of the ISR-FSR interference in the $e^+e^- \to \mu^+\mu^-\gamma$ and $e^+e^- \to \pi^+\pi^-\gamma$ processes at BABAR

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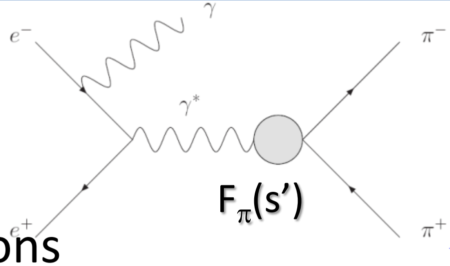
Outline

- Introduction
- Theoretical prediction/model and charge asymmetry
- Measurement of the charge asymmetry
- Results
- Summary

Introduction

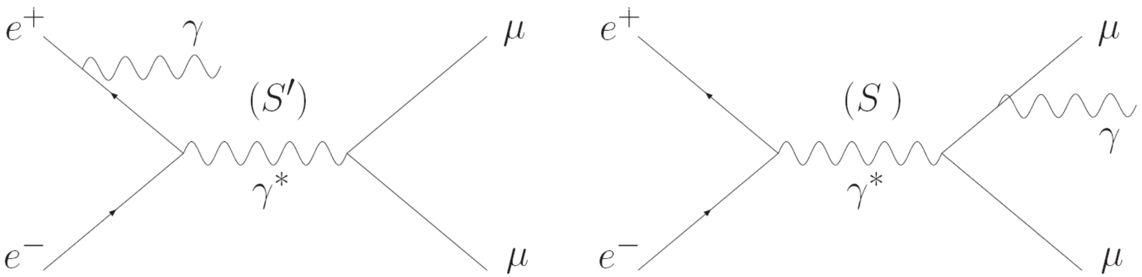
$e^+ e^- \to X \gamma$ (X: QED or hadronic final states)



- ISR method used to measure hadronic cross sections (e.g. $\pi^+\pi^-$)

  =>calculations of vacuum polarization through dispersion relations

- The radiation can be from initial state (ISR) or final state (FSR)



interference=0 for charge-symmetric detector

- $|FSR|^2$ contribution from theoretical predictions
  - LO QED for $\mu\mu\gamma_{FSR}$
  - model-dependent estimation for $\pi\pi\gamma_{FSR}$: negligible
- FSR for $\pi\pi\gamma$ small:
  - hard to do direct measurement, but interference more sensitive

Interference and charge asymmetry

- Cross section for $e^+e^- \to x^+x^-\gamma$ (x=μ or π)

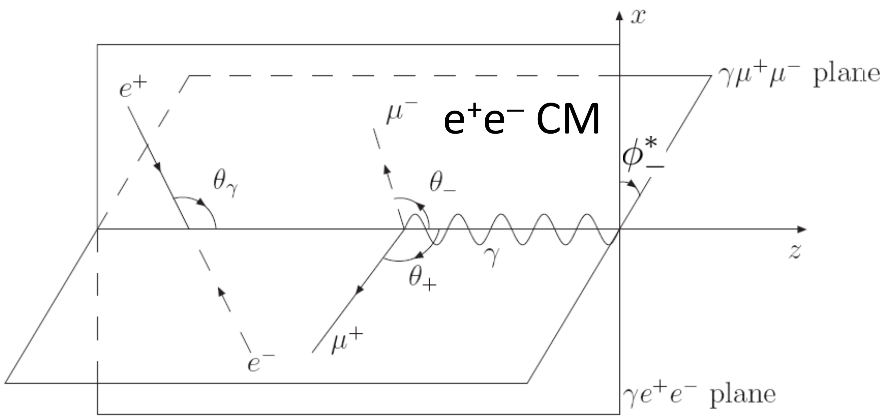
$$\sigma \propto |\mathcal{M}|^2 = |\mathcal{M}_{\rm ISR}|^2 + |\mathcal{M}_{\rm FSR}|^2 + \underline{2\mathcal{R}e(\mathcal{M}_{\rm ISR}\mathcal{M}^*_{\rm FSR})}$$

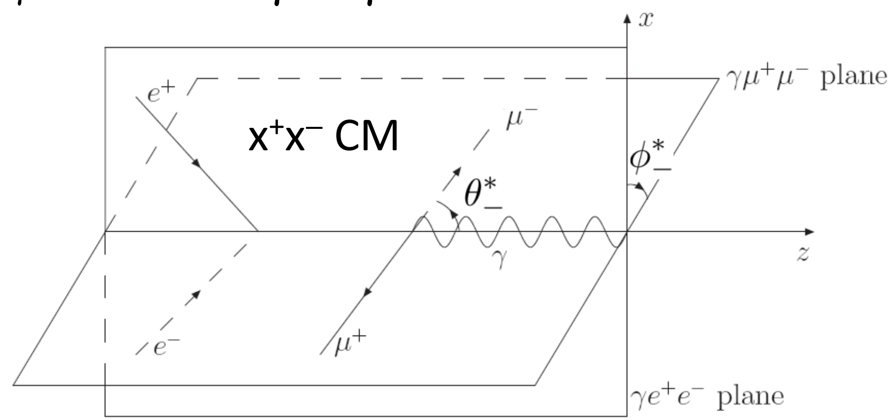
x^+x^- pair: C=–1 C=+1 sign change if $x^+ \leftrightarrow x^-$

- charge asymmetry

$$A = \frac{|\mathcal{M}|^2 - |\mathcal{M}_{x^+\leftrightarrow x^-}|^2}{|\mathcal{M}|^2 + |\mathcal{M}_{x^+\leftrightarrow x^-}|^2} = \frac{2\mathcal{R}e(\mathcal{M}_{\rm ISR}\mathcal{M}^*_{\rm FSR})}{|\mathcal{M}_{\rm ISR}|^2 + |\mathcal{M}_{\rm FSR}|^2}$$

- kinematic variables (leaving aside ϕ_γ): $\mathbf{m_{xx}}$(or $\mathbf{E_\gamma}$), $\boldsymbol{\theta_\gamma}$, $\boldsymbol{\phi^*}$ and $\boldsymbol{\theta^*}$



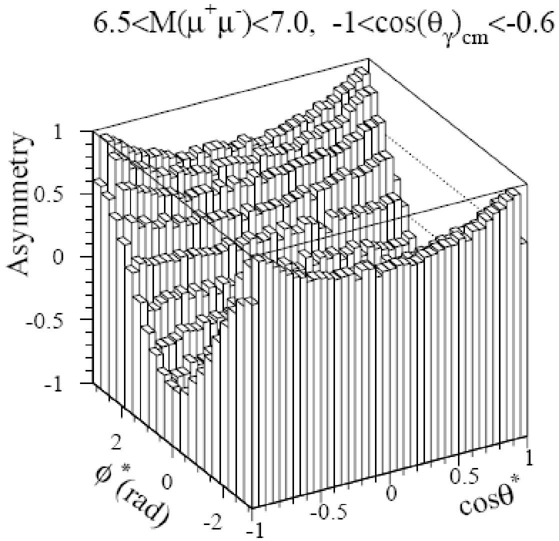


Predicted charge asymmetry for $\mu^+\mu^-\gamma$

- LO massless limit   (*R. Gastmans and T. T. Wu, 'The Ubiquitous Photon', Oxford (1990)*)

$$A_{e^+e^-\to\mu^+\mu^-\gamma}(m_{\mu\mu},\theta_\gamma,\theta^*,\phi^*) = -\frac{2\sqrt{s}\,m_{\mu\mu}\sin\theta_\gamma\sin\theta^*\boxed{\cos\phi^*}}{s\,\sin^2\theta^* + m_{\mu\mu}^2\sin^2\theta_\gamma}$$

- MC by AfkQED which uses LO ee→μμγ QED prediction
  (*A.B. Arbuzov et al., J. High Energy Phys. 9710, 001 (1997)*)
  additional ISR by structure function,
  additional FSR by PHOTOS
  ($20<\theta_\gamma<160$ degree)



$$A(\cos\theta^*,\phi^*) = \frac{\sigma_-(\cos\theta^*,\phi^*) - \sigma_+(\cos\theta^*,\phi^*)}{\sigma_-(\cos\theta^*,\phi^*) + \sigma_+(\cos\theta^*,\phi^*)}$$

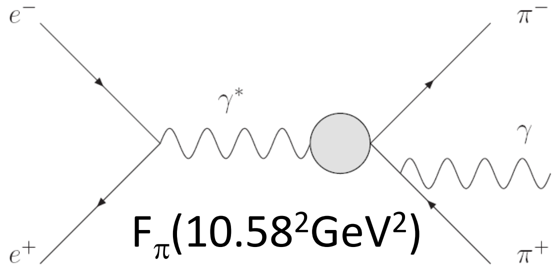
$$\sigma_\pm(\cos\theta^*,\phi^*) = \sigma(\cos\theta^*_\pm = \cos\theta^*, \phi^*_\pm = \phi^*)$$

- For a given mass interval, after an integration over θ* symmetrically and θ_γ the charge asymmetry shows a simple pattern:

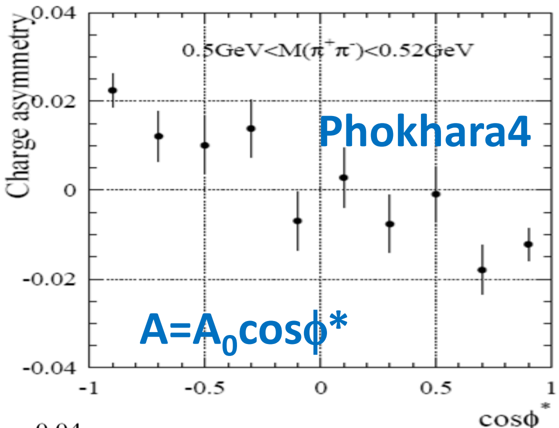
$$\boxed{A(\phi^*) = A_0\cos\phi^*} \qquad \phi^* \in [0,\pi)$$

FSR models for $e^+e^- \to \pi^+\pi^-\gamma$

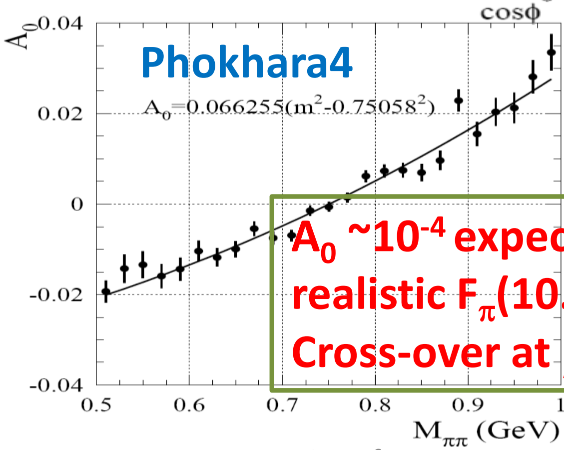
FSR model 1 (point-like pion)



$F_\pi(10.58^2\text{GeV}^2)$

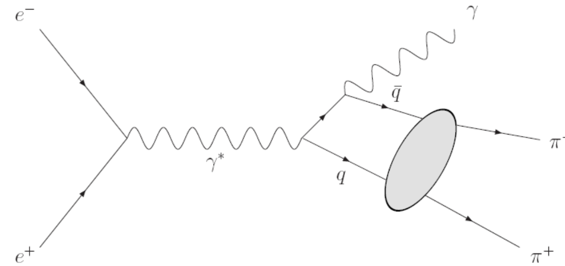


x100

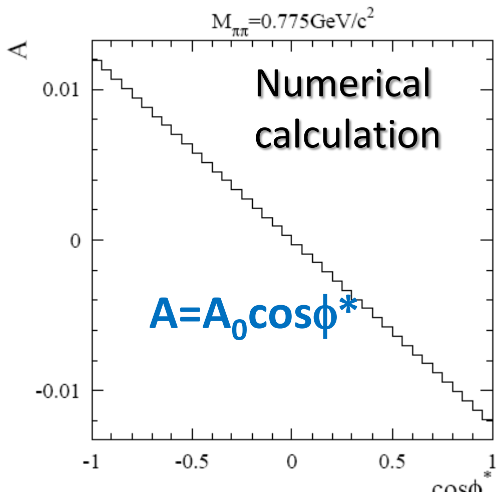


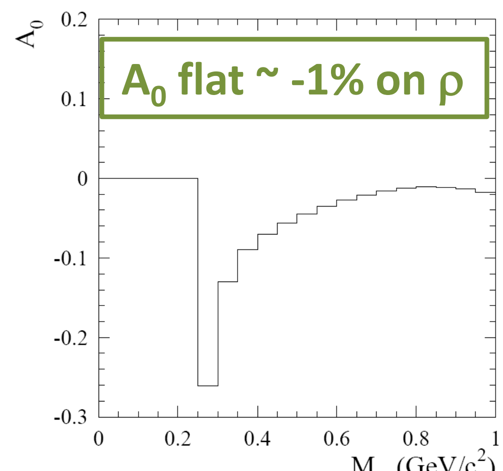
A_0 ~10^{-4} expected with realistic $F_\pi(10.58^2\text{GeV}^2)$
Cross-over at ρ mass

FSR model 2 (FSR from quarks)

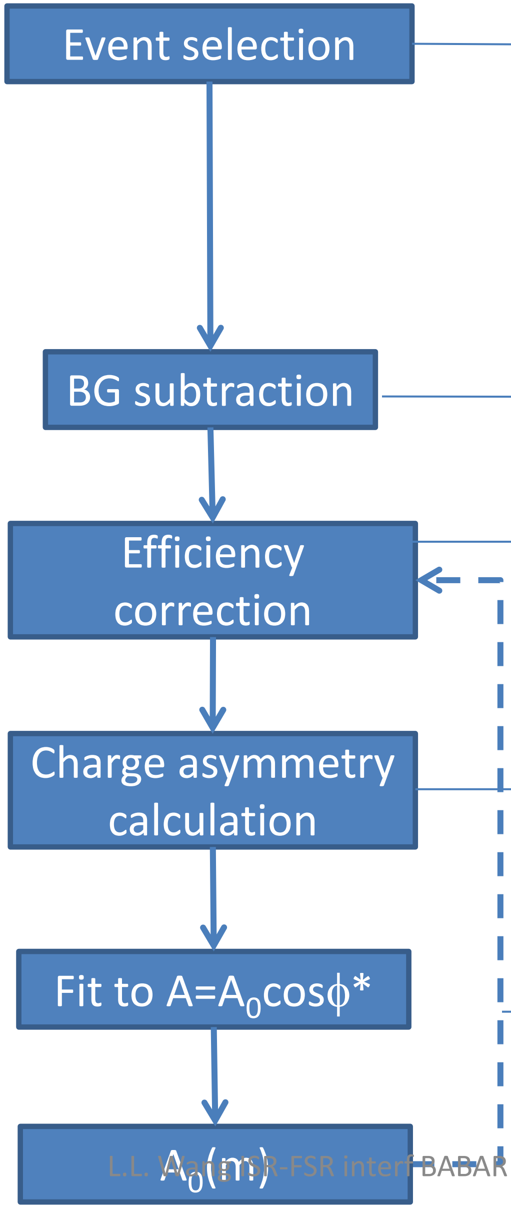


Differential cross sections:
Z. Lu and I. Schmidt, PRD 73, 094021 (2006);
Erratum, PRD 75, 099902(E) (2007)
C-even part 2-pion state described by amplitudes $\Phi^+_{u,d}$:
M. Diehl, T. Gousset and B. Pire, PRD 62, 073014 (2000)
S-wave $f_0(600)$ + D-wave $f_2(1270)$





Measurement of charge asymmetry



$N_{\pm}^{\rm obs}(\cos\phi^*)$ 232fb^{-1} data @ 10.58GeV at BABAR

Trigger, radiative Bhabha veto, ISR photon (>3GeV), 2 good charged tracks (>1GeV/c), mu-ID or pi-ID, active area of detectors, kinematic fit χ^2<15

$$N_{\pm}(\cos\phi^*) = N_{\pm}^{\rm obs}(\cos\phi^*) - N_{\pm}^{\rm BG}(\cos\phi^*)$$ (MC)

$\epsilon_{\pm}(\cos\phi^*)$ Full simulation (physical A dependent) + data/MC difference correction (trigger, tracking, PID ***PRD 86, 032013 (2012)***)

$$A(\cos\phi^*) = \frac{N_-(\cos\phi^*)/\epsilon_-(\cos\phi^*) - N_+(\cos\phi^*)/\epsilon_+(\cos\phi^*)}{N_-(\cos\phi^*)/\epsilon_-(\cos\phi^*) + N_+(\cos\phi^*)/\epsilon_+(\cos\phi^*)}$$

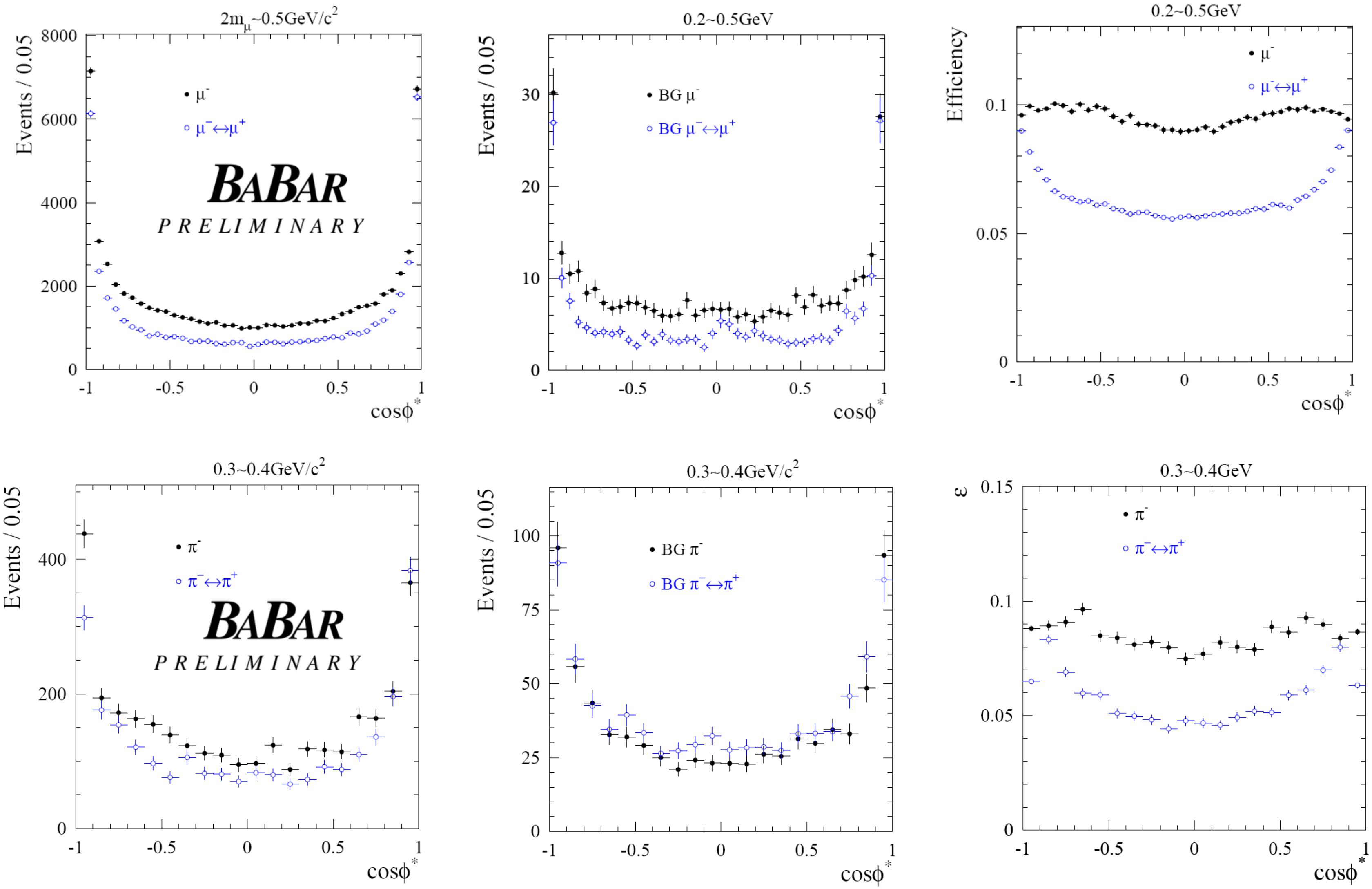
iterative MC reweighting

cosϕ* distributions

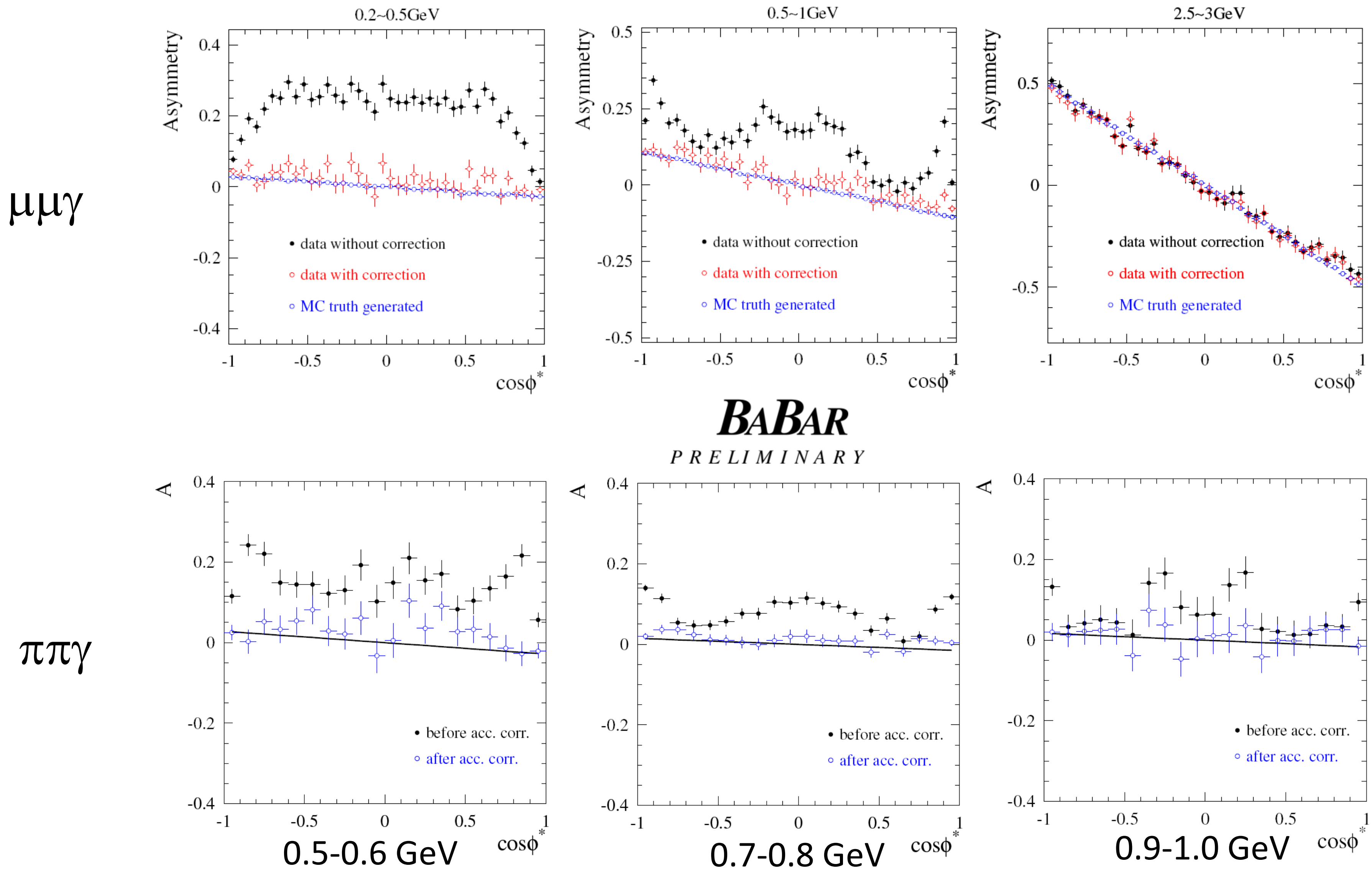
$N^{\mathrm{obs}}_{\pm}(\cos\phi^*)$ | $N^{\mathrm{BG}}_{\pm}(\cos\phi^*)$ | $\epsilon_{\pm}(\cos\phi^*)$

μμγ

ππγ

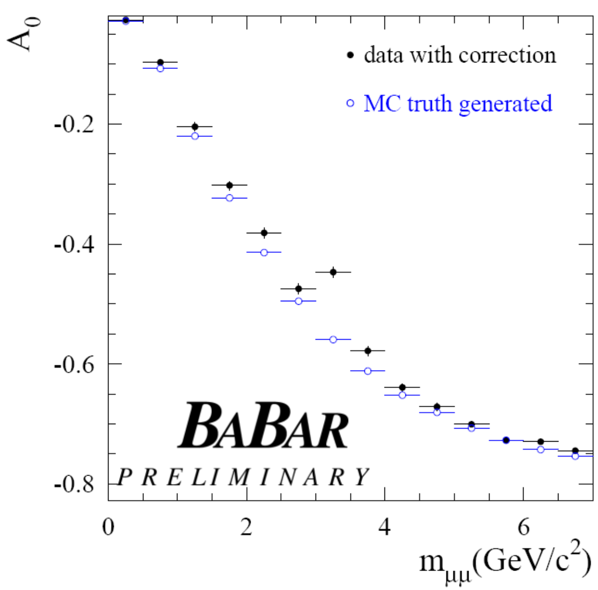


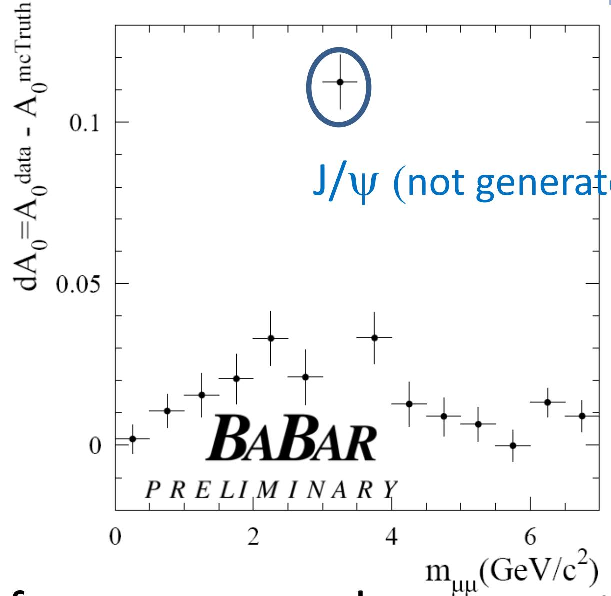
Measured asymmetry



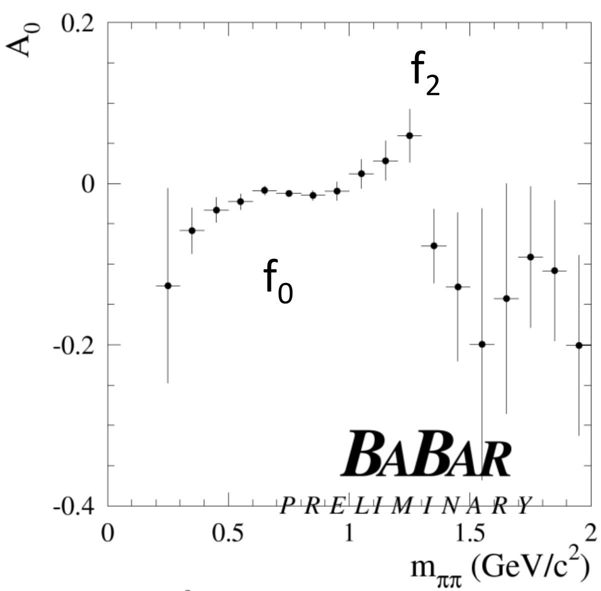
Slope of the charge asymmetry A_0

$\mu\mu\gamma$





$\pi\pi\gamma$



- for $\mu\mu\gamma$: general agreement with the large LO QED asymmetry
- a significant deviation (2-3%) in 2-4 GeV region
- exp syst uncertainty <1%
- test on J/ψ (pure ISR): $A_0=(1.3\pm1.6)\%$
- NLO effect ?

- for $\pi\pi\gamma$: qualitative agreement with FSR model 2 in the ρ region

Revised FSR model 2 for $e^+e^-\to\pi^+\pi^-\gamma$

$$\Phi_u^+(z, m_{\pi\pi}^2, \cos\theta^*) = \Phi_d^+(z, m_{\pi\pi}^2, \cos\theta^*)$$

Diehl et al. PRD 62, 073014 (2000)

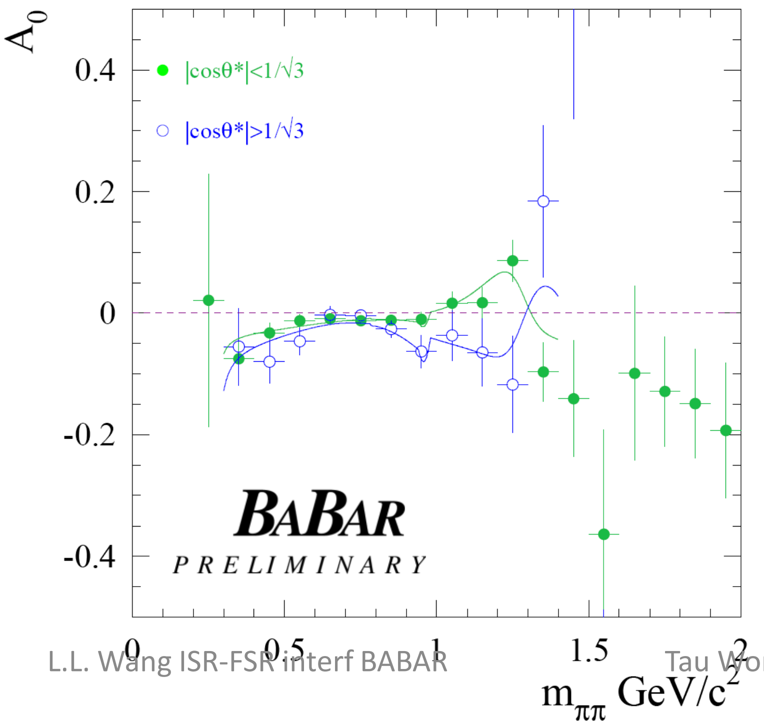
$$= 10z(1-z)(2z-1)R_\pi\left[-\frac{3-\beta^2}{2}e^{i\delta_0(m_{\pi\pi}^2)} + \beta^2 e^{i\delta_2(m_{\pi\pi}^2)}P_2(\cos\theta^*)\right]$$

S-wave ('f_0' phase shift) D-wave (f_2 helicity 0)

- Fit with our revised model:

$$10z(1-z)(2z-1)\left[c_0\frac{3-\beta^2}{2}e^{i\delta_0(m_{\pi\pi}^2)} + c_2\beta^2 \mathrm{BW}_{f_2}(m_{\pi\pi})P_2(\cos\theta^*)\right]$$

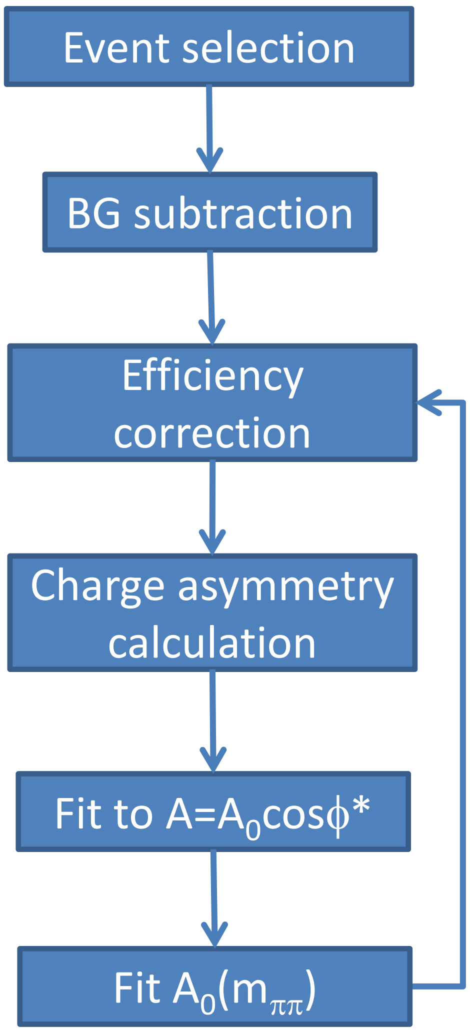
- $P_2(\cos\theta^*) = (3\cos^2\theta^* - 1)/2$ => on f_2: charge asymmetry changes sign at $|\cos\theta^*| = 1/\sqrt{3}$ (≈ 0.58)



- Analyses are done below and above cosθ*=1/sqrt(3) separately (standard selection for >1/sqrt(3), modified selection for <1/sqrt(3), BG sub, efficiency correction)
- **charge asymmetry has opposite sign around f_2**
- Fit A_0 between 0.3 and 1.4GeV with the revised FSR model

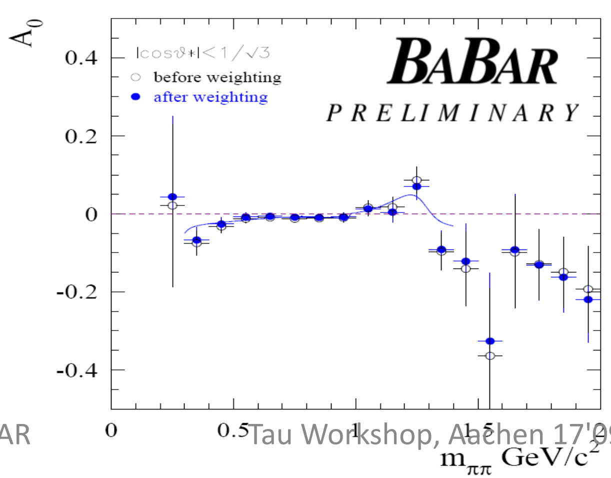
	|cosθ*|<1/sqrt(3)	|cosθ*|>1/sqrt(3)
c_0	-1.13 ± 0.24	-1.59 ± 0.36
c_2	-5.4 ± 1.8	-5.4 ± 3.1

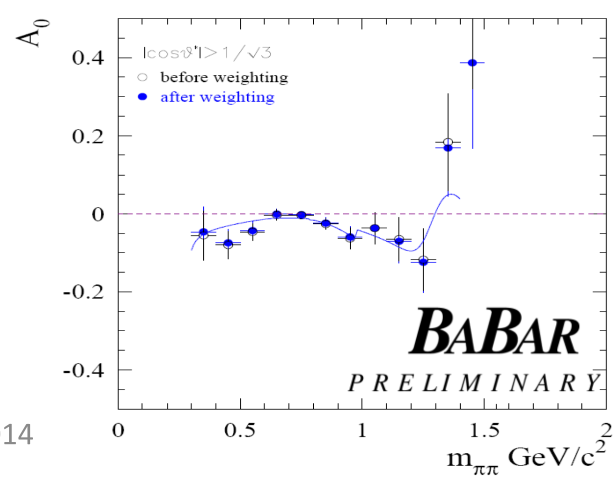
Monte Carlo reweighting and iterations



- Measured A_0 could be different from value in MC: extreme case for ee→$\pi\pi\gamma$ where AfkQed has null A_0
  => iterations to recompute efficiencies
- Reweight MC event by event with
  $$w=(ISR^2+FSR^2+\text{interference})/ISR^2$$
- Fit results stable after 2 iterations

iteration	c_0	c_2
0	-1.27	-5.4
1	-0.866	-4.31
2	-0.959	-4.54
3	-0.93 ± 0.20	-4.48 ± 1.56





Systematic uncertainties

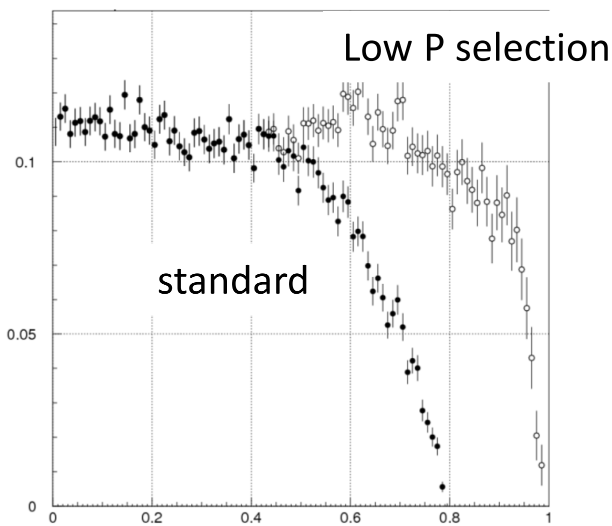
- **Background subtraction** (20% precision for the BG estimation)
  $\sim 10^{-4}$ around f_0 mass, $\sim 10^{-3}$ around f_2 mass
- **Data/MC difference**
  - trigger, tracking and π-ID
  - all the corrections are studied with data with 10% precision

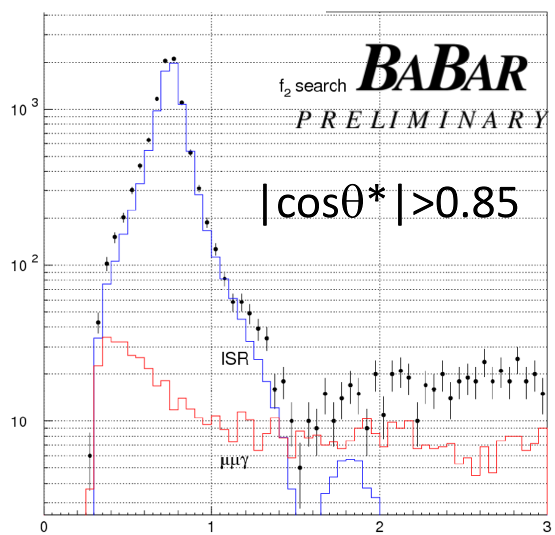
  => $<10^{-4}$ negligible !
- **Physical charge asymmetry**
  - null charge asymmetry in MC
  - result obtained with iteration method
  - difference between the last two iterations is taken as systematic uncertainty (2.7% for c_0, 1.3% for c_2)

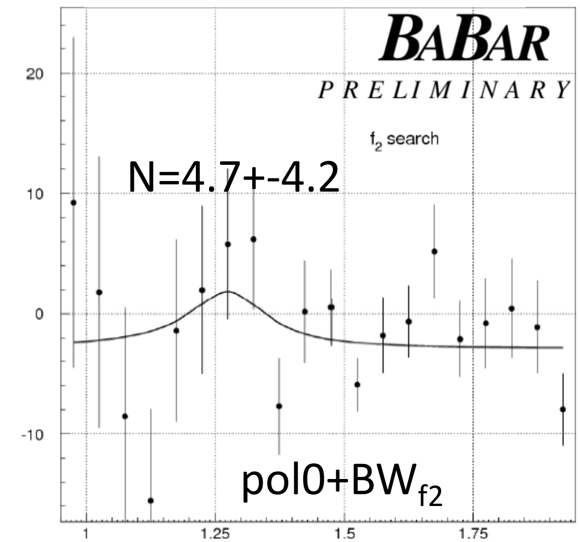
Sum of all systematic uncertainties small << statistical errors

Direct search for $ee \to f_2(\pi\pi)\gamma_{FSR}$

- ➢Relaxed event selection (also for $A(|\cos\theta^*|>1/\mathrm{sqrt}(3))$)
  - for high-momentum track: harder π-ID, Ecal/p<0.6,
  - for low-momentum track: **no 1GeV cut**, no PID except dE/dx cut against e
  - $|\cos\theta^*|>0.85$ as $P_2(\cos\theta^*)$ peaks at $|\cos\theta^*|\sim 1$ vs $ISR^2 \sim \sin^2\theta^*$



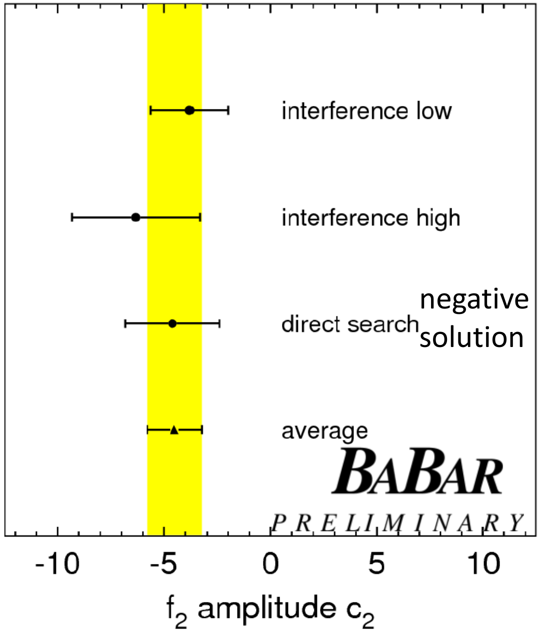




After efficiency correction => $|f_2|^2/(|ISR|^2+|f_2|^2) = 0.22 \pm 0.15$ for $0.8<|\cos\theta^*|<1$

=> $|c_2| = 4.6 \pm 2.2$

Results for ee→$f_{0,2}(\pi\pi)\gamma_{FSR}$



- 3 independent consistent c_2 measurements
- average $|c_2| = 4.5 \pm 1.3$
- 3.6σ significance for $e^+e^- \to f_2\,\gamma_{FSR}$

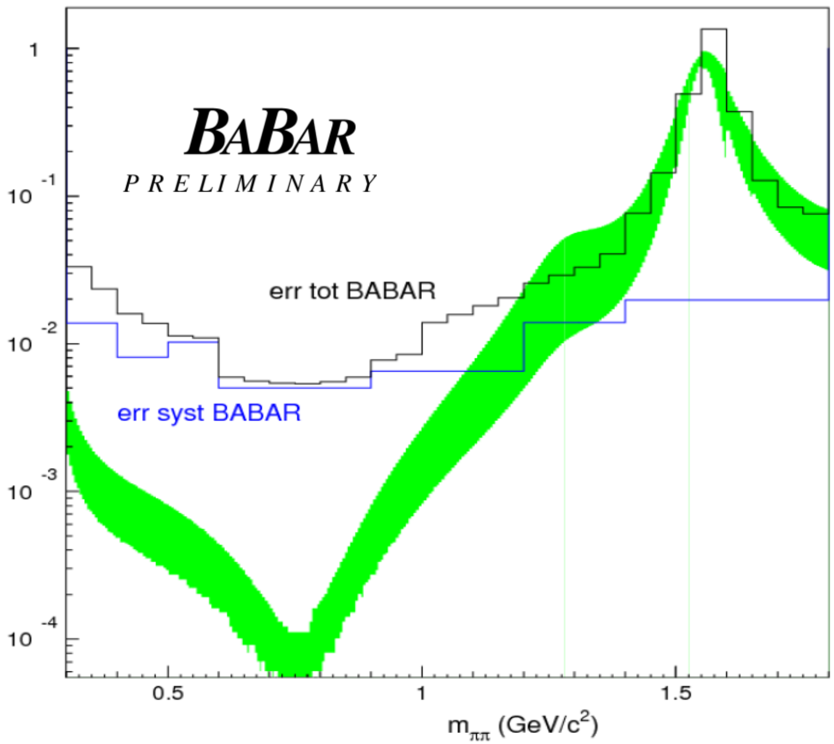
BABAR PRELIMINARY	c_0	c_2
This work	**-0.93±0.20**	**-4.5±1.3**
Diehl et al. 2000	-0.5± 0.5	+0.5± 0.5
Chernyak (private com .)	–	|c2|=2.2± 1.1

⇒ S wave consistent with Diehl et al., but not the D wave (sign and magnitude)

⇒ f_2 consistent with Chernyak in magnitude

Consequence for $\sigma(e^+e^- \to \pi^+\pi^-)$ and a_μ

$FSR^2 / (ISR^2+FSR^2)$



- **Relative contribution of FSR** estimated with the FSR model 2 using our measured f_0 and f_2 parameters
- FSR contribution negligible in ρ mass region
- comparable with the total error of the cross section measurement above 1.2GeV
- Measured F_π between 1.5 and 1.6 GeV very small ~0

	$\pi\pi$ ($2m_\pi$~1.8GeV)	$\pi\pi$ FSR($2m_\pi$~1.8GeV)
a_μ^{had} (10^{-10})	$514.09 \pm 2.22 \pm 3.11$	0.26 ± 0.12

negligible effect

Summary and Conclusions

- ISR-FSR interference in $e^+e^- \to \mu^+\mu^-\gamma$ and $e^+e^- \to \pi^+\pi^-\gamma$ has been studied at 10.6 GeV through measurements of the charge asymmetry $\Rightarrow$ FSR contribution

- Simple pattern $A=A_0\cos\phi^*$ (ϕ^* angle between decay plane and eeγ plane)

- With LO radiation assumption, A_0 measured as a function of $\mu\mu/\pi\pi$ mass

- For $e^+e^- \to \mu^+\mu^-\gamma$, the results are consistent generally with LO QED (large A_0) 2~3% deviation in the 2-4 GeV region beyond experimental syst. errors

- For $e^+e^- \to \pi^+\pi^-\gamma$, the measurement with dominant statistical error shows a charge asymmetry $A_0 \sim -1\%$ around ρ mass and interesting pattern around f_2 mass, which are fitted to a model of FSR from quarks

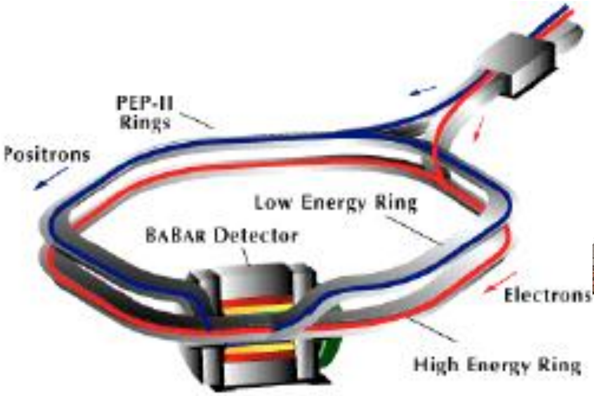
- From this measurement the $|FSR|^2$ contribution is negligible in the cross section measurement (except in f_2 region) and in the $a_\mu^{\pi\pi}$ calculation

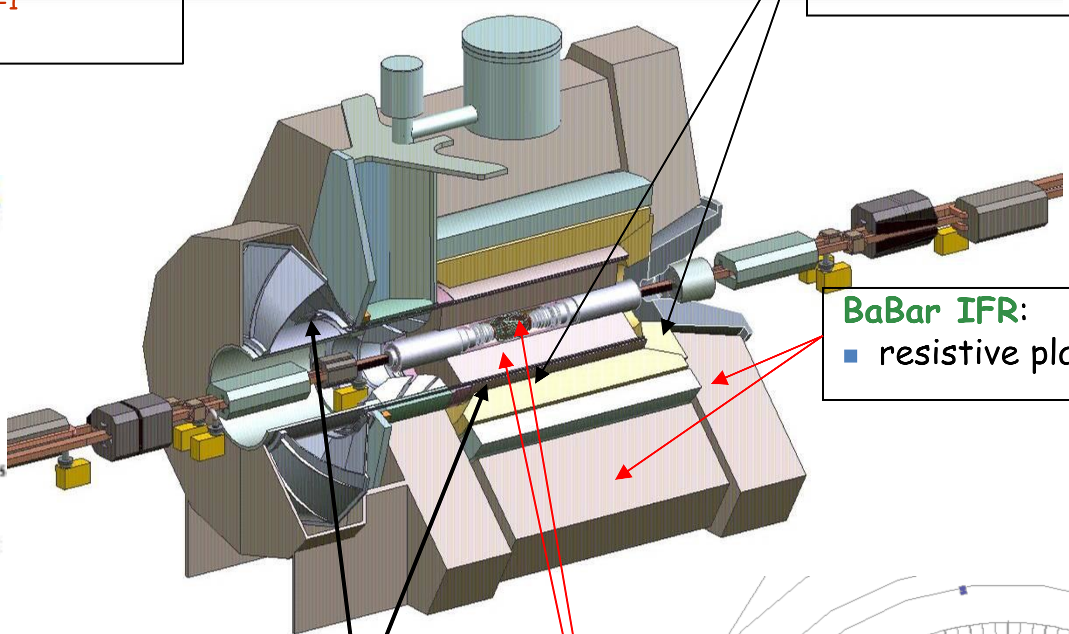
Thanks

backups

BaBar at PEP II

- **PEP-II** is an asymmetric e^+e^- collider operating at CM energy of $\Upsilon(4S)$.
- Integrated luminosity = 531 fb^{-1}



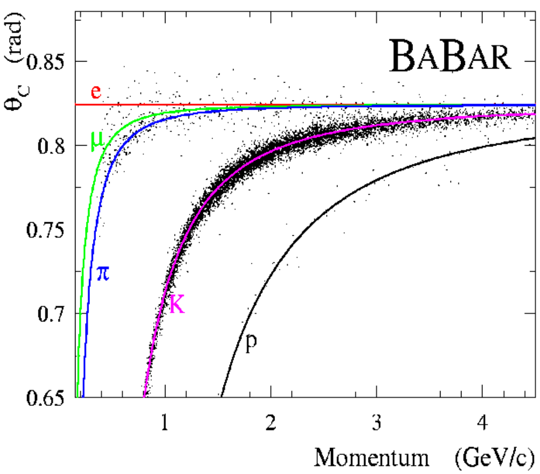


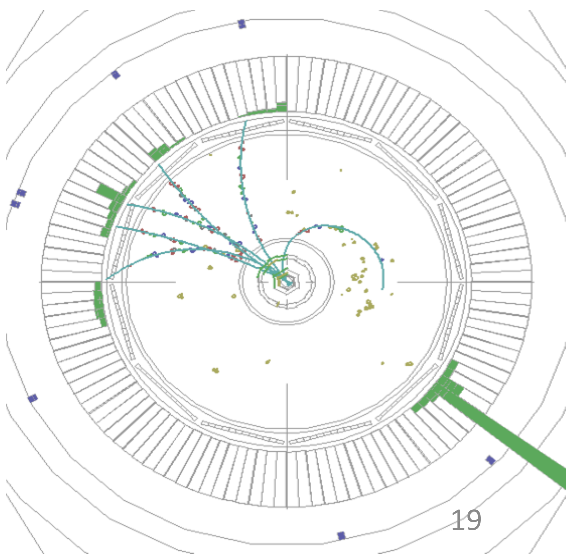
BaBar EMC:
- 6580 CsI(Tl) crystals, resolution ~1-2 % high E.

BaBar IFR:
- resistive plate chambers

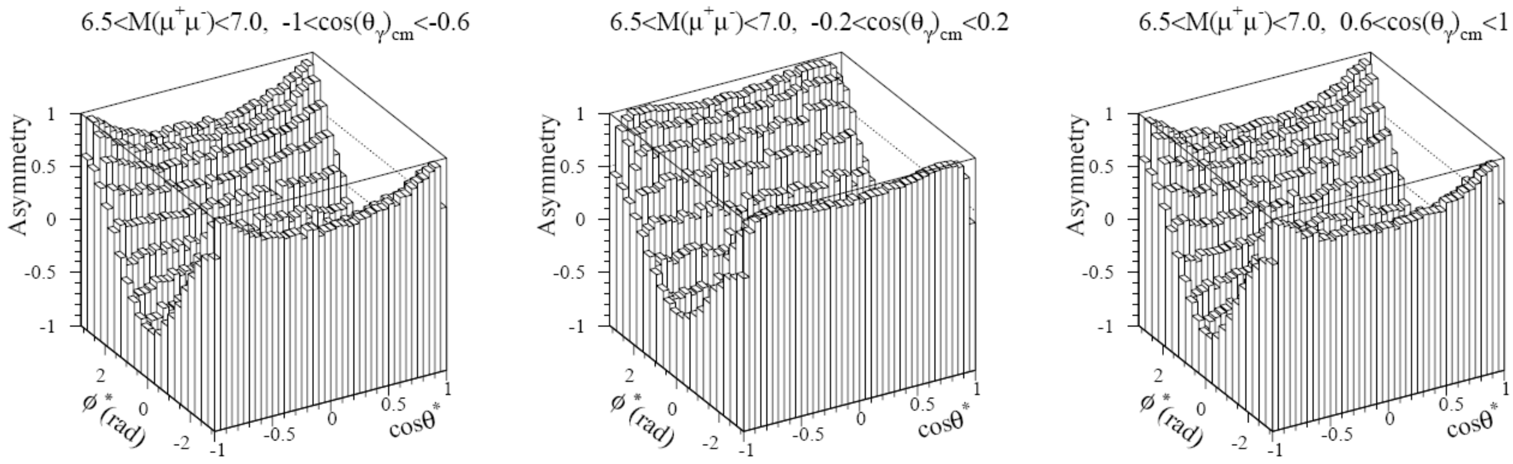
BaBar DIRC
- particle ID up to 4-5 GeV/c

BaBar SVT and DCH
- precision tracking





ϕ* vs cosθ* (μμγ MC)



QED prediction

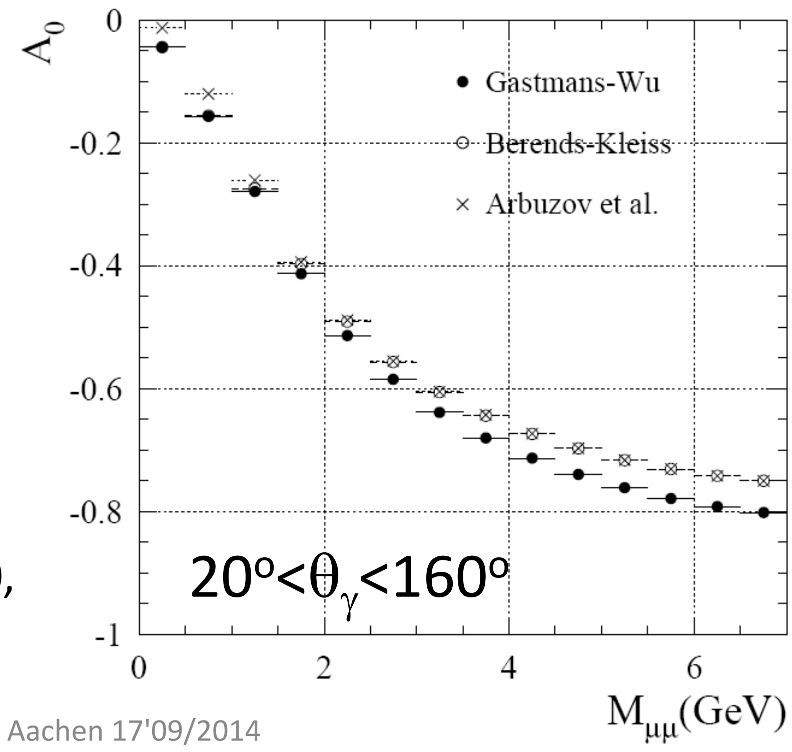
- LO massless limit:

(R. Gastmans and T. T. Wu, ‘The Ubiquitous Photon’, Oxford (1990))

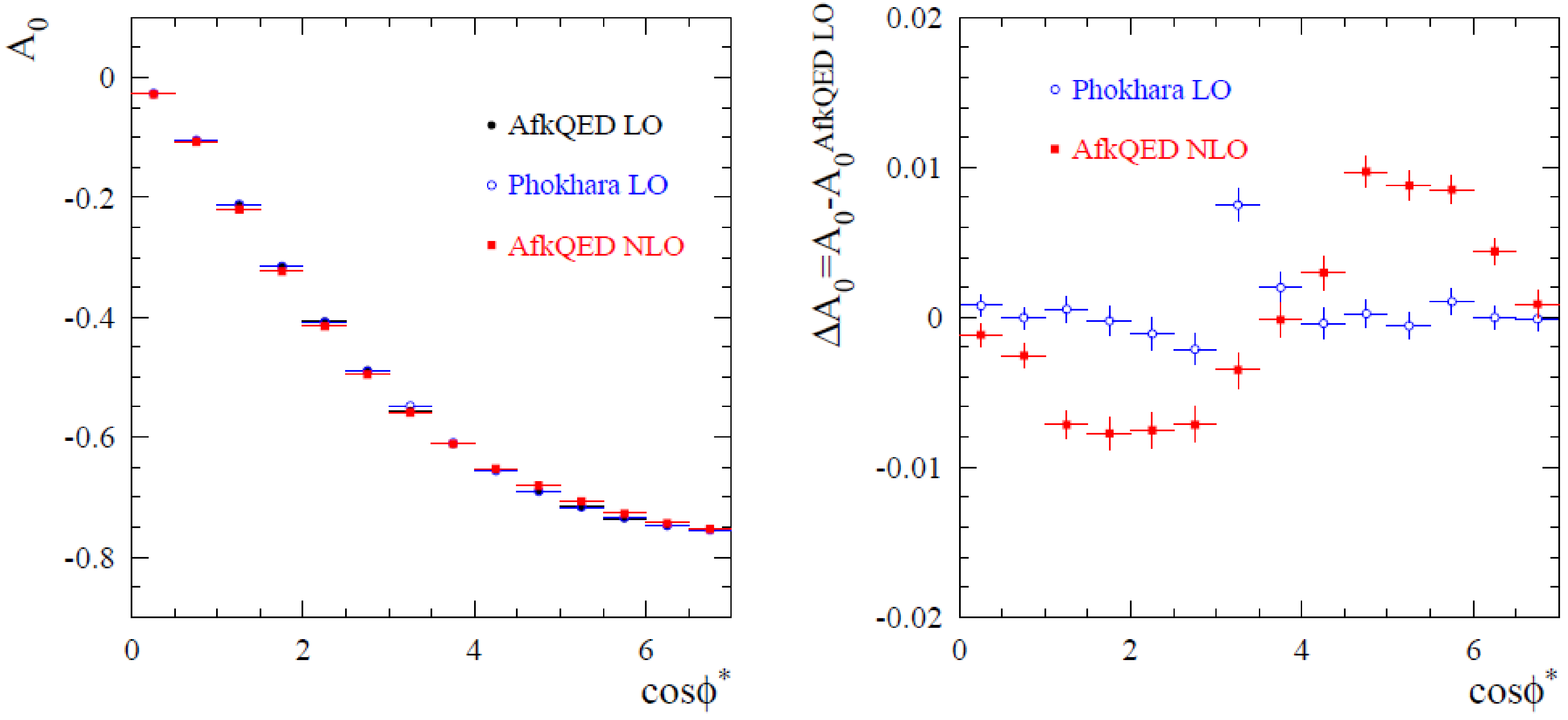
$$A_{e^+e^-\to\mu^+\mu^-\gamma}(m_{\mu\mu},\theta_\gamma,\theta^*,\phi^*) = -\frac{2\sqrt{s}\,m_{\mu\mu}\sin\theta_\gamma\sin\theta^*\cos\phi^*}{s\,\sin^2\theta^* + m_{\mu\mu}^2\sin^2\theta_\gamma}$$

- LO massive case:

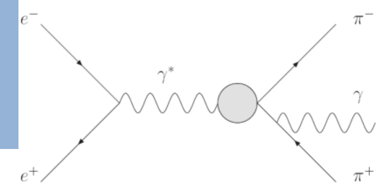
Numerical integration =>



F.A. Berends and R. Kleiss, Nucl. Phys. B 177, 237 (1981).
R. B. Arbuzov et al., J. High Energy Phys. 9710, 001 (1997).

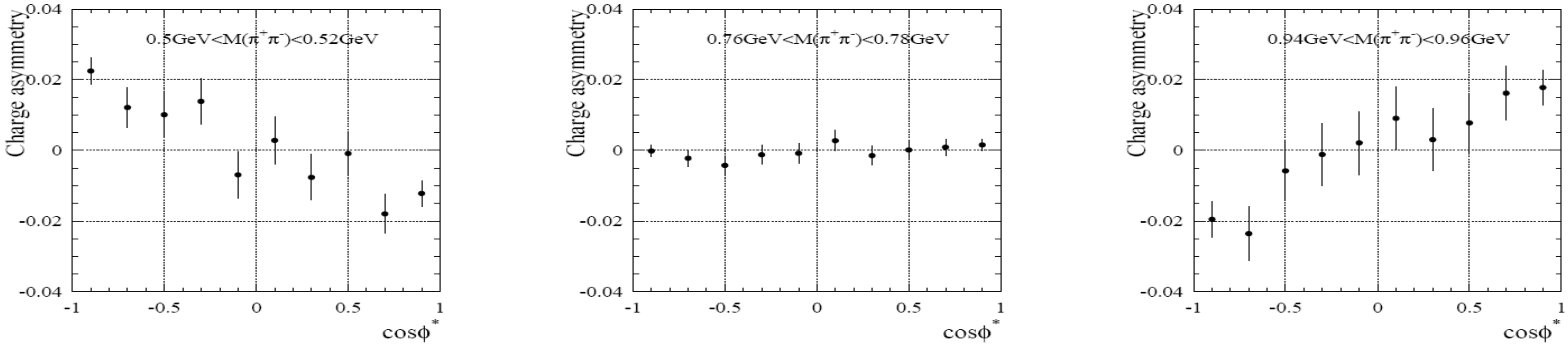

A_0
0
-0.2
-0.4
-0.6
-0.8
AfkQED LO
Phokhara LO
AfkQED NLO
0
2
4
6
$\cos\phi^*$
$\Delta A_0 = A_0 - A_0^{AfkQED\ LO}$
0.02
0.01
0
-0.01
-0.02
Phokhara LO
AfkQED NLO
0
2
4
6
$\cos\phi^*$

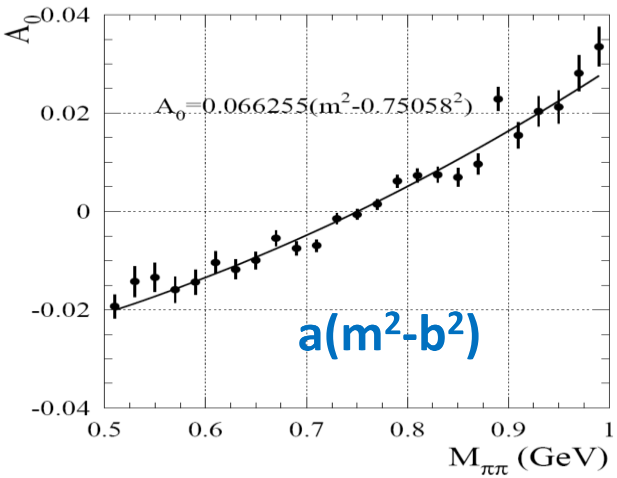
FSR model 1 for ee→$\pi\pi\gamma$



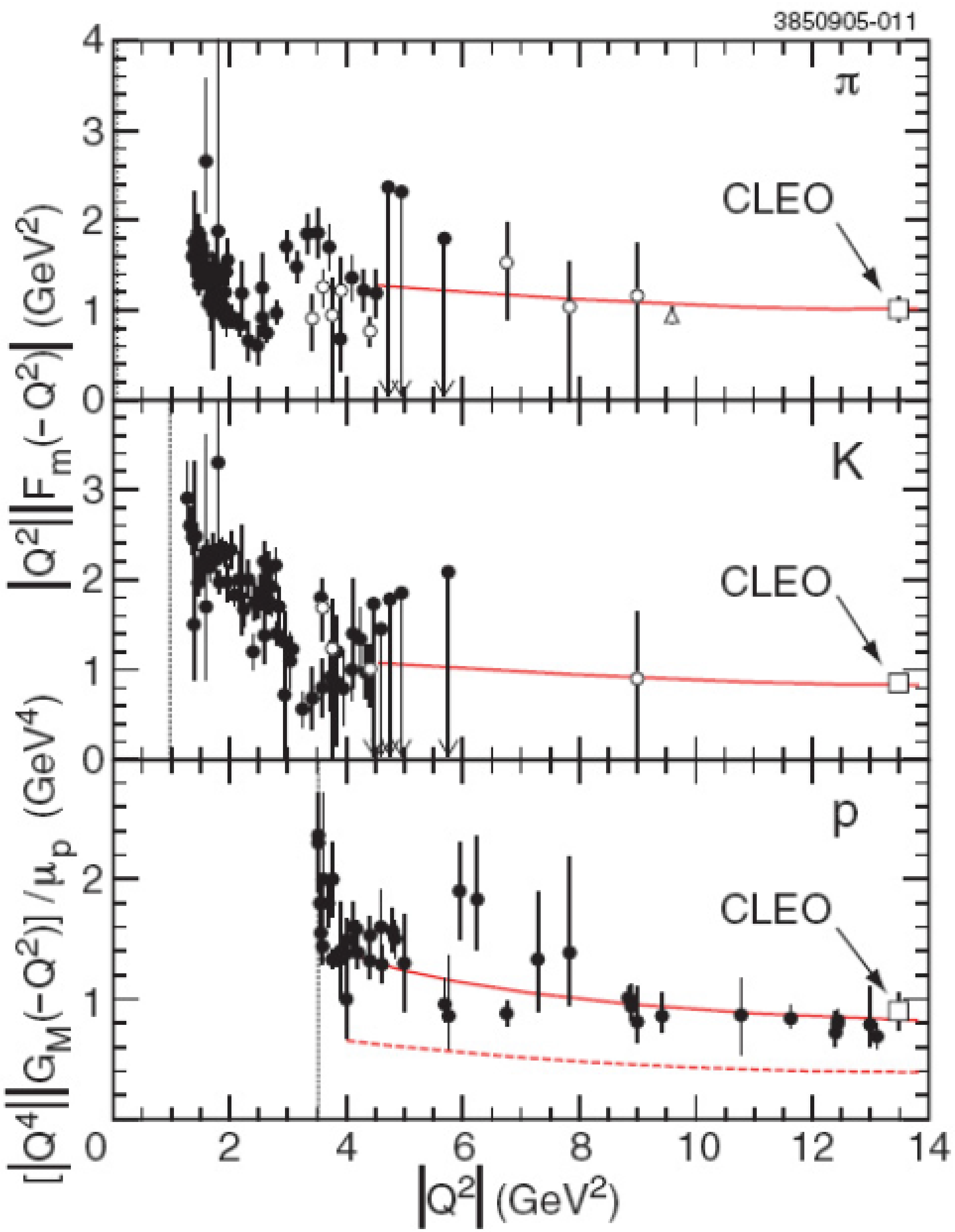
M_{ISR}~$F_\pi(s')$~$BW_\rho(s')$
M_{FSR}~$F_\pi(s=10.58^2 GeV^2)$~-0.00336+0.00042i (**Phokhara4**) *100 to study the feature

➡ $A(s')=2Re(M_{ISR}M^*_{FSR})/(|M_{ISR}|^2+|M_{FSR}|^2)$~$BW_\rho(s')/|BW_\rho(s')|^2$~$(s'-m_\rho^2)$=**($m^2_{\pi\pi}-m^2_\rho$)**





- Pattern **$A=A_0\cos\phi^*$**
- A_0 ~ $(m^2_{\pi\pi}-m^2_\rho)$
- $F_\pi(10.58^2 GeV^2)$ in simulation is 30 times larger than the estimation from CLEO and BaBar data => A_0 ~10^{-4} expected (too small to be measured)


3850905-011
π
CLEO
K
CLEO
p
CLEO
$|Q^2||F_m(-Q^2)|$ (GeV^2)
$[|Q^4||G_M(-Q^2)|]/\mu_p$ (GeV^4)
$|Q^2|$ (GeV^2)
0
1
2
3
4
0
1
2
3
0
1
2
0
2
4
6
8
10
12
14

FSR model 2 for ee→$\pi\pi\gamma$

Z. Lu and I. Schmidt, Phys. Rev. D **73**, 094021 (2006); Erratum, Phy. Rev. D **75**, 099902(E) (2007)

$$\frac{d\sigma^{\rm ISR}_{e^+e^-\to\pi^+\pi^-\gamma}}{dm^2_{\pi\pi}d(\cos\theta_\gamma)d(\cos\theta^*)d\phi^*} = \frac{\alpha^3\beta^3}{16\pi s^2 m^2_{\pi\pi}(s-m^2_{\pi\pi})}|F_\pi(m^2_{\pi\pi})|^2$$

→ From BaBar measurement

$$\times\left\{(s^2+m^4_{\pi\pi})\frac{1+\cos^2\theta_\gamma}{\sin^2\theta_\gamma}\sin^2\theta^* + 4sm^2_{\pi\pi}\cos^2\theta^* + 2\sqrt{s}m_{\pi\pi}(s+m^2_{\pi\pi})(\tan\theta_\gamma)^{-1}\sin 2\theta^*\cos\phi^* - 2sm^2_{\pi\pi}\sin^2\theta^*\cos 2\phi^*\right\}$$

$$\frac{d\sigma^{\rm FSR}_{e^+e^-\to\pi^+\pi^-\gamma}}{dm^2_{\pi\pi}d(\cos\theta_\gamma)d(\cos\theta^*)d\phi^*} = \frac{\alpha^3\beta(s-m^2_{\pi\pi})}{64\pi s^3}(1+\cos^2\theta_\gamma)|V(m^2_{\pi\pi},\theta^*)|^2$$

$$\frac{d\sigma^{I}_{e^+e^-\to\pi^+\pi^-\gamma}}{dm^2_{\pi\pi}d(\cos\theta_\gamma)d(\cos\theta^*)d\phi^*} = \frac{\alpha^3\beta^2}{16\pi s^2\sqrt{s}m_{\pi\pi}}\mathcal{R}e\{F^*_\pi(m^2_{\pi\pi})V(m^2_{\pi\pi},\theta^*)\}$$

$$\times\left\{-\sqrt{s}m_{\pi\pi}\cos\theta_\gamma\cos\theta^* + [(1+\cos^2\theta_\gamma)s + m^2_{\pi\pi}\sin^2\theta_\gamma]\frac{\sin\theta^*\cos\phi^*}{2\sin\theta_\gamma}\right\}$$

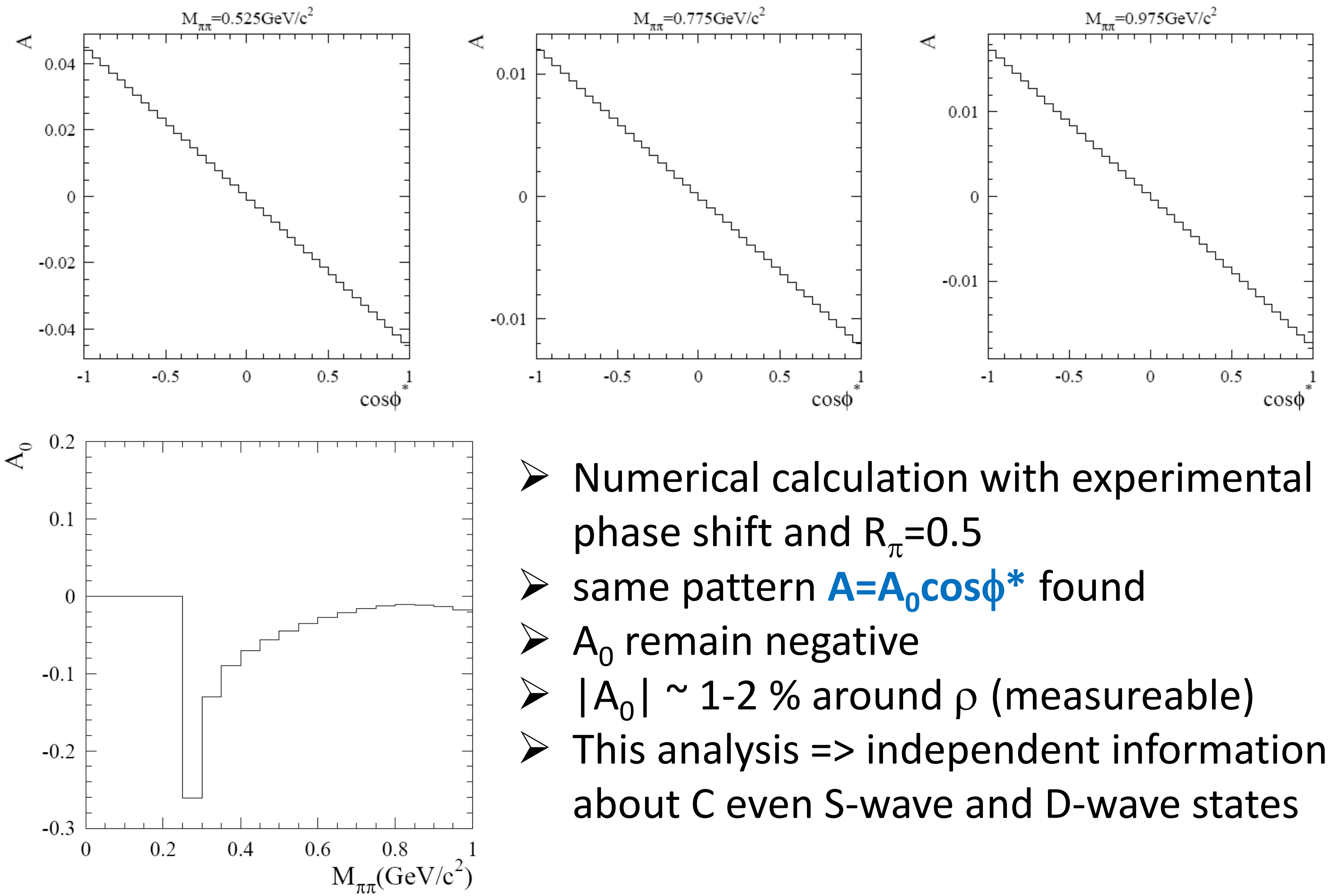
$$V = \sum_q e_q^2 V_q = \sum_q e_q^2\int_0^1 dz\frac{2z-1}{z(1-z)}\Phi_q^+(z,m^2_{\pi\pi},\cos\theta^*)\quad (q=u,d)$$

C-even part of 2-pion generalized distribution amplitudes (GDA):

$$\Phi_u^+(z,m^2_{\pi\pi},\cos\theta^*) = \Phi_d^+(z,m^2_{\pi\pi},\cos\theta^*)$$
$$= 10z(1-z)(2z-1)R_\pi\left[-\frac{3-\beta^2}{2}e^{i\delta_0(m^2_{\pi\pi})} + \beta^2 e^{i\delta_2(m^2_{\pi\pi})}P_2(\cos\theta^*)\right]$$

~0.5 S-wave (f_0) D-wave (f_2) Helicity 0 dominant

FSR model 2 for ee→$\pi\pi\gamma$ (cont.)

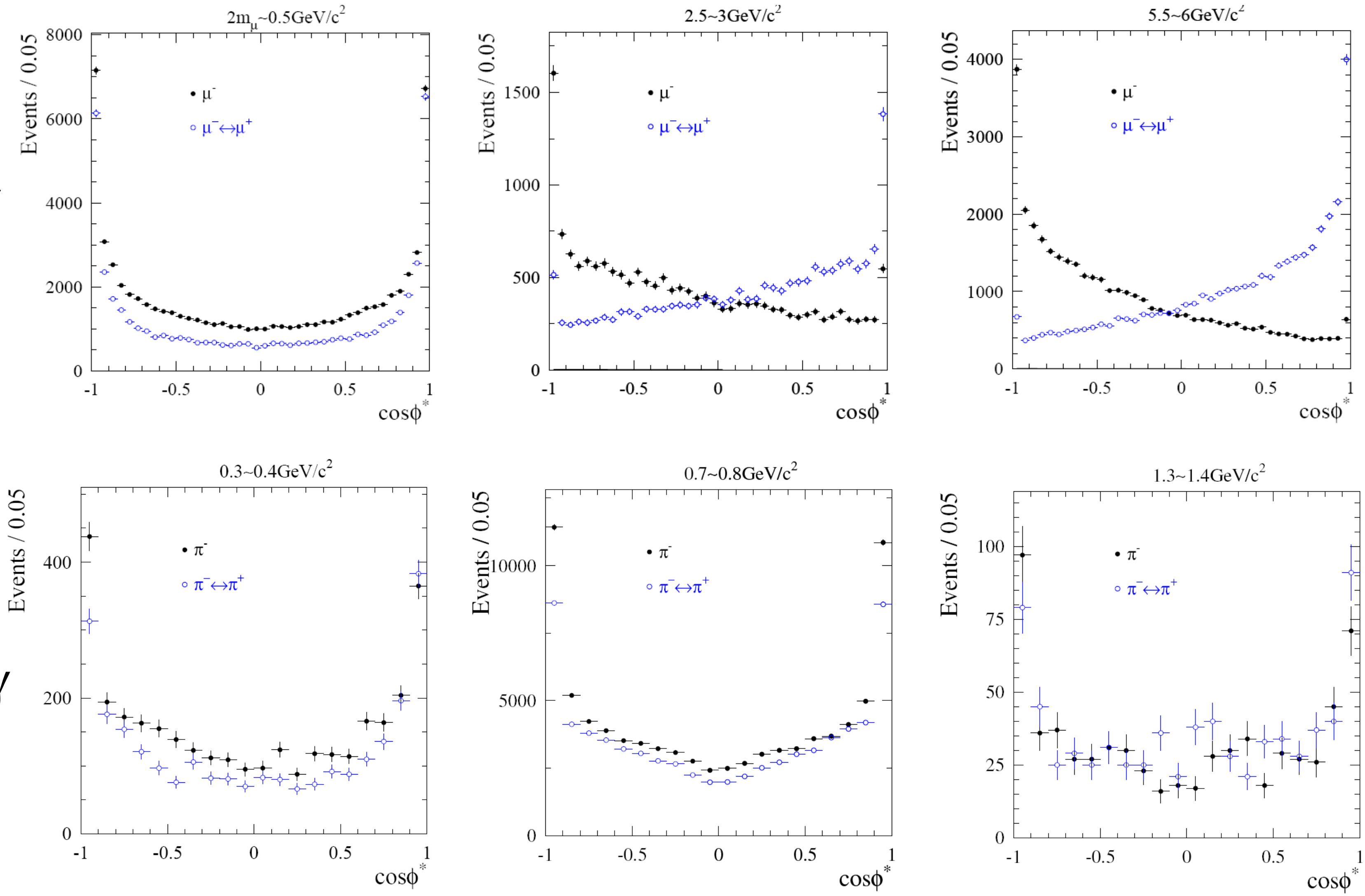


- Numerical calculation with experimental phase shift and R_π=0.5
- same pattern **$A=A_0\cos\phi^*$** found
- A_0 remain negative
- $|A_0|$ ~ 1-2 % around ρ (measureable)
- This analysis => independent information about C even S-wave and D-wave states

cosϕ* distributions

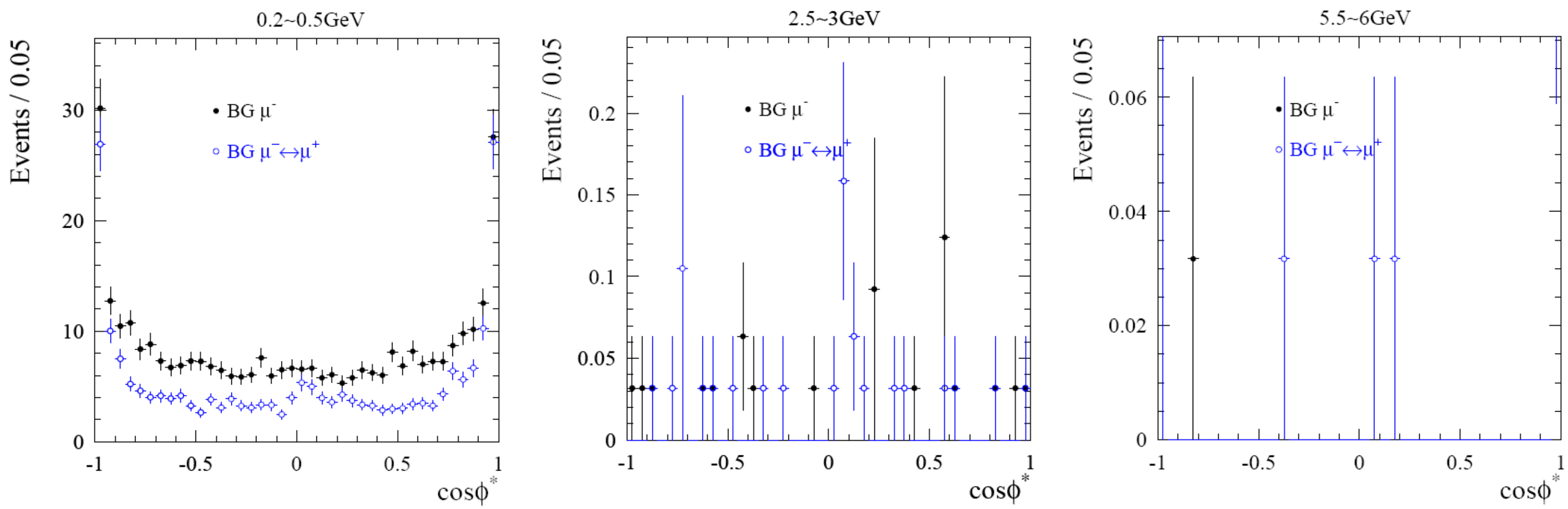
μμγ

ππγ

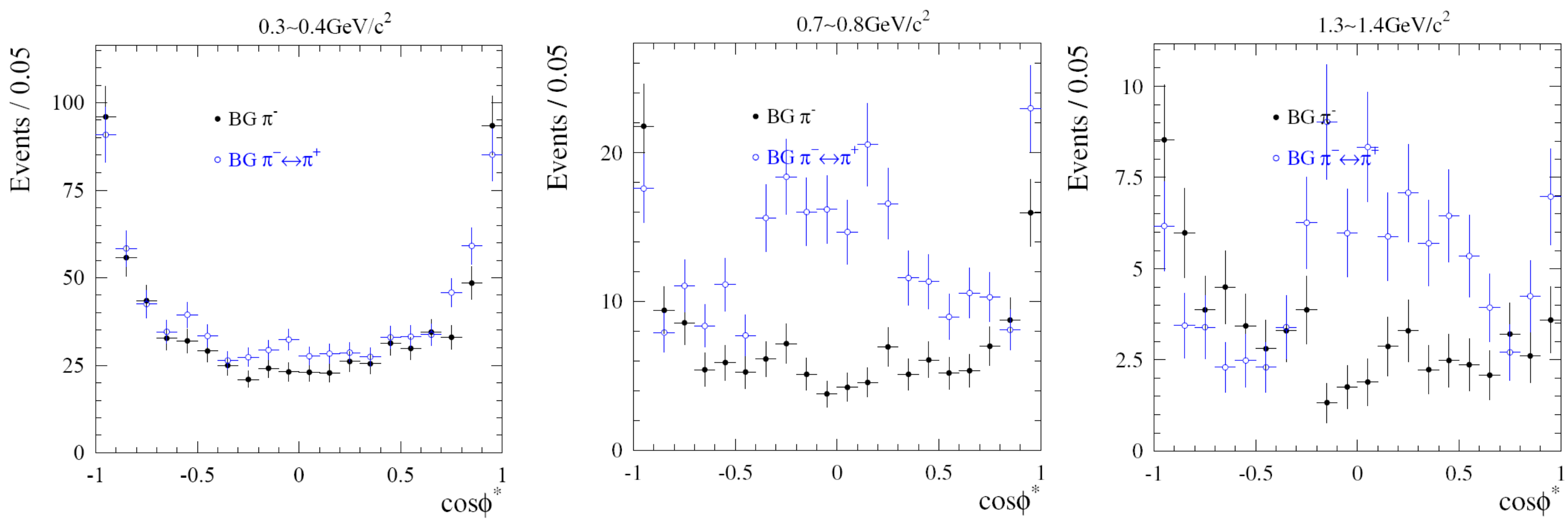


Background estimation

μμγ

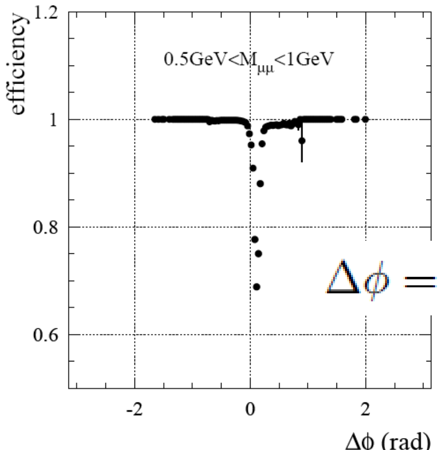


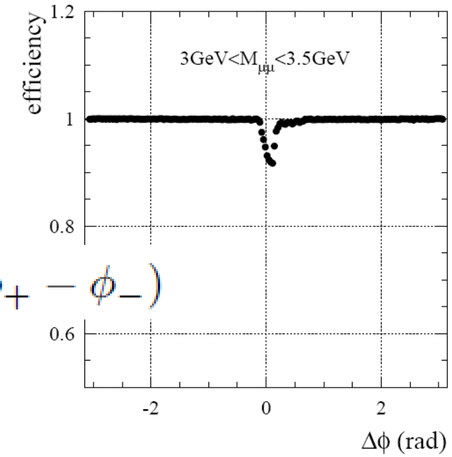
ππγ



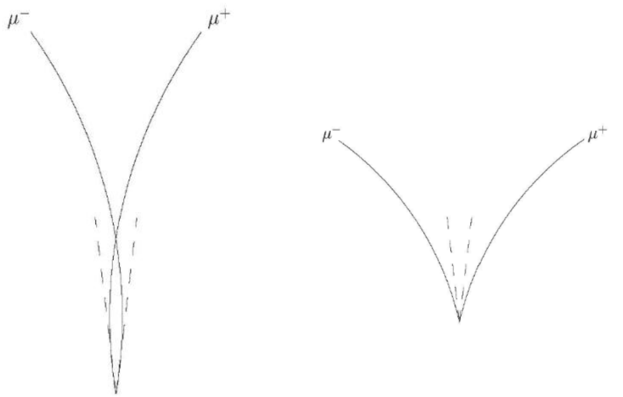
Detector effects and data/simulation

- Charge asymmetry effects matter in this analysis
- Charged tracks overlap in DCH (trigger, tracking) => data/simulation correction

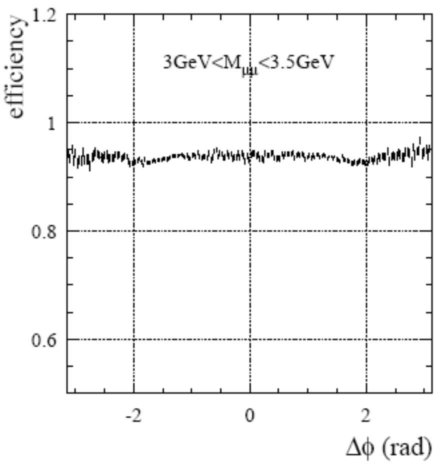


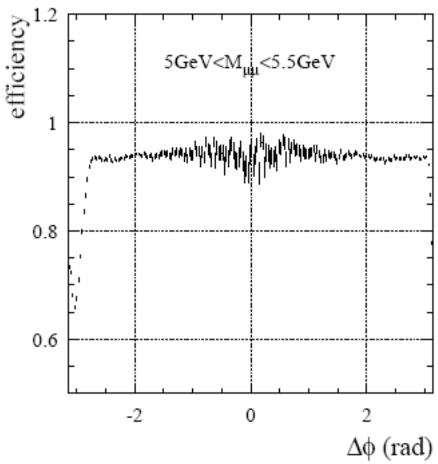


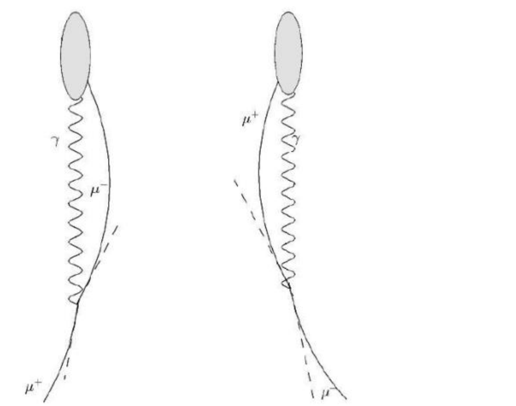
$$\Delta\phi = (\phi_+ - \phi_-)$$



- Track photon overlap in EMC (photon reconstruction and mu-ID) => remove overlap events



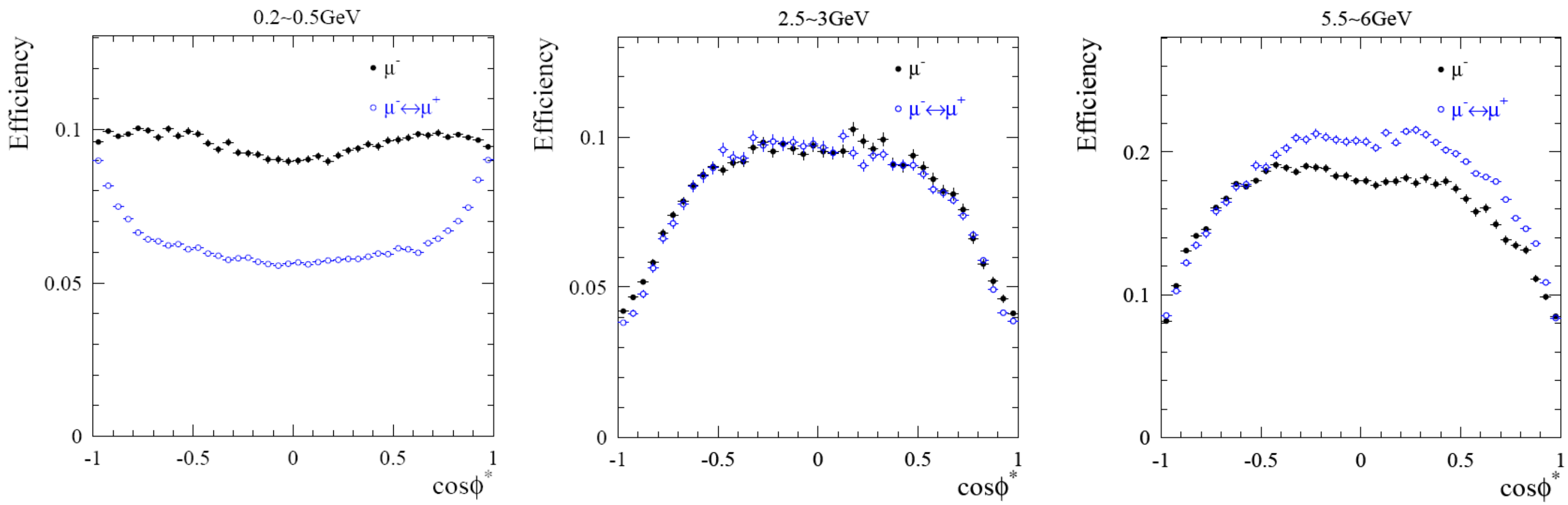




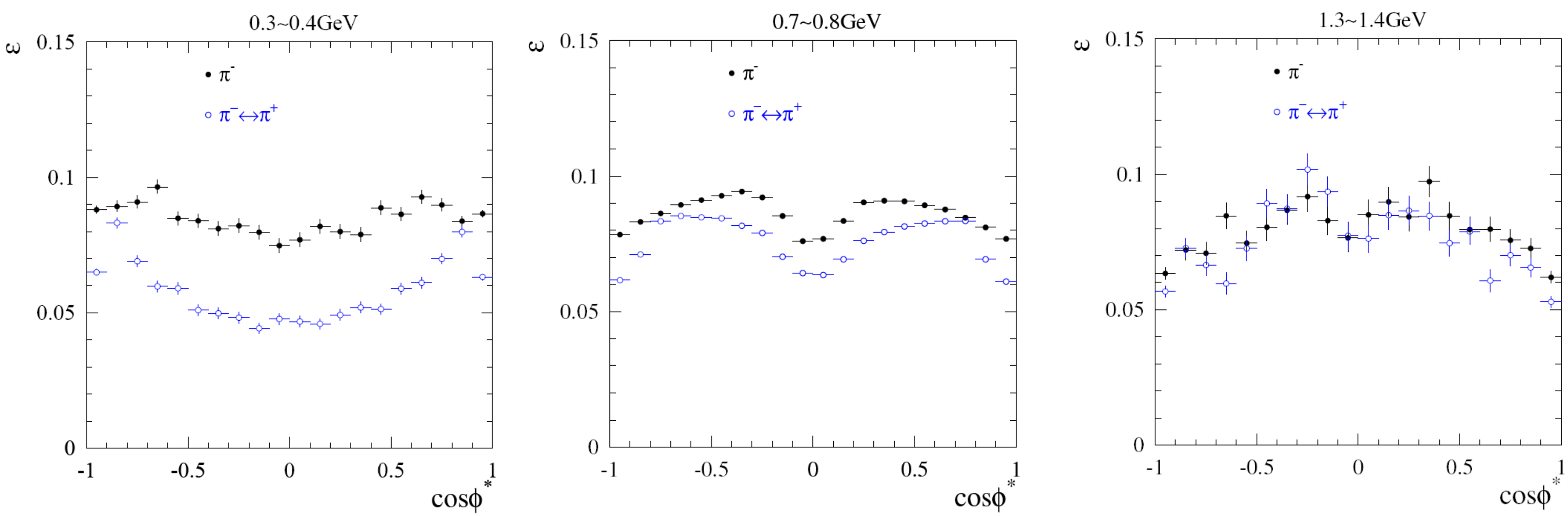
- Track overlap in IFR (mu-ID) => remove overlap events (run1,2)

Efficiency

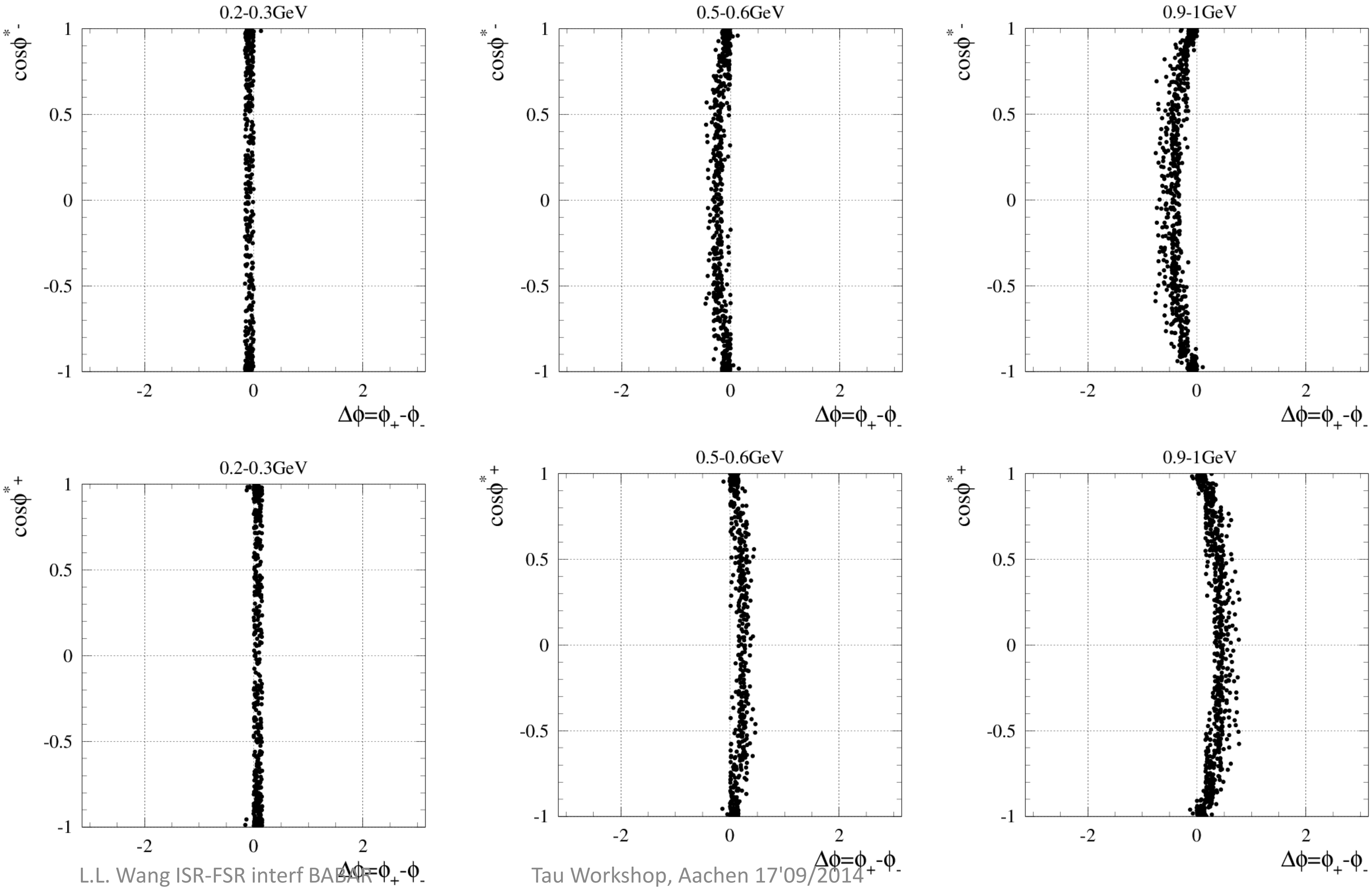
μμγ



ππγ

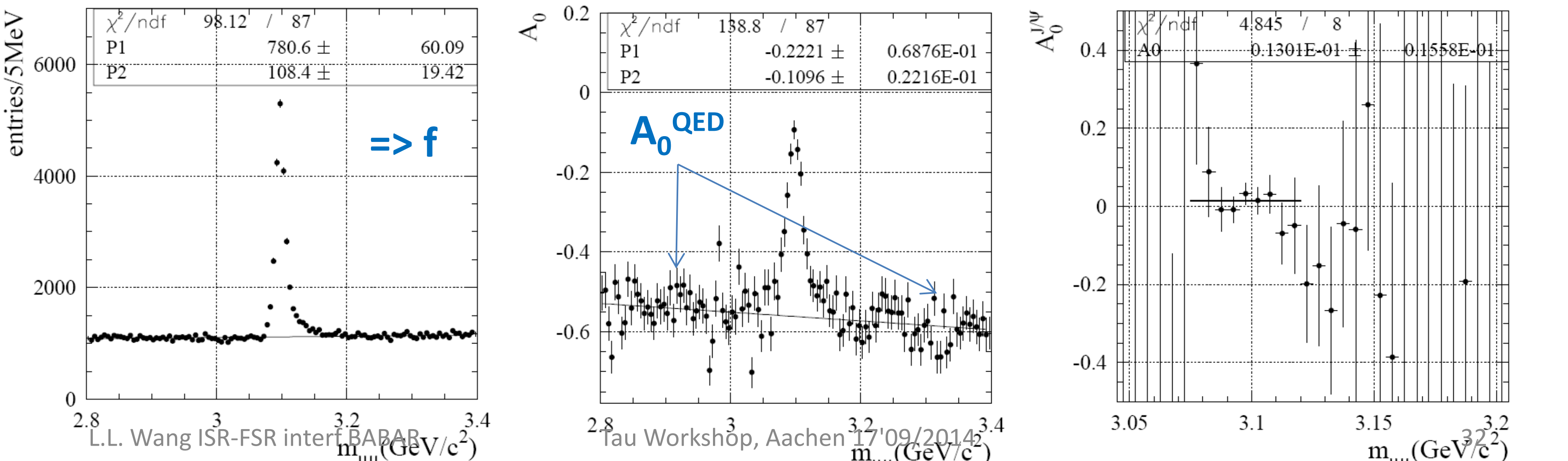


cosϕ* vs Δϕ


0.2-0.3GeV
0.5-0.6GeV
0.9-1GeV
0.2-0.3GeV
0.5-0.6GeV
0.9-1GeV
$\cos\phi^{*}_{-}$
$\cos\phi^{*}_{+}$
$\Delta\phi=\phi_{+}-\phi_{-}$
1
0.5
0
-0.5
-1
-2
0
2

Test of the charge asymmetry with J/ψ→μμ

- pure ISR process ee→γJ/ψ→γμμ => $A_0^{J/\psi}=0$
- looser event selection: no mu-ID, no active IFR requirement
  => statistics gain by 4
- $$A_0 = \frac{A_0^{\mathrm{QED}} N_{\mathrm{QED}} + A_0^{J/\psi} N_{J/\psi}}{N_{\mathrm{QED}} + N_{J/\psi}}$$ => $$A_0^{J/\psi} = \frac{A_0 - A_0^{\mathrm{QED}} f}{1-f}$$ $$f = \frac{N_{\mathrm{QED}}}{N_{\mathrm{QED}}+N_{J/\psi}}$$
- up limit for systematic for null A0 measurement: $A_0^{J/\psi} = (1.3 \pm 1.6)\%$



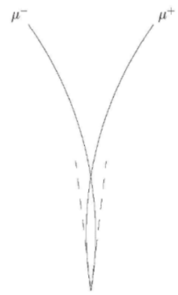
Discussion on possible bias (μμγ)

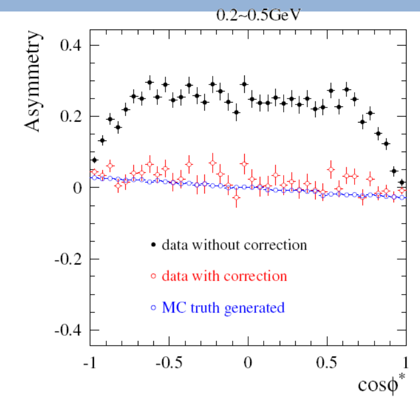
- Detector charge asymmetry effects
  - μ+/μ- bend in different directions in magnetic field => detector charge asymmetry (overlap vs no overlap)
  - Change measured A(cosϕ*) => no linear
  - in fact detector asymmetries depending on Δϕ are rather symmetric in cosϕ*
  - measuring A_0 (anti-symmetric part of A in cosϕ*) we are much less dependent on detector asymmetries

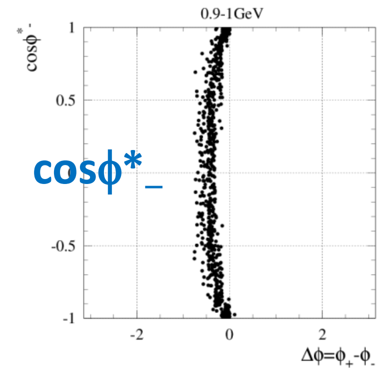
⇨ A_0 a good estimator of the interference robust with respect to detector asymmetries

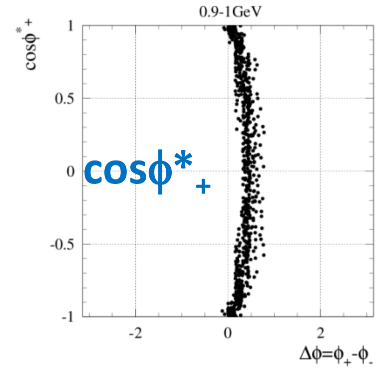
- The A deviation between data/MC (1.5-4GeV) has a linear cosϕ* dependence

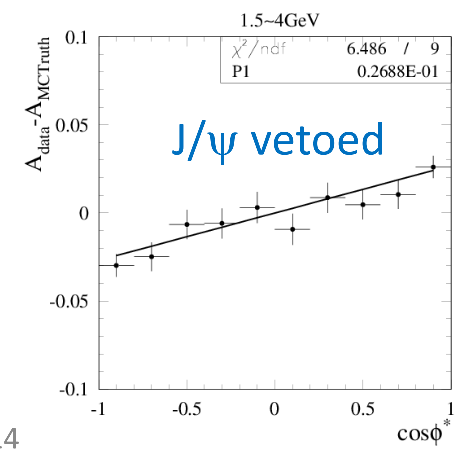
- J/ψ test









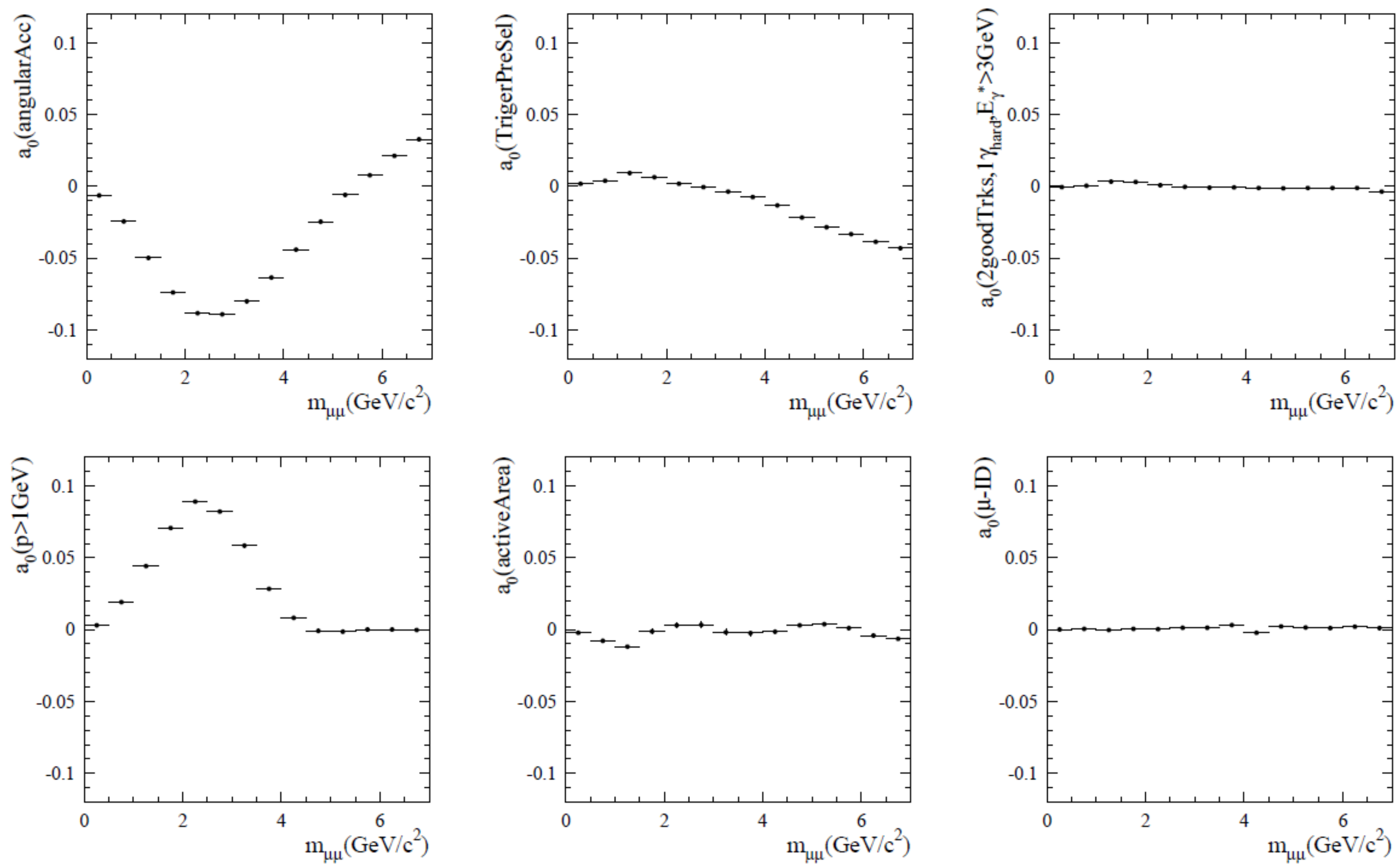


Indication: deviation from physical charge asymmetry

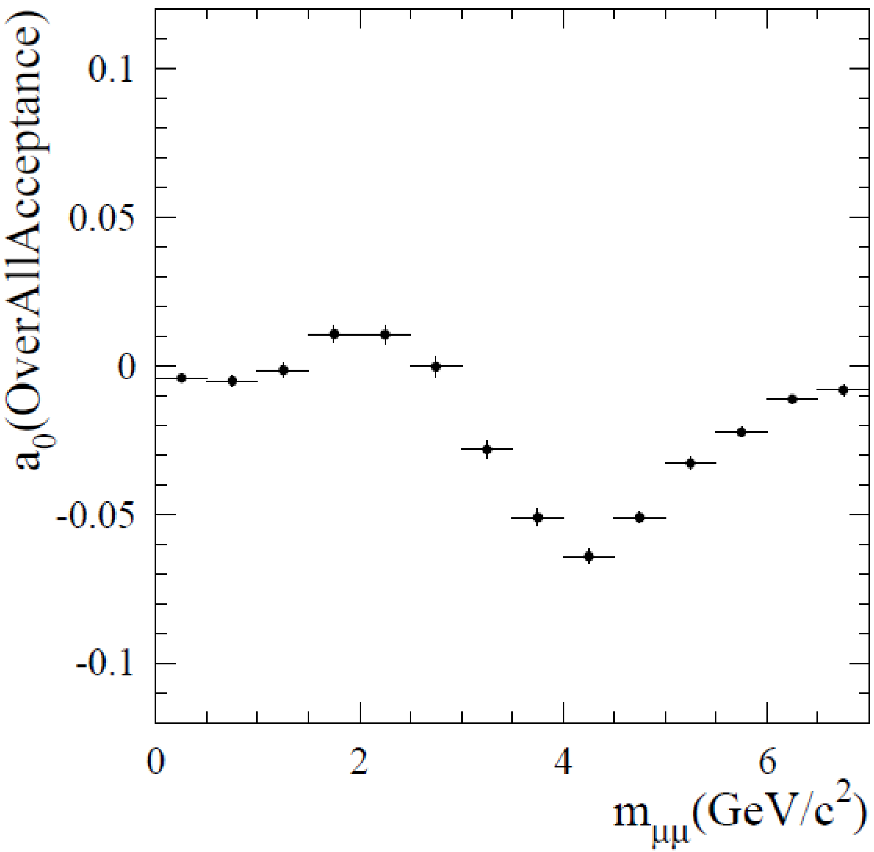
Asymmetry bias from analysis cuts ($\mu\mu\gamma$)

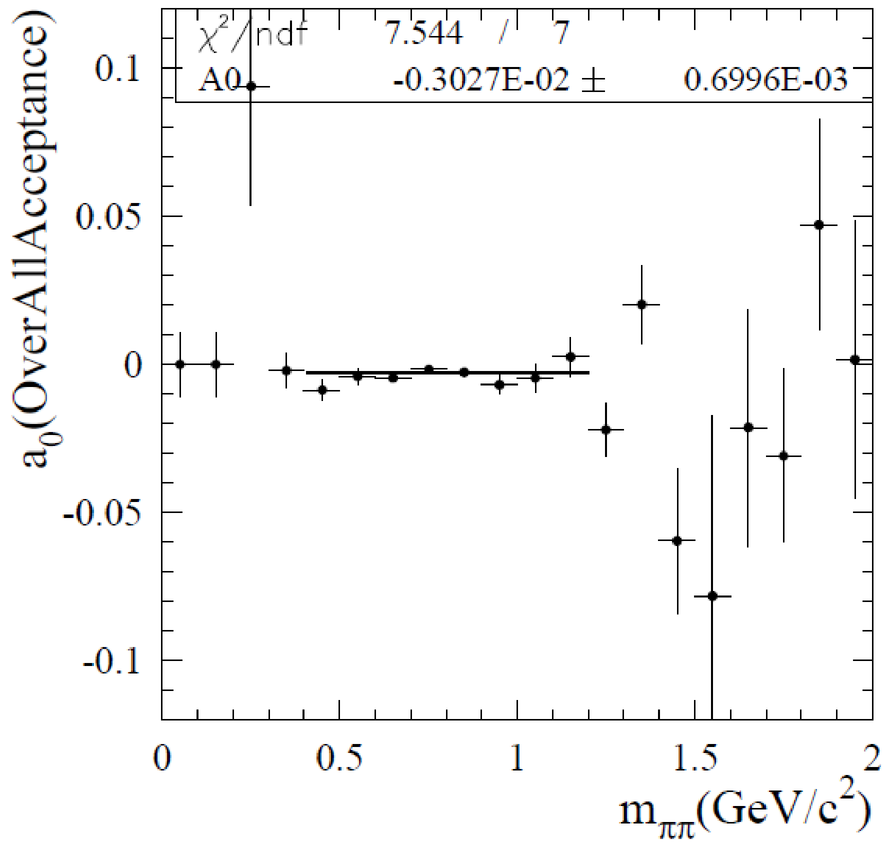
$$a(\cos\phi^*) = (\Delta A(\cos\phi^*) - \Delta A(-\cos\phi^*))/2$$

Fit to $a_0 \cos\phi^*$

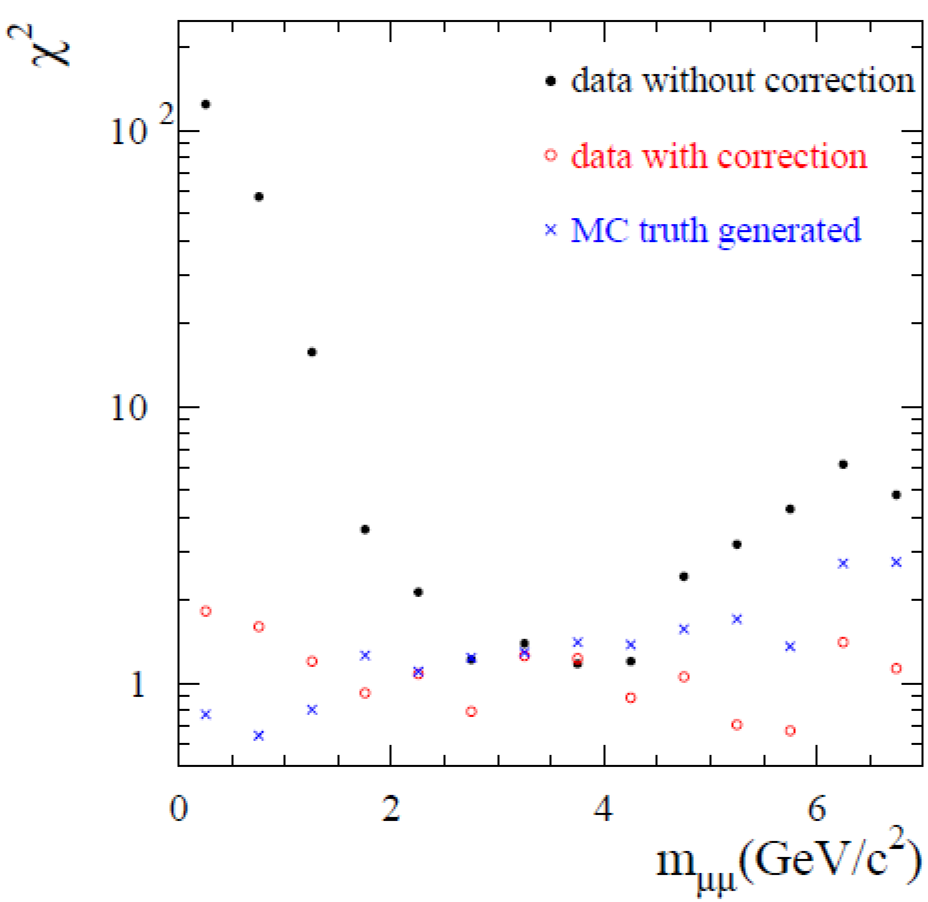


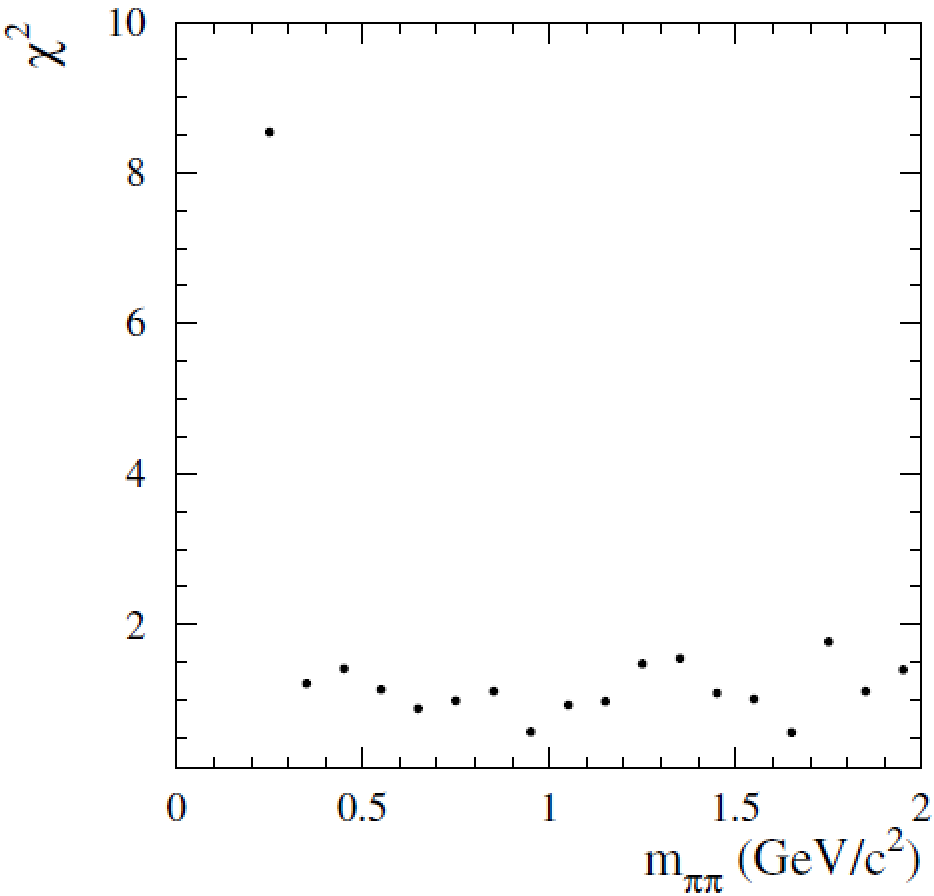
Overall bias (cross effect acceptance/physical A_0)



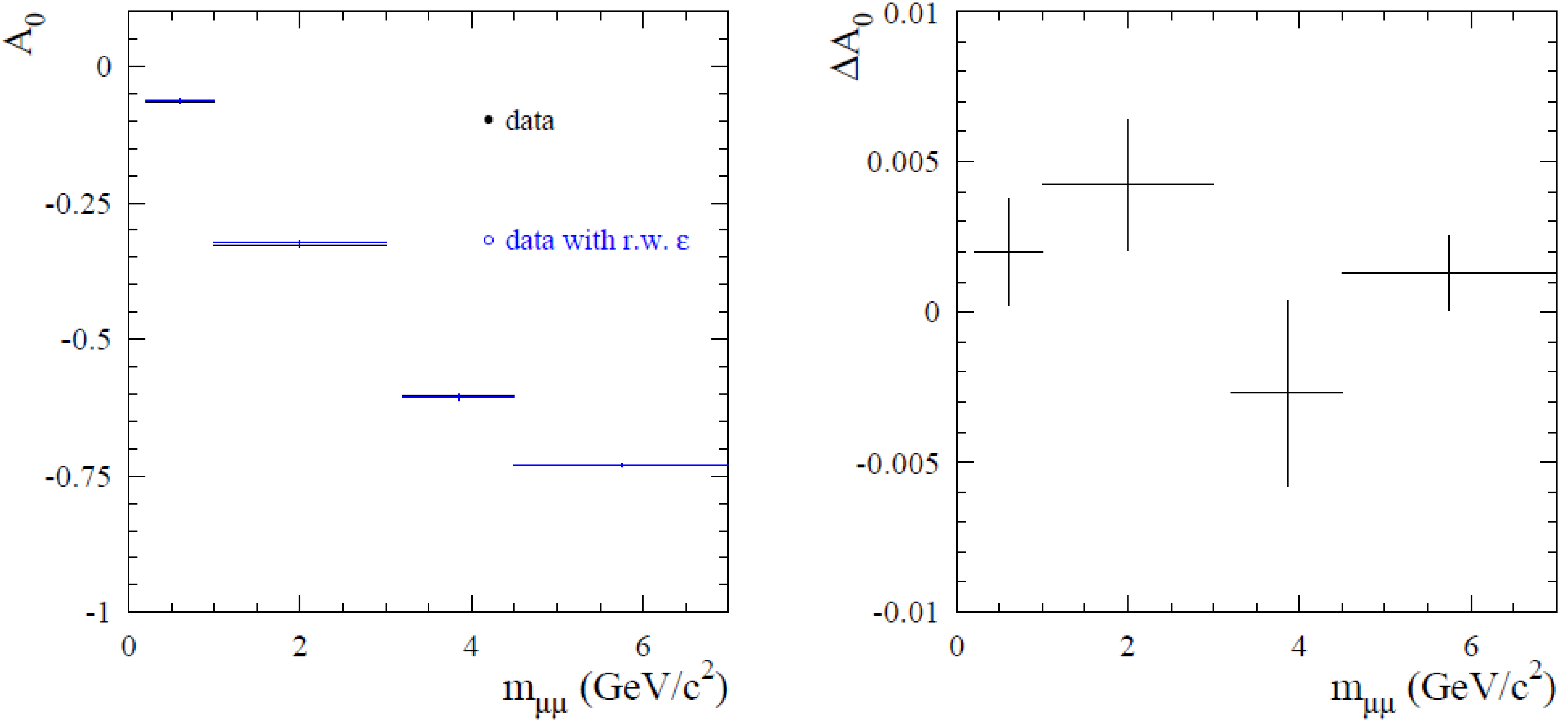


χ^2 of cosϕ* fit before/after corrections

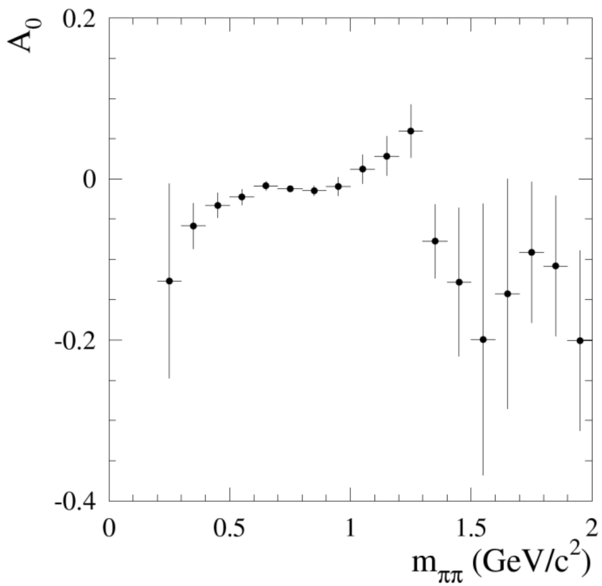


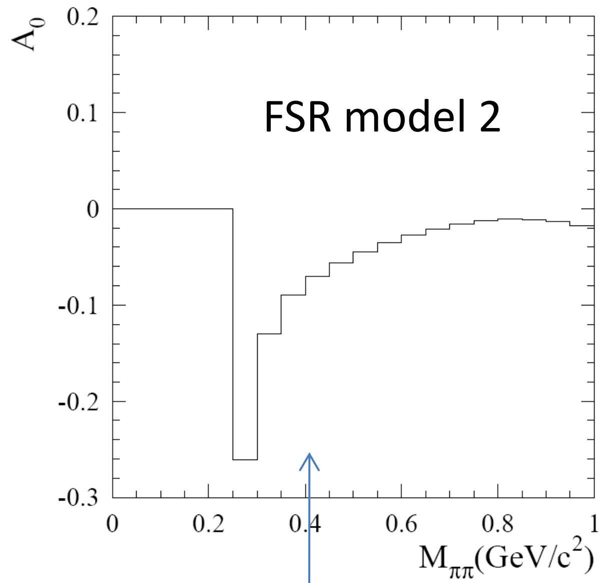


A_0 change after MC reweighting ($\mu\mu\gamma$)



Revised FSR model for ee→ππγ





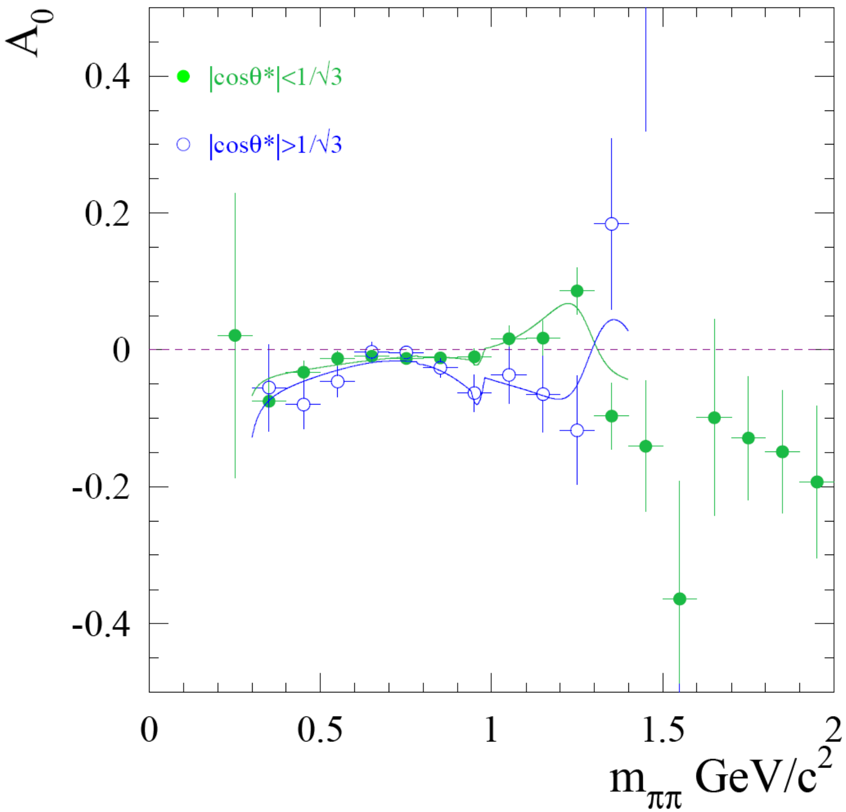
$$\Phi_u^+(z, m_{\pi\pi}^2, \cos\theta^*) = \Phi_d^+(z, m_{\pi\pi}^2, \cos\theta^*)$$

$$= 10z(1-z)(2z-1)R_\pi\left[-\frac{3-\beta^2}{2}e^{i\delta_0(m_{\pi\pi}^2)} + \beta^2 e^{i\delta_2(m_{\pi\pi}^2)}P_2(\cos\theta^*)\right]$$

- Fit with the revised model: $$10z(1-z)(2z-1)\left[c_0\frac{3-\beta^2}{2}e^{i\delta_0(m_{\pi\pi}^2)} + c_2\beta^2 \mathrm{BW}_{f_2}(m_{\pi\pi})P_2(\cos\theta^*)\right]$$

- $P_2(\cos\theta^*) = (3\cos^2\theta^* - 1)/2$ => Around f_2: Charge asymmetry changes sign at $|\cos\theta^*| = 1/\sqrt{3}$ (≈ 0.58)

Charge asymmetry below and above cosθ*=1/sqrt(3) for ee→ππγ

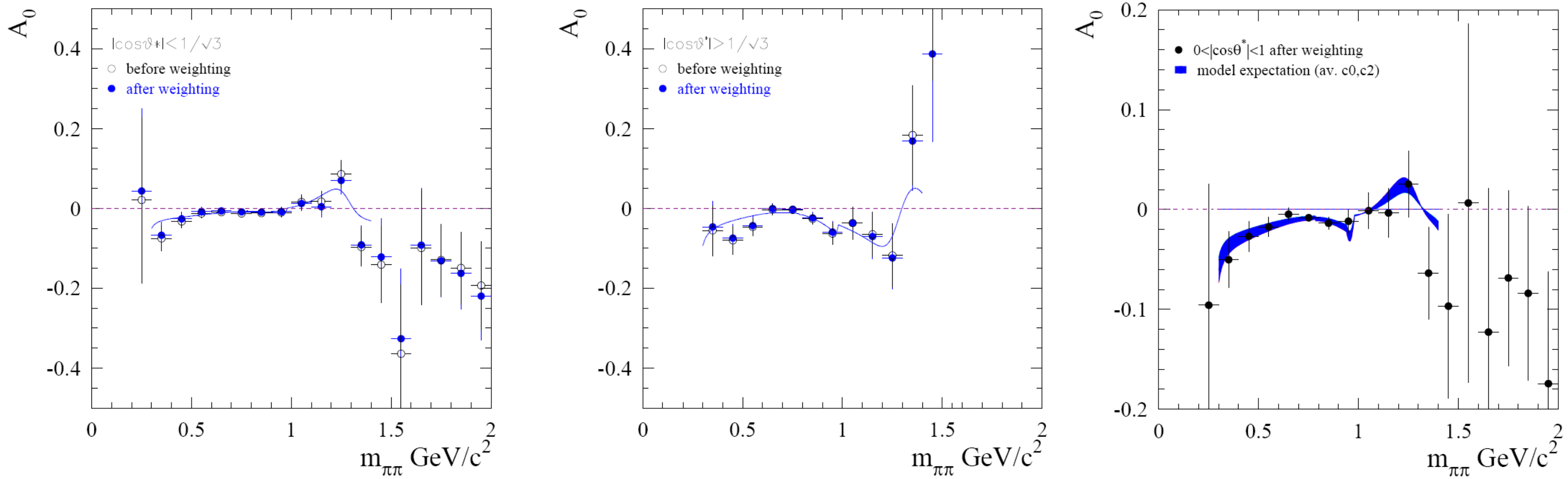


- Analyses are done below and above cosθ*=1/sqrt(3) separately (standard selection for >1/sqrt(3), modified selection for <1/sqrt(3), BG sub, efficiency correction)

- charge asymmetry has opposite sign around f2
- Fit A_0 between 0.3 and 1.4GeV with the revised FSR model

	cosθ*<1/sqrt(3)	cosθ*>1/sqrt(3)
c_0	-1.13+-0.24	-1.59+-0.36
c_2	-5.4+-1.8	-5.4+-3.1

Converged result for ee→$\pi\pi\gamma_{FSR}$



parameter	$0. < \lvert\cos\theta^*\rvert < 1/\sqrt{3}$	$\lvert\cos\theta^*\rvert > 1/\sqrt{3}$	average	$0. < \lvert\cos\theta^*\rvert < 1.$
c_0	-0.84 ± 0.24	-1.13 ± 0.35	-0.93 ± 0.20	-0.87 ± 0.20
c_2	-3.82 ± 1.81	-6.33 ± 3.03	-4.48 ± 1.56	-3.41 ± 4.25

- **results from different |cosθ*| consistent with each other**
- **combined result from the two complementary ranges**
  opposite sign => clear confirmation of f_2 contribution
  more sensitive to ee→$f_2\gamma_{FSR}$