

Dotacje na innowacje Inwestujemy w waszą przyszłość





Study of the tau meson decay with with Monte Carlo generator TAUOLA. Status and perspectives.

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OUTLINE

Introduction and motivation

Fitting to BaBar $\tau^- \rightarrow \pi^- \pi^+ \pi^- \nu_{\tau}$ data. Strategy and results

Preliminary results for $\tau^- \rightarrow K^- \pi^- K^+ \nu_{\tau}$ and $\tau^- \rightarrow \pi^- \pi^0 \nu_{\tau}$

Conclusion and plans

Tau lepton
$$m_{\tau} = 1.78 \text{ GeV} \rightarrow \text{decays in hadrons}$$

Br (
$$\tau \rightarrow \text{hadrons}$$
) = 64.8%

Precise measurements of the hadronic tau decay modes allow the low and intermediate energy study:

$$\mathcal{M}(\tau^- \to \nu_\tau h^-) = \frac{G_F}{\sqrt{2}} \,\mathcal{H}_h^\mu \,\left[\overline{\nu}_\tau \gamma_\mu (1 - \gamma_5)\tau\right]$$

$$\mathcal{H}_{h}^{\mu} \equiv \langle h^{-} | \left(V_{ud}^{*} \, \overline{d} \, \gamma^{\mu} (1 - \gamma_{5}) u + V_{us}^{*} \, \overline{s} \, \gamma^{\mu} (1 - \gamma_{5}) u \right) | 0 \rangle$$

* hadronization mechanism (pQCD does not work, ChPT low tail)

* Wess-Zumino anomaly (ex. K K π)

* resonance parameters

* Okuba-Zweig-Iizuka suppressed modes (ex. ∮ K)

* second class currents (ex. $\pi \eta$)

* measure |V_us| CKM matrix (modes with K)

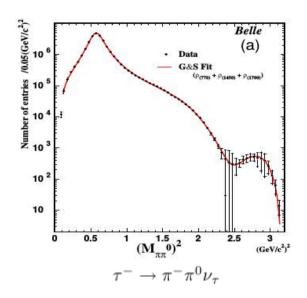
Nonetheless.... Knowledge of the dynamics is important for Higgs polarization measurement and agreement MC/data, searched for beyond SM physics

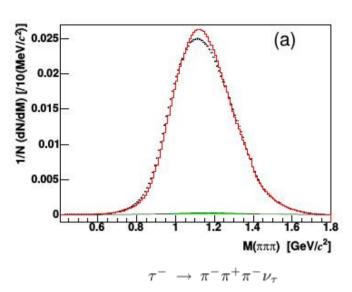
Precise analysis of available data for 2 pion + 3 pion modes

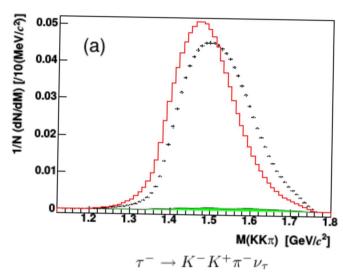
BaBar / Belle data

~44% hadr Br to check

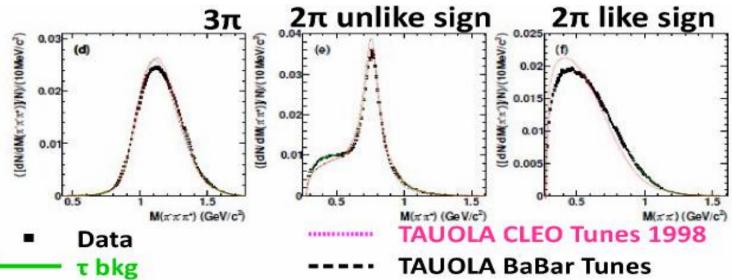
Belle MC/data







BaBar MC/data



A. Lusiani, PhiPsi 13

TAUOLA BaBar Tunes

MC = private versions of Tauola

TAUOLA (Monte Carlo generator for tau decay modes)

CPC version R. Decker, S.Jadach, M.Jezabek, J.H.Kuhn, Z. Was, Comp. Phys. Comm. 76 (1993) 361

<u>Cleo version</u> Alain Weinstein: http://www.cithep.caltech.edu/~ajw/korb_doc.html#files

- * BaBar version
- * Belle version

Aleph version B. Bloch, private communications

Features of all versions:

- * based on VMD, i.e. 3 scalar modes BW(V1)*BW(V2), reproduces LO ChPT limit
- * wrong normalization for 2 scalar modes, except 2π , only vector FF, no scalar FF
- * not correct low energy behaviour of the vector part for $KK\pi$ modes
- * 3 scalar mode results are not able to reproduce experimental data

Belle (2π , $K\pi$) spectra, BaBar 3 meson invariant mass spectra published

Hadronic currents for two and three meson decay modes

$$J_{\mu}$$
= μ e^{iS} _{α} |0>= Σ_{i} (Lorentz Structure)ⁱ F_{i} (Q²,s_j)

Hadronic form factors are:

- Model: Resonance Chiral Lagrangian (Chiral lagrangian with the explicit inclusion of resonances, G.Ecker et al., Nucl. Phys B321(1989)311)
 - * The resonance fields $(V_{\mu\nu}, A_{\mu\nu}$ antisymmetric tensor field) is added by explicit way
 - * Reproduces NLO prediction of ChPT (at least)
 - * Correct high energy behaviour of form factors → relation between model parameters

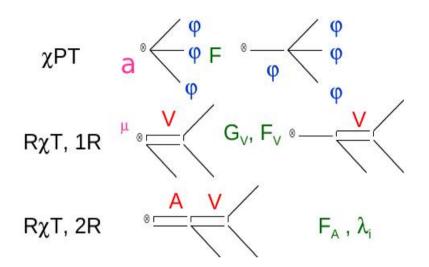
Finite numbers of parameters (one octet: f_{π} , $F_{V'}$, $G_{V'}$, F_{A})

Modes: $2\pi\tau$, $2K\tau$, $K\pi\tau$, $3\pi\tau$, $KK\pi\tau \rightarrow 88\%$ of tau hadronic width self consistent results within RChL for TAUOLA

We will start with $\tau^- \to \pi^- \pi^+ \pi^- \nu_{\tau}$ Br($\tau^- \to \pi^- \pi^+ \pi^- \nu_{\tau}$)/Br($\tau^- \to \text{hadrons } \nu_{\tau}$) = 14.1%

Three pion modes: $\tau \rightarrow \pi^+ \pi^- \pi^+ \nu_{\tau}$

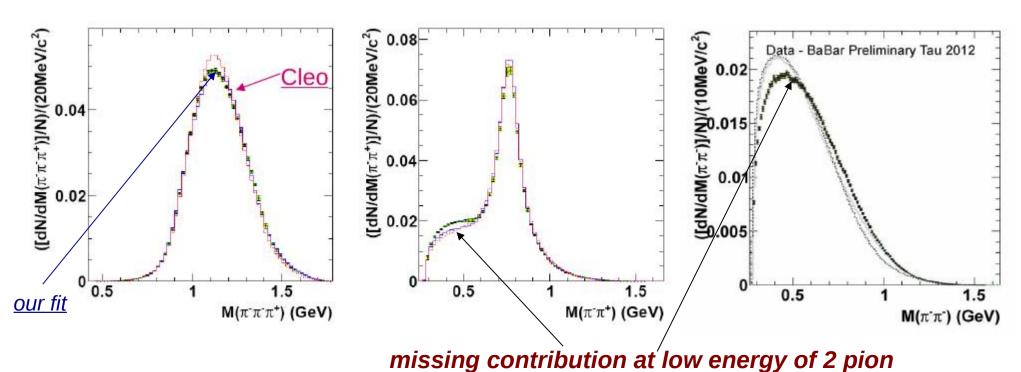
$$J^{\mu} = N \left\{ T^{\mu}_{\nu} \left[c_{1} (p_{2} - p_{3})^{\nu} F_{1} + c_{2} (p_{3} - p_{1})^{\nu} F_{2} + c_{3} (p_{1} - p_{2})^{\nu} F_{3} \right] + c_{4} q^{\nu} F_{4} - \frac{i}{4\pi^{2} F^{2}} c_{5} \varepsilon^{\mu\nu\rho\sigma} p_{1\nu} p_{2\rho} p_{3\sigma} F_{5} \right\}$$



For 3 pion modes
$$F_5 = 0$$
; $F4 \sim m_\pi^2/q^2$; $F_2(q^2, s_1, s_2) = F_1(q^2, s_2, s_1)$ $A = a_1$; $V = \rho$; ρ '

D. Gomez Dumm et al, 0911.4436

Tauola 2012: Implementation + technical tests



Modification of RChL \rightarrow inclusion of σ meson

- * σ meson is not in RChL scheme
- * BW approach
- * the RChL current structure (single and double resonance contributions)

$$\begin{split} F_{1}^{\mathrm{R}} & \to & F_{1}^{\mathrm{R}} + \frac{\sqrt{2}F_{V}G_{V}}{3F^{2}} \left[\alpha_{\sigma}BW_{\sigma}(s_{1})F_{\sigma}(q^{2},s_{1}) + \beta_{\sigma}BW_{\sigma}(s_{2})F_{\sigma}(q^{2},s_{2}) \right] \\ F_{1}^{\mathrm{RR}} & \to & F_{1}^{\mathrm{RR}} + \frac{4F_{A}G_{V}}{3F^{2}} \frac{q^{2}}{q^{2} - M_{a_{1}}^{2} - iM_{a_{1}}\Gamma_{a_{1}}(q^{2})} \left[\gamma_{\sigma}BW_{\sigma}(s_{1})F_{\sigma}(q^{2},s_{1}) + \delta_{\sigma}BW_{\sigma}(s_{2})F_{\sigma}(q^{2},s_{2}) \right] \\ BW_{\sigma}(x) & = & \frac{m_{\sigma}^{2}}{m_{\sigma}^{2} - x - im_{\sigma}\Gamma_{\sigma}(x)} \quad \Gamma_{\sigma}(x) = \Gamma_{\sigma}\frac{\sigma_{\pi}(x)}{\sigma_{\pi}(m_{\sigma}^{2})} \quad F_{\sigma}(q^{2},x) = \exp\left[\frac{-\lambda(q^{2},x,m_{\pi}^{2})R_{\sigma}^{2}}{8q^{2}} \right] \end{split}$$

Fit parameters 15 parameters:

$$\underbrace{M_A,\ M_\rho,\ M_{\rho^{'}},\ F_V,F_A,\ \beta_\rho,\ F,\ \Gamma_{\rho^{'}}}_{\text{RChL parameters}} + \alpha_\sigma,\ \beta_\sigma,\ \gamma_\sigma,\ \delta_\sigma,\ R_\sigma,M_\sigma,\Gamma_\sigma$$

Fitting strategy

* Fit functions is semi-analytic functions, distributions on s_1 , s_3 , q^2

d Γ /dq2 ds1 ds2 -> 1d dimensional distributions (s1, s2, q2) to fit to BaBar data

$$\frac{G_F^2|V_{ud}|^2}{128(2\pi)^5 M_\tau F^2} \left(\frac{M_\tau^2}{q^2} - 1\right)^2 \left[W_{SA} + \frac{1}{3}\left(1 + 2\frac{q^2}{M_\tau^2}\right)(W_A + W_B)\right]^{-2}$$

$$W_A = -(V_1^{\mu}F_1 + V_2^{\mu}F_2 + V_3^{\mu}F_3)(V_{1\mu}F_1 + V_{2\mu}F_2 + V_{3\mu}F_3)^* \longrightarrow \text{resonances}$$

To smooth integrand

$$\int_{x_1}^{x_2} f(x)dx = \int_0^1 g'(t)f(g(t))dt \qquad x = g(t) = A^2 + AB \operatorname{tg}(y_1 + t(y_2 - y_1)) \qquad y_1 = \operatorname{arctg}\left(\frac{x_1 - A^2}{AB}\right)$$

$$A = 0.77, B = 1.8$$

- * 16 point Gaussian integration was done using Fortran Cernlib
- * Minuit from Cernlib for fitting, Hesse for stat uncertainty
- * C wrapper for Tauola
- * integration over bin / the central bin value
- * a1 width: using the g-function approximation / calculated at some q^2 + extrapolation using a polynomial

Calculation of the a width

$$\Gamma_{a_1}(q^2) = 2\Gamma_{a_1}^{\pi}(q^2)\theta \left(q^2 - 9m_{\pi}^2\right) + 2\Gamma_{a_1}^{K^{\pm}}(q^2)\theta \left(q^2 - (m_{\pi} + 2m_K)^2\right) + \Gamma_{a_1}^{K^0}(q^2)\theta \left(q^2 - (m_{\pi} + 2m_K)^2\right)$$

$$\Gamma_{a_{1}}^{\pi,K}(q^{2}) = \frac{-S}{192(2\pi)^{3}F_{A}^{2}F^{2}M_{a_{1}}} \left(\frac{M_{a_{1}}^{2}}{q^{2}} - 1\right)^{2} \tag{***}$$

$$\int dsdt \left(V_{1}^{\mu}F_{1} + V_{2}^{\mu}F_{2} + V_{3}^{\mu}F_{3}\right)^{\pi,K} \left(\left(V_{1\mu}F_{1} + V_{2\mu}F_{2} + V_{3\mu}F_{3}\right)^{\pi,K}\right)^{*} V_{i}^{\mu} = c_{i}T^{\mu\nu}(p_{j} - p_{k})_{\nu}$$

1. g-function approach for 3 pion piece and a fixed KK π part (*Table 4, arXiv 1203.3955*)

(**) calculated at 9 points, other q2 extrapolated as

$$g(q^2) = \begin{cases} (q^2 - 9m_\pi^2)^3(a - b(q^2 - 9m_\pi^2) + c(q^2 - 9m_\pi^2)^2), & 9m_\pi^2 < q^2 < (M_\rho + m_\pi)^2, \\ q^2(d - e/q^2 + f/q^4 - g/q^6), & (M_\rho + m_\pi)^2 < q^2 < 3(M_\rho + m_\pi)^2, \\ h + 2p\frac{q^2 - 3(M_\rho + m_\pi)^2}{(M_\rho + m_\pi)^2}, & 3(M_\rho + m_\pi)^2 < q^2 < m_\tau^2, \end{cases}$$

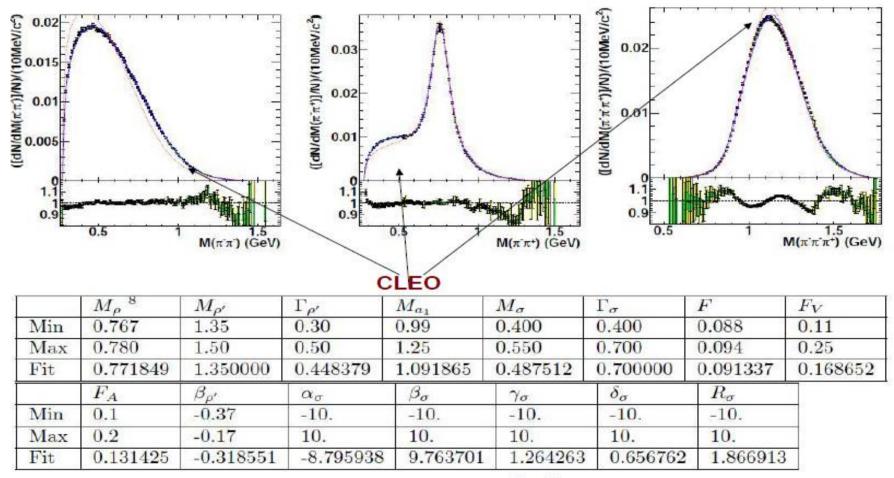
... random scanning

2. 3 pion contribution calculated at some q^2 - points (**) + extrapolation using a polynomial ... *final fit; parallelization*

Fit results

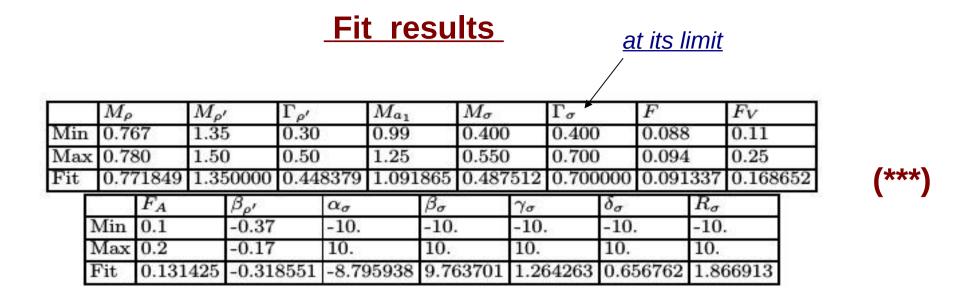
BaBar data * 10 MeV/bin (twice decreased)

* separated statistical and systematical errors



$$\chi^2/\text{ndf} = 6658/401 \text{ stat}$$

$$\chi^2/\text{ndf} = 889/401 \text{ stat+syst} - 2262 / 132$$
 (Tauola2012)
$$\Gamma_{\tau^- \to \pi^- \pi^- \pi^+ \nu_\tau} = 2.0001 \cdot 10^{-13} \text{ GeV}$$



$$\alpha_{\sigma} = \beta_{\sigma} \qquad \gamma_{\sigma} = \delta_{\sigma}$$

	$M_{ ho}$	$M_{\rho'}$	§	$\Gamma_{\rho'}$		M_{a_1}		M_{σ}		Γ_{σ}		F		F_V
Min	0.767	1.35		0.30		0.99	93	0.400)	0.400)	0.088	3	0.11
Max	0.780	1.50)	0.50		1.25		0.550)	0.700)	0.094	1	0.25
Fit	0.772774	1.35	0000	0.41	0404	1.116	6400	0.495	5353	0.465	5367	0.089	9675	0.167130
			F_A		$\beta_{\rho'}$		α_{σ}		γ_{σ}		R_{σ}			
		Min	0.1		-0.37	131	-10.	,	-10.	0	-10.			
		Max 0.2		-0.17		88	10.		10.	10.		15		
		Fit	0.146	6848	-0.30	1847	1.09	4981	0.58	2533	0.00	0315		

 \sim as in Cleo; = 0

- * Statistical errors and correlations between model parameters
- * Convergence of the fitting procedure
- * Toy MC studies to check of behaviour near the minimum
- * Estimation of systematic uncertainties

* Statistical errors and correlations between model parameters - Hesse algorithm of Minuit package

	α_{σ}	β_{σ}	γ_{σ}	δ_{σ}	R_{σ}	$M_{ ho}$	$M_{\rho'}$	$\Gamma_{\rho'}$	M_{a_1}	M_{σ}	Γ_{σ}	F_{π}	F_V	F_A	$\beta_{\rho'}$
α_{σ}	1	0.60	0.36	-0.29	-0.41	-0.69	0.46	0.68	-0.77	-0.09	0.02	0.78	0.76	0.52	-0.78
β_{σ}	0.60	1	0.44	-0.39	-0.42	-0.75	0.55	0.79	-0.89	-0.16	0.04	0.89	0.88	0.58	-0.88
γ_{σ}	0.36	0.44	1	-0.56	-0.22	-0.59	0.16	0.37	-0.47	-0.28	0.00	0.49	0.45	0.30	-0.45
δ_{σ}	-0.29	-0.39	-0.56	1	0.46	0.46	-0.24	-0.42	0.49	0.01	0.01	-0.49	-0.47	-0.31	0.47
R_{σ}	-0.41	-0.42	-0.22	0.46	1	0.42	-0.33	-0.56	0.62	0.34	0.02	-0.53	-0.56	-0.42	0.48
M_{ρ}	-0.69	-0.75	-0.59	0.46	0.42	1	-0.27	-0.64	0.79	0.29	-0.02	-0.83	-0.74	-0.48	0.75
$M_{\rho'}$	0.46	0.55	0.16	-0.24	-0.33	-0.27	1	0.67	-0.61	-0.13	0.03	0.61	0.66	0.37	-0.65
$\Gamma_{\rho'}$	0.68	0.79	0.37	-0.42	-0.56	-0.64	0.67	1	-0.88	-0.24	0.03	0.86	0.88	0.57	-0.88
M_{a_1}	-0.77	-0.89	-0.47	0.49	0.62	0.79	-0.61	-0.88	1	0.28	-0.03	-0.96	-0.97	-0.62	0.95
M_{σ}	-0.09	-0.16	-0.28	0.01	0.34	0.29	-0.13	-0.24	0.28	1	-0.02	-0.30	-0.29	-0.20	0.30
Γ_{σ}	0.02	0.04	0.00	0.01	0.02	-0.02	0.03	0.03	-0.03	-0.02	1	0.03	0.03	0.03	-0.04
F_{π}	0.78	0.89	0.49	-0.49	-0.53	-0.83	0.61	0.86	-0.96	-0.30	0.03	1	0.95	0.55	-0.97
F_V	0.76	0.88	0.45	-0.47	-0.56	-0.74	0.66	0.88	-0.97	-0.29	0.03	0.95	1	0.63	-0.96
F_A	0.52	0.58	0.30	-0.31	-0.42	-0.48	0.37	0.57	-0.62	-0.20	0.03	0.55	0.63	1	-0.56
$\beta_{\rho'}$	-0.78	-0.88	-0.45	0.47	0.48	0.75	-0.65	-0.88	0.95	0.30	-0.04	-0.97	-0.96	-0.56	1

*

- Convergence of the fitting procedure
 to verify that the found minimum is a global minimum
- start with random scan of 210 K points
- select 1K with the best chi2
- from them select 20 points with maximum distance
- use them as a start point for the full fit and apply the full fit procedure
- > 50% converge to the minimum (others falls with number of parameters at their limits, converge to local minimum with higher chi2)

Indicates that the found minimum point is a global minimum and the fitting procedure does not depend on an initial point

*

*

- * Toy MC studies to check of behaviour near the minimum
 - 8 MC samples (different seeds) of 20 million generated with
 - (I) the fit parameter values ('global minimum'), i.e. difference is "statistical error", a set "Toy"
 - (II) the set "Toy" is fitted
 - (a) the starting point is the 'global' minimum
 - (b) the starting point is the initial parameter values

The results of fit are consistent, i.e. the fitting procedure is stable

*

*

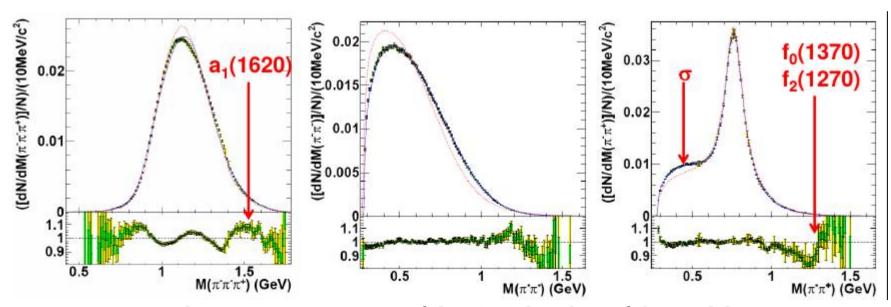
* Estimation of systematic uncertainties

Used systematical covariance matrix from BaBar experiment to include the correlations between bins

Limitations of the model



TAUOLA 2014



The resonance structure of data is richer than of the model

Future extension? Can it be pursued with 1dim histo fit?

No data for $\pi 0 \pi 0 \pi$ -!!!!

... and will be not available in near future.

Difference is with the sigma meson contribution fit SIGMA parameters to $\pi 0$ $\pi 0$ $\pi -$ BaBar data

$$F_{1}^{\mathrm{R}} \rightarrow F_{1}^{\mathrm{R}} + \frac{\sqrt{2}F_{V}G_{V}}{3F^{2}}\alpha_{\sigma}^{0}BW_{\sigma}(s_{3})F_{\sigma}(q^{2}, s_{3}),$$

$$F_{1}^{\mathrm{RR}} \rightarrow F_{1}^{\mathrm{RR}} + \frac{4F_{A}G_{V}}{3F^{2}}\frac{q^{2}}{q^{2}-M_{a_{1}}^{2}-iM_{a_{1}}\Gamma_{a_{1}}(q^{2})}\gamma_{\sigma}^{0}BW_{\sigma}(s_{3})F_{\sigma}(q^{2}, s_{3}).$$

$$\boldsymbol{\pi}+\boldsymbol{\pi}-\boldsymbol{\pi}- \qquad \alpha_{\sigma}=\beta_{\sigma}, \ \gamma_{\sigma}=\delta_{\sigma}$$

$$\alpha_{\sigma}=1.139486, \ \gamma_{\sigma}=0.889769, \ R_{\sigma}=0.000013, \ M_{\sigma}=0.550 \quad \Gamma_{\sigma}=0.700.$$

$$\alpha_{\sigma}^{0}=\alpha_{\sigma}\cdot Scaling_{factor}^{\gamma}$$

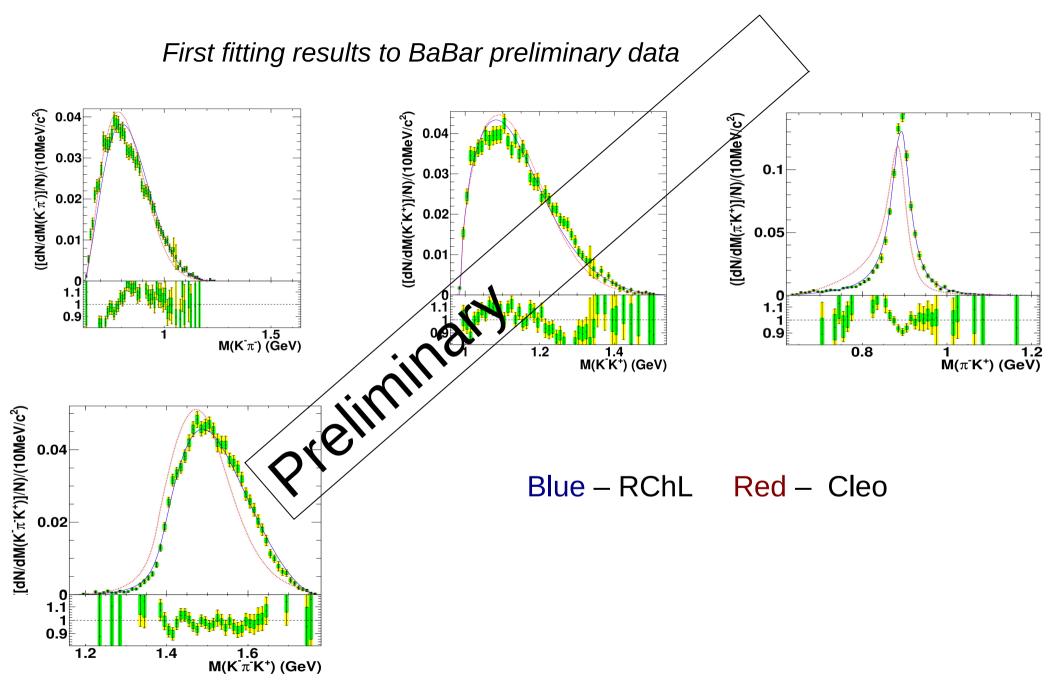
$$\mathbf{CLEO}$$

$$Scaling_{factor}^{\gamma}=2.1/3.35=0.63$$

$$\Gamma=(2.1440\pm0.02\%)\cdot10^{-13}$$

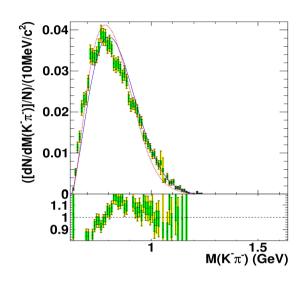
2.1% higher than the PDG value

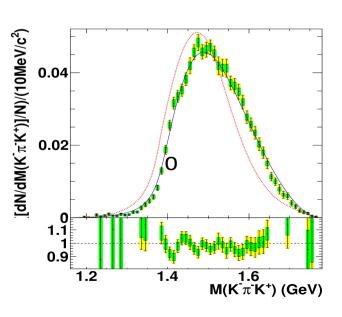
$\tau^- \rightarrow K^+ K^- \pi^- \nu$

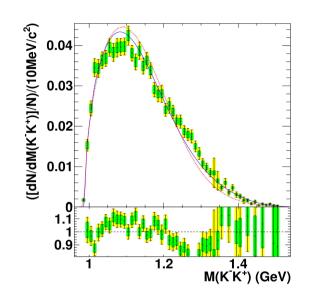


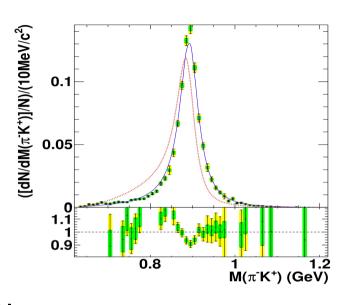
$\tau^- \rightarrow K^+ K^- \pi^- \nu$

First fitting results to BaBar preliminary data









Blue – RChL Red – Cleo some parameters on their limits ...

- * generalization of 3 pion fit strategy
- * in contrary to 3 pion case, no discussion of experimental systematic errors yet
- * the a1 width table corresponds to 3pion parameter values, not re-tabulated

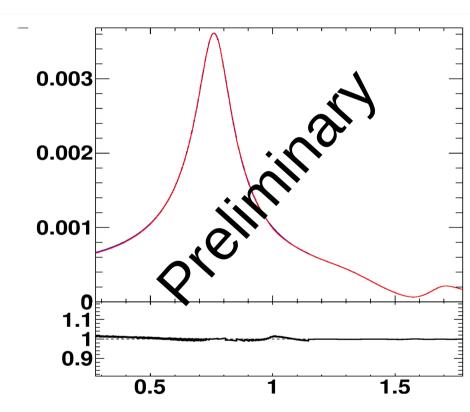
... common fit to $\pi^+\pi^-\pi^-$ and $K^+K^-\pi^-$

$$\tau \rightarrow \pi^0 \pi^- V$$

Two meson modes:

$$J^{\mu} = N [(p_1 - p_2)^{\mu} F^V(s) + (p_1 + p_2)^{\mu} F^S(s)]$$

$$= 0 \text{ (for } \pi^o \pi)$$



Vector FF: D.Gomez Dumm & P. Roig

s> 1.35 GeV 2

$$F_{V}^{\pi}(s) = \frac{M_{\rho}^{2} + (\alpha' e^{i\phi'} + \alpha'' e^{i\phi''}) s}{M_{\rho}^{2} \left[1 + \frac{s}{96\pi^{2}F_{\pi}^{2}} \left(A_{\pi}(s) + \frac{1}{2}A_{K}(s)\right)\right] - s} - \frac{\alpha' e^{i\phi'} s}{M_{\rho'}^{2} \left[1 + s C_{\rho'} A_{\pi}(s)\right] - s} - \frac{\alpha'' e^{i\phi''} s}{M_{\rho''}^{2} \left[1 + s C_{\rho''} A_{\pi}(s)\right] - s}$$

s< 1.35 GeV 2

$$F_V^{\pi}(s) = \exp\left[\alpha_1 s + \frac{\alpha_2}{2} s^2 + \frac{s^3}{\pi} \int_{s_{\text{thr}}}^{\infty} ds' \frac{\delta_1^1(s')}{(s')^3 (s' - s - i\epsilon)}\right]$$

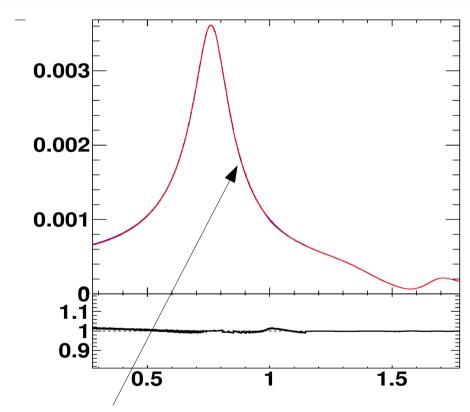
Fit to Belle analytical FF

$$\tau \rightarrow \pi^0 \pi^- \nu$$

Two meson modes:

$$J^{\mu} = N [(p_1 - p_2)^{\mu} F^V(s) + (p_1 + p_2)^{\mu} F^S(s)]$$

$$= 0 \text{ (for } \pi^0 \pi)$$



Vector FF: D.Gomez Dumm & P. Roig

s> 1.35 GeV ²

$$F_{V}^{\pi}(s) = \frac{M_{\rho}^{2} + (\alpha' e^{i\phi'} + \alpha'' e^{i\phi''}) s}{M_{\rho}^{2} \left[1 + \frac{s}{96\pi^{2} F_{\pi}^{2}} \left(A_{\pi}(s) + \frac{1}{2} A_{K}(s)\right)\right] - s} - \frac{\alpha' e^{i\phi'} s}{M_{\rho'}^{2} \left[1 + s C_{\rho'} A_{\pi}(s)\right] - s} - \frac{\alpha'' e^{i\phi''} s}{M_{\rho''}^{2} \left[1 + s C_{\rho''} A_{\pi}(s)\right] - s}$$

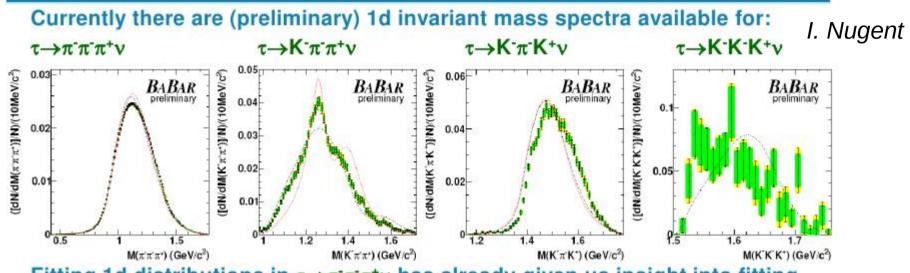
s< 1.35 GeV ²

$$F_V^{\pi}(s) = \exp\left[\alpha_1 s + \frac{\alpha_2}{2} s^2 + \frac{s^3}{\pi} \int_{s_{\text{thr}}}^{\infty} ds' \frac{\delta_1^1(s')}{(s')^3 (s' - s - i\epsilon)}\right]$$
$$\tan \delta_1^1(s) = \frac{\Im m F_V^{\pi(0)}(s)}{\Re e F_V^{\pi(0)}(s)}$$

fit to the Belle analytical function

... fit to the Belle data; study of the results

Conclusion and plans

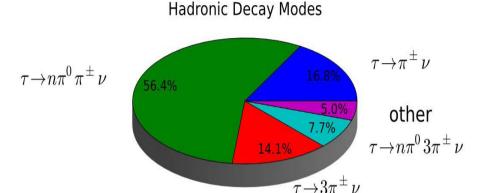


Fitting 1d distributions in $\tau \to \pi^-\pi^-\pi^+\nu$ has already given us insight into fitting models of low energy QCD (RCHL):

- Information on missing resonances
- Problems and with multi-dimensional fitting data provided by collaborations
- * 1d projection → multi-dimentional fit for 3 pion mode
- * KK π mode fit
- 2 pion RChL current fit to Belle data
- 4 pion RChL current in Tauola and fit to BaBar data
- * general fitting strategy for 3 meson modes

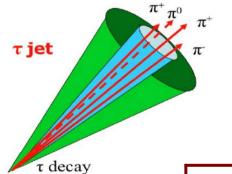
Backup Slides

The tau invariant mass distributions are essential to update the simulation of tau decays for LHC, SuperB factories...



Tau Lepton Properties

- m_{τ} = 1.78 GeV \longrightarrow decays in hadrons
- cτ = 87 μm
- BR($\tau \to l\nu\nu$) = 35.2%
 - BR(τ —hadrons) = 64.8%



Typical detector signature

- one or three charged tracks
- collimated calorimeter energy deposits
- large leading track momentum fraction
- possible secondary vertex reconstruction

<u>Precise analysis of available data</u> <u>for 2 pion + 3 pion modes</u>

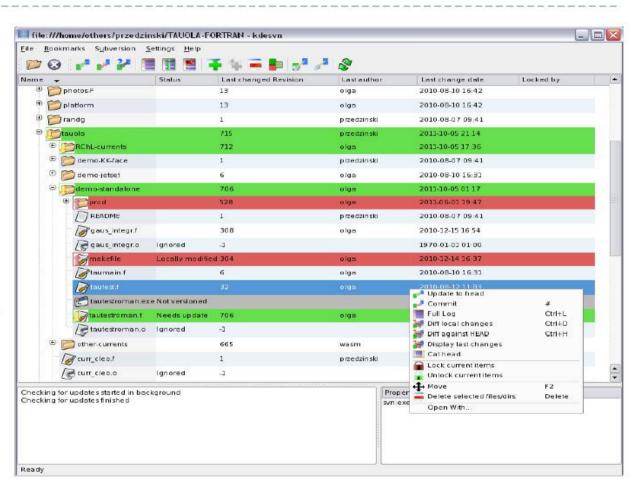
BaBar / Belle

Knowledge of the dynamics is important for Higgs polarization measurement and agreement MC/data, searched for beyond SM physics

TAUOLA repository

Code management

- SVN revision control system
 - displaying recent changes
 - branching different approaches
 - tagging milestones and stable revisions
 - when bug is found –"blame" to checkwho and when
 - GUI: kdesvn



Z. Was, T. Przedzinski, O.Shekhovtsova

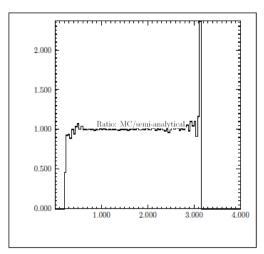
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Technical tests to check stability of MC

for every mode

- * to check phase space MC / analytical F1 = 1, others = 0
- * MC / analytical for total width
- * MC / analytical result for qq spectrum
- * analytical result (Gauss integration) compared with linear interpolated spectrum

An example: three pions $(\tau \to \pi^-\pi^-\pi^+\nu_\tau)$:



$$F_1 = F$$
, $F_{others} = 0$ to check phase space

$$F_1 = physical, F_{others} = 0$$

-
$$F_{all}$$
 = physical

linear interpolation $\sim 0.1\%$ for whole spectrum except for ends

MC (6e6):
$$(2.1013\pm0.016\%)\cdot10^{-13}$$
GeV;

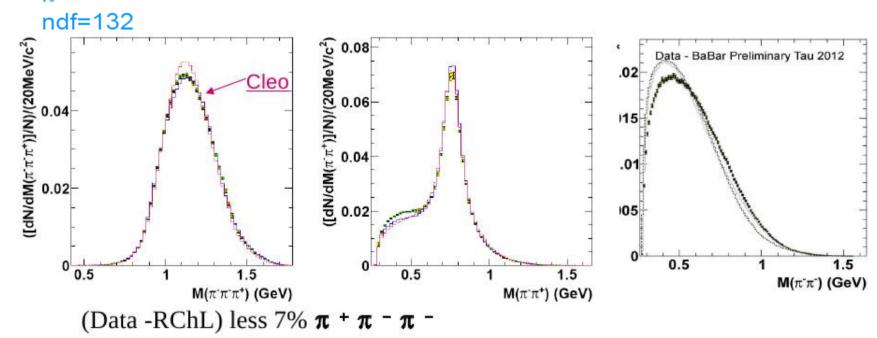
... fit for $\pi - \pi - \pi +$ to BaBar data

Fit to BaBar 20Mev/bin $\pi - \pi - \pi + \text{data (SLAC-R-936)}$

non trivial dynamics, the simplest from the 3 hadronic modes

Fit Parameters											
	M _ρ ,	$\Gamma_{\rho'}$	M _{a1}	F	F_{\lor}	F _A	$\beta_{\rho'}$				
Min.	1.44	0.32	1.00	0.0920	0.12	0.10	-0.36				
Max.	1.48	0.39	1.24	0.0924	0.24	0.20	-0.18				
Default	1.453	0.40	1.12	0.0924	0.18	0.149	-0.25				
Fit	1.4302	0.376061	1.21706	0.092318	0.121938	0.11291	-0.208811				

 χ^2 = 2262.12



(Data -RChL) less 12% π + π -

main contribution from low energy two pion inavariant mass region!!

RChL without extension cannot describe the data

Three meson modes the widths of the resonances:

$$\begin{split} \Gamma_{\rho}(q^2) &= \frac{M_{\rho}q^2}{96\pi F^2} \bigg[\sigma_{\pi}^3(q^2)\theta(q^2 - 4m_{\pi}^2) + \frac{1}{2}\sigma_{K}^3(q^2)\theta(q^2 - 4m_{K}^2) \bigg] \\ \Gamma_{\rho'}(q^2) &= \Gamma_{\rho'} \frac{q^2}{M_{\rho'}^2} \frac{\sigma_{\pi}^3\left(q^2\right)}{\sigma_{\pi}^3\left(M_{\rho'}^2\right)} \,\theta\left(q^2 - 4m_{\pi}^2\right) \\ \sigma_{P}(q^2) &\equiv \sqrt{1 - 4m_{P}^2/q^2} \end{split} \qquad \qquad \\ SU(2) \ \text{limit} \\ m_{\pi^2} = m_{\pi^0} \\ m_{K^2} = m_{K^0} \end{split}$$

a, resonance:

$$\Gamma_{a_1}(q^2) = 2\Gamma_{a_1}^{\pi}(q^2)\theta \left(q^2 - 9m_{\pi}^2\right) + 2\Gamma_{a_1}^{K^{\pm}}(q^2)\theta \left(q^2 - (m_{\pi} + 2m_K)^2\right) + \Gamma_{a_1}^{K^0}(q^2)\theta \left(q^2 - (m_{\pi} + 2m_K)^2\right)$$

$$\Gamma_{a_{1}}^{\pi,K}(q^{2}) = \frac{-S}{192(2\pi)^{3}F_{A}^{2}F^{2}M_{a_{1}}} \left(\frac{M_{a_{1}}^{2}}{q^{2}} - 1\right)^{2} \qquad V_{i}^{\mu} = c_{i}T^{\mu\nu}(p_{j} - p_{k})_{\nu}, \quad i \neq j \neq k = 1, 2, 3$$

$$\int dsdt \left(V_{1}^{\mu}F_{1} + V_{2}^{\mu}F_{2} + V_{3}^{\mu}F_{3}\right)^{\pi,K} \left(\left(V_{1\mu}F_{1} + V_{2\mu}F_{2} + V_{3\mu}F_{3}\right)^{\pi,K}\right)^{*} \qquad S = 1/n!$$

a1 width ($\Gamma_{a_1}(q^2)$) is tabulated to avoid problem with triple integration, linear interpolation

TAUOLA update, main test done, results PRD Phys.Rev. D86 (2012) 113008