Towards a determination of the tau lepton dipole moments

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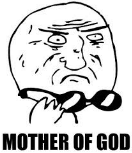
hep-ph: 1310.1081

work in collaboration with:
S. Eidelman, D. Epifanov, L. Mercolli, M. Passera.

Problem:



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$$a_e = 1\,159\,652\,180.\,73\,(28) \cdot 10^{-12}$$

0.24 parts per billion! Hanneke et al, PRL100 (2008) 120801

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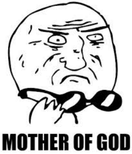

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0.54 parts per million! E821–Final Rep: PRD73 (2006) 072003

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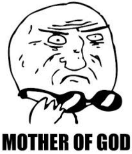

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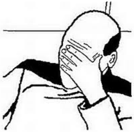


Tau

$$-0.052 \leq a_\tau \leq 0.013$$

Not even a test of LO! $\alpha/(2\pi) \approx 0.00116$
DELPHI - EPJC 35 (2004) 159

Outline

$g-2$ and EDM

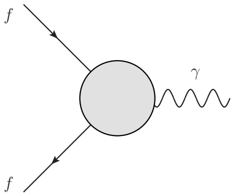
Radiative Leptonic Decays

Feasibility study

Conclusions

Form Factors & Dipole Moments

Vertex Function:



$$\Gamma^{\mu}_{ff\gamma}\left(q^2\right) = \gamma^{\mu}\left[\; F_{1\,V}\left(q^2\right) \;+\gamma_5\, F_{1\,A}(q^2)\right]$$
$$+\frac{\sigma^{\mu\nu}}{2m_f}q_{\nu}\left[\; i\,F_{2\,V}\left(q^2\right) \;+\; F_{2\,A}\left(q^2\right)\,\gamma_5 \;\right]$$

Form Factors & Dipole Moments

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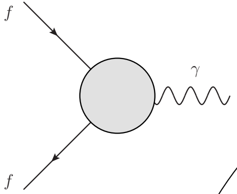

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- Charge: $F_{1\,V}\left(q^2 = 0\right) = Q_f$

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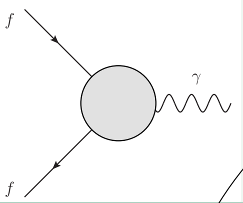

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- $g-2$: $F_{2V}\left(q^2 = 0\right) = a_f\, Q_f$

Form Factors & Dipole Moments

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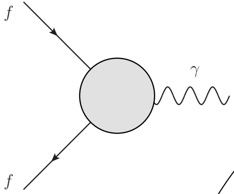

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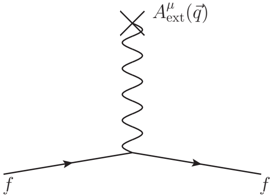
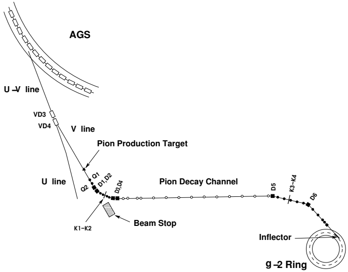
- $g-2$: $F_{2V}\left(q^2 = 0\right) = a_f Q_f$
- EDM: $F_{2A}\left(q^2 = 0\right) = d_f(2m_f)/e$

Direct and Indirect Measurement

- Direct measurements





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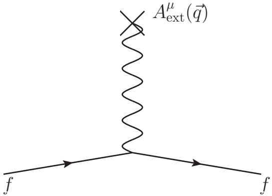
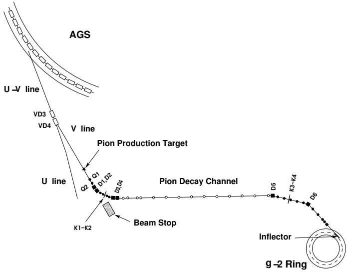
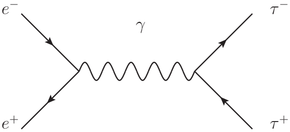
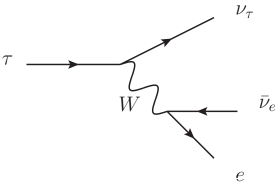




- Indirect measurements



$$q^2 \neq 0$$



Effective Lagrangian Approach

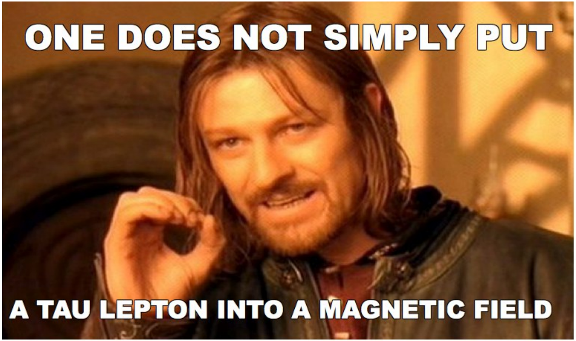
$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \left[c_a \frac{eQ_\tau}{4\Lambda} \overline{\tau} \sigma^{\mu\nu} \tau - c_d \frac{i}{2\Lambda} \overline{\tau} \sigma^{\mu\nu} \gamma_5 \tau \right] F_{\mu\nu}$$

- The new terms arise after integrating out the heavy degrees of freedom associated to possible NP.

$$a_\tau = a_\tau^{\text{SM}} + c_a \frac{m_\tau}{\Lambda} + \ldots \qquad d_\tau = d_\tau^{\text{SM}} + c_d \frac{1}{\Lambda} + \ldots$$

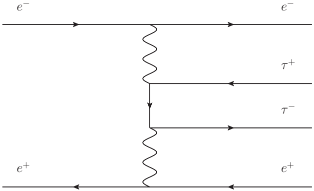
- The contributions from the two effective operators to a_τ and d_τ are the same for $q^2 = 0$ and $q^2 \neq 0 \ll \Lambda^2$.
- Only higher dimensional operators would give rise to a difference between these two cases

Radiative Leptonic Decays



Current bounds

- $e^+e^- \to e^+e^-\tau^+\tau^-$ at $\sqrt{s}$ between 183 and 208 GeV at LEP2 (the PDG value)
- The 95% C.L. limit



$$-0.052 \leq a_\tau \leq 0.013$$ DELPHI - EPJC35 (2004) 159

- EDM SM estimate $|d_\tau^{\text{SM}}| \leq 10^{-35} e \cdot \text{cm}$!
- EDM current 95% C.L. limits from $e^+e^- \to \tau^+\tau^-$:

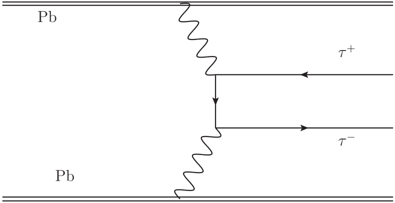
$$-2.2 \leq \text{Re}(d_\tau) \leq 4.5\,(10^{-17} e \cdot \text{cm})$$
$$-2.5 \leq \text{Im}(d_\tau) \leq 0.8\,(10^{-17} e \cdot \text{cm})$$

Belle coll. PLB 551 (2003) 16.

Determination of a_τ: Proposals

- Heavy ions collisions at LHC

$$\mathrm{Pb\ Pb} \to \mathrm{Pb\ Pb}\ \gamma\gamma \to \mathrm{Pb\ Pb}\ \tau\tau$$



F. del Aguila et al, PLB 271 (1991) 256

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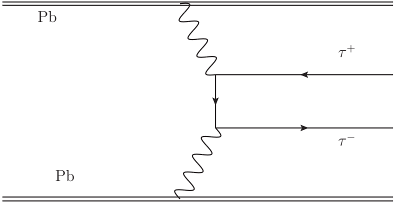
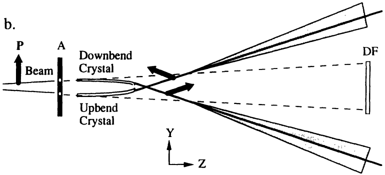


F. del Aguila et al, PLB 271 (1991) 256

- Channeling of polarized τs in a bent crystal.



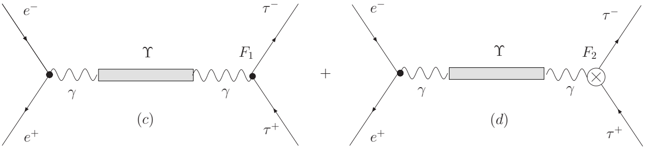
Chen et al. PRL 69 (1992) 3286

Samuel et al. PRL 67 (1991) 668

Determination of a_τ: Proposals

Bernabéu et al. propose the measurement of $F_{2V}(q^2 = M_\Upsilon^2)$ from $e^+e^- \to \Upsilon \to \tau^+\tau^-$ production at B factories.

Bernabéu et al. NPB 790 (2008) 160.



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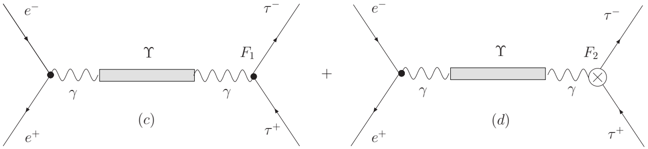
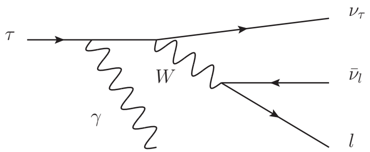


- Expected sensitivity for Babar+Belle: $4.6 \cdot 10^{-6}$ with $\mathcal{L} = 2$ ab^{-1}.
- At Belle and Belle II the visible cross section is dominated by non-resonant interaction due to the beam energy spread.

$$\Gamma_{\Upsilon(1S),\Upsilon(2S),\Upsilon(3S)} \sim \mathcal{O}(10 \text{ keV}), \quad \sigma_\varepsilon \sim 3 \text{ MeV}$$

Leptonic Radiative Decays of the Tau



$$\tau^{\pm} \to \gamma\, l^{\pm}\, \nu_\tau \nu_l$$

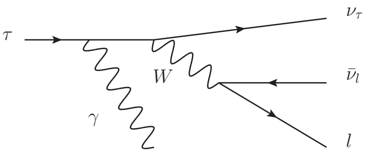
with $l = e, \mu$

- Suggested by Laursen et al. to search for the a_τ in radiative leptonic τ decays using the phenomenon of radiation zero.
- The SM tree-level amplitude vanishes in the phase space region:

$$\cos(l, \gamma) = -1, \quad E_l = \frac{m_\tau^2 + m_l^2}{2 m_\tau} \qquad \text{(in the tau r.f.)}$$

M. L. Laursen et al. PRD 29 (1984) 2652

Leptonic Radiative Decays of the Tau



$$\tau^\pm \to \gamma\, l^\pm\, \nu_\tau \nu_l$$

with $l = e, \mu$

- Extend the strategy to d_τ,
- Probe at $\mathcal{O}(10^{-3})$ the parameters

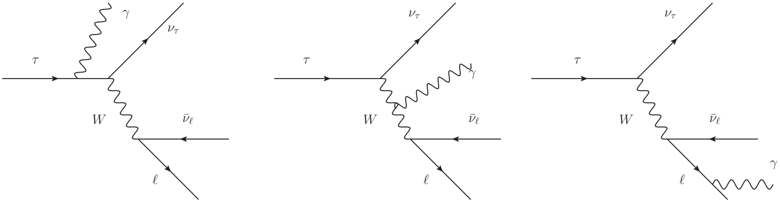
$$\tilde{a}_\tau \equiv c_a \frac{m_\tau}{\Lambda} \text{ and } \tilde{d}_\tau \equiv c_d \frac{1}{\Lambda}$$

- Provide the theoretical framework for such measurement.

QED NLO Corrections to Tau Radiative Decay

- Polarized differential decay rate $d\Gamma$ up to $\mathcal{O}(10^{-3})$.
- Dependence on E_l, E_γ, Ω_l and Ω_γ

$$d\Gamma = d\Gamma_{\text{LO}} + \left(\frac{m_\tau}{M_W}\right)^2 d\Gamma_W + \tilde{a}_\tau\, d\Gamma_{\text{a}} + \tilde{d}_\tau\, d\Gamma_{\text{d}} + \frac{\alpha}{\pi} d\Gamma_{\text{NLO}}$$



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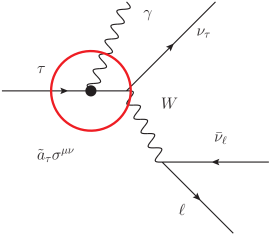

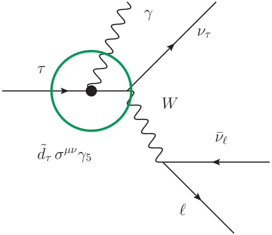

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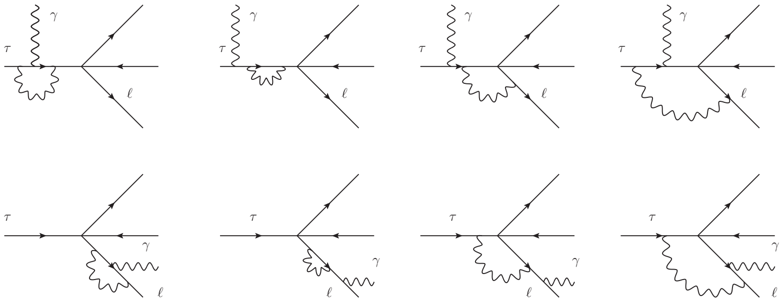

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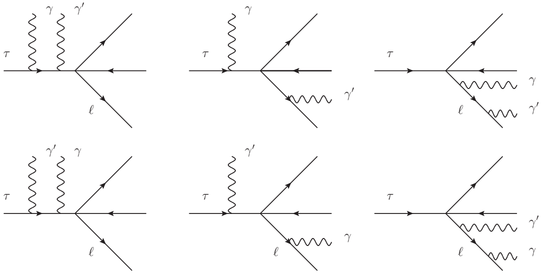

Branching Ratios: $E_\gamma > 10$ MeV

process	LO %	NLO %	MW %
$\tau^- \to e^- \bar{\nu}_e \nu_\tau \gamma$	1.836	-0.183	0.0006
$\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau \gamma$	0.367	-0.009	0.0001

process	B.R. %	exp. B.R. % (PDG)
$\mu^+ \to e^+ \nu_e \bar{\nu}_\mu \gamma$	1.3	1.4 (4) *
$\tau^- \to e^- \bar{\nu}_e \nu_\tau \gamma$	1.653	$1.75 \pm 0.06 \pm 0.17$ †
$\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau \gamma$	0.358	$0.361 \pm 0.016 \pm 0.035$ †

*Crittenden et al. PR 121 (1964) 1823, †CLEO Coll. PRL 84 (2000) 830

process	B.R. %	exp. B.R. %
$\tau^- \to e^- \bar{\nu}_e \nu_\tau \gamma$	1.653	$1.847 \pm 0.015 \pm 0.052$ *
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*BaBar preliminary results, B. Oberhof

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$3.6\,\sigma$

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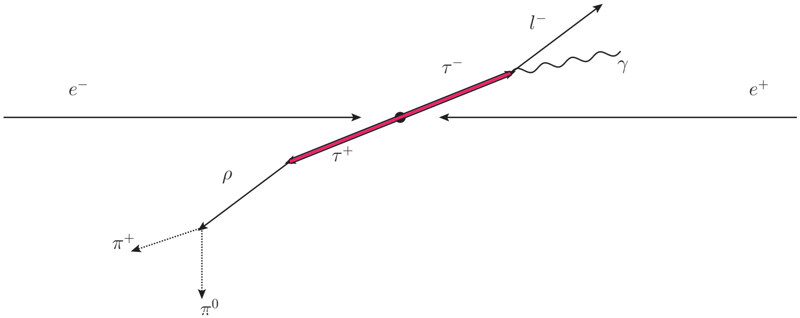
Feasibility study



Approach 1: Radiation Zero

- Study of the radiation zero point.
- A set of $\tau^+\tau^-$ events analyzed. $(\tau^\pm \to l_1^\pm \nu\nu\gamma, \tau^\mp \to l_2^\mp \nu\nu)$
- Final state: $(l_1^\pm \gamma, l_2^\mp)$ with $l_1, l_2 = e, \mu$ and $l_1 \neq l_2$

$$\cos(l_1, \gamma) < -0.9, \quad 0.1 < \cos(l_2, \gamma), \text{ and } E_\gamma > 0.5 \text{ GeV}$$

- With the whole Belle statistics $(0.9 \times 10^9\ \tau$ pairs),
  the upper $\tilde{a}_\tau$ upper bound is

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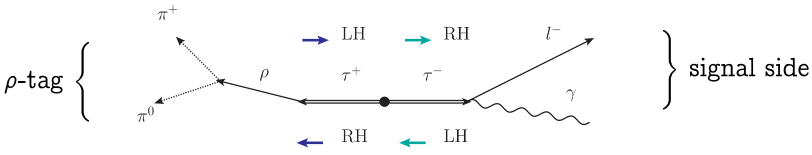
- With the whole Belle statistics (0.9×10^9 τ pairs), the upper $\tilde{a}_\tau$ upper bound is

$$\text{U.L. } (\tilde{a}_\tau) \simeq 2$$

Approach 2: Unbinned Maximum Likelihood

- Take advantage of $\tau^{\pm} \to \rho^{\pm}\nu \to \pi^{\pm}\pi^{0}\nu$ as a spin analyzer.
- In $\tau^{\mp} \to l^{\mp}\nu\nu\gamma$ we are sensitive to the spin dependent part.
- 12-dimensional phase space analysis $(l^{\mp}, \gamma, \pi^{\pm}, \pi^{0})$.



- Developed special generator of the $(l^{\mp}\nu\nu\gamma, \pi^{\pm}\pi^{0}\nu)$ events
- Fit the generated event samples corresponding to the amount of data available at Belle and expected at Belle II.

Results

- ρ-*tag* mode, BR= 25.5% — $\tau^\pm \to \rho^\pm \nu$ only
- *full-tag* mode, BR= 90% — $\tau^\pm \to \rho^\pm\nu, \tau^\pm \to \pi^\pm\nu, \tau^\pm \to \pi^\pm\pi^0\pi^0\nu,$ $\tau^\pm \to \pi^\pm\pi^+\pi^-\nu, \tau^\pm \to e^\pm\nu\nu, \tau^\pm \to \mu^\pm\nu\nu$

Sensitivity to $\tilde{a}_\tau$ and $\tilde{d}_\tau$

	$\mathrm{Re}(\tilde{a}_\tau)$	$\mathrm{Im}(\tilde{a}_\tau)$	$\mathrm{Re}(\tilde{d}_\tau)$	$\mathrm{Im}(\tilde{d}_\tau)$
Belle (ρ-tag)	0.16	0.16	0.15	0.046
Belle-II (ρ-tag)	0.023	0.023	0.021	0.007
Belle (full tag)	0.085	0.085	0.080	0.024
Belle-II (full tag)	0.012	0.012	0.011	0.003
DELPHI†	0.017	—	—	—
Belle*	—	—	0.0015	0.0008

†DELPHI - EPJC35 (2004) 159

*Belle coll. PLB 551 (2003) 16.

Summary and Outlook

- Radiative leptonic τ decays can probe τ DipM.
- We provided with the polarized differential decay rate at NLO in QED plus small W-boson effects.
  We found some discrepancies with previous results.
- The upper limit achievable at Belle via radiation zero phenomenon is only $a_\tau \sim 1$.
- Analysis in the full phase-space is required.
- Feasibility study shows that Belle II can ameliorate the current DELPHI result for a_τ.
- The extraction of τ DipM from $e^+e^- \to \tau^+\tau^-$ is not excluded.
  A careful theoretical reanalysis is needed.
- A possible dedicated experiment (bent crystal)?
- What are the prospects at the LHC?

Thanks!

Backup slides

QED NLO Corrections to Tau Radiative Decay

The total differential decay for a polarized τ lepton in the tau r.f. is

$$\frac{d^6\Gamma^{\text{NLO}}}{dx\, dy\, d\Omega_l\, d\Omega_\gamma} = \frac{\alpha\, G_F^2 m_\tau^5}{(4\pi)^6} \frac{x\beta}{1+\delta_{\text{W}}(m_\mu, m_e)} \Bigg[G(x,y,c)$$

$$+\; x\beta\, \hat{n}\cdot\hat{p}_l\, J(x,y,c) + y\, \hat{n}\cdot\hat{p}_\gamma\, K(x,y,c) + y\, x\beta\, \hat{n}\cdot(\hat{p}_l\times\hat{p}_\gamma)\; L(x,y,c)\Bigg]$$

where $x = 2E_l/m_\tau$, $y = 2E_\gamma/m_\tau$, $c = \cos\theta_{l\gamma}$. The tau polarization vector $n = (0, \vec{n})$ satisfies $n^2 = -1$ and $n\cdot p_\tau = 0$. The function $G(x,y,c)$, and similarly for J and K, is given by

$$G(x,y,c) = \frac{4}{3yz^2}\left[g_{\text{LO}}(x,y,z) + \frac{\alpha}{\pi}\, g_{\text{NLO}}(x,y,z;y_{\text{min}}) + \left(\frac{m_\tau}{M_W}\right)^2 g_{\text{W}}(x,y,z)\right]$$

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Compared with previous work A. B. Arbuzov PLB 597 (2004) 285

QED NLO Corrections to Tau Radiative Decay

- Polarized differential decay rate $d\Gamma$ up to $\mathcal{O}(10^{-3})$.
- Dependence on E_l, E_γ, Ω_l and Ω_γ

$$d\Gamma = d\Gamma_{\text{LO}} + \left(\frac{m_\tau}{M_W}\right)^2 d\Gamma_W + \tilde{a}_\tau \, d\Gamma_{\text{a}} + \tilde{d}_\tau \, d\Gamma_{\text{d}} + \frac{\alpha}{\pi} d\Gamma_{\text{NLO}}$$

- Analytic expression implemented in Fortran

```
subroutine gnlo(resgnlo)
      implicit none
      external NLOfunctions
      double precision resgnlo

                              ⋮
```

Feasibility study: Unbinned maximum likelihood

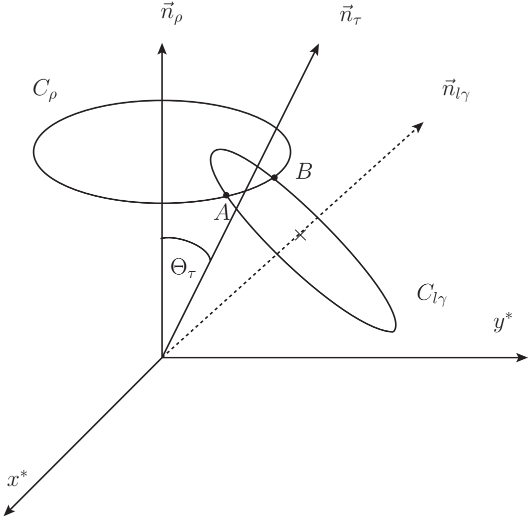
- Phase space point $X = (p_l, \Omega_l, p_\gamma, \Omega_\gamma, p_\rho, \Omega_\rho, m^2_{\pi\pi}, \tilde{\Omega}_\pi)$
- The PDF $\mathcal{P}(\vec{X})$ is constructed from the differential cross section

$$\frac{d\sigma}{d\mathrm{PS}}(e^+e^- \to \tau^\mp\tau^\pm \to (l^\mp\nu\nu\gamma, \pi^\pm\pi^0\nu))$$

- Unbinned maximum likelihood of the generated events

$$\mathcal{P}(\vec{X}|\tilde{a}_\tau, \tilde{d}_\tau) = \frac{\mathcal{F}_{\tilde{a}_\tau,\tilde{d}_\tau}(\vec{X})}{\int \mathcal{F}_{\tilde{a}_\tau,\tilde{d}_\tau}(\vec{X})d\vec{X}}$$

  where $\mathcal{F}_{\tilde{a}_\tau,\tilde{d}_\tau}(\vec{X})$ is the visible differential cross section.
- Developed special generator of the $(l^\mp\nu\nu\gamma, \pi^\pm\pi^0\nu)$ events
- Fit of generated event samples corresponding to the amount of data available at Belle and expected at Belle II.



- $\tau \to \rho\nu$: $(p_\tau - p_\rho)^2 = 0$
- $\tau \to \gamma\ell\nu\bar{\nu}$: $M_{\nu\bar{\nu}} > 0$