Resonance Chiral Lagrangians and alternative approaches to hadronic tau decays

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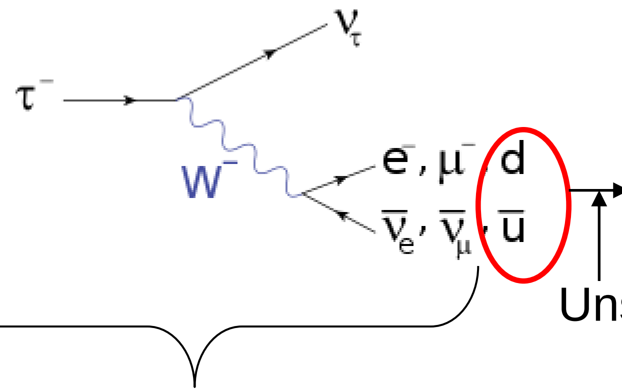
INTRODUCTION



Clean and under control

Hadronize to different final meson states

Unsolved problem

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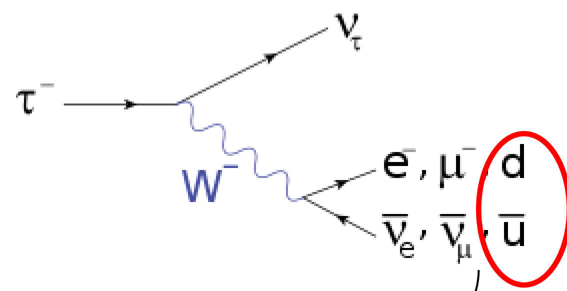

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$$H_\mu = \left\langle H \left| (\mathcal{V}_\mu - \mathcal{A}_\mu) e^{iL_{QCD}} \right| 0 \right\rangle = \sum_i \underbrace{(\ldots\ldots\ldots)^i_\mu}_{\text{Lorentz structure}} \overbrace{F_i\left(q^2, \ldots\right)}^{\text{Form Factor}}$$

Only decay constant for ***H*** = π or K

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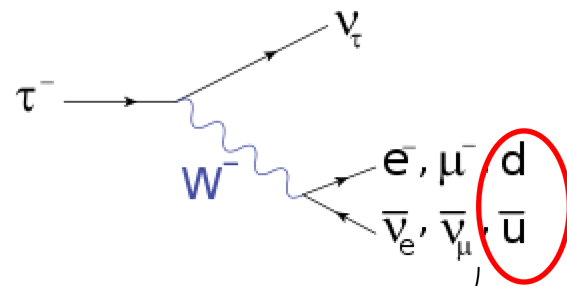

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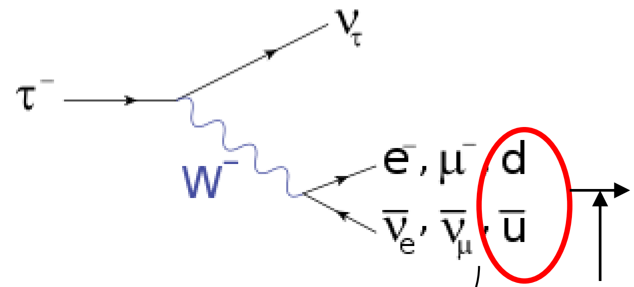

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→ $F_i\left(q^2, \ldots\right)$ →

Data analysis using **MC Generators**: TAUOLA Was et. al. (1990,1993)

See next talks of Zbigniew and Olga (TAUOLA), Philip (PYTHIA8) and Holger (Sherpa and Herwig)

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Of interest both for **theorist and experimentalists**; for **flavor factories** and for **high-E** applications!!

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Resonances

Masses

Widths

Couplings

$\Rightarrow$ **PDG**

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<u>Pole positions should be used to avoid model dependence</u>

(although potentially problematic cases, like K_{1A})

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Generalized Kühn-Santamaría model (GKS)

[Kühn, Santamaría, 1990] [Gounaris, Sakurai, 1968]

1) χ**PT** $\mathcal{O}(p^2)$

2) $F(q^2, s, \ldots) = f(\alpha_i, BW_i)$

$$BW_R^{KS} = \frac{M_R^2}{M_R^2 - s - i\sqrt{s}\,\Gamma_R(s)}$$

$$BW_R^{GS} = \frac{M_R^2 + d\cdot M_R\Gamma_R\left(M_R^2\right)}{M_R^2 - s + f(s) - i\sqrt{s}\,\Gamma_R(s)}$$

Ex.

$$F_V(s) = \frac{BW_\rho\left(\frac{1+\alpha\, BW_\omega}{1+\alpha}\right) + \beta\, BW_{\rho'} + \gamma\, BW_{\rho''} + \ldots}{1+\beta+\gamma+\ldots}$$

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[Kühn, Santamaría, 1990] [Gounaris, Sakurai, 1968]

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[Gómez Dumm, Pich, Portolés, 2004]

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RχL intend to interpolate between the two known extreme regimes of **QCD**:
χPT for E<<M_ρ and the **OPE** of QCD for E>>M_ρ.

Ecker et. al. ’89, Ruiz-Femenía et. al. ’03, Gómez-Dumm et. al. ’04, Cirigliano et. al. ’04, ’05, ’06, Kampf and Novotny ’11, Roig and Sanz-Cillero ’14.

Moussallam ’95, ’97, Knecht et. al. ’97, ‘99, Bijnens ’03.

INTRODUCTION

RχL intend to interpolate between the two known extreme regimes of **QCD**:
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→ Complementary aproach: Using **dispersion relations** to obtain the form factors. In this way, **analyticity and unitarity** are automatically fulfilled and the poorest known (**high-E**) **region** is **suppressed by** the **subtractions** of the dispersive integrals. Consequently, results are also **less sensitive to** the precise **short-distance** QCD **constraints.**

These ideas will be quickly illustrated with the two-pion vector form factor

$$\tau^- \to \pi^-\pi^0\nu_\tau$$

$$\left\langle \pi^-(p)\ \pi^0(p')\left|\bar{d}\gamma_\mu u\right|0\right\rangle = \sqrt{2}\,(p-p')_\mu\, F_V(s)\ , \qquad s=(p+p')^2$$

<u>*Major present interest*</u> **: ~ 65% of $(g\text{-}2)_\mu$**

Example: Vector form factor of the pion

[Ecker et al, 1989]

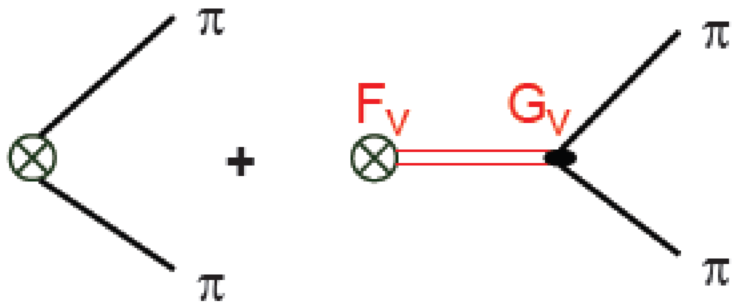
$$\mathcal{L}_{R\chi T} = \mathcal{L}_\chi^{(2)} + \mathcal{L}_V$$

$$\mathcal{L}_\chi^{(2)} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+ \rangle$$

$$\mathcal{L}_V = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + i\frac{G_V}{\sqrt{2}}\langle V_{\mu\nu} u^\mu u^\nu \rangle$$

Note : Antisymmetric tensor formulation (J=1)

$F_\pi(q^2) =$



$$= 1 + \frac{F_V G_V}{F^2}\frac{q^2}{M_V^2 - q^2}$$

Asymptotic behaviour of Form Factors

[Brodsky, Lepage, 1981]

$$F_\pi(q^2) \underset{q^2 \to \infty}{\sim} \frac{const.}{q^2}$$

$$F_V G_V = F^2$$

Vector form factor of the pion

Gómez-Dumm and Roig ‘13

$$F_V^\pi(s) = \exp\left[\alpha_1\, s + \frac{\alpha_2}{2}\, s^2 + \frac{s^3}{\pi}\int_{s_{\rm thr}}^\infty ds' \,\frac{\delta_1^1(s')}{(s')^3(s'-s-i\epsilon)}\right]$$

$$\tan\delta_1^1(s) = \frac{\Im m F_V^{\pi(0)}(s)}{\Re e F_V^{\pi(0)}(s)} \,,$$

$$F_V^{\pi\,(0)}(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_\pi(s)+\frac{1}{2}A_K(s)\right)\right]-s}$$

$$= \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\Re e\left(A_\pi(s)+\frac{1}{2}A_K(s)\right)\right]-s-iM_\rho\Gamma_\rho(s)} \,,$$

$$\Gamma_\rho(s) = -\frac{M_\rho\, s}{96\,\pi^2\,F_\pi^2}\Im m\left[A_\pi(s)+\frac{1}{2}A_K(s)\right] \,,$$

$$\Gamma_\rho(s) = \frac{s\,M_\rho}{96\,\pi\,F_\pi^2}\left[\theta\left(s-4m_\pi^2\right)\,\sigma_\pi^3(s) + \frac{1}{2}\,\theta\left(s-4m_K^2\right)\,\sigma_K^3(s)\right]$$

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Analogous constructs have been employed in Pich & Portolés ’01, Bernard et. al. ’07, ’09, ’10, ‘13, Escribano et. al. ’08, ’10, ’13, ’14 (also for Kπ & kη), Celis et. al. ’13.

→ <u>Improved determination of K*(1410) pole parameters</u>

Dispersive form factors for Kππ modes have been worked out by Moussallam ’08.

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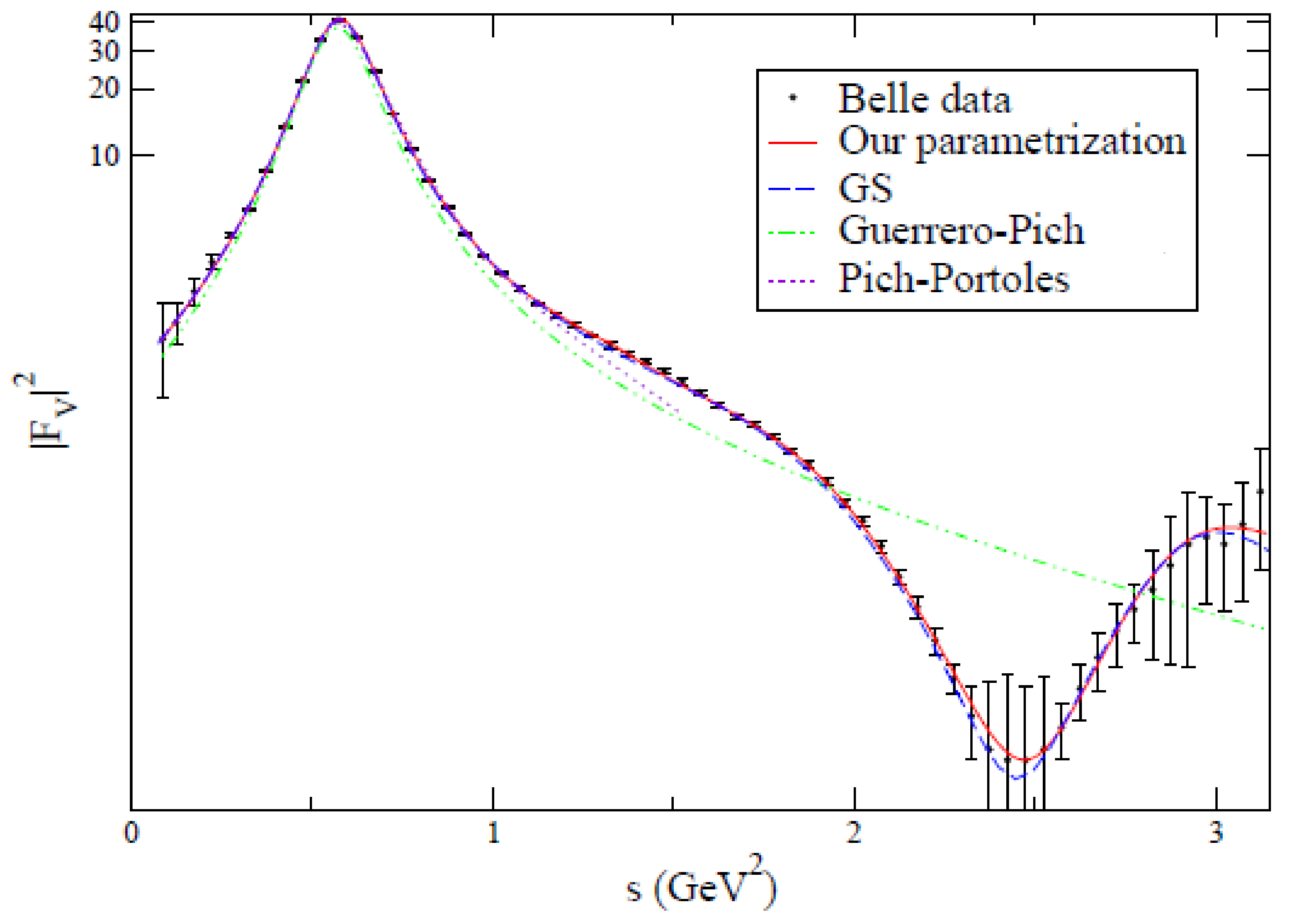

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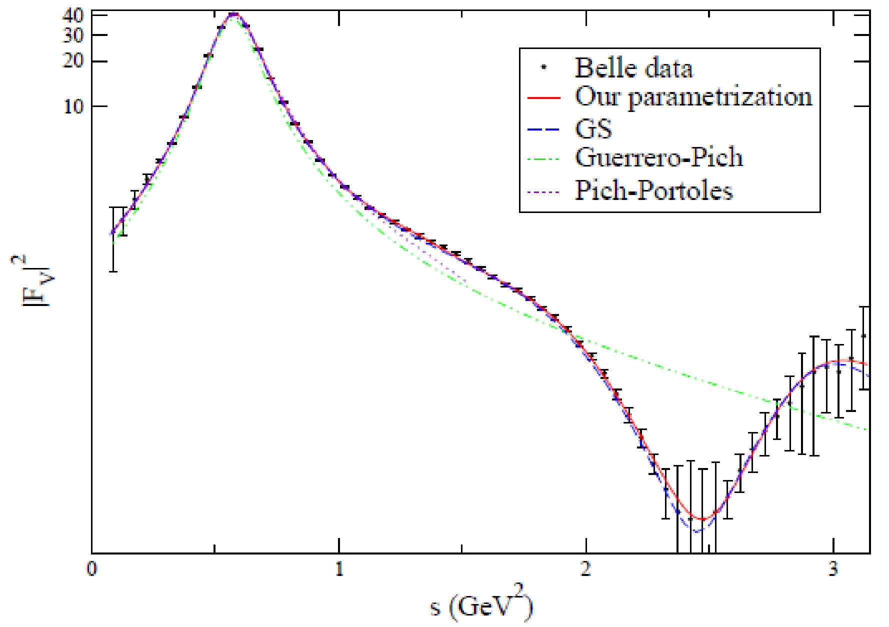

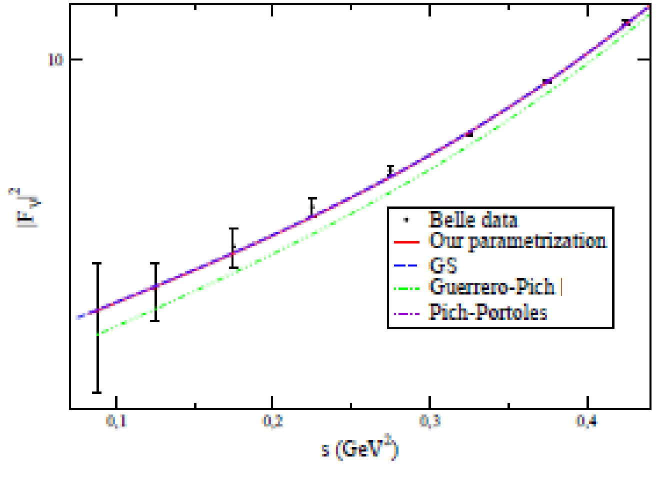

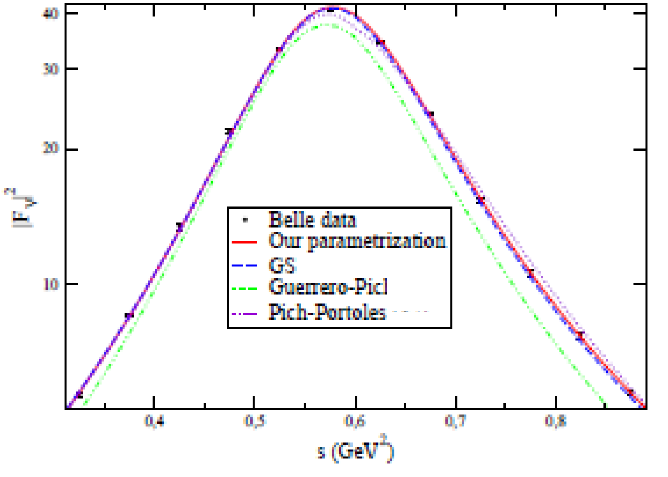

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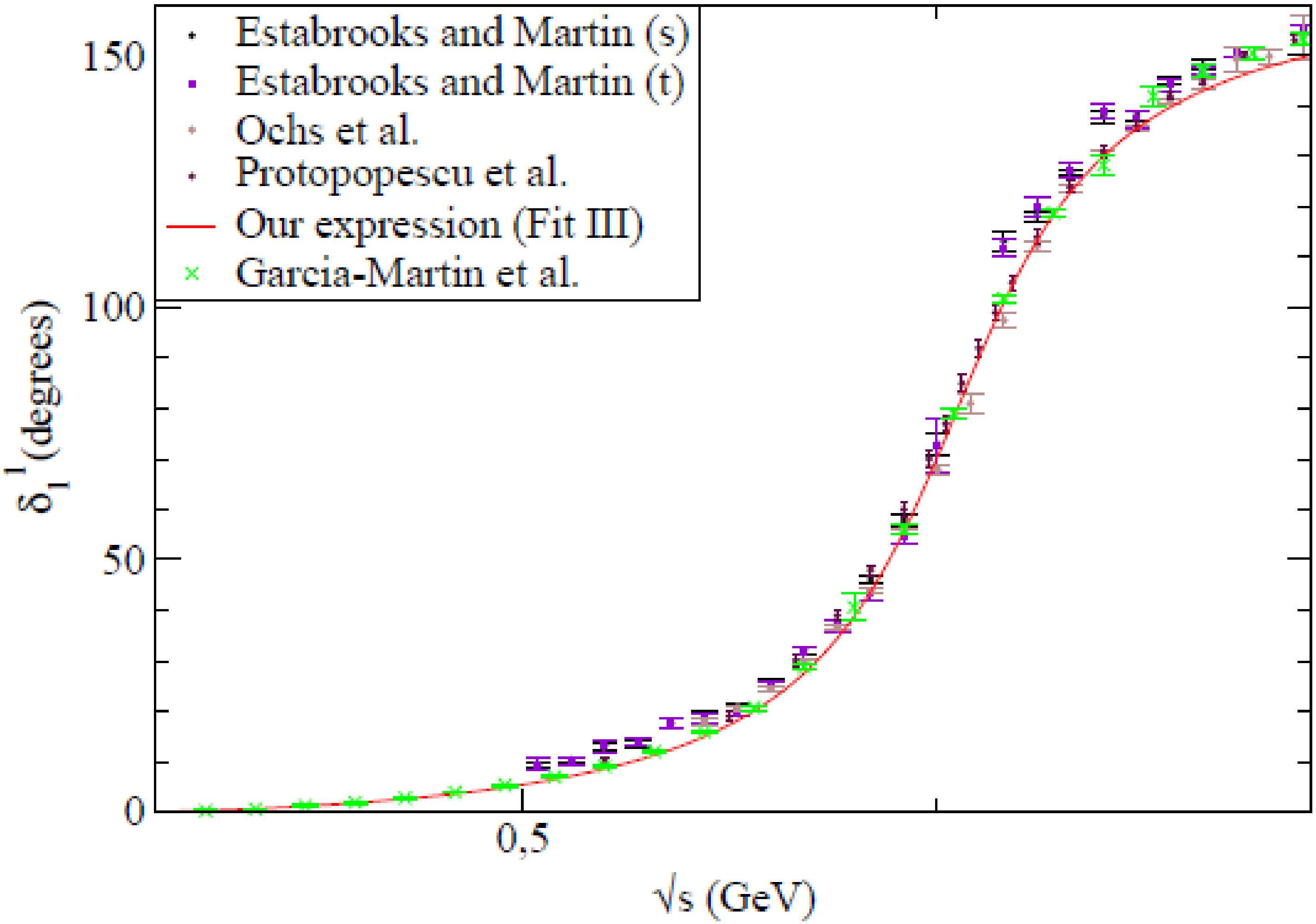

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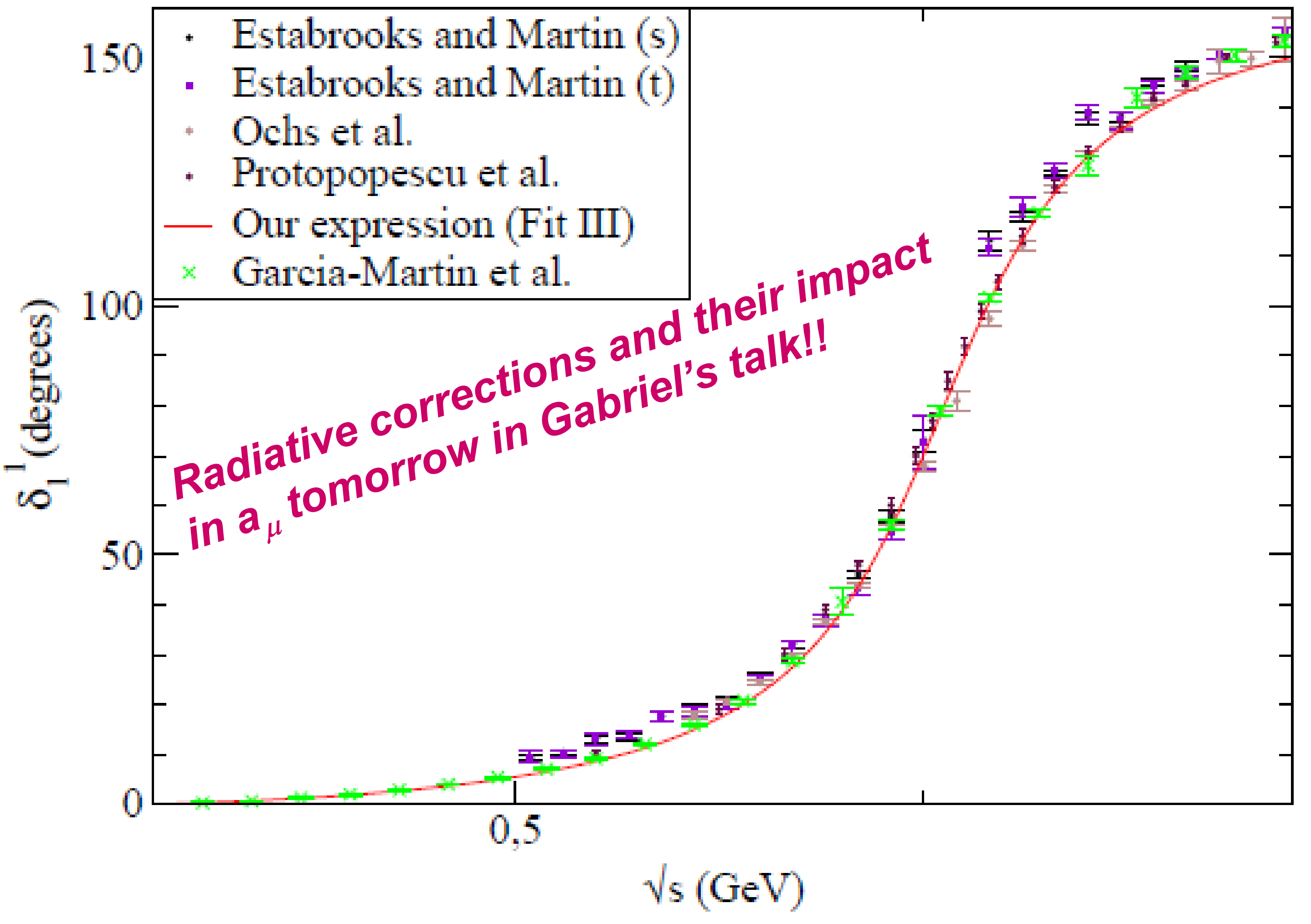

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$$\sqrt{s_{\mathrm{pole}}} = M_\rho^{\mathrm{pole}} - \frac{i}{2}\Gamma_\rho^{\mathrm{pole}}$$

Reference	M_ρ^{pole}	$\Gamma_\rho^{\mathrm{pole}}$	Data	Analysis
Sanz-Cillero *et al.* [36]	$764.1^{+4.8}_{-3.7}$	$148.2^{+2.5}_{-6.2}$	τ & e^+e^-	DSE
Ananthanarayan *et al.* [70]	762.5 ± 2	142 ± 7	$\pi\pi \to \pi\pi$	RE
Feuillat *et al.* [71]	758.3 ± 5.4	145.1 ± 6.3	τ & e^+e^-	SMA
Peláez [72]	754 ± 18	148 ± 20	$\pi\pi \to \pi\pi$	UχPT
Zhou *et al.* [73]	763.0 ± 0.2	139.0 ± 0.5	$\pi\pi \to \pi\pi$	χU
Masjuan *et al.* [65]	763.7 ± 1.2	144 ± 3	τ	RA
Results from our fit I	759 ± 2	146 ± 6	τ	DR
Results from our fit III	760 ± 2	147 ± 6	τ	DR
Results from GS model	760.9 ± 0.6	142.2 ± 1.6	τ	GS

Table 2: Comparison between different results for the pole mass and width of the $\rho(770)$ meson (values are in MeV). Abbreviations for the type of analysis carried out are DSE: Dyson-Schwinger equations; RE: Roy equations; SMA: S matrix approach; UχPT: Unitarized Chiral Perturbation Theory; χU: Chiral unitarization; RA: Rational approximants; DR: Dispersive representation; GS: Gounaris-Sakurai parametrization.

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Three meson modes

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See Olga's talk next !!

Multi-meson modes

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• For **higher multiplicity modes**, the biggest challenge is to go beyond Ecker and Zauner's '07 parametrization in $\pi\pi\pi\pi$ mode.

•For **multi-meson modes** there are parametrizations from **GKS** (Kühn & Was '06) and **isospin relations** (Rougé '96, '98 and Sobie '95, '99), but experiment goes ahead.

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- RχL and dispersion relations are the best theoretical approaches to deal with hadronization in these decays.
- Two-meson decay modes are well understood, so the problem is now on radiative corrections.
- Among multi-meson modes, only the $\pi\pi\pi$ channels are described satisfactorily. Dedicated effort is needed from the collaboration between experiment & theory through MC Gens, specially with forthcoming flavor facilities in mind.