HFAG-Tau Report

with Theory Introduction for $|V_{us}|$

Alberto Lusiani
INFN and Scuola Normale Superiore
Pisa

(on behalf of the **HFAG-Tau group**)

HFAG-Tau sub-group

- since 2008, the Heavy Flavor Averaging Group (HFAG) includes a Tau sub-group
- mandate
  - compute tau world averages following HFAG prescriptions
  - make best use of often systematic-dominated B-factories' results
- members
  - **Swagato Banerjee** (Univ. of Victoria, BaBar)
  - **Kiyoshi Hayasaka** (Nagoya University, Belle)
  - **Hisaki Hayashii** (Nara Women's University, Belle)
  - **A. L.**, convener
  - **J. Michael Roney** (Univ. of Victoria, BaBar)
  - **Boris Shwartz** (Budker Institute of Nuclear Physics, Belle)
- this report: collaboration of **Marcin Chrząszcz** [University of Zurich (CH), Polish Academy of Sciences (PL)]
- `http://www.slac.stanford.edu/xorg/hfag/org/index.html`

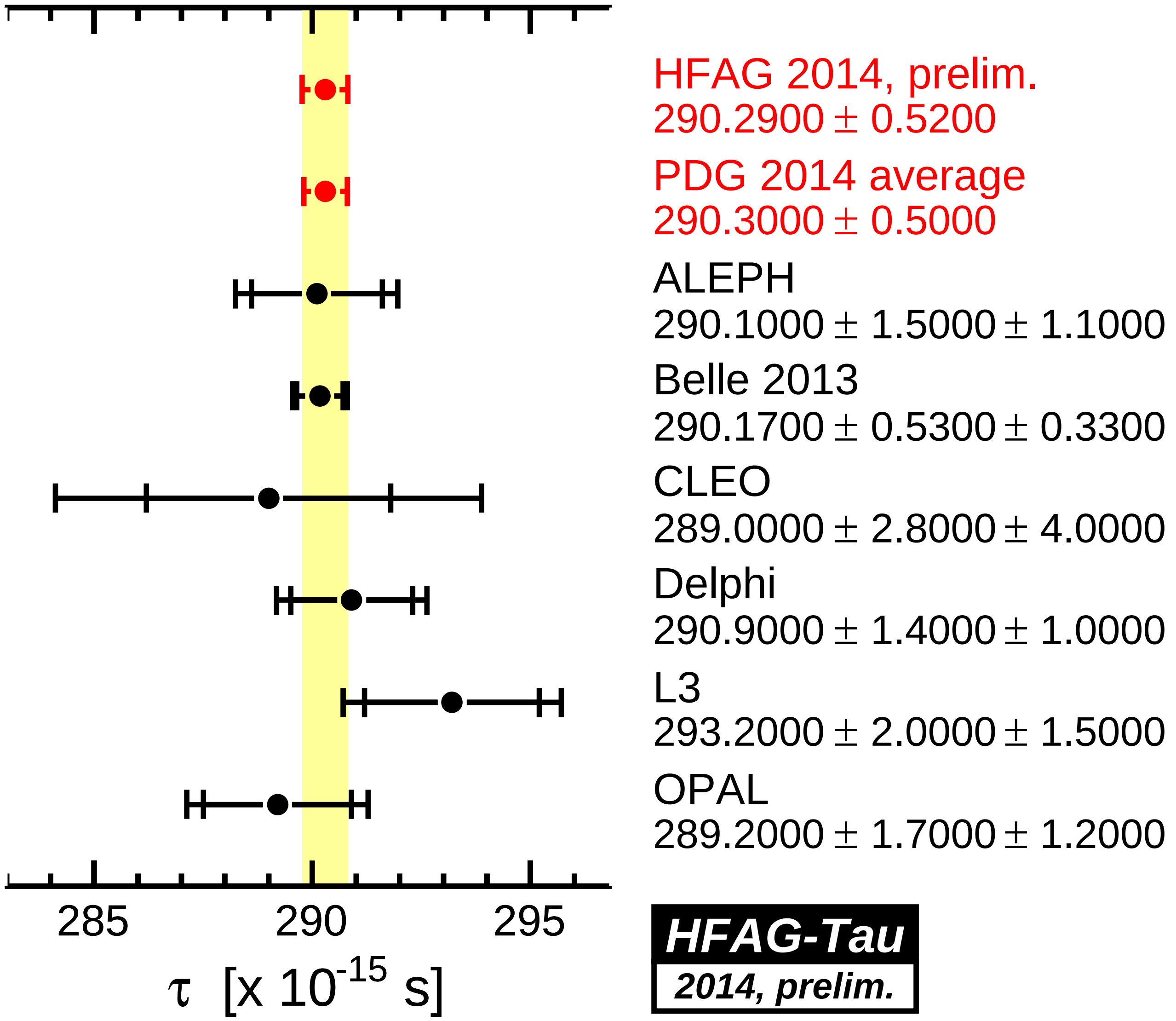
HFAG-Tau: work done

- **world averages of tau results following HFAG prescriptions**
  - use available published and recent preliminary results
  - report on arXiv every ~ 2 years, intermediate undates on the web
  - avoid PDG-style error scale factors taking better account of
    - statistical and systematic correlations between different results
    - dependencies on common external parameters (e.g. tau pair cross-section)
  - quote confidence level
- **tau branching fractions fit: averages, their uncertainties and correlation matrix**
  - lepton universality tests
  - universality improved $B(\tau \rightarrow e\nu\overline{\nu})$ and R^{τ}_{had}
  - $|V_{us}|$
- averages of other tau properties (tau mass, lifetime)
- compilation of tau LFV upper limits (2010, 2012)
- **since 2014: combinations of LFV limits** (Marcin Chrząszcz)
- HFAG report 2010, arXiv:1010.1589; HFAG report 2012, arXiv:1207.1158
- **HFAG report 2014, preliminary**

What is new since 2012

- Belle 2013 tau lifetime measurement, most precise ever
- added ***BaBar*** **PRD 86, 092010 (2012)**, Study of high-multiplicity 3-prong and 5-prong tau decays at BABAR
- added ***BaBar*** **PRD 86, 092013 (2012)**, The branching fraction of $\tau \to \pi^- K^0_S K^0_S (\pi^0) \nu$ decays
- added **Belle PRD 89, 072009 (2014)**, Measurements of Branching Fractions of τ decays with ≥1 K^0_S
- superseeded
  - one Belle 2007 result on $B(\tau^- \to \pi^- \overline{K}^0 \nu_\tau)$, from the 2014 paper
  - one *BaBar* 2008 result on $B(\tau^- \to \pi^- \pi^- \pi^+ \eta \nu_\tau$ (ex.K^0)), from the 2012 high multiplicity paper
  - preliminary *BaBar* and Belle results now in the above papers
- replaced ALEPH inclusive $B(\tau^- \to \pi^- K^0 \overline{K}^0 \pi^0 \nu_\tau)$ result with exclusive $B(\tau^- \to \pi^- \pi^0 K^0_S K^0_L \nu_\tau)$
  - better combine with new results
- removed wrong constraint on τ decay with η (negligible effect on $|V_{us}|$, universality tests, unitarity check)
- added several new constraints for the new modes in the above papers
- revised **unitarity constraint BRs** and $\boldsymbol{B(\tau^- \to X_s^- \nu_\tau)}$ **definition**

Tau Lifetime Average



What is new since 2012, *BABAR* high multiplicity paper

$\Gamma_{811} = B(\tau^- \to \pi^- 2\pi^0 \omega \nu_\tau \text{ (ex. } K^0))$	$(7.3 \pm 1.2 \pm 1.2) \cdot 10^{-5}$
$\Gamma_{812} = B(\tau^- \to 2\pi^- \pi^+ 3\pi^0 \nu_\tau \text{ (ex. } K^0, \eta, \omega, f_1))$	$(0.1 \pm 0.08 \pm 0.30) \cdot 10^{-4}$
$\Gamma_{821} = B(\tau^- \to 3\pi^- 2\pi^+ \nu_\tau \text{ (ex. } K^0, \omega, f_1))$	$(7.68 \pm 0.04 \pm 0.40) \cdot 10^{-4}$
$\Gamma_{822} = B(\tau^- \to K^- 2\pi^- 2\pi^+ \nu_\tau \text{ (ex. } K^0))$	$(0.6 \pm 0.5 \pm 1.1) \cdot 10^{-6}$
$\Gamma_{831} = B(\tau^- \to 2\pi^- \pi^+ \omega \nu_\tau \text{ (ex. } K^0))$	$(8.4 \pm 0.4 \pm 0.6) \cdot 10^{-5}$
$\Gamma_{832} = B(\tau^- \to 3\pi^- 2\pi^+ \pi^0 \nu_\tau \text{ (ex. } K^0, \eta, \omega, f_1))$	$(0.36 \pm 0.03 \pm 0.09) \cdot 10^{-4}$
$\Gamma_{833} = B(\tau^- \to K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau \text{ (ex. } K^0))$	$(1.1 \pm 0.4 \pm 0.4) \cdot 10^{-6}$
$\Gamma_{910} = B(\tau^- \to 2\pi^- \pi^+ \eta \nu_\tau (\eta \to 3\pi^0) \text{ (ex. } K^0))$	$(8.27 \pm 0.88 \pm 0.81) \cdot 10^{-5}$
$\Gamma_{911} = B(\tau^- \to \pi^- 2\pi^0 \eta \nu_\tau (\eta \to \pi^+ \pi^- \pi^0) \text{ (ex. } K^0))$	$(4.57 \pm 0.77 \pm 0.50) \cdot 10^{-5}$
$\Gamma_{920} = B(\tau^- \to \pi^- f_1 \nu_\tau (f_1 \to 2\pi^- 2\pi^+))$	$(5.20 \pm 0.31 \pm 0.37) \cdot 10^{-5}$
$\Gamma_{930} = B(\tau^- \to 2\pi^- \pi^+ \eta \nu_\tau (\eta \to \pi^+ \pi^- \pi^0) \text{ (ex. } K^0))$	$(5.39 \pm 0.27 \pm 0.41) \cdot 10^{-5}$
$\Gamma_{944} = B(\tau^- \to 2\pi^- \pi^+ \eta \nu_\tau (\eta \to \gamma\gamma) \text{ (ex. } K^0))$	$(8.26 \pm 0.35 \pm 0.51) \cdot 10^{-5}$

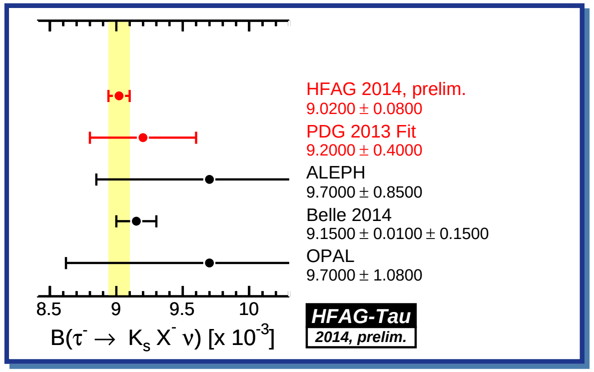
What is new since 2012, *BABAR* $\tau \to \pi^- K^0_S K^0_S (\pi^0) \nu$

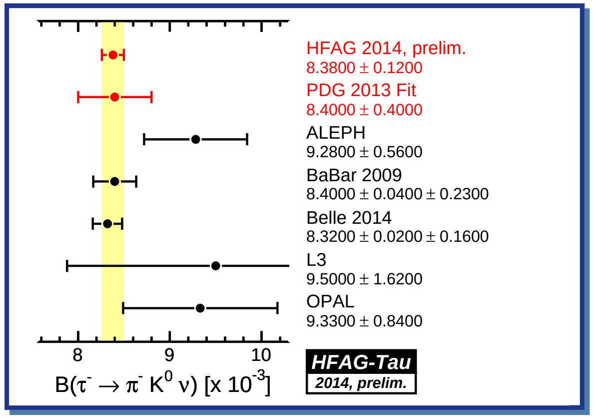
$\Gamma_{47} = B(\tau^- \to \pi^- K^0_S K^0_S \nu_\tau)$	$(2.31 \pm 0.04 \pm 0.08) \cdot 10^{-4}$
$\Gamma_{50} = B(\tau^- \to \pi^- \pi^0 K^0_S K^0_S \nu_\tau)$	$(1.60 \pm 0.20 \pm 0.22) \cdot 10^{-5}$

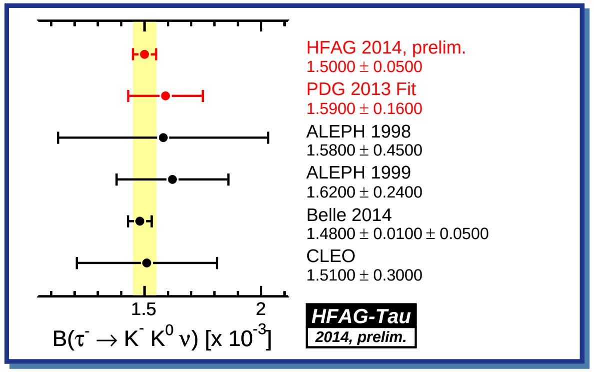
What is new since 2012, Belle $\tau \rightarrow$ one or more K^0_S

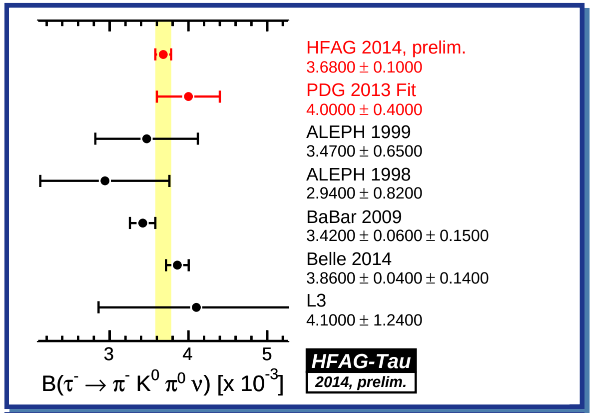
$\Gamma_{33} = B(\tau^- \to K^0_S(\text{particles})^- \nu_\tau)$	$(9.15 \pm 0.01 \pm 0.15) \cdot 10^{-3}$
$\Gamma_{35} = B(\tau^- \to \pi^- \overline{K}^0 \nu_\tau)$	$(8.32 \pm 0.02 \pm 0.16) \cdot 10^{-3}$
$\Gamma_{37} = B(\tau^- \to K^- K^0 \nu_\tau)$	$(14.8 \pm 0.14 \pm 0.54) \cdot 10^{-4}$
$\Gamma_{40} = B(\tau^- \to \pi^- \overline{K}^0 \pi^0 \nu_\tau)$	$(3.86 \pm 0.04 \pm 0.14) \cdot 10^{-3}$
$\Gamma_{42} = B(\tau^- \to K^- \pi^0 K^0 \nu_\tau)$	$(14.96 \pm 0.20 \pm 0.74) \cdot 10^{-4}$
$\Gamma_{47} = B(\tau^- \to \pi^- K^0_S K^0_S \nu_\tau)$	$(2.33 \pm 0.03 \pm 0.09) \cdot 10^{-4}$
$\Gamma_{50} = B(\tau^- \to \pi^- \pi^0 K^0_S K^0_S \nu_\tau)$	$(2.00 \pm 0.22 \pm 0.20) \cdot 10^{-5}$

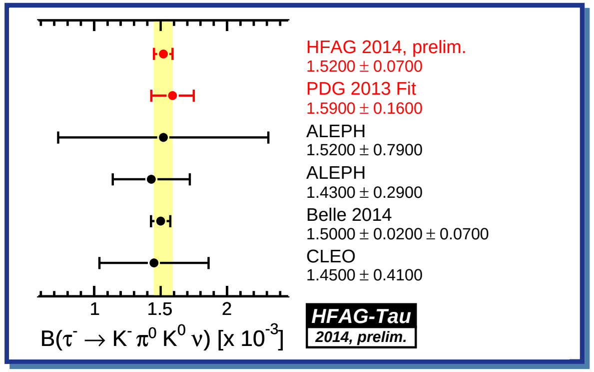
New K^0_S BRs

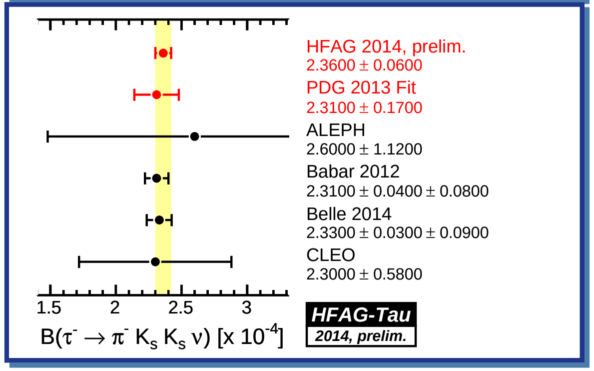

HFAG 2014, prelim.
9.0200 ± 0.0800
PDG 2013 Fit
9.2000 ± 0.4000
ALEPH
9.7000 ± 0.8500
Belle 2014
9.1500 ± 0.0100 ± 0.1500
OPAL
9.7000 ± 1.0800
8.5
9
9.5
10
$B(\tau^- \to K_S X^- \nu)$ [x 10^{-3}]
HFAG-Tau
2014, prelim.

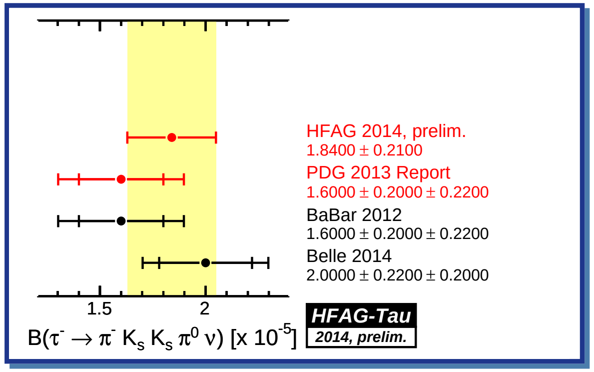

HFAG 2014, prelim.
8.3800 ± 0.1200
PDG 2013 Fit
8.4000 ± 0.4000
ALEPH
9.2800 ± 0.5600
BaBar 2009
8.4000 ± 0.0400 ± 0.2300
Belle 2014
8.3200 ± 0.0200 ± 0.1600
L3
9.5000 ± 1.6200
OPAL
9.3300 ± 0.8400
8
9
10
$B(\tau^- \to \pi^- K^0 \nu)$ [x 10^{-3}]
HFAG-Tau
2014, prelim.


HFAG 2014, prelim.
1.5000 ± 0.0500
PDG 2013 Fit
1.5900 ± 0.1600
ALEPH 1998
1.5800 ± 0.4500
ALEPH 1999
1.6200 ± 0.2400
Belle 2014
1.4800 ± 0.0100 ± 0.0500
CLEO
1.5100 ± 0.3000
1.5
2
$B(\tau^- \to K^- K^0 \nu)$ [x 10^{-3}]
HFAG-Tau
2014, prelim.


HFAG 2014, prelim.
3.6800 ± 0.1000
PDG 2013 Fit
4.0000 ± 0.4000
ALEPH 1999
3.4700 ± 0.6500
ALEPH 1998
2.9400 ± 0.8200
BaBar 2009
3.4200 ± 0.0600 ± 0.1500
Belle 2014
3.8600 ± 0.0400 ± 0.1400
L3
4.1000 ± 1.2400
3
4
5
$B(\tau^- \to \pi^- K^0 \pi^0 \nu)$ [x 10^{-3}]
HFAG-Tau
2014, prelim.


HFAG 2014, prelim.
1.5200 ± 0.0700
PDG 2013 Fit
1.5900 ± 0.1600
ALEPH
1.5200 ± 0.7900
ALEPH
1.4300 ± 0.2900
Belle 2014
1.5000 ± 0.0200 ± 0.0700
CLEO
1.4500 ± 0.4100
1
1.5
2
$B(\tau^- \to K^- \pi^0 K^0 \nu)$ [x 10^{-3}]
HFAG-Tau
2014, prelim.


HFAG 2014, prelim.
2.3600 ± 0.0600
PDG 2013 Fit
2.3100 ± 0.1700
ALEPH
2.6000 ± 1.1200
Babar 2012
2.3100 ± 0.0400 ± 0.0800
Belle 2014
2.3300 ± 0.0300 ± 0.0900
CLEO
2.3000 ± 0.5800
1.5
2
2.5
3
$B(\tau^- \to \pi^- K_S K_S \nu)$ [x 10^{-4}]
HFAG-Tau
2014, prelim.


HFAG 2014, prelim.
1.8400 ± 0.2100
PDG 2013 Report
1.6000 ± 0.2000 ± 0.2200
BaBar 2012
1.6000 ± 0.2000 ± 0.2200
Belle 2014
2.0000 ± 0.2200 ± 0.2000
1.5
2
$B(\tau^- \to \pi^- K_S K_S \pi^0 \nu)$ [x 10^{-5}]
HFAG-Tau
2014, prelim.

What is new since 2012, constraints

removed

$$\Gamma_{136} = \Gamma_{104} \cdot \Gamma_{\eta\to\pi^+\pi^-\pi^0} + \Gamma_{78} \cdot \Gamma_{\eta\to 3\pi^0}$$

new

$$\begin{aligned}
\Gamma_{13} &= \Gamma_{14} + \Gamma_{16}\\
\Gamma_{33} &= \Gamma_{35} \cdot \Gamma_{<\overline{K}^0|K_S>} + \Gamma_{40} \cdot \Gamma_{<\overline{K}^0|K_S>} + \Gamma_{42} \cdot \Gamma_{<K^0|K_S>} + \Gamma_{47} + \Gamma_{48} + \Gamma_{50} + \Gamma_{51} + \Gamma_{37} \cdot \Gamma_{<K^0|K_S>} + \Gamma_{132} \cdot (\Gamma_{<\overline{K}^0|K_S>} \cdot \Gamma_{\eta\to\text{neutral}}) +\\
&\quad \Gamma_{44} \cdot \Gamma_{<\overline{K}^0|K_S>} + \Gamma_{801} \cdot \Gamma_{\phi\to K_S K_L} / (\Gamma_{\phi\to K^+K^-} + \Gamma_{\phi\to K_S K_L})\\
\Gamma_{49} &= \Gamma_{50} + \Gamma_{51} + \Gamma_{806}\\
\Gamma_{78} &= \Gamma_{810} + \Gamma_{50} \cdot 2 \cdot \Gamma_{K_S\to\pi^+\pi^-} \cdot \Gamma_{K_S\to\pi^0\pi^0} + \Gamma_{132} \cdot (\Gamma_{<\overline{K}^0|K_S>} \cdot \Gamma_{K_S\to\pi^+\pi^-} \cdot \Gamma_{\eta\to 3\pi^0})\\
\Gamma_{103} &= \Gamma_{820} + \Gamma_{822} + \Gamma_{831} \cdot \Gamma_{\omega\to\pi^+\pi^-}\\
\Gamma_{104} &= \Gamma_{930} + \Gamma_{833} + \Gamma_{831} \cdot \Gamma_{\omega\to\pi^+\pi^-\pi^0}\\
\Gamma_{806} &= \Gamma_{50}\\
\Gamma_{810} &= \Gamma_{910} + \Gamma_{911} + \Gamma_{811} + \Gamma_{812}\\
\Gamma_{820} &= \Gamma_{920} + \Gamma_{821}\\
\Gamma_{830} &= \Gamma_{930} + \Gamma_{831} \cdot \Gamma_{\omega\to\pi^+\pi^-\pi^0} + \Gamma_{832}\\
\Gamma_{910} &= \Gamma_{136} \cdot \Gamma_{\eta\to 3\pi^0}\\
\Gamma_{930} &= \Gamma_{136} \cdot \Gamma_{\eta\to\pi^+\pi^-\pi^0}\\
\Gamma_{944} &= \Gamma_{136} \cdot \Gamma_{\eta\to\gamma\gamma}\\
\Gamma_{911} &= \Gamma_{945} \cdot \Gamma_{\eta\to\pi^+\pi^-\pi^0}
\end{aligned}$$

revised

$$\begin{aligned}
\Gamma_{110} &= \Gamma_{10} + \Gamma_{16} + \Gamma_{23} + \Gamma_{28} + \Gamma_{35} + \Gamma_{40} + \Gamma_{128} + \Gamma_{802} + \Gamma_{803} + \Gamma_{151} + \Gamma_{130} + \Gamma_{132} + \Gamma_{44} + \Gamma_{53} + \Gamma_{801} + \Gamma_{822} + \Gamma_{833}\\
\Gamma_{\text{All}} &= \Gamma_{3}+\Gamma_{5}+\Gamma_{9}+\Gamma_{10}+\Gamma_{14}+\Gamma_{16}+\Gamma_{20}+\Gamma_{23}+\Gamma_{27}+\Gamma_{28}+\Gamma_{30}+\Gamma_{35}+\Gamma_{37}+\Gamma_{40}+\Gamma_{42}+\Gamma_{46}+\Gamma_{62}+\Gamma_{70}+\Gamma_{77}+\Gamma_{78}+\Gamma_{93}+\Gamma_{94}+\Gamma_{104}+\Gamma_{126}+\\
&\quad \Gamma_{128}+\Gamma_{802}+\Gamma_{803}+\Gamma_{800}+\Gamma_{151}+\Gamma_{130}+\Gamma_{132}+\Gamma_{44}+\Gamma_{53}+\Gamma_{50}\cdot(1-2\cdot\Gamma_{K_S\to\pi^+\pi^-}\cdot\Gamma_{K_S\to\pi^0\pi^0})+\Gamma_{51}+\Gamma_{806}+\Gamma_{805}+\Gamma_{801}+\Gamma_{152}+\Gamma_{103}
\end{aligned}$$

What is new since 2012, number of results in the global BR fit

- **Summer 2010** HFAG, arXiv:1010.1589v3 [hep-ex],
  D. Asner et al., “Averages of b-hadron, c-hadron, and tau-lepton Properties”
  - **12 *BaBar* and 10 Belle measurement added to 124 former measurements in PDG**
  - 3 $|V_{us}|$ fits, lepton universality tests
- **Early 2012** HFAG, arXiv:1207.1158 [hep-ex],
  Y. Amhis et al., “Averages of b-hadron, c-hadron, and tau-lepton properties as of early 2012”
  - updated external parameters to PDG 2011 and CODATA 2006
  - drop old ALEPH and CLEO $B(\tau^- \to K^- \eta \nu_\tau)$ measurements
  - added ALEPH: $B(\tau^- \to \pi^- K^0 \overline{K}^0 \nu_\tau)$ and $B(\tau^- \to \pi^- K^0 \overline{K}^0 \pi^0 \nu_\tau)$
  - **total of 157 measurements and 47 constraint equations to fit 86 quantities**
- **HFAG 2014 prelim. fit**
- updated external parameters to **PDG 2013** and **CODATA 2010**
  - added 14 new *BaBar* results, 7 new Belle results, related constraints
  - **total of 175 measurements and 57 constraint equations to fit 104 quantities**

results by collaboration

collaboration	measurements	collaboration	measurements	collaboration	measurements
ALEPH	39	ARGUS	2	BaBar	26
Belle	16	CELLO	1	CLEO	36
CLEO3	6	DELPHI	14	HRS	2
L3	11	OPAL	19	TPC	3

Tau Branching Fractions Fit

- 175 measurements, 57 constraint equations
- fit 104 quantities: 47 BRs, 57 derived quantities, ratios of linear combinations of BR
- χ^2/d.o.f. = 143.8/128, CL = 16.10% (was 5.5% in 2012)
- no unitarity constraint enforced (reduce "pollution" from hadronic to leptonic modes)
- 5.44 error scale factor for inconsistent *BABAR* and Belle $B(\tau^- \to K^- K^- K^+ \nu_\tau)$ as in 2012
- consistent with unitarity, per mill precision, residual = $(9.937 \pm 9.849) \cdot 10^{-4}$

BR fit, leptonic branching fractions

$\Gamma_5 = B(\tau^- \to e^- \overline{\nu}_e \nu_\tau)$	0.17817 ± 0.00041	HFAG 2014 prelim. fit
0.17837 ± 0.00080 ± 0.00000	ALEPH	Schael:2005am
0.17760 ± 0.00180 ± 0.00000	CLEO	Anastassov:1996tc
0.17877 ± 0.00155 ± 0.00000	DELPHI	Abreu:1999rb
0.17806 ± 0.00129 ± 0.00000	L3	Acciarri:2001sg
0.17810 ± 0.00108 ± 0.00000	OPAL	Abbiendi:1998cx
$\Gamma_3 = B(\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau)$	0.17392 ± 0.00040	HFAG 2014 prelim. fit
0.17319 ± 0.00077 ± 0.00000	ALEPH	Schael:2005am
0.17325 ± 0.00122 ± 0.00000	DELPHI	Abreu:1999rb
0.17342 ± 0.00129 ± 0.00000	L3	Acciarri:2001sg
0.17340 ± 0.00108 ± 0.00000	OPAL	Abbiendi:2002jw
$\frac{\Gamma_3}{\Gamma_5} = \frac{B(\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau)}{B(\tau^- \to e^- \overline{\nu}_e \nu_\tau)}$	0.97611 ± 0.00278	HFAG 2014 prelim. fit
0.99700 ± 0.05315 ± 0.00000	ARGUS	Albrecht:1991rh
0.97960 ± 0.00390 ± 0.00053	BaBar	Aubert:2009qj
0.97770 ± 0.01074 ± 0.00000	CLEO	Anastassov:1996tc

BR Fit: unitarity constraint branching fractions

BR	HFAG fit
$\Gamma_{3} = \mu^{-}\overline{\nu}_{\mu}\nu_{\tau}$	$(17.3916 \pm 0.0396) \cdot 10^{-2}$
$\Gamma_{5} = e^{-}\overline{\nu}_{e}\nu_{\tau}$	$(17.8172 \pm 0.0409) \cdot 10^{-2}$
$\Gamma_{9} = \pi^{-}\nu_{\tau}$	$(10.8139 \pm 0.0527) \cdot 10^{-2}$
$\Gamma_{10} = K^{-}\nu_{\tau}$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$
$\Gamma_{14} = \pi^{-}\pi^{0}\nu_{\tau}$	$(25.5040 \pm 0.0917) \cdot 10^{-2}$
$\Gamma_{16} = K^{-}\pi^{0}\nu_{\tau}$	$(0.4331 \pm 0.0149) \cdot 10^{-2}$
$\Gamma_{20} = \pi^{-}2\pi^{0}\nu_{\tau}$ (ex. K^{0})	$(9.2389 \pm 0.0997) \cdot 10^{-2}$
$\Gamma_{23} = K^{-}2\pi^{0}\nu_{\tau}$ (ex. K^{0})	$(0.0630 \pm 0.0220) \cdot 10^{-2}$
$\Gamma_{27} = \pi^{-}3\pi^{0}\nu_{\tau}$ (ex. K^{0})	$(1.0297 \pm 0.0749) \cdot 10^{-2}$
$\Gamma_{28} = K^{-}3\pi^{0}\nu_{\tau}$ (ex. K^{0}, η)	$(0.0419 \pm 0.0216) \cdot 10^{-2}$
$\Gamma_{30} = h^{-}4\pi^{0}\nu_{\tau}$ (ex. K^{0}, η)	$(0.1103 \pm 0.0391) \cdot 10^{-2}$
$\Gamma_{35} = \pi^{-}\overline{K}^{0}\nu_{\tau}$	$(0.8378 \pm 0.0123) \cdot 10^{-2}$
$\Gamma_{37} = K^{-}K^{0}\nu_{\tau}$	$(0.1500 \pm 0.0050) \cdot 10^{-2}$
$\Gamma_{40} = \pi^{-}\overline{K}^{0}\pi^{0}\nu_{\tau}$	$(0.3680 \pm 0.0103) \cdot 10^{-2}$
$\Gamma_{42} = K^{-}\pi^{0}K^{0}\nu_{\tau}$	$(0.1528 \pm 0.0070) \cdot 10^{-2}$
$\Gamma_{44} = \pi^{-}\overline{K}^{0}\pi^{0}\pi^{0}\nu_{\tau}$	$(0.0124 \pm 0.0204) \cdot 10^{-2}$
$\Gamma_{46} = \pi^{-}K^{0}\overline{K}^{0}\nu_{\tau}$	$(0.1329 \pm 0.0110) \cdot 10^{-2}$
$\Gamma_{50} = \pi^{-}\pi^{0}K_{S}^{0}K_{S}^{0}\nu_{\tau}$	$(0.0018 \pm 0.0002) \cdot 10^{-2}$
$\Gamma_{51} = \pi^{-}\pi^{0}K_{S}^{0}K_{L}^{0}\nu_{\tau}$	$(0.0253 \pm 0.0105) \cdot 10^{-2}$
$\Gamma_{53} = \overline{K}^{0}h^{-}h^{-}h^{+}\nu_{\tau}$	$(0.0222 \pm 0.0202) \cdot 10^{-2}$
$\Gamma_{62} = \pi^{-}\pi^{-}\pi^{+}\nu_{\tau}$ (ex. K^{0}, ω)	$(8.9800 \pm 0.0511) \cdot 10^{-2}$
$\Gamma_{70} = \pi^{-}\pi^{-}\pi^{+}\pi^{0}\nu_{\tau}$ (ex. K^{0}, ω)	$(2.7672 \pm 0.0710) \cdot 10^{-2}$
$\Gamma_{77} = h^{-}h^{-}h^{+}2\pi^{0}\nu_{\tau}$ (ex. K^{0}, ω, η)	$(0.0971 \pm 0.0354) \cdot 10^{-2}$

BR	HFAG fit
$\Gamma_{78} = h^{-}h^{-}h^{+}3\pi^{0}\nu_{\tau}$	$(0.0215 \pm 0.0030) \cdot 10^{-2}$
$\Gamma_{93} = \pi^{-}K^{-}K^{+}\nu_{\tau}$	$(0.1436 \pm 0.0027) \cdot 10^{-2}$
$\Gamma_{94} = \pi^{-}K^{-}K^{+}\pi^{0}\nu_{\tau}$	$(0.0061 \pm 0.0018) \cdot 10^{-2}$
$\Gamma_{103} = 3h^{-}2h^{+}\nu_{\tau}$ (ex. K^{0})	$(0.0834 \pm 0.0024) \cdot 10^{-2}$
$\Gamma_{104} = 3h^{-}2h^{+}\pi^{0}\nu_{\tau}$ (ex. K^{0})	$(0.0129 \pm 0.0007) \cdot 10^{-2}$
$\Gamma_{126} = \pi^{-}\pi^{0}\eta\nu_{\tau}$	$(0.1386 \pm 0.0072) \cdot 10^{-2}$
$\Gamma_{128} = K^{-}\eta\nu_{\tau}$	$(0.0153 \pm 0.0008) \cdot 10^{-2}$
$\Gamma_{130} = K^{-}\pi^{0}\eta\nu_{\tau}$	$(0.0048 \pm 0.0012) \cdot 10^{-2}$
$\Gamma_{132} = \pi^{-}\overline{K}^{0}\eta\nu_{\tau}$	$(0.0093 \pm 0.0015) \cdot 10^{-2}$
$\Gamma_{151} = K^{-}\omega\nu_{\tau}$	$(0.0410 \pm 0.0092) \cdot 10^{-2}$
$\Gamma_{152} = h^{-}\pi^{0}\omega\nu_{\tau}$	$(0.4051 \pm 0.0418) \cdot 10^{-2}$
$\Gamma_{800} = \pi^{-}\omega\nu_{\tau}$	$(1.9539 \pm 0.0647) \cdot 10^{-2}$
$\Gamma_{801} = K^{-}\phi\nu_{\tau}(\phi \to KK)$	$(0.0037 \pm 0.0014) \cdot 10^{-2}$
$\Gamma_{802} = K^{-}\pi^{-}\pi^{+}\nu_{\tau}$ (ex. K^{0}, ω)	$(0.2923 \pm 0.0068) \cdot 10^{-2}$
$\Gamma_{803} = K^{-}\pi^{-}\pi^{+}\pi^{0}\nu_{\tau}$ (ex. K^{0}, ω, η)	$(0.0411 \pm 0.0143) \cdot 10^{-2}$
$\Gamma_{805} = a_{1}^{-}(\to \pi^{-}\gamma)\nu_{\tau}$	$(0.0400 \pm 0.0200) \cdot 10^{-2}$
$\Gamma_{806} = \pi^{-}\pi^{0}K_{L}^{0}K_{L}^{0}\nu_{\tau}$	$(0.0018 \pm 0.0002) \cdot 10^{-2}$
$\Gamma_{998} = 1 - \Gamma_{\mathrm{All}}$	$(0.0994 \pm 0.0985) \cdot 10^{-2}$

Lepton Universality tests

Standard Model (Marciano 1988):

$$\Gamma(L \to \nu_L \ell \overline{\nu}_\ell(\gamma)) = \frac{B(L \to \nu_L \ell \overline{\nu}_\ell)}{\tau_L} = \frac{G_L G_\ell m_L^5}{192\pi^3} f\left(\frac{m_\ell^2}{m_L^2}\right) r_W^L r_\gamma^L ,$$

where

$$G_\ell = \frac{g_\ell^2}{4\sqrt{2} M_W^2} \qquad f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$$

$$r_W^L = 1 + \frac{3}{5}\frac{m_L^2}{M_W^2} \qquad r_\gamma^L = 1 + \frac{\alpha(m_L)}{2\pi}\left(\frac{25}{4} - \pi^2\right)$$

Using: $r_\gamma^\tau = 1 - 43.2 \cdot 10^{-4}$ and $r_\gamma^\mu = 1 - 42.4 \cdot 10^{-4}$ (Marciano 1988), M_W from PDG 2013

Proper ratios of the above partial widths:

$$\left(\frac{g_\tau}{g_\mu}\right) = 1.0011 \pm 0.0015 , \qquad \left(\frac{g_\tau}{g_e}\right) = 1.0029 \pm 0.0015 , \qquad \left(\frac{g_\mu}{g_e}\right) = 1.0018 \pm 0.0014 .$$

- **precision improved** from 0.2% (HFAG 2012) to **0.14%** thanks to the Belle tau lifetime result

Lepton Universality tests (2)

Standard Model:

$$\left(\frac{g_\tau}{g_\mu}\right)^2 = \frac{B(\tau \to h\nu_\tau)}{B(h \to \mu\bar{\nu}_\mu)} \frac{2m_h m_\mu^2 \tau_h}{(1+\delta_h)m_\tau^3 \tau_\tau} \left(\frac{1-m_\mu^2/m_h^2}{1-m_h^2/m_\tau^2}\right)^2 \qquad (h = \pi \text{ or } K)$$

rad. corr. $\delta_\pi = (0.16 \pm 0.14)\%$, $\delta_K = (0.90 \pm 0.22)\%$ (Decker 1994)

$$\left(\frac{g_\tau}{g_\mu}\right)_\pi = 0.9963 \pm 0.0027\,, \qquad \left(\frac{g_\tau}{g_\mu}\right)_K = 0.9858 \pm 0.0071\,.$$

electron tests less precise because hadron two body decays to electrons are helicity-suppressed

Averaging the three g_τ/g_μ ratios:

$$\left(\frac{g_\tau}{g_\mu}\right)_{\tau+\pi+K} = 1.0001 \pm 0.0014\,,$$

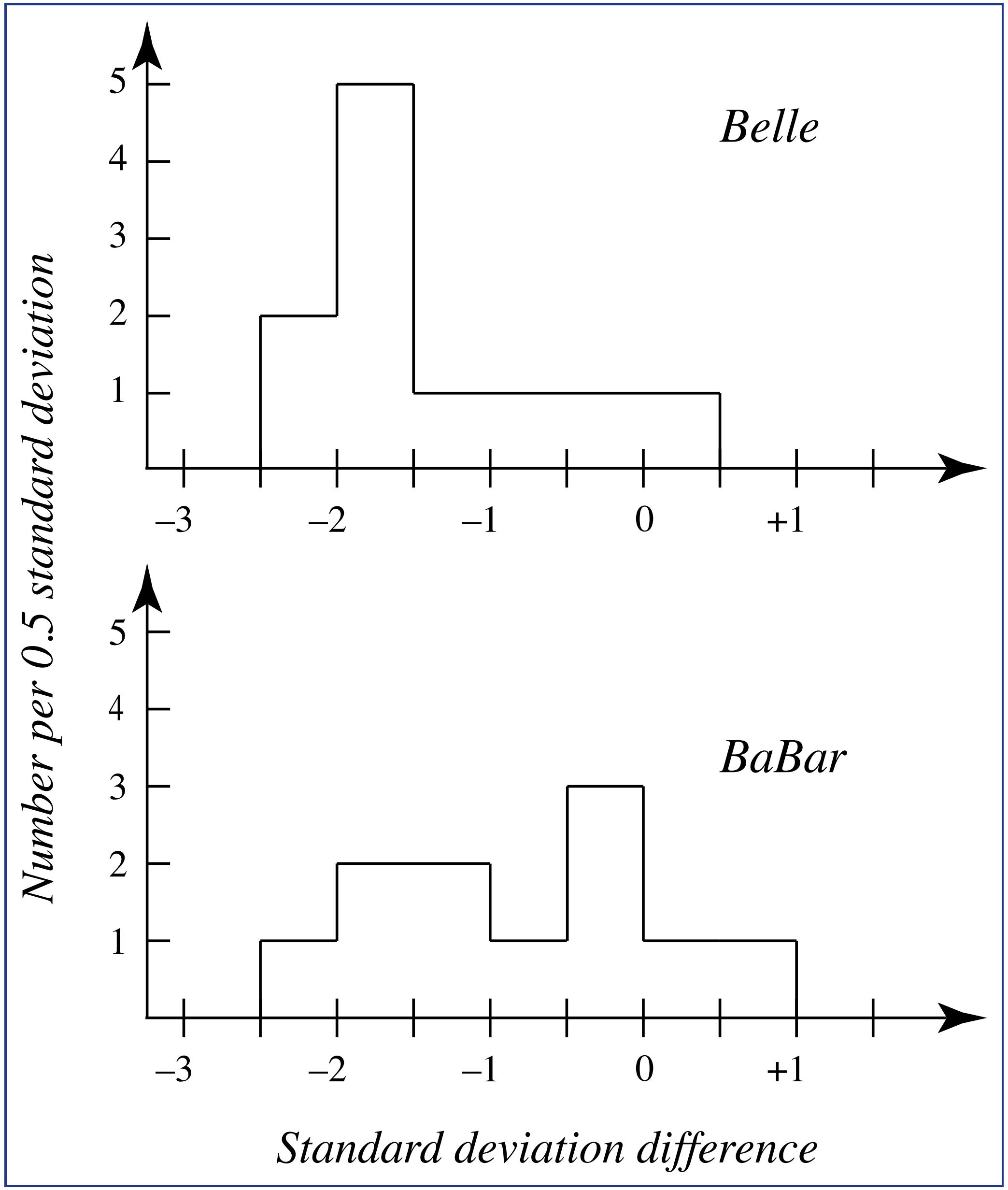
accounting for statistical correlations.

- **precision improved** also for universality from hadronic processes thanks to the Belle tau lifetime result

Universality improved $B(\tau \to e\nu\bar{\nu})$ and R_{had}

- (Davier 2005): assume lepton universality to improve $B_e = B(\tau \to e\bar{\nu}_e\nu_\tau)$
  - $B_e = B_\mu \cdot f(m_e^2/m_\tau^2)/f(m_\mu^2/m_\tau^2)$
  - $B_e = B(\mu \to e\bar{\nu}_e\nu_\mu) \cdot (\tau_\tau/\tau_\mu) \cdot (m_\tau/m_\mu)^5 \cdot f(m_e^2/m_\tau^2)/f(m_e^2/m_\mu^2) \cdot (\delta_\gamma^\tau\delta_W^\tau)/(\delta_\gamma^\mu\delta_W^\mu)$
    $(B(\mu \to e\bar{\nu}_e\nu_\mu) = 1)$
- HFAG 2014 prelim. fit:
  - $\mathbf{B_e^{\text{univ}} = (17.815 \pm 0.023)\%}$
  - $$R_{\text{had}} = \frac{\Gamma(\tau \to \text{hadrons})}{\Gamma(\tau \to e\nu\bar{\nu})} = 3.6314 \pm 0.0081$$

B-factories measure on average lower BRs



PDG 2014 review

- ♦ -0.75σ *BaBar* (11 measurements)
- ♦ -1.41σ Belle (11 measurements)

BaBar vs. Belle discrepancies from PDG 2014 review (no change since 2012)

mode	*BaBar* – Belle in σ
$\pi^-\pi^-\pi^+\nu_\tau$ (ex.K^0)	+1.4
$K^-\pi^-\pi^+\nu_\tau$ (ex.K^0)	–2.9
$\overline{K}^0 h^- h^- h^+ \nu_\tau$	–2.9
$K^-K^-K^+\nu_\tau$	–5.4
$K^-\eta\nu_\tau$	–1.0
$\tau \to \phi K \nu$	–1.3

Determination of $|V_{us}|$ from experimental data

- from kaon decays
  - $\Gamma(K\to\pi\ell\bar{\nu}_\ell[\gamma]) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2\, S_{EW}^K \left(|V_{us}| f_+^{K\pi}(0)\right)^2 I_K^\ell \left(1+\delta_{EM}^{K\ell}+\delta_{SU(2)}^{K\pi}\right)^2$
  - $\frac{\Gamma(K^\pm\to\ell^\pm\nu)}{\Gamma(\pi^\pm\to\ell^\pm\nu)} = \frac{|V_{us}|^2}{|V_{ud}|^2}\frac{f_K^2}{f_\pi^2}\frac{m_K(1-m_\ell^2/m_K^2)^2}{m_\pi(1-m_\ell^2/m_\pi^2)^2}\left(1+\delta_{EM}\right)$
- from tau decays
  - $\frac{R(\tau\to X_{\text{strange}})}{{V_{us}}^2} - \frac{R(\tau\to X_{\text{non-strange}})}{V_{ud}^2} = \delta R_{\tau,\text{SU3 breaking}}, \quad R(\tau\to X) = \frac{\Gamma(\tau\to X)}{\Gamma(\tau\to e\nu\bar{\nu})},$ **inclusive**
  - $\frac{B(\tau^-\to K^-\nu_\tau)}{B(\tau^-\to\pi^-\nu_\tau)} = \frac{f_K^2|V_{us}|^2}{f_\pi^2\,|V_{ud}|^2}\frac{\left(1-m_K^2/m_\tau^2\right)^2}{\left(1-m_\pi^2/m_\tau^2\right)^2}\frac{r_{LD}(\tau^-\to K^-\nu_\tau)}{r_{LD}(\tau^-\to\pi^-\nu_\tau)}$
  - $B(\tau^-\to K^-\nu_\tau) = \frac{G_F^2 f_K^2|V_{us}|^2 m_\tau^3\tau_\tau}{16\pi\hbar}\left(1-\frac{m_K^2}{m_\tau^2}\right)^2 S_{EW}$
  - $\Gamma(\tau\to\bar{K}\pi\nu_\tau[\gamma]) = \frac{G_F^2 m_\tau^5}{96\pi^3}\, C_K^2\, S_{EW}^\tau \left(|V_{us}| f_+^{K\pi}(0)\right)^2 I_K^\tau \left(1+\delta_{EM}^{K\tau}+\delta_{SU(2)}^{K\pi}\right)^2$

$|V_{us}|$ from inclusive tau partial width to strange

E. Gamiz at CKM 2012

\# Sizeable corrections in the semi-inclusive τ-decay width into Cabbibo-suppressed modes due to $SU(3)$ breaking.

$$\delta R_\tau \equiv \frac{R_{\tau,V+A}}{|V_{ud}|^2} - \frac{R_{\tau,s}}{|V_{us}|^2}$$

Dominated by m_s (flavour indep. uncertainties drop out in the difference)

→ extraction of the strange quark mass **Pich and Prades**, hep-ph/9909244

* Strong dependence of m_s on $|V_{us}|$

⟹ **Determination of $|V_{us}|$ from a fixed m_s**

E.G., Jamin, Pich, Prades, Schwab, hep-ph/0212230,0408044

$$|V_{us}|^2 = \frac{R^{exp}_{\tau,S}}{\frac{R^{exp}_{\tau,V+A}}{|V_{ud}|^2} - \delta R^{theor}_\tau}$$

Advantage: Final error dominated by experimental uncertainty

→ potential to be competitive with best determinations

$|V_{us}|$ from inclusive tau partial width to strange

- δR_{theory} can be determined with OPE, complemented with phenomenology where convergence is slowest
  - E.Gamiz, M.Jamin, A.Pich, J.Prades, F.Schwab, arXiv:hep-ph/0408044
  - E.Gamiz, M.Jamin, A.Pich, J.Prades, F.Schwab, arXiv:0709.0282 [hep-ph]
  - K.Maltman, arXiv:1011.6391 [hep-ph]
- some disagreement in the estimate of theory uncertainties

$|V_{us}|$ from inclusive tau partial width to strange

$$|V_{us}| = \sqrt{R_s \Big/ \left[\frac{R_{VA}}{|V_{ud}|^2} - \delta R_\text{theory}\right]}; \quad R_i = \frac{\Gamma_i}{\Gamma_e^\text{univ}} = \frac{B_i}{B_e^\text{univ}}; \quad R_\text{had} = R_s + R_{VA};$$

- $\delta R_\text{theory} = 0.239 \pm 0.030$ (Gamiz et al, arXiv:hep-ph/0612154) QCD sum rules & scattering data
  - uncertainty between Gamiz et al, arXiv:0709.0282 [hep-ph] and Maltman, arXiv:1011.6391 [hep-ph]
- $|V_{ud}| = 0.97425 \pm 0.00022$ (Hardy & Towner 2008, also PDG 2013)
- often $\boldsymbol{B_\text{had} = 1 - B_e - B_\mu}$ (or similar expressions based on B_e^univ)
- since HGAG-Tau 2012: B_had, B_{VA} directly from hadronic tau BRs
  - no statistical loss (other expr. more correlated to B_e^univ)
  - R_{VA} will not absorb effect of unobserved hadronic decay modes
- $\boxed{|V_{us}|_{\tau s} = 0.2176 \pm 0.0021}$ (3.4σ lower than $|V_{us}|_\text{uni} = \sqrt{1 - |V_{ud}|^2} = 0.22547 \pm 0.00095$)
  - $B_s = (2.882 \pm 0.047)\%$; $B_{VA} = (61.81 \pm 0.10)\%$)

Tau branching fractions to strange final states

Branching fraction	HFAG 2014 prelim. fit
$\Gamma_{10} = K^- \nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$
$\Gamma_{16} = K^- \pi^0 \nu_\tau$	$(0.4331 \pm 0.0149) \cdot 10^{-2}$
$\Gamma_{23} = K^- 2\pi^0 \nu_\tau$ (ex. K^0)	$(0.0630 \pm 0.0220) \cdot 10^{-2}$
$\Gamma_{28} = K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	$(0.0419 \pm 0.0216) \cdot 10^{-2}$
$\Gamma_{35} = \pi^- \overline{K}^0 \nu_\tau$	$(0.8378 \pm 0.0123) \cdot 10^{-2}$
$\Gamma_{40} = \pi^- \overline{K}^0 \pi^0 \nu_\tau$	$(0.3680 \pm 0.0103) \cdot 10^{-2}$
$\Gamma_{44} = \pi^- \overline{K}^0 \pi^0 \pi^0 \nu_\tau$	$(0.0124 \pm 0.0204) \cdot 10^{-2}$
$\Gamma_{53} = \overline{K}^0 h^- h^- h^+ \nu_\tau$	$(0.0222 \pm 0.0202) \cdot 10^{-2}$
$\Gamma_{128} = K^- \eta \nu_\tau$	$(0.0153 \pm 0.0008) \cdot 10^{-2}$
$\Gamma_{130} = K^- \pi^0 \eta \nu_\tau$	$(0.0048 \pm 0.0012) \cdot 10^{-2}$
$\Gamma_{132} = \pi^- \overline{K}^0 \eta \nu_\tau$	$(0.0093 \pm 0.0015) \cdot 10^{-2}$
$\Gamma_{151} = K^- \omega \nu_\tau$	$(0.0410 \pm 0.0092) \cdot 10^{-2}$
$\Gamma_{801} = K^- \phi \nu_\tau (\phi \to KK)$	$(0.0037 \pm 0.0014) \cdot 10^{-2}$
$\Gamma_{802} = K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	$(0.2923 \pm 0.0068) \cdot 10^{-2}$
$\Gamma_{803} = K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	$(0.0411 \pm 0.0143) \cdot 10^{-2}$
$\Gamma_{822} = K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. K^0)	$(0.0001 \pm 0.0001) \cdot 10^{-2}$
$\Gamma_{833} = K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0)	$(0.0001 \pm 0.0001) \cdot 10^{-2}$
$\Gamma_{110} = X_s^- \nu_\tau$	$(2.8816 \pm 0.0470) \cdot 10^{-2}$

$|V_{us}|$ from $B(\tau \to K\nu)/B(\tau \to \pi\nu)$

$$\frac{B(\tau^- \to K^-\nu_\tau)}{B(\tau^- \to \pi^-\nu_\tau)} = \frac{f_K^2\,|V_{us}|^2}{f_\pi^2\,|V_{ud}|^2}\frac{\left(1-m_K^2/m_\tau^2\right)^2}{\left(1-m_\pi^2/m_\tau^2\right)^2}\frac{r_{\mathrm{LD}}(\tau^- \to K^-\nu_\tau)}{r_{\mathrm{LD}}(\tau^- \to \pi^-\nu_\tau)}\,.$$

- $\boxed{|V_{us}|_{\tau K/\pi} = 0.2232 \pm 0.0019}$ 1.0σ below CKM unitarity prediction
- details on rad. corrections in the report
- $f_K/f_\pi = 1.1920 \pm 0.0050$ FLAG 2013 2+1
  - lattice uncertainty similar to 2012

$|V_{us}|$ from $B(\tau \to K\nu)$

$$B(\tau^- \to K^- \nu_\tau) = \frac{G_F^2 f_K^2 \, |V_{us}|^2 \, m_\tau^3 \tau_\tau}{16\pi\hbar} \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 S_{EW} \, ,$$

- $\boxed{|V_{us}|_{\tau K} = 0.2212 \pm 0.0020}$ 1.9σ below CKM unitarity prediction
- details on rad. corrections in the report
- $f_K = 156.3 \pm 0.9$ MeV FLAG 2013 2+1
  - lattice uncertainty similar to 2012
- using CODATA 2010

$|V_{us}|$ from $B(\tau \rightarrow K\pi\nu)$

- $\Gamma(\tau \rightarrow \overline{K}\pi\nu_\tau[\gamma]) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau \left(|V_{us}| f_+^{K\pi}(0)\right)^2 I_K^\tau \left(1 + \delta_{EM}^{K\tau} + \delta_{SU(2)}^{K\pi}\right)^2$
- M.Antonelli, V.Cirigliano, A.L., E.Passemar, arXiv:1304.8134 [hep-ph]

  updated presentation by E.Passemar at CKM 2014
  - compute the phase space integrals, I_K^ℓ using $K\pi$ form factors
    - from $\tau \rightarrow K\pi\nu_\tau$ Belle '08 $K_S^0\pi$ data
    - $K_{\ell 3}$ data may also be used for the low energy end of the integral
  - first estimate of the long-distance electromagnetic corrections ($\delta_{EM}^{K\tau}$) to $\tau \rightarrow K\pi\nu_\tau$
  - isospin breaking corrections ($\delta_{SU(2)}^{K\pi}$) for $\tau \rightarrow K^-\pi^0\nu_\tau$ vs. $\tau \rightarrow K_S^0\pi\nu_\tau$
  - $f_+^{K\pi}(0)$ from FLAG 2013
  - $f_+^{K\pi}(0)\,|V_{us}| = 0.2141 \pm 0.0014_{IK\tau} \pm 0.0021_{\text{exp}}$ E. Passemar, CKM 2014
  - **$|V_{us}| = 0.2216 \pm 0.0027$** E. Passemar, CKM 2014
- V-Bernard, arXiv:1311.2569 [hep-ph]

  First determination of $f_+(0)|V_{us}|$ from a combined analysis of $\tau \rightarrow K\pi\nu_\tau$ decay and πK scattering with constraints from $K_{\ell 3}$ decays
  - global fit of tau and K data

$|V_{us}|$ summary

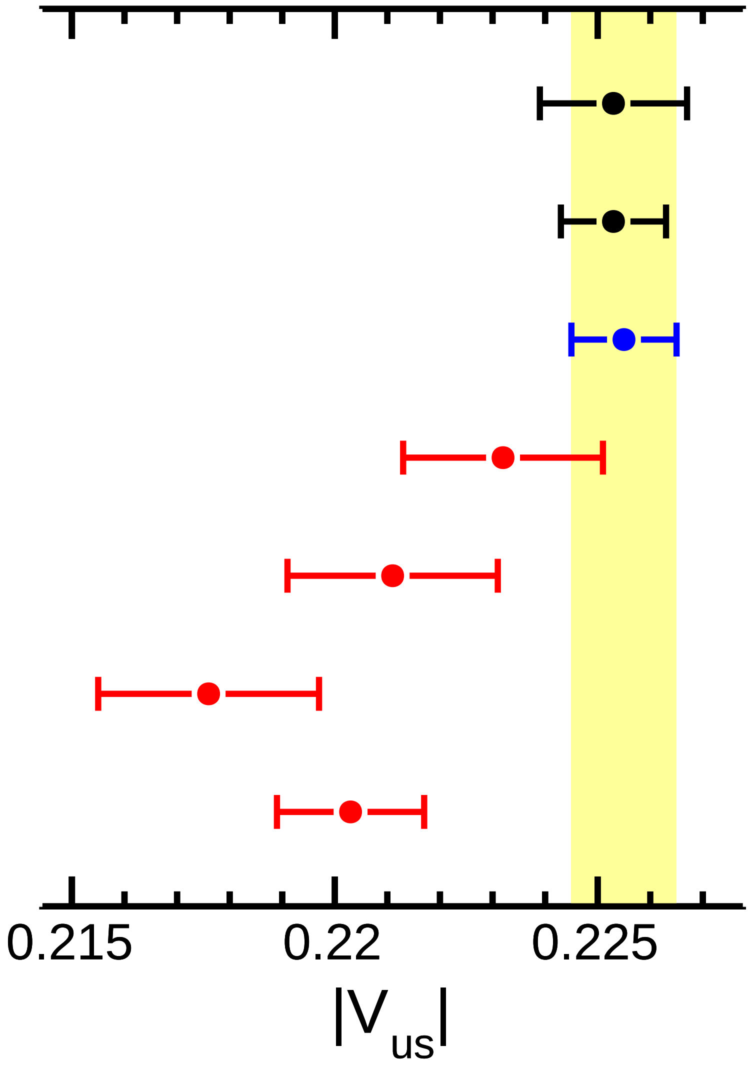
Summary of $|V_{us}|$ measurements, with discrepancies w.r.t. CKM unitarity

$$
\begin{array}{llll}
|V_{us}|_{\text{uni}} & = 0.22547 \pm 0.00095 & & \text{from } \sqrt{1-|V_{ud}|^2} \text{ (CKM unitarity) ,} \\
|V_{us}|_{\tau s} & = 0.2176 \pm 0.0021 & -3.4\sigma & \text{from } \Gamma(\tau^- \to X_s^- \nu_\tau) \text{ ,} \\
|V_{us}|_{\tau K/\pi} & = 0.2232 \pm 0.0019 & -1.0\sigma & \text{from } \Gamma(\tau^- \to K^- \nu_\tau)/\Gamma(\tau^- \to \pi^- \nu_\tau) \text{ ,} \\
|V_{us}|_{\tau K} & = 0.2212 \pm 0.0020 & -1.9\sigma & \text{from } \Gamma(\tau^- \to K^- \nu_\tau) \text{ .}
\end{array}
$$

averaging the three $|V_{us}|$ tau determinations

- $\boxed{|V_{us}|_\tau = 0.2204 \pm 0.0014}$ -2.9σ below CKM unitarity
- all significant correlations included
- there is some uncertainty on the correlations of the lattice results (see report for details)
- **no significant change with respect to previous HFAG reports**

$|V_{us}|$ summary, comparison HFAG 2014 prelim. with PDG 2013



K_{l3} decays, PDG 2013
0.2253 ± 0.0014

K_{l2} decays, PDG 2013
0.2253 ± 0.0010

CKM unitarity, PDG 2013
0.2255 ± 0.0010

$\tau \to K\nu$ / $\tau \to \pi\nu$, HFAG 2014
0.2232 ± 0.0019

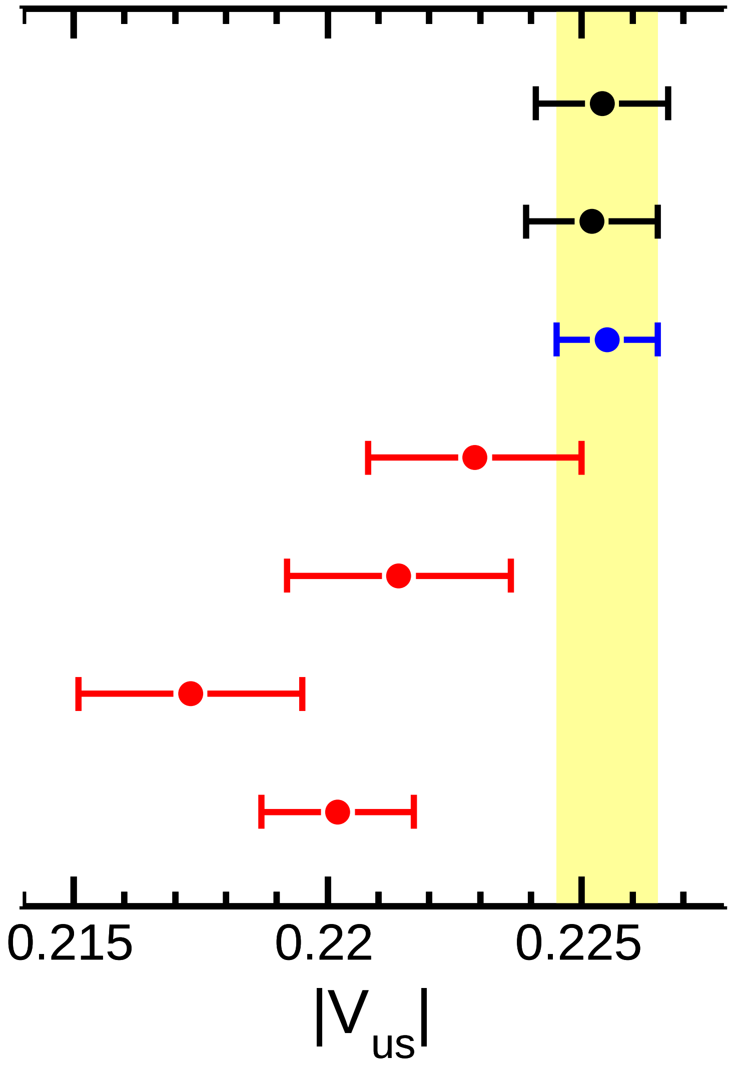
$\tau \to K\nu$, HFAG 2014
0.2211± 0.0020

$\tau \to s$ inclusive, HFAG 2014
0.2176 ± 0.0021

τ average, HFAG 2014
0.2203 ± 0.0014

HFAG-Tau
2014, prelim.

$|V_{us}|$ summary, comparison HFAG 2012 with FlaviaNet 2010



K_{l3} decays, FlaviaNet 2010
0.2254 ± 0.0013

K_{l2} decays, FlaviaNet 2010
0.2252 ± 0.0013

CKM unitarity
0.2255 ± 0.0010

$\tau \to K\nu / \tau \to \pi\nu$, HFAG 2012
0.2229 ± 0.0021

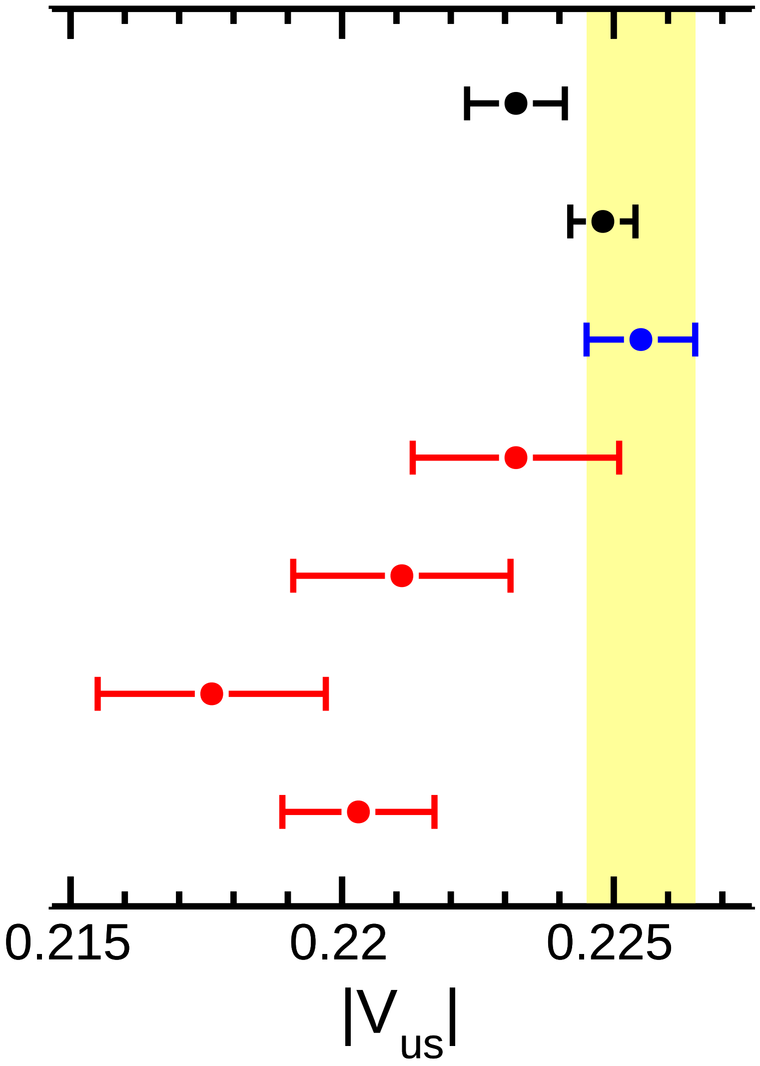
$\tau \to K\nu$, HFAG 2012
0.2214 ± 0.0022

$\tau \to$ s inclusive, HFAG 2012
0.2173 ± 0.0022

τ average, HFAG 2012
0.2202 ± 0.0015

HFAG-Tau
Winter 2012

$|V_{us}|$ summary, comparison HFAG 2014 prelim. with Moulson CKM 2014



K_{l3} decays, Moulson CKM 2014
0.2232 ± 0.0009

K_{l2} decays, Moulson CKM 2014
0.2248 ± 0.0006

CKM unitarity, PDG 2013
0.2255 ± 0.0010

$\tau \to K\nu$ / $\tau \to \pi\nu$, HFAG 2014
0.2232 ± 0.0019

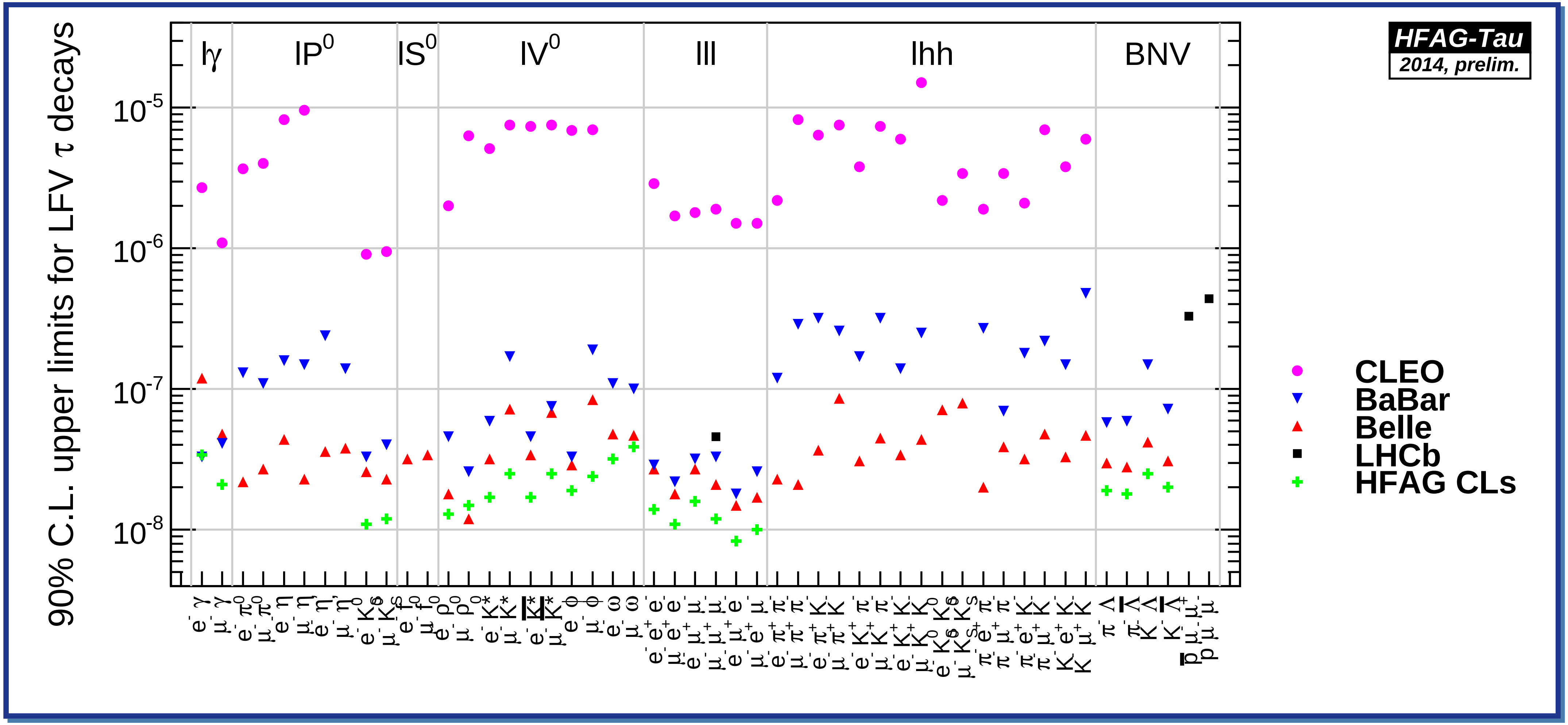
$\tau \to K\nu$, HFAG 2014
0.2211± 0.0020

$\tau \to s$ inclusive, HFAG 2014
0.2176 ± 0.0021

τ average, HFAG 2014
0.2203 ± 0.0014

HFAG-Tau
2014, prelim.

tau LFV upper limits



tau LFV upper limits: some documentation

Marcin Chrząszcz

- limits combinations with CLs method (`http://pdg.lbl.gov/2011/reviews/rpp2011-rev-statistics.pdf`)
- code from T.Junk (CDF) `http://www-cdf.fnal.gov/~trj/mclimit/production/mclimit.html`
  with documentation: `http://www-cdf.fnal.gov/~trj/mclimit/mclimit_csm.pdf`
  - cross-check with CLs implementation in ROOSTATS
- outline of the combination procedure
  - get number of expected $\tau \to$ LFV candidates using $N = 2\sigma \cdot \mathcal{L}_{\text{int}} B \tau \to$ LFV
  - assume that systematics of a single measurements are uncorrelated
  - 2 nuisance parameters, one for signal one for background
  - combined CLs is the product of single measurements CLs
  - scan BR from 10^{-10} to 10^{-7} with 1M toy MC
  - 90% CL upper limit set for CLs < 0.1

Conclusions and prospects

- **Belle 2013 tau lifetime measurement improved lepton universality tests**
- had not time to include BESIII tau mass result, modest improvements expected on lepton universality
- **$|V_{us}|$ from τ still lower than other determinations**
  - tau inclusive $|V_{us}|$ is inconsistent at the 3.4σ level
  - exclusive determinations are statistically compatible
  - somewhat reduced $|V_{us}|$ discrepancy w.r.t. Kaons
- may get additional precision results from B-factories
  - *BABAR* will eventually release complex **study of $\tau \to K n\pi^0\nu$, $n = 0$–3**