g-2 Theory: Update

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DESY

Workshop on Tau Lepton Physics, Aachen, 2014

Outline

Lepton anomalous magnetic moment

Best experimentally measured and theoretically predicted quantity

- electron

$$a_e|_{\mathrm{exp}} = 0.001\,159\,652\,180\,73(28)$$
$$a_e|_{\mathrm{theo}} = 0.001\,159\,652\,181\,78(6)(4)(3)(77)$$

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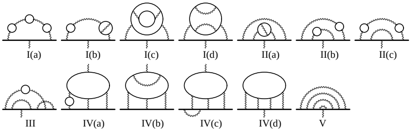
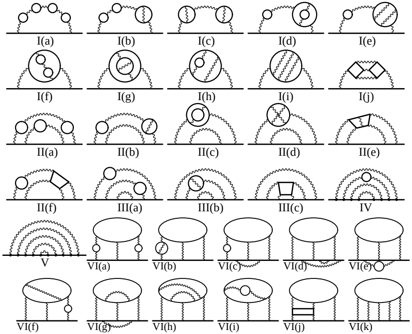
$$a_\mu|_{\mathrm{exp}} - a_\mu|_{\mathrm{theo}} = 240 \times 10^{-11} \qquad 2.9\sigma \text{ diff.}$$

Pure Leptonic Contributions

- analytical results
  - one loop: $a_\mu^{(1)} = \frac{1}{2}$ [Schwinger 1948]
  - two loop [Petermann; Sommerfeld 1957]
  - three loop [Laporta, Remiddi 1996]
  - $\geq$ four loop: see this talk
- numerical results
  - four loop [Kinoshita et al]
  - five loop [Aoyama,Hayakawa,Kinoshita,Nio 2012]

QED contributions @ 4 and 5 loops





$$a_\mu^{4\ell} = 130.8796(63) \left(\frac{\alpha}{\pi}\right)^4 \qquad a_\mu^{5\ell} = 753.29(1.04) \left(\frac{\alpha}{\pi}\right)^5$$

$$\begin{aligned} a_\mu(QED, \alpha(Rb)) &= 116584718951(9)(19)(7)(77) \times 10^{-14} \\ a_\mu(QED, \alpha(a_e)) &= 116584718845(9)(19)(7)(30) \times 10^{-14} \end{aligned}$$

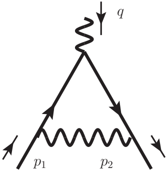
[Aoyama,Hayakawa,Kinoshita,Nio 2012]

Contributions from different orders

order	with $\alpha^{-1}(\text{Rb})[\times 10^{-11}]$	with $\alpha^{-1}(a_e)[\times 10^{-11}]$
2	116 140 973.318 (77)	116 140 973.212 (30)
4	413 217.6291 (90)	413 217.6284 (89)
6	30 141.902 48 (41)	30 141.902 39 (40)
8	381.008 (19)	381.008 (19)
10	5.0938 (70)	5.0938 (70)
a_μ(QED)	116 584 718.951 (80)	116 584 718.845 (37)

Note: four-loop (eighth order contribution) is larger than the difference between SM prediction and measured value.

Definition + Calculation

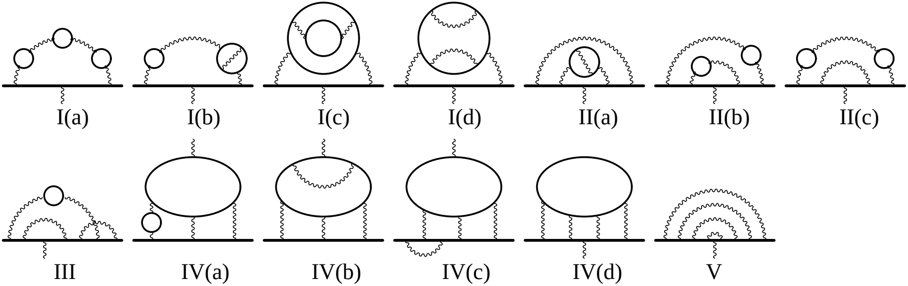


$$= \quad (-ie)\overline{u}(p_2)\left\{\gamma^\mu F_E(q^2) + i\frac{\sigma^{\mu\nu}q^\nu}{2m}F_M(q^2)\right\}u(p_1)$$

$$a_\mu = F_M(0)$$

Contributions from diagrams with electrons and/or tau leptons can be calculated using asymptotic expansions $\Rightarrow$ reduction to single-scale problem.

Diagram classes at four loops



[Aoyama et al ’2012]

Results for Tau contribution

group	$10^2 \cdot A_{2,\mu}^{(8)}(M_\mu/M_\tau)$	
	[Kurz et al]	[Aoyama et al]
I(a)	0.00324281(2)	0.0032(0)
I(b) + I(c) + II(b) + II(c)	−0.6292808(6)	−0.6293(1)
I(d)	0.0367796(4)	0.0368(0)
III	4.5208986(6)	4.504(14)
II(a) + IV(d)	−2.316756(5)	−2.3197(37)
IV(a)	3.851967(3)	3.8513(11)
IV(b)	0.612661(5)	0.6106(31)
IV(c)	−1.83010(1)	−1.823(11)

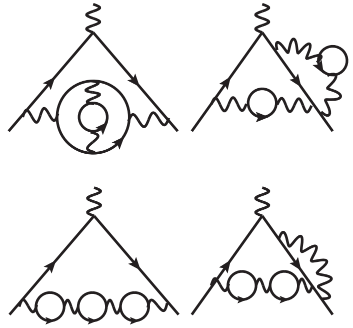
fast convergence

$$A_{2,\mu}^{(8)}(M_\mu/M_\tau) = 0.0421670 + 0.0003257 + 0.0000015 = 0.0424941(2)(53)$$

[Kurz et al]

Contributions involving electrons

n_e^3 and n_e^2 part



- leading term in expansion in m_e/m_μ calculable using massless electrons
- logarithmic terms can be recovered by going form the $\overline{\mathrm{M}S}$ scheme to the onshell scheme.

Results: $n_e^3, n_e^2 n_\mu, n_e^2$

$$a_\mu^{(43)} = \frac{1}{54} L_{\mu e}^3 - \frac{25}{108} L_{\mu e}^2 + \left(\frac{317}{324} + \frac{\pi^2}{27}\right) L_{\mu e} - \frac{2\zeta_3}{9} - \frac{25\pi^2}{162} - \frac{8609}{5832}$$
$$\approx 7.196\,66\,,$$

[Lee et al; Laporta; Aguilar,Greynat,De Rafael]

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$$a_\mu^{(42)} = a_\mu^{(42)a} + n_h a_\mu^{(42)b}$$

$$a_\mu^{(42)a} = L_{\mu e}^2 \left[\pi^2 \left(\frac{5}{36} - \frac{\log 2}{6}\right) + \frac{\zeta_3}{4} - \frac{13}{24}\right] + \ldots \approx -3.624\,27\,,$$

[Lee et al]

$$a_\mu^{(42)a}\Big|_{\text{num}} = -3.642\,04(1\,12)\,,$$

[Aoyama,Hayakawa,Kinoshita,Nio 2012]

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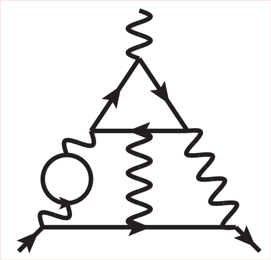
$$a_\mu^{(42)b} = \left(\frac{119}{108} - \frac{\pi^2}{9}\right)L_{\mu e}^2 + \left(\frac{\pi^2}{27} - \frac{61}{162}\right)L_{\mu e} - \frac{4\pi^4}{45} + \frac{13\pi^2}{27} + \frac{7627}{1944}$$
$$\approx 0.494\,05$$

[Lee et al; Laporta; Aguilar,Greynat,De Rafael]

Leptonic light-by-light contributions



- calculation requires proper asymptotic expansion
- structure of result:

$$C_0 + C_1 \ln \frac{m_e}{m_\mu} + C_2 \ln^2 \frac{m_e}{m_\mu}$$

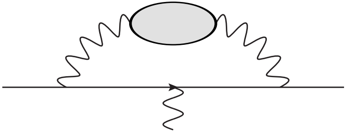
Preliminary results

agreement with results from Aoyama et al.

Vacuum polarization insertions: Method

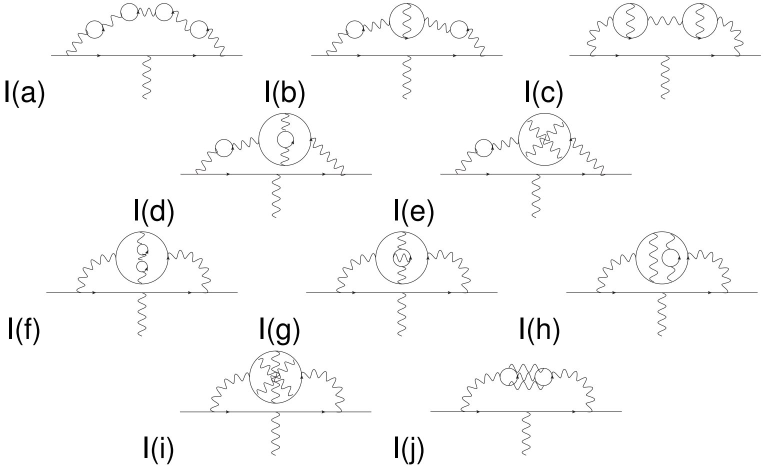
- Certain contributions to g-2 can be obtained by integration over the vacuum polarization function $\Pi(q^2)$

$$a_\mu = \frac{\alpha}{\pi}\int_0^1 \mathrm{d}x(1-x)\,[-\Pi(s_x)]\ ,\ s_x = -\frac{x^2}{1-x}m_\mu^2\,.$$



- analysis @ four loops by Baikov and Broadhurst '95 was used to verify results by Kinoshita et al '92
- use a suitable approximating function for $\Pi(q^2)$ at four loops and extend analysis to five loops
- N.B.: There are no additional bound state contributions

Accessible classes @ five loops



Vacuum polarization function $\Pi(q^2)$

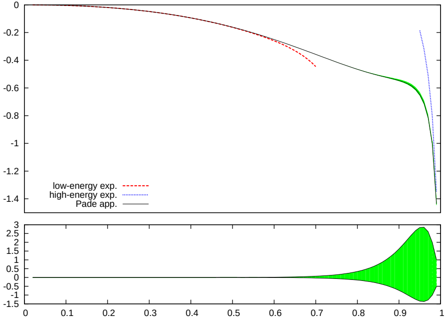
- first step: take only leading term in high-energy expansion

[Baikov et al 2013]

- Improve by using all available information in the low- and high-energy and the threshold region to obtain best possible approximation for $\Pi(q^2)$ in form of a Padé approximation

[Baikov et al 2013]

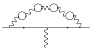
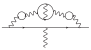
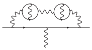
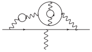
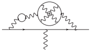
- To obtain an error estimate constructed $\approx$ 800 Padé approximants

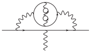
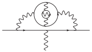
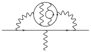
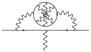
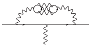


Results

	Baikov et al	Baikov et al	Aoyama et al
I(a)	20.142 813	20.183 2	20.142 93(23)
I(b)	27.690 061	27.718 8	27.690 38(30)
I(c)	4.742 149	4.817 59	4.742 12(14)
I(d)+I(e)	6.241 470	6.117 77	6.243 32(101)(70)
I(e)	-1.211 249	-1.331 41	-1.208 41(70)
I(f)+I(g)+I(h)	$4.446\,8^{+6}_{-4}$	4.391 31	4.446 68(9)(23)(59)
I(i)	$0.074\,6^{+8}_{-19}$	0.252 37	0.087 1(59)
I(j)	$-1.246\,9^{+4}_{-3}$	-1.214 29	-1.247 26(12)

Improved agreement with previous works by Aoyama et al.

I(a)  I(b)  I(c)  I(d)  I(e) 

I(f)  I(g)  I(h)  I(i)  I(j) 

Electro-Weak Contributions

complete up to two loops + logarithmic three-loop contributions

$$
\begin{aligned}
a_\mu^{EW,(1)} &= (194.80 \pm 0.01) \times 10^{-11}, \\
a_{\mu,\mathrm{bos}}^{EW,(2)} &= -(19.97 \pm 0.03) \times 10^{-11}, \\
a_{\mu,f_r est,H}^{EW,(2)} &= -(1.50 \pm 0.01) \times 10^{-11}, \\
a_\mu^{EW,(2)}(e,\mu,u,c,d,s) &= -(6.91 \pm 0.20 \pm 0.30) \times 10^{-11}, \\
a_\mu^{EW,(2)}(\tau,t,b) &= -(8.21 \pm 0.10) \times 10^{-11}, \\
a_{\mu,f_r est,noH}^{EW,(2)} &= -(4.64 \pm 0.10) \times 10^{-11}, \\
a_\mu^{EW,\geq 3\ell} &= (0 \pm 0.20) \times 10^{-11}
\end{aligned}
$$

$$
a_\mu^{EW} = (153.6 \pm 1.0) \times 10^{-11}
$$

[Fargnoli et al; Czarnecki et al]

Hadronic vacuum-polarization insertions

Can be obtained by integrating experimentally measured

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma_{pt}}$$

over a kernel function

$$a_\mu^{(1)} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{m_\pi^2}^{\infty} \mathrm{d}s \frac{R(s)}{s} K^{(1)}(s)$$

with

$$K^{(1)}(s) = \int_0^1 \mathrm{d}x \frac{x^2(1-x)}{x^2 + (1-x)\frac{s}{M_\mu^2}}$$

For higher-order corrections need higher-order kernels.

Hadronic vacuum polarization insertions

- Analysis limited by measurement of $R(s)$, results to about half of the total error of a_μ
- LO analysis

$$a_\mu^{LO}(had.v.p.) = 6949.1(37.2)_{\rm exp}(21.0)_{\rm rad} \times 10^{-11}$$

[Hagiwara et al]

- NLO analysis

$$a_\mu^{\rm NLO}(had.v.p.) = -98.4(0.6)_{\rm exp}(0.4)_{\rm rad} \times 10^{-11}$$

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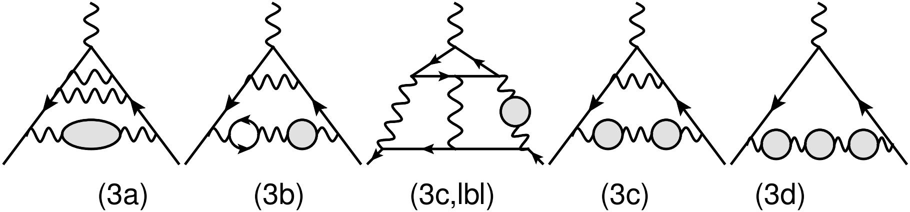
- NNLO analysis

$$a_\mu^{\rm NNLO}(had.v.p.) = 12.4 \pm 0.1 \times 10^{-11}$$

slight reduction of discrepancy by 0.2σ [Kurz et al]

Hadronic vacuum polarization insertion @ NNLO



$$a_\mu^{(3a)} = 8.0 \times 10^{-11}$$
$$a_\mu^{(3b)} = -4.1 \times 10^{-11}$$
$$a_\mu^{(3b,\mathrm{lbl})} = 9.1 \times 10^{-11}$$
$$a_\mu^{(3c)} = -0.6 \times 10^{-11}$$
$$a_\mu^{(3d)} = 0.005 \times 10^{-11}$$

first appearance of light-by-light diagrams leads to an enhanced NNLO contribution.

Conclusions

- Pure QED contribution well known up to five loops, so far no deviation from known results found
- Electro-weak corrections known up to two loop including Higgs-mass effects
- NNLO hadronic vacuum polarization contribution quite large and should not be neglected.