Study of an anomalous tau lepton decay using a chiral Lagrangian with vector mesons

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Contents of my talk

1. **Intrinstic parity violating process in τ decays.**
   $\tau^- \to \eta\pi^-\pi^0\nu_\tau$

2. **Form factors using Chiral Lagrangian with vector mesons**

3. **Numerical results**

4. **Conclusions**

Intrinstic Parity violating process

$\langle \pi^- \pi^0 \eta | \bar{d} \gamma_\mu u | 0 \rangle \simeq$ Intrinsic Parity violation

$\langle \pi^- \pi^0 \eta | \bar{d} \gamma_\mu \gamma_5 u | 0 \rangle \simeq$ G Parity violation $\sim (m_u - m_d)$

PDG(Chin.Phys.C)	$(1.39 \pm 0.10) \times 10^{-3}$
Belle (PLB 672, 09)	$(1.35 \pm 0.03 \pm 0.07) \times 10^{-3}$

Interaction

$$\mathcal{L}_{int} = -\frac{G_F}{\sqrt{2}} V_{ud}^{*} \overline{\nu}_{\tau} \gamma_{\mu} (1 - \gamma_5) \tau \bar{d} \gamma_{\mu} (1 - \gamma_5) u$$

Phase space Phase space

$$d\Gamma = \frac{1}{2m_\tau}|T|^2 dPs^4$$

$$T = < \nu_\tau \eta(p^\eta)\pi^-(p^-)\pi^0(p^0)|T|\tau^- >$$

dPs^4: Phase space:

$$\frac{dPs^4}{dM_{\pi^-\pi^0}dM_{\pi^-\eta}dM_{\pi^-\pi^0\eta}} = \frac{1}{(2\pi)^5}\frac{m_\tau^2 - M_{\pi^-\pi^0\eta}^2}{8m_\tau^2}$$

$$\times \frac{M_{\pi^-\pi^0}M_{\pi^-\eta}}{M_{\pi^-\pi^0\eta}} \times \frac{d\cos\theta}{2}\frac{d\cos\beta}{2}\frac{d\alpha}{2\pi}\frac{d\gamma}{2\pi}$$

Decay distributions

$$\frac{d^3\Gamma}{dM_{\pi^-\pi^0}dM_{\pi^-\eta}dM_{\pi^-\pi^0\eta}} = \frac{G_F^2|V_{ud}|^2}{(2\pi)^5}\frac{m_\tau^2 - M_{\pi^-\pi^0\eta}^2}{32m_\tau^3}$$

$$\frac{M_{\pi^-\pi^0}M_{\pi^-\eta}}{M_{\pi^-\pi^0\eta}}L_{\mu\nu}H^{\mu\nu} \times \frac{d\alpha}{2\pi}\frac{d\gamma}{2\pi}\frac{d\cos\theta}{2}\frac{d\cos\beta}{2}$$

Hadoronic and Leptonic tensor

$$H^{\mu\nu} = J^\mu J^{\nu *}$$

$$L_{\mu\nu} = \bar{u}\gamma_\mu(1-\gamma_5)u_\tau\overline{u_\tau}\gamma_\nu(1-\gamma_5)u$$

Hadoronic current

$$J_\mu = < \pi^- \pi^0 \eta | \bar{d} \gamma_\mu (1 - \gamma_5) u | 0 >$$

$$J_\mu = V_{1\mu} F_1 + V_{2\mu} F_2 - i V_{3\mu} F_3 + V_{4\mu} F_4$$

$$V_{1\mu} = (p_{\pi^-} - p_{\pi^0})_\mu - Q_\mu \frac{(p_{\pi^-} - p_{\pi^0}) \cdot Q}{Q^2}$$

$$V_{2\mu} = (p_{\pi^0} - p_\eta)_\mu - Q_\mu \frac{(p_{\pi^0} - p_\eta) \cdot Q}{Q^2}$$

$$V_{3\mu} = \epsilon_{\mu\nu\rho\sigma} p^\nu_{\pi^-} p^\rho_{\pi^0} p^\sigma_\eta$$

$$V_{4\mu} = Q_\mu \equiv (p_{\pi^0} + p_\eta + p_{\pi^-})_\mu$$

F_i (i=1-4)are form factors and functions of invariant masses such as $M_{\pi^-\pi^0}, M_{\pi^-\eta}$ and $M_{\pi^-\pi^0\eta}$

Contruction of Lorentz indices

$$L_{\mu\nu}H^{\mu\nu} = 2(m_\tau^2 - M_{\pi^-\pi^0\eta}^2)\sum \bar{L}_X W_X$$

$X = A, B, ...$ 16 independent structures (see Khun and Mirkes ZPC. 56(1992)). After taking average over angle variables α, β, γ, only thee $X = A, B, SA$ survive

$$< \bar{L}_A > = < \bar{L}_B > = \frac{2}{3} + \frac{m_\tau^2}{3Q^2} + P\cos\theta(\frac{m_\tau^2}{Q^2} - \frac{2}{3})$$

$$< \bar{L}_{SA} > = \frac{m_\tau^2}{Q^2}(1 + P\cos\theta)$$

Correspondingly Hadronic tensor W_A, W_B, W_{SA} are given as follows

$$W_A = |x_1 F_1 + x_2 F_2|^2 + x_3^2 \Big[|F_1 - \frac{F_2}{2}|^2 + \frac{3|F_2|^2}{4} \Big]$$

$$W_B = x_4^2 |F_3|^2, \quad W_{SA} = Q^2 |F_4|^2$$

Distribution functions of three invariant masses

$$\frac{d^3\Gamma}{dM_{\pi^-\pi^0}dM_{\pi^-\eta}dM_{\pi^-\pi^0\eta}} = \frac{G_F^2|V_{ud}|^2}{(2\pi)^5}\frac{M_{\pi^-\pi^0}M_{\pi^-\eta}}{M_{\pi^-\pi^0\eta}}$$

$$\frac{(m_\tau^2 - M_{\pi^-\pi^0\eta}^2)^2}{16m_\tau^3} \times \sum_{X=A,B,SA} < \bar{L}_X > W_X$$

$$\sum_{X=A,B,SA} < \bar{L}_X > W_X = (\frac{2}{3} + \frac{m_\tau^2}{3Q^2})(W_A + W_B)$$

$$+ \frac{m_\tau^2}{Q^2}W_{SA} \quad (Q^2 = M_{\pi^-\pi^0\eta}^2)$$

Chiral Lagrangian with vector mesons including isospin and intrinsic parity violating effects

$$\mathcal{L} = \frac{f^2}{4}\mathrm{Tr}(D_L U D_L U^\dagger) + B\mathrm{Tr}[M_q(U + U^\dagger)]$$
$$- i g_{2p}\mathrm{Tr}(\xi M_q \xi - \xi^\dagger M_q \xi^\dagger) \cdot \eta_0$$
$$+ \frac{1}{2}\partial_\mu \eta_0 \partial^\mu \eta_0 - \frac{M_0^2}{2}\eta_0^2 + M_V^2 \mathrm{Tr}\left[(V_\mu - \frac{\alpha_\mu}{g})^2\right]$$

$$\xi = \exp(i\frac{\pi}{f}) \quad U = \xi^2 \quad D_{L\mu}U = (\partial_\mu + iA_{L\mu})$$

Isospin breaking $m_u \neq m_d$ leads to mixings π^0 η, η'

$$M_q = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

M^2 neutral pseudoscalar mass matrix in (π^0, η_8, η_0) basis.

$\Delta M_K = M^2_{K^+} - M^2_{K^0}, \Sigma M_K = M^2_{K^+} + M^2_{K^0}$

$$\begin{pmatrix} M^2_{\pi^+} & * & * \\ \dfrac{\Delta M_K}{\sqrt{3}} & \dfrac{2(\Sigma M_K) - M^2_{\pi^+}}{3} & * \\ -\hat{g}_{2p}(\Delta M_K) & \dfrac{\hat{g}_{2p}(\Sigma M_K - 2M^2_{\pi^+})}{\sqrt{3}} & M^2_{00} \end{pmatrix}$$

With $\mathrm{Tr}[M^2] = M^2_{\pi^0} + M^2_{\eta} + M^2_{\eta'}$, one can determine $\hat{g}_{2p}$ so that it reproduces mass eigenvalues.

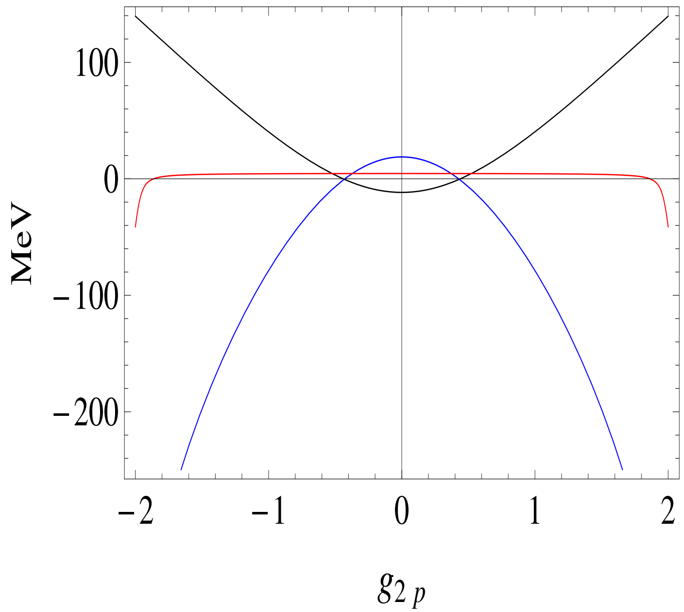


Figure 1: Δm=eigenvalue−physical value fo neutral meson masses. (Black for η', Blue for η, and Red is for π^0) The horizontal axis is the coupling constant $\hat{g}_{2p}$.

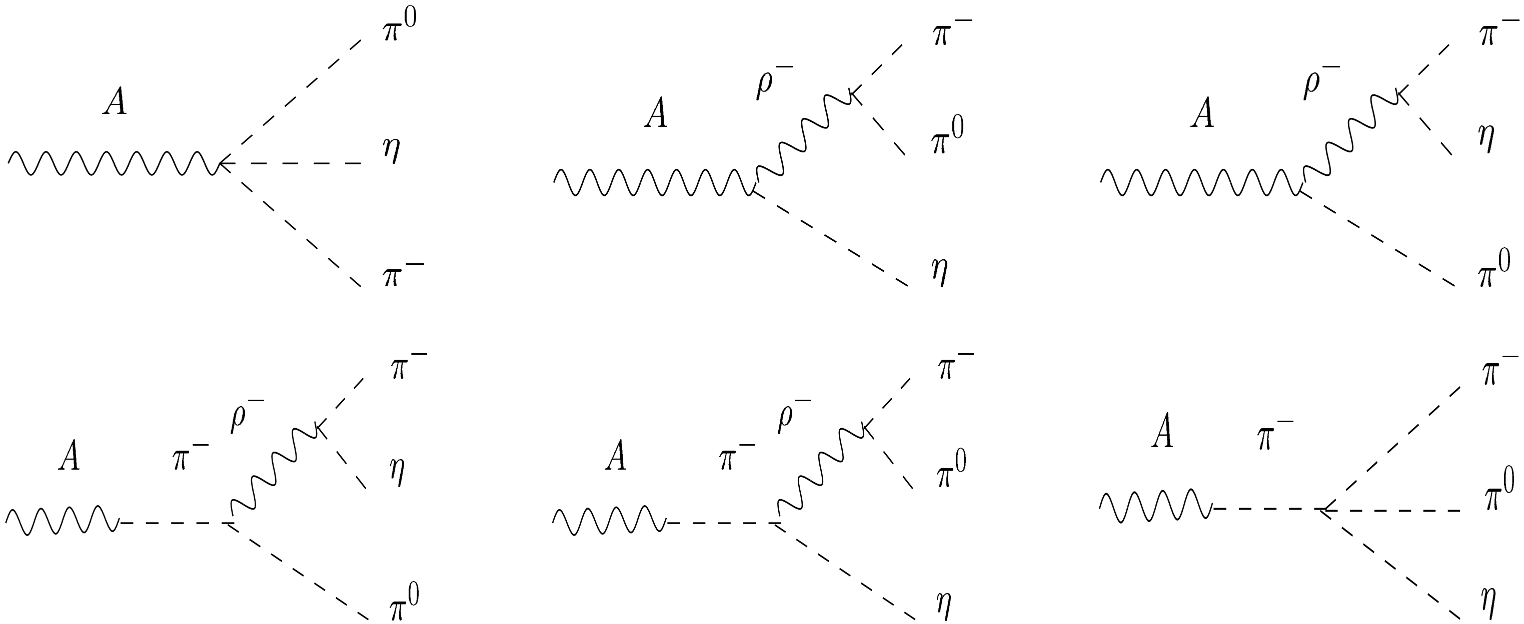
$|\hat{g}_{2p}| = 0.43 \sim 0.44$ leads to a good fit for masses η' and η. $\Delta m_\pi \simeq 5$ (MeV) remains. Below ,we show the orthogonal matrix O for $\hat{g}_{2p} = -0.43$.

$$O = \begin{pmatrix} 0.999959 & -0.00900404 & -0.00137294 \\ 0.00859435 & 0.982629 & -0.185383 \\ 0.00301846 & 0.185363 & 0.982665 \end{pmatrix}$$

$$\begin{pmatrix} \pi^0 \\ \eta_8 \\ \eta^0 \end{pmatrix} = \begin{pmatrix} O_{11} & O_{12} & O_{13} \\ O_{21} & O_{22} & O_{23} \\ O_{31} & O_{32} & O_{33} \end{pmatrix} \begin{pmatrix} \Pi^0 \\ \eta \\ \eta' \end{pmatrix}$$

Contribution to Form factors for axial current

$$< \pi^0\pi^-\eta|\bar{d}\gamma_\mu\gamma_5 d|0 >= V_{1\mu}F_1 + V_{2\mu}F_2 + V_{4\mu}F_4$$



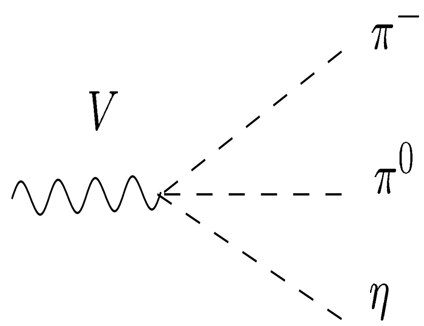
Intrisic Parity violating terms:

(1) Wess Zumino term of chiral anomaly:

$$\mathcal{L}_{WZ} = \frac{i}{\pi^2 f^3}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}V_\mu\partial_\nu\pi\partial_\rho\pi\partial_\sigma\pi.$$

$$\langle\pi^-\pi^0\eta|\bar{d}\gamma^\mu u|0\rangle = \frac{(O_{11}O_{22} - O_{12}O_{21})}{2\sqrt{6}\pi^2 f^3}\epsilon^{\mu\nu\rho\sigma}p^-_\nu p^0_\rho p^\eta_\sigma.$$

contributes to the from factor F_3 of the vector current.



Other intrinsic parity violating interactions such as $\rho \longrightarrow \pi\pi\eta, V\rho\eta$

Fujiwara et.al. PTP.73(1985) with corrections.

$$\mathcal{L}^{IPV} = \sum_{i=1,2,4} C_i \mathcal{L}_i$$

$$\mathcal{L}_1 = i\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}[\alpha_{L\mu}\alpha_{L\nu}\alpha_{L\rho}\alpha_{R\sigma} - (R \leftrightarrow L)],$$

$$\mathcal{L}_2 = i\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}[\alpha_{L\mu}\alpha_{R\nu}\alpha_{L\rho}\alpha_{R\sigma}],$$

$$\mathcal{L}_4 = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}F_{V\mu\nu}[\alpha_{L\rho}\alpha_{R\sigma} - (R \leftrightarrow L)].$$

definition

$$\alpha_{L\mu} = \alpha_\mu + \alpha_{\perp\mu} - gV_\mu$$

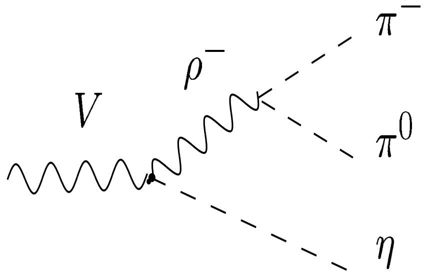
$$\alpha_{R\mu} = \alpha_\mu - \alpha_{\perp\mu} - gV_\mu$$

$$\alpha_\mu = \frac{1}{2i}(\xi^\dagger D_{L\mu}\xi + \xi\partial_\mu\xi^\dagger)$$

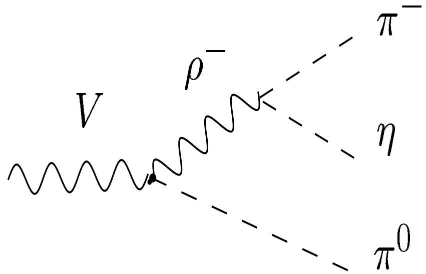
$$\alpha_{\perp\mu} = \frac{1}{2i}(\xi^\dagger D_{L\mu}\xi - \xi\partial_\mu\xi^\dagger)$$

$$F_{V\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + ig[V_\mu, V_\nu]$$

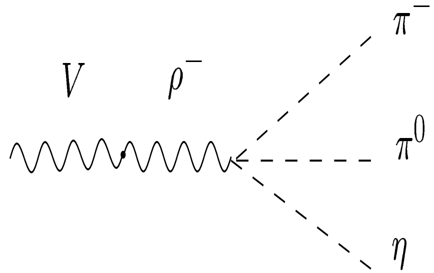
$\boldsymbol{V\rho\eta}$



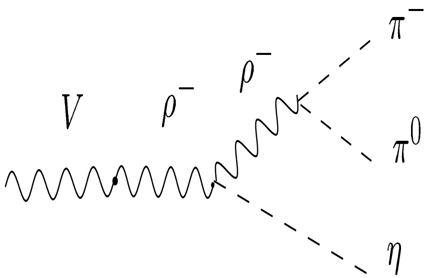
$\boldsymbol{V\rho\pi}$



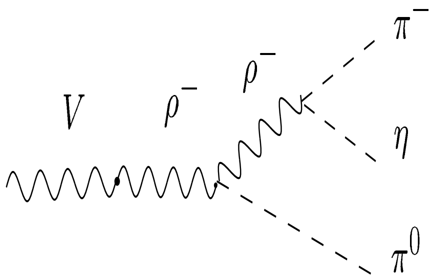
$\boldsymbol{\rho\pi\pi\eta}$



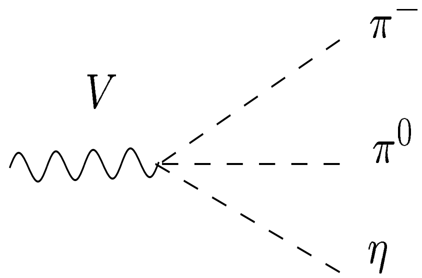
$\boldsymbol{\rho\rho\eta}$



$\boldsymbol{\rho\rho\pi}$



$\boldsymbol{V\pi\pi\eta}$



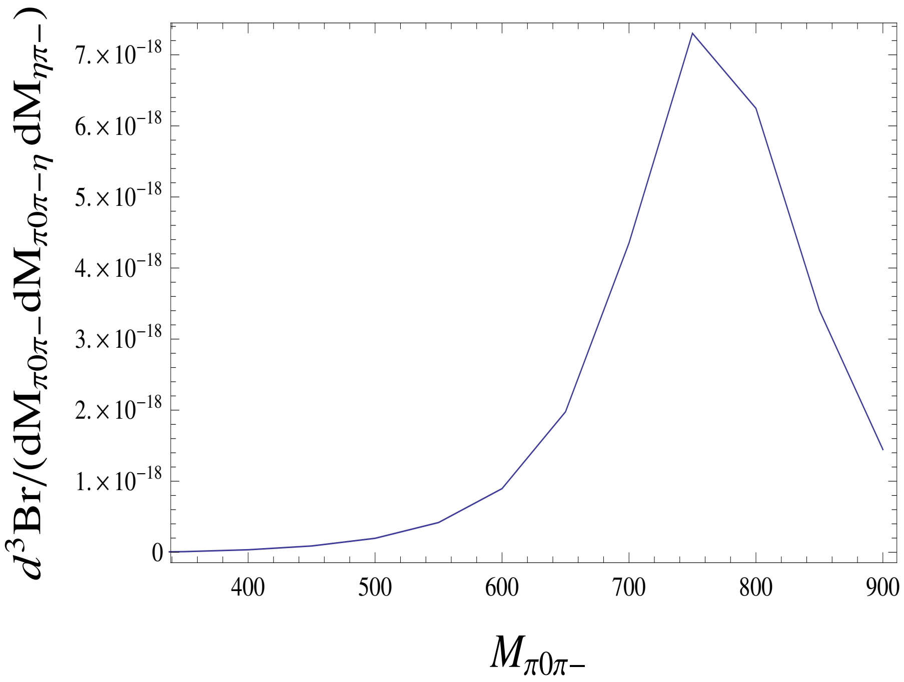


Figure 2: Differential branching distribution for $\tau^- \longrightarrow \eta\pi^0\pi^-$. The unit is MeV^{-3}. $M_{\pi^0\pi^-\eta} = 1500$ (MeV) $M_{\eta\pi^-} = 900$(MeV) 300(MeV)$<$ $M_{\pi^0\pi^-} < 900$(MeV). $C_1 = C_2 = C_4 = 1$ is chosen.

1 We study intrinsic parity violating process $\tau^- \longrightarrow \eta\pi^-\pi^0\nu$.

2 Chiral Lagrangian with explicit isospin breaking and intrinsic parity violation is adopted. With this Lagrangian, the non-vanishing contribution to the axial vector form factor (F_1, F_2, F_4) are computed. We add intrinsic parity violating interactions and compute the vector form factor F_3. (Three free parameters C_1, C_2, C_4.)

3 We show the differential rate for three invariant masses for the case $C_i = 1$ and found the peak

of ρ^- meson for the distribution with respect to $M_{\pi^-\pi^0}$. More serious comparison and fitting with Belle's data will be shown in the near future.