Radiative corrections to di-pion τ decays

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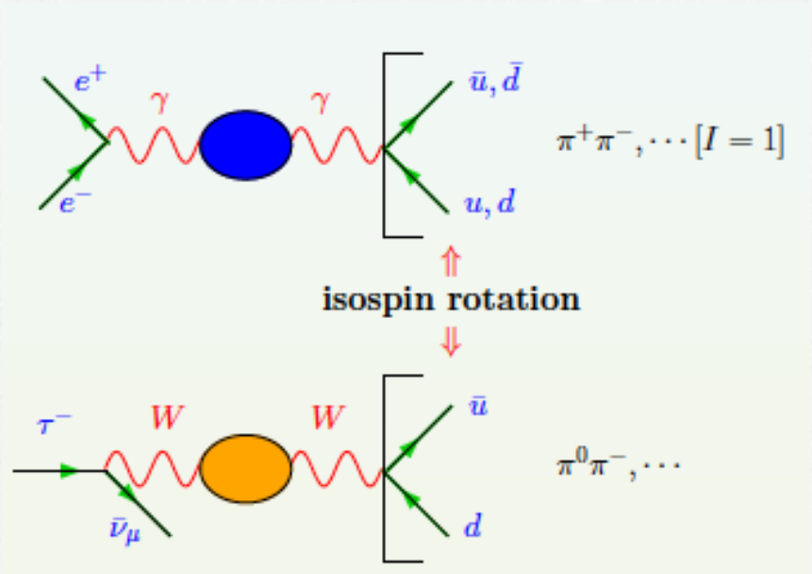
LD radiative corrections to exclusive $\tau \to X \nu$ decays

channel X	accuracy (PDG 2014)	Interest	RadCorr $O(\alpha)$
π^-	0.6%	$e-\mu$ univ, V_{us}	ratio K/π [1]
K^-	1.4%	$e-\mu$ univ, V_{us}	ratio K/π [1]
$\pi^-\pi^0$	0.4%	CVC, a_μ	$t-$distrib, BR [2,3]
$K^-\pi^0$	3.5%	V_{us}	Estim. BR [4,5]
$K^0\pi^-$	4.8%	V_{us}	Estim. BR [4]
$\pi^-\pi^0\pi^0$	1.2%	-	no
$\pi^+\pi^-\pi^-$	0.6%	-	no

[1] Decker & Finkemeier, NPB 1994; PLB 1994
[2] Cirigliano, Ecker & Neufeld PLB 2001; JHEP 2002
[3] Flores-Baez et al, PRD 2006
[4] M. Antonelli et al, JHEP 2013
[5] Flores-Baez & Morones, PRD 2013

CVC hypothesis (isospin symmetry)



$$v_{\pi^+\pi^-}(s) \quad = \quad v_{\pi^-\pi^0}(s) \cdot R_{IB}(s)$$

$$R_{IB}(s) \quad = \quad \frac{\mathrm{FSR}(s)}{G_{EM}(s)} \frac{\beta_0^3(s)}{\beta_-^3(s)} \left| \frac{F_0(s)}{F_-(s)} \right|^2$$

Remaining CVC puzzles (after IB corrections):

- Measured values of $B(\pi^-\pi^0)$ and predictions from e+e- data, differ by $\simeq 2\sigma$
- τ vs. e+e- measured spectral functions differ for $E > m_{\rho(770)}$
- predictions of $a_\mu[\pi\pi, \mathrm{LO}]$ based on τ and e+e- data differ by 1.5σ

2π invariant-mass spectrum

$$\frac{d\Gamma[\pi\pi(\gamma)]}{dt} = \frac{d\Gamma^0[\pi\pi]}{dt} G_{EM}(t)$$

$$G_{EM}(t) = G^0_{EM}(t) + G^{\rm rest}_{EM}(t)$$

virtual+real (infrared) photon corrections

remaining real-photon corrections

$a_\mu(\pi\pi)$: remove radiative corrections encoded in $G_{EM}(s)$ from photon inclusive di-pion spectrum;

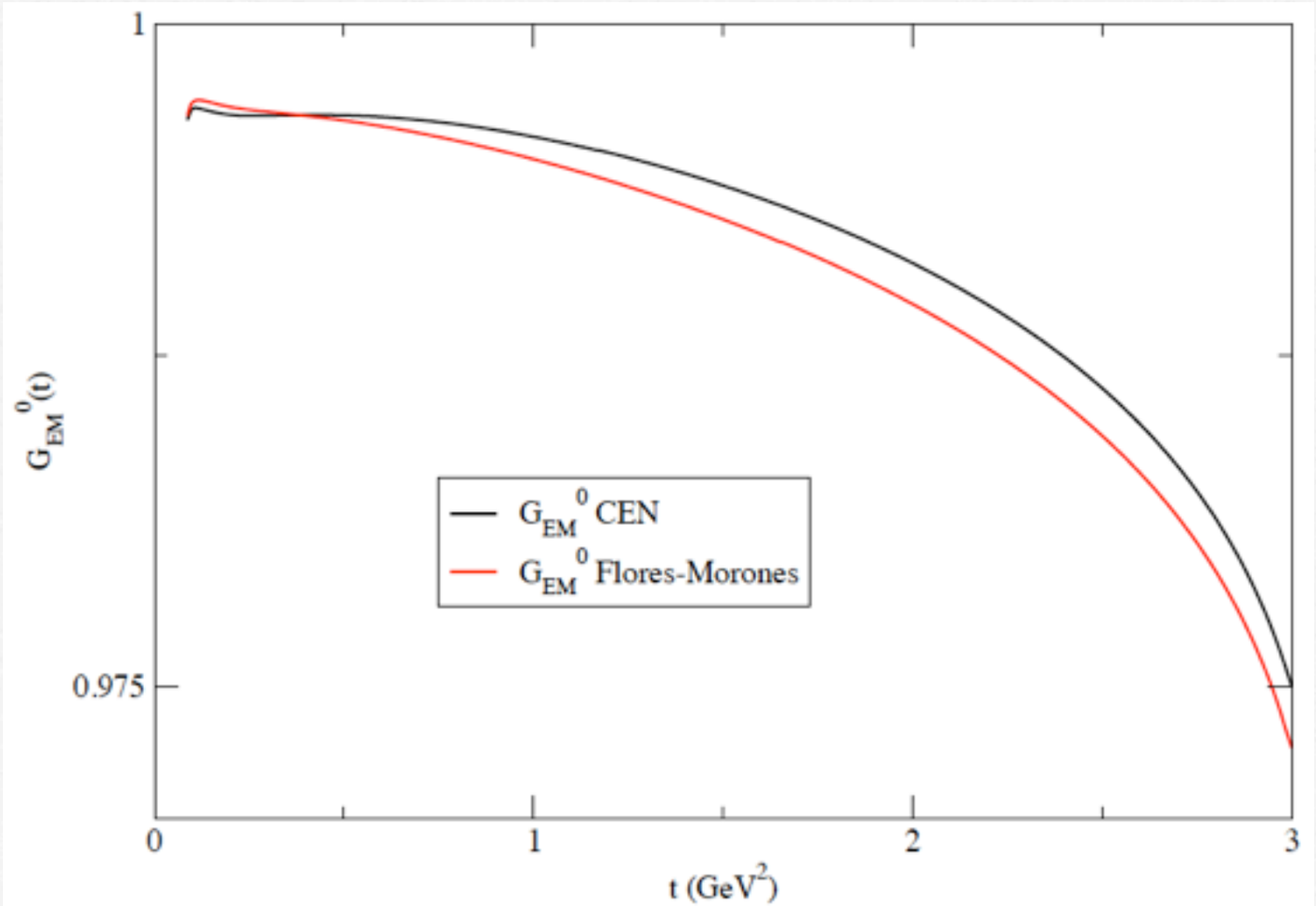
BR($\pi\pi$): add $G_{EM}(s)$ to e+e- spectral function to compare to measured photon inclusive BR.

Virtual-photon corrections:

- ChPT with virtual photons and leptons + relevante resonance d.o.f.

  (Cirigliano, Ecker & Neufeld, PLB 2001)
- Model-Independent corrections [photon exchange among external

  lines + SQED a la Sirlin (1967)], (Flores-Baez & Morones, PRD 2013)



Effects on BR

$$\mathrm{BR}_\tau(\pi^-\pi^0(\gamma)) = \mathrm{BR}^0_\tau(\pi^-\pi^0)[1+\delta^0_{LD}]$$

$$\delta^0_{LD} = -0.004$$

CEN & Flores-Baez, Morones

Real-photon corrections $\tau^-(p)\to\pi^-(p_-)\pi^0(p_0)\nu(q)\gamma(\varepsilon,k)$

- Low's low-energy theorem (F. E. Low 1958) $\mathcal{M}^\gamma = \frac{A}{k} + Bk^0 + \cdots$

$$\mathcal{M}^\gamma(\pi\pi) = eG_F V_{ud}\left[L(p,p_-)\bar{Q}_\nu f_+ + X_\nu + \frac{if_+}{2p\cdot k}\varepsilon_{\mu\nu\rho\sigma}\epsilon^\mu \bar{Q}^\rho k^\sigma\right] l^\nu$$

$$L(a,b) \equiv \frac{a.\epsilon}{a.k} - \frac{b.\epsilon}{b.k} \equiv \frac{L_\beta(a)b^\beta}{b.k}$$

$$X_\nu = -L(p_-,Q)(f'_+ - f_+)\left[\bar{Q}_\nu + \frac{p_-.k}{Q.k}Q_\nu\right] + L_\nu(p_-)f'_+ + L_\nu(\bar{Q})\frac{\bar{Q}.k}{2p.k}f_+$$

Extra term of VMD w.r.t. RχT (we tag it with a coeff. δ; Lopez Castro, Roig & Toledo in progress)

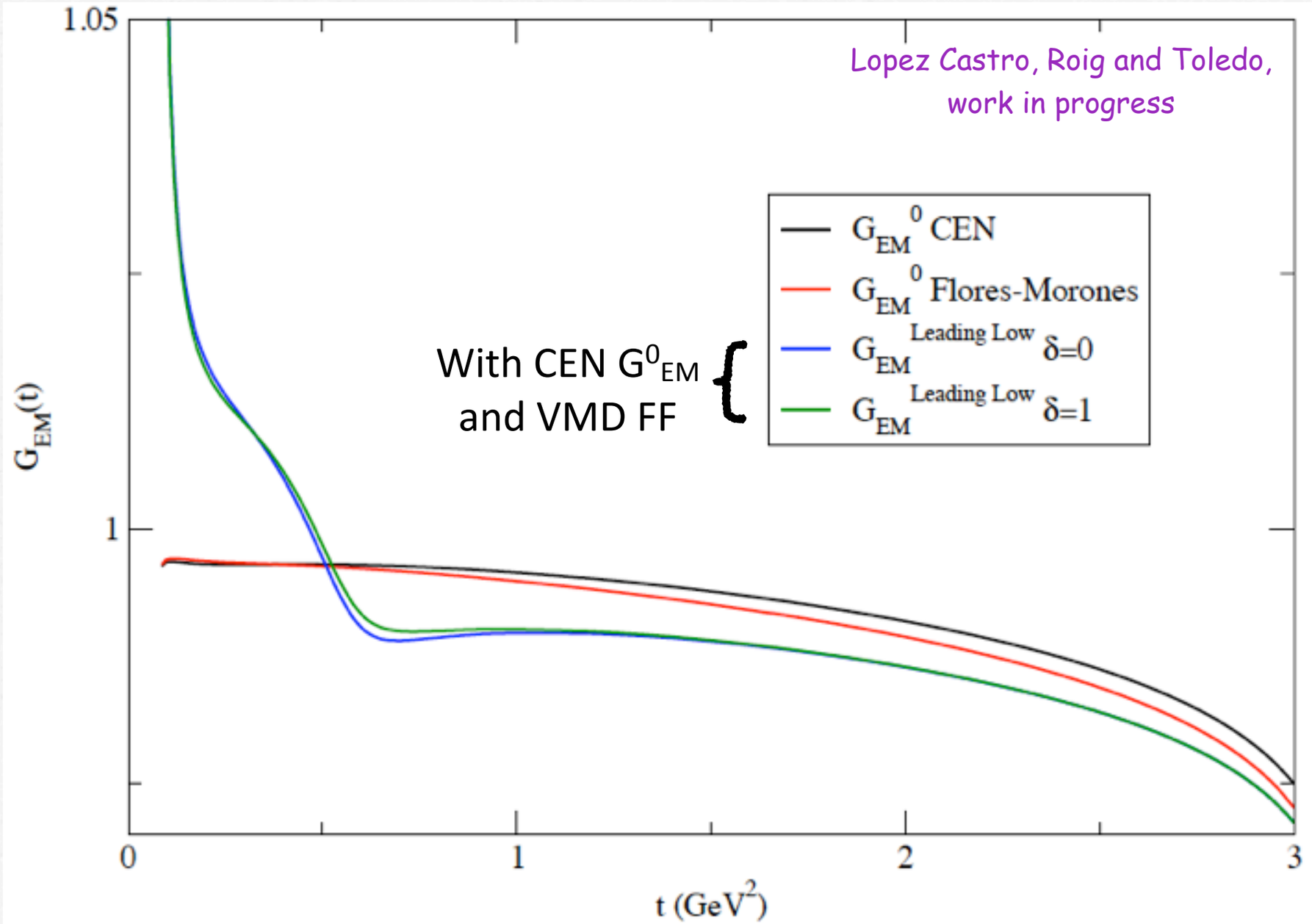
$$Q = p_- + p_0;\ \bar{Q} = p_0 - p_-;\ f_+^{(')} = f_+(t^{(')});\quad t' = (Q+k)^2 = t + 2Q.k$$

- Burnett and Kroll's theorem (T. Burnett and N. Kroll, 1968)

$$\sum_{\text{spin}} |\mathcal{M}^\gamma| = e^2 \sum_{\text{spin}} |\mathcal{M}^0(k=0)|^2 \left|\frac{p_\tau\cdot\epsilon}{p_\tau\cdot k} - \frac{p_{\pi^-}\cdot\epsilon}{p_{\pi^-}\cdot k}\right|^2 + \mathcal{O}(k^0) + \cdots$$

BK not satisfied!; remaining terms of $O(k^0)$ not fixed completely by gauge-invariance requirements?

Virtual+(leading-Low) real photon corrections



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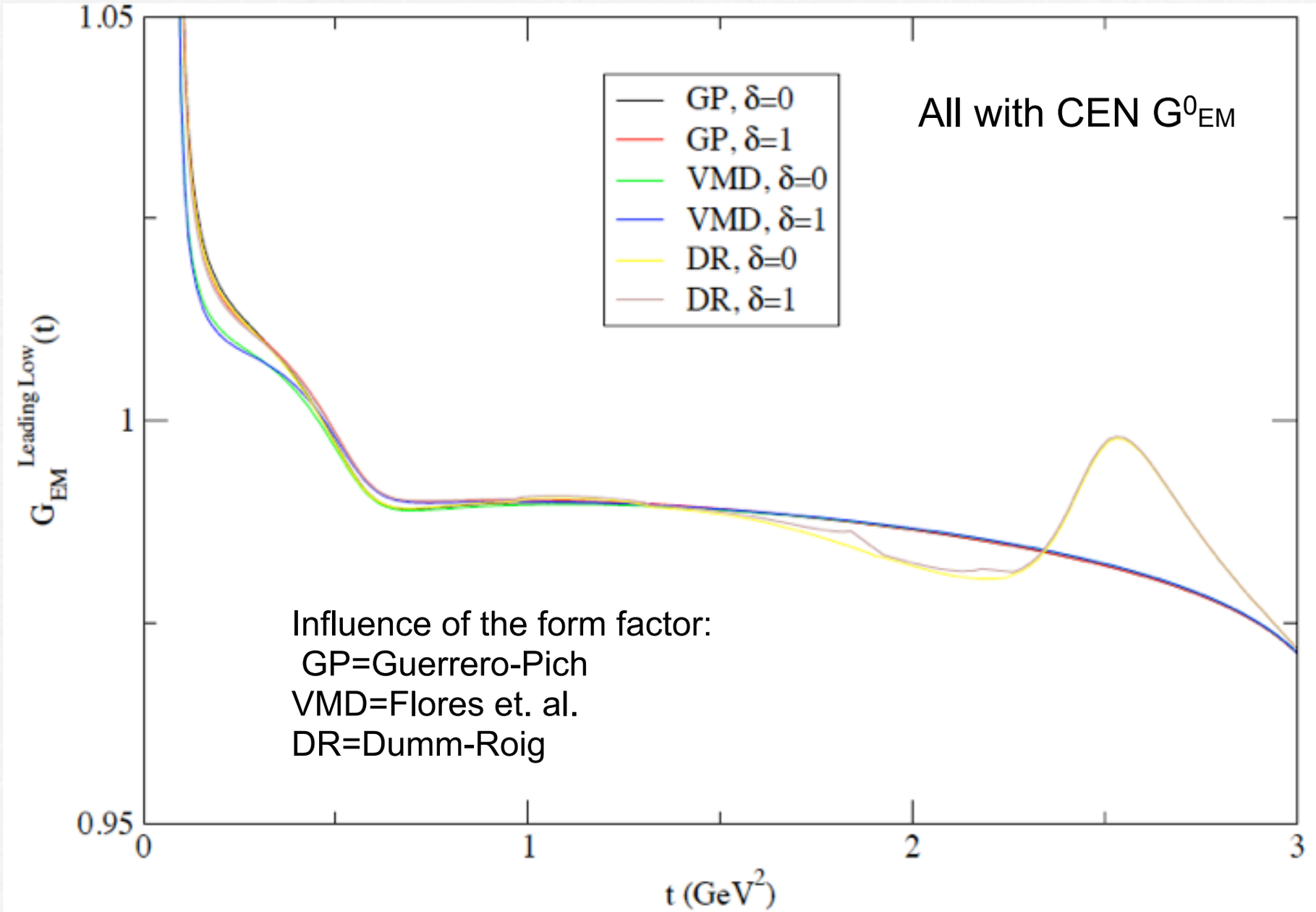

López Castro, Roig & Toledo, in progress

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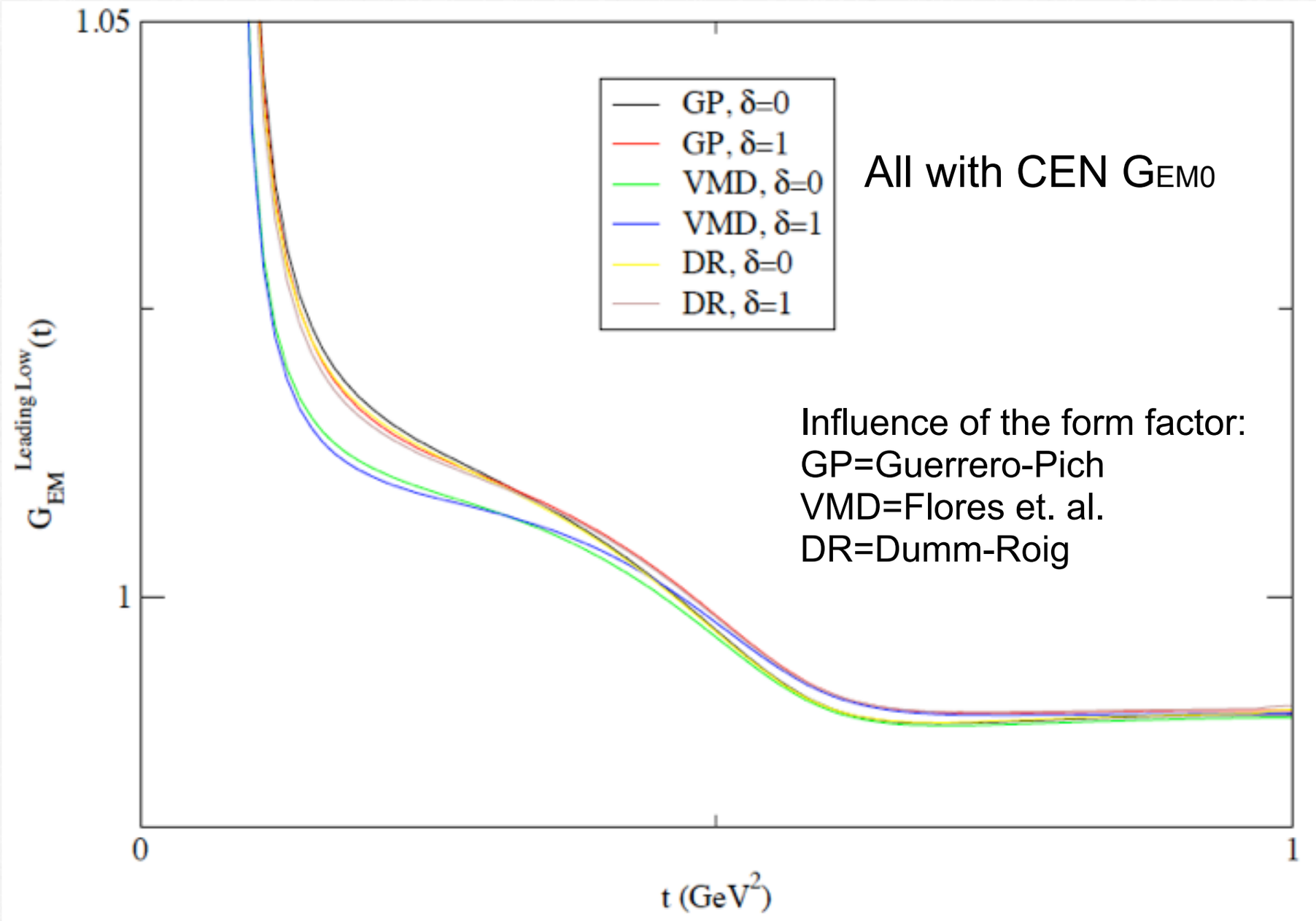
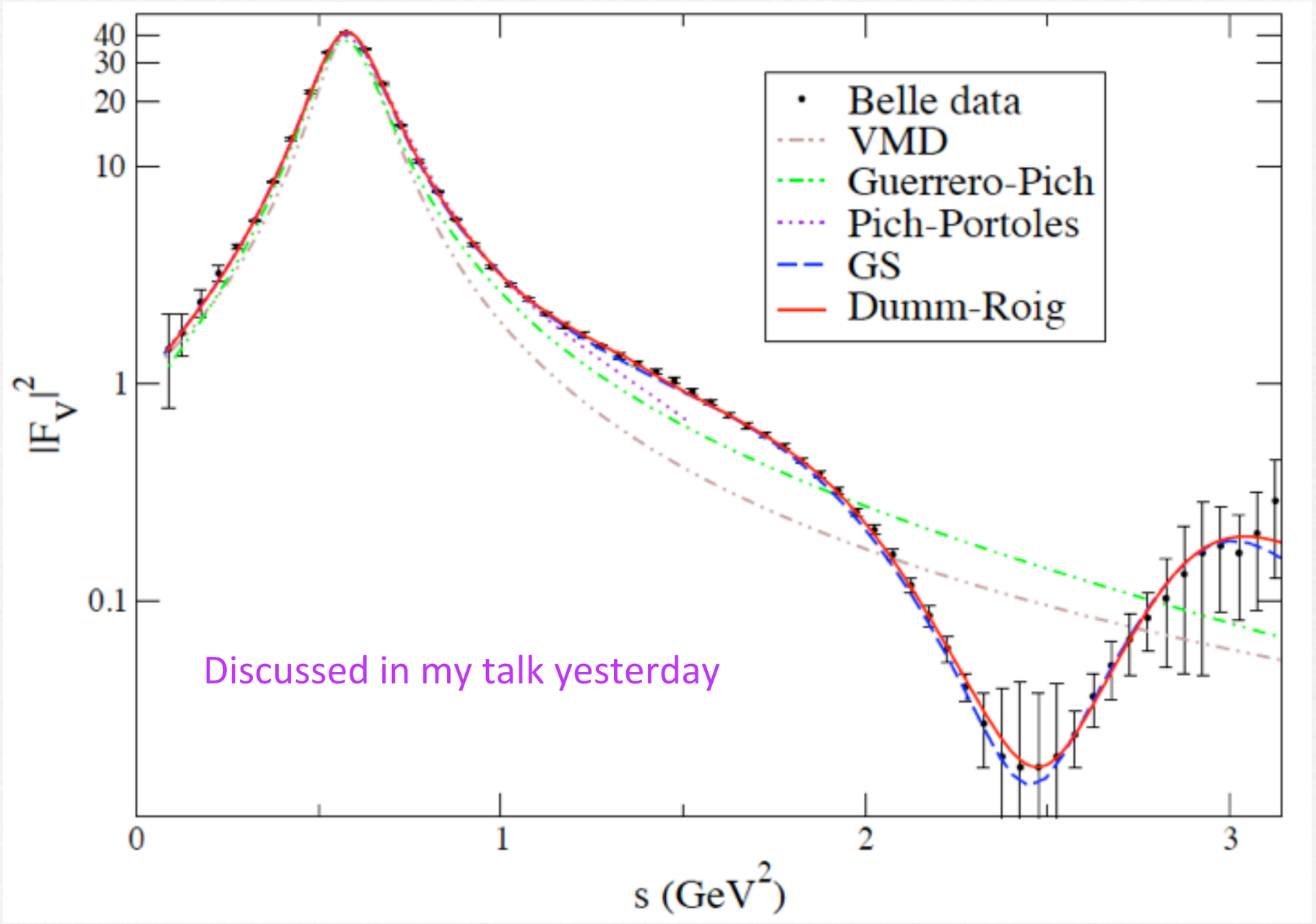


López Castro, Roig & Toledo, in progress)

Form factor from non-radiative 2-pion tau decays



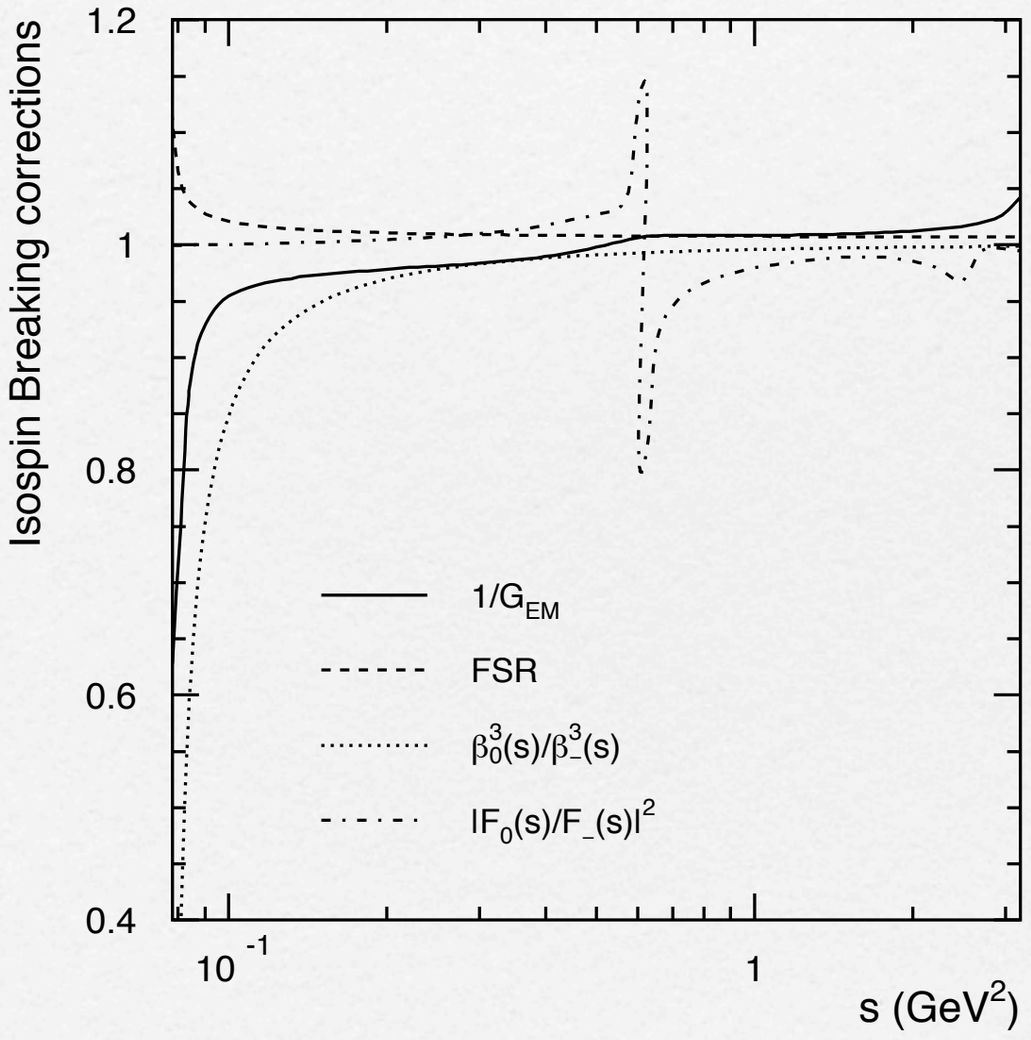
Discussed in my talk yesterday

IB corrections to τ data and associated uncertainties

$$\Delta^{\rm IB} a_\mu^{\rm LO,had}[\pi\pi,\tau] = \frac{\alpha^2 m_\tau^2}{6\,|V_{ud}|^2\pi^2}\,\frac{\mathcal{B}_{\pi\pi^0}}{\mathcal{B}_e}\int_{4m_\pi^2}^{m_\tau^2} ds\,\frac{K(s)}{s}$$
$$\times\,\frac{dN_X}{N_X\,ds}\left(1-\frac{s}{m_\tau^2}\right)^{-2}\left(1+\frac{2s}{m_\tau^2}\right)^{-1}\left[\frac{R_{\rm IB}(s)}{S_{\rm EW}}-1\right]$$

Source	$\Delta a_\mu^{\rm had,LO}[\pi\pi,\tau]$ (10^{-10}) GS model	KS model
$S_{\rm EW}$	-12.21 ± 0.15	
$G_{\rm EM}$	-1.92 ± 0.90	
FSR	$+4.67 \pm 0.47$	
ρ–ω interference	$+2.80 \pm 0.19$	$+2.80 \pm 0.15$
$m_{\pi^\pm} - m_{\pi^0}$ effect on σ	-7.88	
$m_{\pi^\pm} - m_{\pi^0}$ effect on Γ_ρ	$+4.09$	$+4.02$
$m_{\rho^\pm} - m_{\rho^0_{\rm bare}}$	$0.20^{+0.27}_{-0.19}$	$0.11^{+0.19}_{-0.11}$
$\pi\pi\gamma$, electrom. decays	-5.91 ± 0.59	-6.39 ± 0.64
Total	-16.07 ± 1.22	-16.70 ± 1.23
	-16.07 ± 1.85	

M. Davier et al, EPJ 2010



Important below the ρ peak

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- We are after reducing the error in $a_\mu^{HVP,LO}$