# Leptons & QCD

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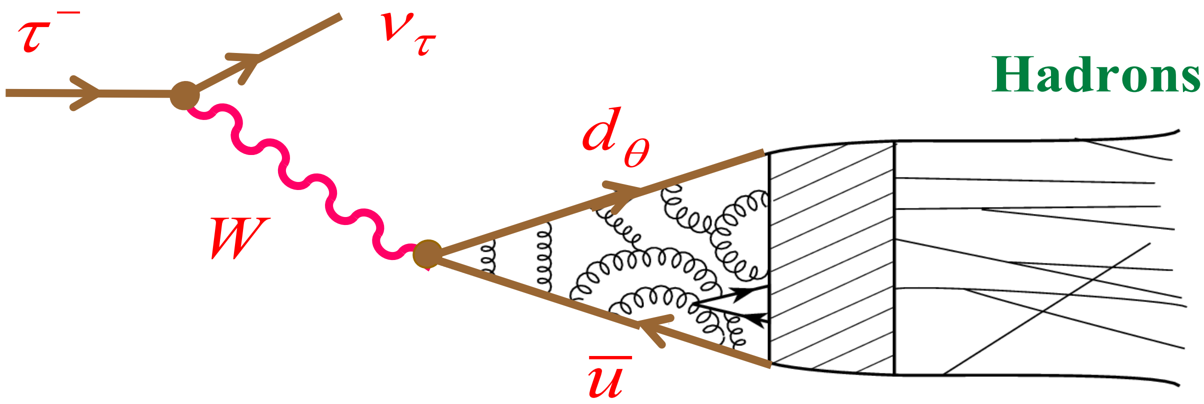
# HADRONIC TAU DECAY




$$d_\theta = V_{ud}\, d + V_{us}\, s$$

**Only lepton massive enough to decay into hadrons**

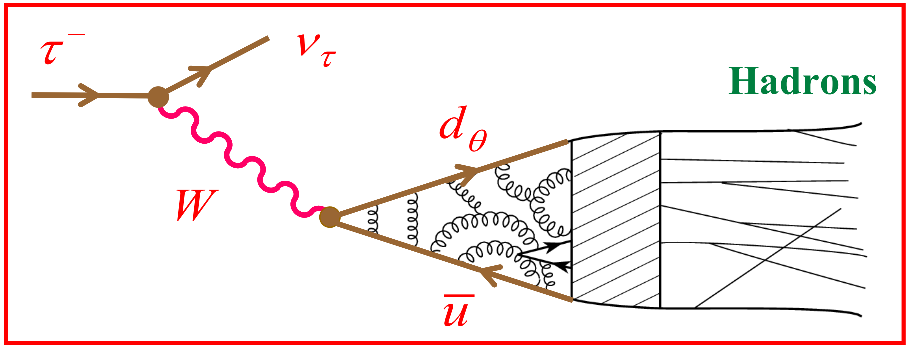
$$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{Hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau\, e^-\, \overline{\nu}_e\right)} \approx N_C \qquad ; \qquad R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.636 \pm 0.011$$

$$R_\tau = \frac{1}{B_e^{\text{univ}}} - 1.97256 = 3.6397 \pm 0.0076 \qquad ; \qquad R_\tau = \frac{\text{Br}\left(\tau^- \to \nu_\tau + \text{Hadrons}\right)}{B_e^{\text{univ}}} = 3.6326 \pm 0.0084$$

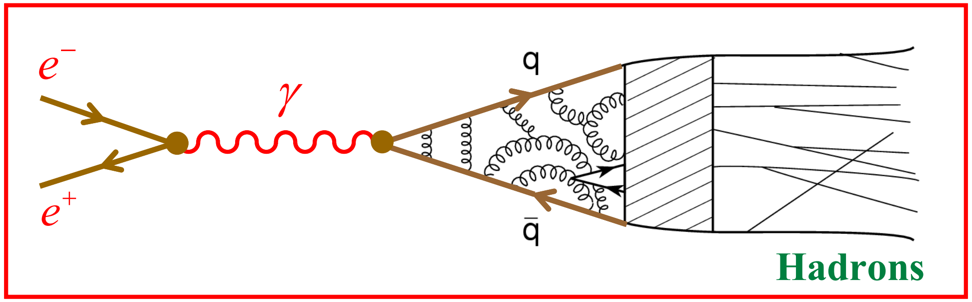
# Only Lepton Massive Enough to Decay into Hadrons

$\tau^- \to \nu_\tau H^-$ **probes the hadronic V-A current**

$$\langle H^- | \bar{d}_\theta \gamma^\mu (1-\gamma_5) u | 0 \rangle$$



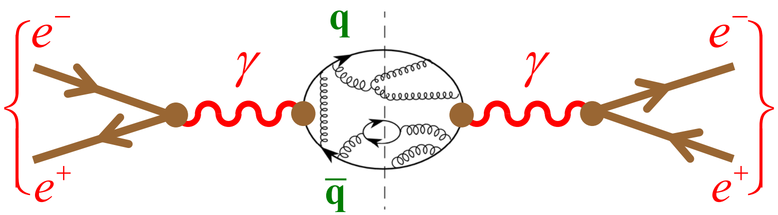

$e^+e^- \to H^0$ **probes the hadronic electromagnetic current**




$$\langle H^0 | \sum_q Q_q \, \bar{q} \gamma^\mu q | 0 \rangle$$

**Isospin:**
$$\frac{\Gamma(\tau^- \to \nu_\tau V^-)}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = \frac{3\cos^2\theta_C}{2\pi\alpha^2} S_{\rm EW} \int_0^1 dx\, (1-x)^2 (1+2x)\, x\; \sigma^{I=1}_{e^+e^- \to V^0}(x m_\tau^2)$$

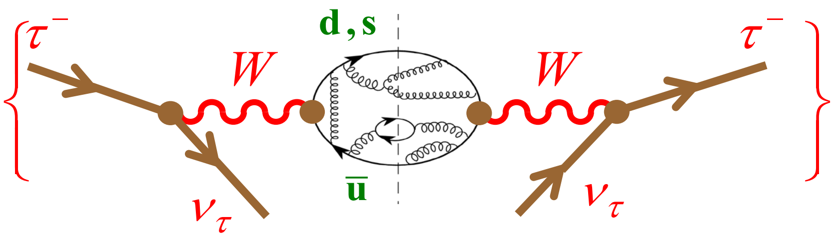
$\sigma \sim \mathrm{Im}$




$$\frac{\sigma(e^+e^-\to \mathrm{had})}{\sigma(e^+e^-\to\mu^+\mu^-)} = 12\pi\, \mathrm{Im}\,\Pi_{\mathrm{em}}(s)$$

$$\Pi^{\mu\nu}_{\mathrm{em}}(q) \equiv i\int d^4x\; e^{iqx}\, \langle 0|T[J^\mu_{\mathrm{em}}(x) J^\nu_{\mathrm{em}}(0)]|0\rangle = \left(-g^{\mu\nu}q^2 + q^\mu q^\nu\right)\Pi_{\mathrm{em}}(q^2)$$

$\Gamma_{\tau\to\nu_\tau+\mathrm{had}} \sim \mathrm{Im}$




$$R_\tau \equiv \frac{\Gamma(\tau^-\to\nu_\tau+\mathrm{had})}{\Gamma(\tau^-\to\nu_\tau e^-\bar{\nu}_e)} = 12\pi\int_0^{m_\tau^2}\frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\,\Pi^{(1)}(s) + \mathrm{Im}\,\Pi^{(0)}(s)\right]$$

$$\Pi^{(J)}(s) \equiv |V_{ud}|^2\left[\Pi^{(J)}_{ud,V}(s)+\Pi^{(J)}_{ud,A}(s)\right] + |V_{us}|^2\left[\Pi^{(J)}_{us,V}(s)+\Pi^{(J)}_{us,A}(s)\right]$$

$$\Pi^{\mu\nu}_{ij,J}(q) \equiv i\int d^4x\; e^{iqx}\,\langle 0|T[J^\mu_{ij}(x) J^\nu_{ij}(0)^\dagger]|0\rangle = \left(-g^{\mu\nu}q^2+q^\mu q^\nu\right)\Pi^{(1)}_{ij,J}(q^2) + q^\mu q^\nu\,\Pi^{(0)}_{ij,J}(q^2)$$

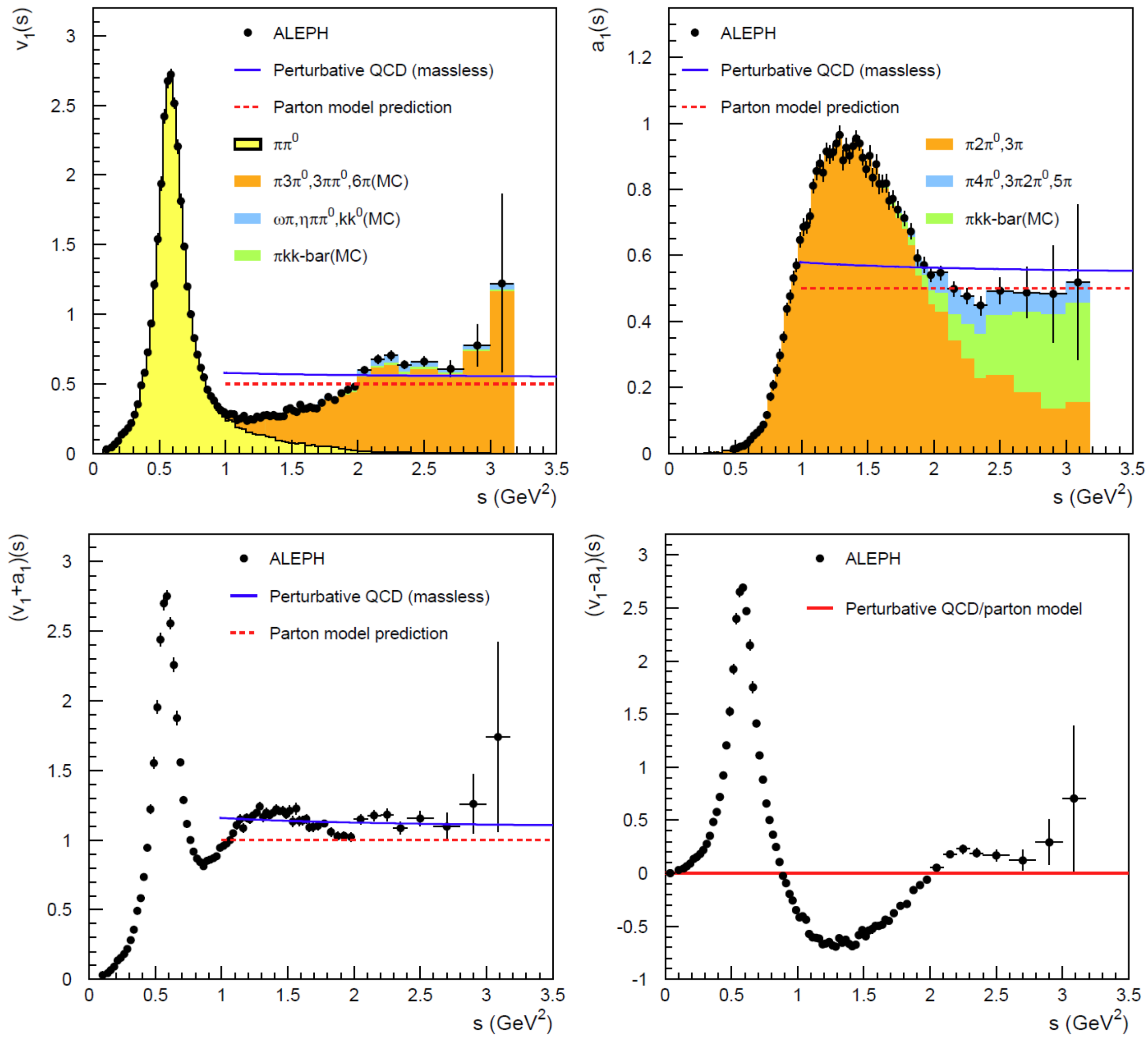
# SPECTRAL FUNCTIONS

Davier et al, 1312.1501

$$v_1(s) = 2\pi \, \mathrm{Im}\, \Pi^{(0+1)}_{ud,V}(s)$$

$$a_1(s) = 2\pi \, \mathrm{Im}\, \Pi^{(0+1)}_{ud,A}(s)$$

**BF data needed**

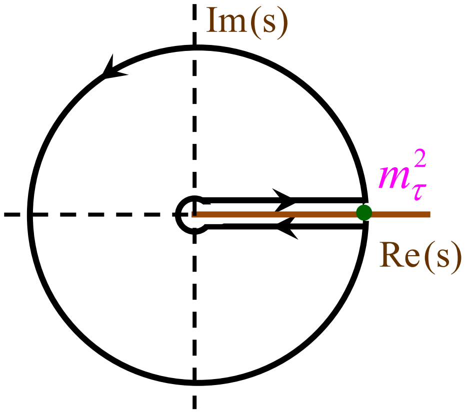


# QCD Prediction of $R_\tau$

Braaten-Narison-Pich'92

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{had})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = 12\pi \int_0^1 dx\,(1-x)^2 \left[(1+2x)\,\mathrm{Im}\,\Pi^{(1)}(x m_\tau^2) + \mathrm{Im}\,\Pi^{(0)}(x m_\tau^2)\right]$$

$x \equiv s/m_\tau^2$




$$R_\tau = 6\pi i \oint_{|x|=1} dx\,(1-x)^2 \left[(1+2x)\,\Pi^{(0+1)}(x m_\tau^2) - 2x\,\Pi^{(0)}(x m_\tau^2)\right]$$

$$\Pi^{(J)}(s) = \sum_{D=2n} \frac{C_D^{(J)}(s,\mu)\,\langle O_D(\mu)\rangle}{(-s)^{D/2}}$$

**OPE**

$$R_\tau = N_C\, S_{\mathrm{EW}} \left(1 + \delta_{\mathrm{P}} + \delta_{\mathrm{NP}}\right) = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

$S_{\mathrm{EW}} = 1.0201\ (3)$ ; $\delta_{\mathrm{NP}} = -0.0064 \pm 0.0013$

**Marciano-Sirlin, Braaten-Li, Erler** — **Fitted from data (Davier et al)**

$\delta_{\mathrm{P}} = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + 127\, a_\tau^4 + \ldots \approx 20\%$ ; $a_\tau \equiv \alpha_s(m_\tau)/\pi$

**Baikov-Chetyrkin-Kühn**

## Perturbative (m$_q$=0)

$$-s\,\frac{d}{ds}\,\Pi^{(0+1)}(s) = \frac{1}{4\pi^2}\sum_{n=0} K_n \left(\frac{\alpha_s(-s)}{\pi}\right)^n$$

$K_0 = K_1 = 1$ , $K_2 = 1.63982$ , $K_3 = 6.37101$ , $K_4 = 49.07570$ **Baikov-Chetyrkin-Kühn '08**

$$\delta_{\mathrm{P}} = \sum_{n=1} K_n\; A^{(n)}(\alpha_s) = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + 127\, a_\tau^4 + \cdots$$

**Le Diberder- Pich '92**

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}(1-2x+2x^3-x^4)\left(\frac{\alpha_s(-s)}{\pi}\right)^n = a_\tau^n + \cdots \quad ; \quad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

## Power Corrections

$$\Pi^{(0+1)}_{\mathrm{OPE}}(s) \approx \frac{1}{4\pi^2}\sum_{n\ge 2}\frac{C_{2n}\langle O_{2n}\rangle}{(-s)^n}$$

**Braaten-Narison-Pich '92**

$$C_4\langle O_4\rangle \approx \frac{2\pi}{3}\langle 0|\alpha_s G^{\mu\nu}G_{\mu\nu}|0\rangle$$

$$\delta_{\mathrm{NP}} \approx \frac{-1}{2\pi i}\oint_{|x|=1}dx\;(1-3x^2+2x^3)\sum_{n\ge 2}\frac{C_{2n}\langle O_{2n}\rangle}{(-x m_\tau^2)^n} = -3\,\frac{C_6\langle O_6\rangle}{m_\tau^6} - 2\,\frac{C_8\langle O_8\rangle}{m_\tau^8}$$

**Suppressed by $m_\tau^6$** **[additional chiral suppression in $C_6\langle O_6\rangle^{V+A}$ ]**

# Perturbative Uncertainty on $\alpha_s(m_\tau)$

$$-s\,\frac{d}{ds}\,\Pi^{(0+1)}(s) = \frac{1}{4\pi^2}\sum_{n=0} K_n\; a(-s)^n$$

$$\delta_P = \underbrace{\sum_{n=1} K_n\; A^{(n)}(\alpha_s)}_{\text{CIPT}} = \underbrace{\sum_{n=0} r_n\; a_\tau^n}_{\text{FOPT}}$$

$$r_n = K_n + g_n$$

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}\,(1-2x+2x^3-x^4)\, a_\tau(-x m_\tau^2)^n = a_\tau^n + \cdots \qquad ; \qquad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

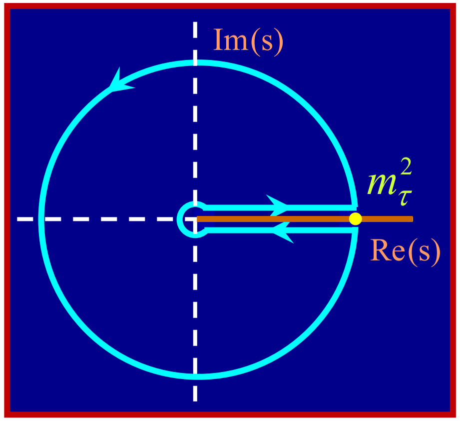
| n | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $K_n$ | 1 | 1.6398 | 6.3710 | 49.0757 | |
| $g_n$ | 0 | 3.5625 | 19.9949 | 78.0029 | 307.78 |
| $r_n$ | 1 | 5.2023 | 26.3659 | 127.079 | |

**The dominant corrections come from the contour integration**

Le Diberder- Pich 1992

**Large running of $a_s$ along the circle** $s = m_\tau^2\ e^{i\varphi}$ , $\varphi \in [0\,,2\pi]$

$$A^{(n)}(a_\tau) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}\,(1-2x+2x^3-x^4)\,a_\tau(-x m_\tau^2)^n \;=\; a_\tau^n + \cdots \qquad ; \qquad a_\tau \equiv \alpha_s(m_\tau)/\pi$$




$$A^{(1)}(a_\tau) = a_\tau - \frac{19}{24}\,\beta_1\,a_\tau^2 + \left[\beta_1^2\left(\frac{265}{288}-\frac{\pi^2}{12}\right)-\frac{19}{24}\,\beta_2\right]a_\tau^3 + \cdots$$

$$a(-s) \simeq \frac{a_\tau}{1-\frac{\beta_1}{2}a_\tau\log\left(-s/m_\tau^2\right)} = \frac{a_\tau}{1-i\frac{\beta_1}{2}a_\tau\phi} = a_\tau\sum_n\left(i\,\frac{\beta_1}{2}a_\tau\,\phi\right)^n \qquad ; \qquad \phi\in[0,2\pi]$$

**FOPT** expansion only convergent if $\alpha_\tau < 0.14$ (0.11) [at 1 (3) loops]

Experimentally $\alpha_\tau \approx 0.11$ ➡ **FOPT should not be used** (divergent series)

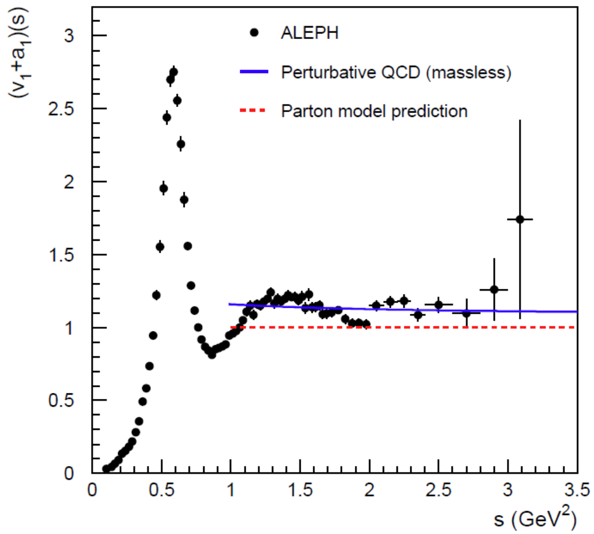
FOPT suffers a large renormalization-scale dependence **(Le Diberder- Pich , Menke)**

The difference between FOPT and CIPT grows at higher orders

# Spectral Function Distribution

- **Moments:**

$$R_\tau^{kl}(s_0) \equiv \int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{dR_\tau}{ds}$$




**Sensitivity to power corrections (k,l)**

**The non-perturbative contribution to $R_\tau$ can be obtained from the invariant-mass distribution of the final hadrons**

$$\delta_{NP} = -0.0064 \pm 0.0013$$ **Davier et al. (ALEPH data)**

- **Fitting the Spectral Function itself:** **Duality Violations**

$\mathrm{Im}\,\Pi(s) = \kappa\, e^{-\gamma s} \sin[\alpha+\beta s]$

$$\delta_{NP} = -0.003 \pm 0.012$$ **Boito et al. (OPAL data)**

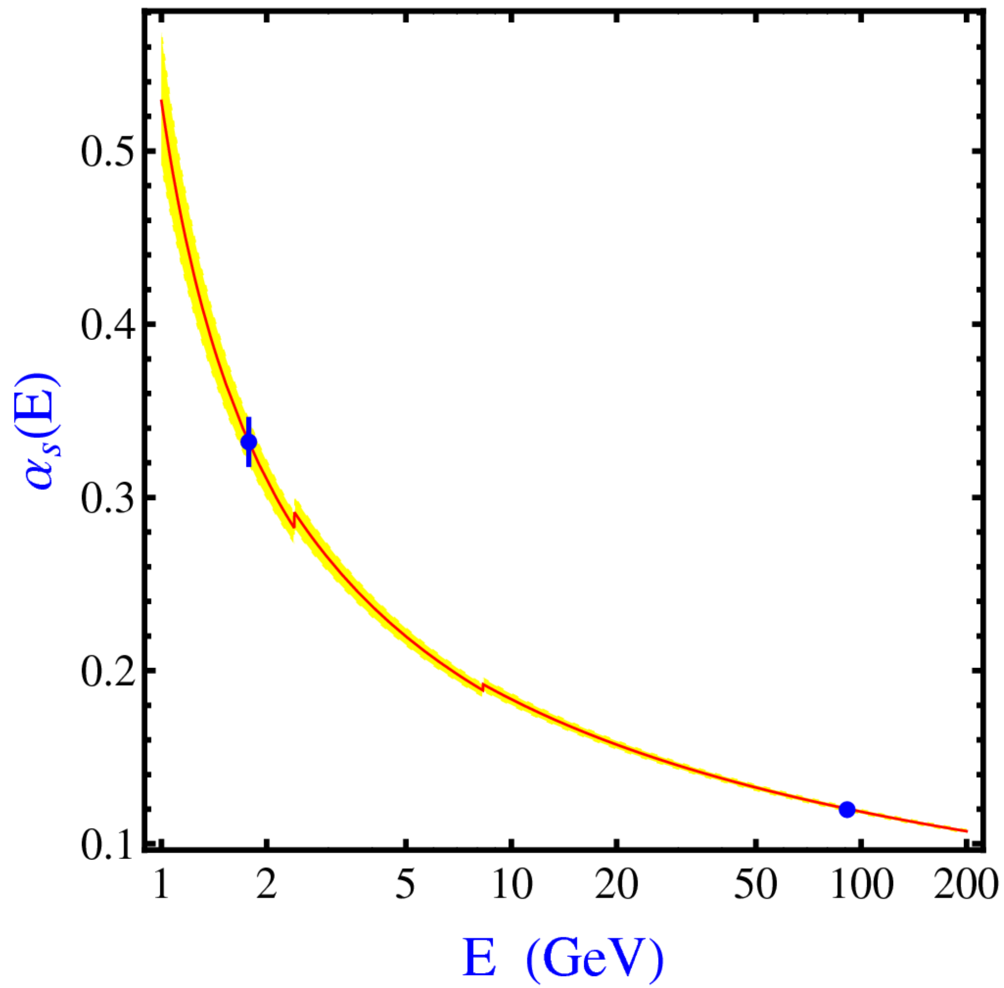
# Recent $\alpha_s(m_\tau)$ Analyses

| Reference | Method | $\delta_{NP}$ | $\delta_P$ | $\alpha_s(m_\tau)$ | $\alpha_s(m_Z)$ |
|---|---|---|---|---|---|
| Baikov et al | CIPT, FOPT | | 0.1998 (43) | 0.332 (16) | 0.1202 (19) |
| **Davier et al'14** | **CIPT, FOPT** | **– 0.0064 (13)** | **–** | **0.332 (12)** | **0.1199 (15)** |
| Beneke-Jamin | BSR + FOPT | – 0.007 (3) | 0.2042 (50) | 0.316 (06) | 0.1180 (08) |
| Maltman-Yavin | PWM + CIPT | + 0.012 (18) | – | 0.321 (13) | 0.1187 (16) |
| Menke | CIPT, FOPT | | 0.2042 (50) | 0.342 (11) | 0.1213 (12) |
| Narison | CIPT, FOPT | | – | 0.324 (08) | 0.1192 (10) |
| Caprini-Fischer | BSR + CIPT | | 0.2037 (54) | 0.322 (16) | – |
| Abbas et al | IFOPT | | 0.2037 (54) | 0.338 (10) | |
| Cvetič et al | $\beta_{exp}$ + CIPT | | 0.2040 (40) | 0.341 (08) | 0.1211 (10) |
| Boito et al | CIPT, DV | – 0.002 (12) | – | 0.347 (25) | 0.1216 (27) |
| | FOPT, DV | – 0.004 (12) | | 0.325 (18) | 0.1191 (22) |
| Pich'14 | CIPT | – 0.0064 (13) | 0.2014 (31) | 0.342 (13) | 0.1213 (14) |
| | FOPT | | | 0.320 (14) | 0.1187 (17) |
| **Pich'14** | **CIPT, FOPT** | **– 0.0064 (13)** | **0.2014 (31)** | **0.332 (13)** | **0.1202 (15)** |

CIPT: Contour-improved perturbation theory
FOPT: Fixed-order perturbation theory
BSR: Borel summation of renormalon series
IFOPT Improved FOPT

$\beta_{exp}$: Expansion in derivatives of $\alpha_s$ ($\beta$ function)
PWM: Pinched-weight moments
CIPTm: Modified CIPT (conformal mapping)
DV: Duality violation (OPAL only)

# Present Status




$$\alpha_s(m_\tau^2) = 0.332 \pm 0.013$$

$$\alpha_s(M_Z^2) = 0.1202 \pm 0.0015$$

$$\alpha_s(M_Z^2)_{Z\,\text{width}} = 0.1197 \pm 0.0028$$

**The most precise test of Asymptotic Freedom**

$$\alpha_s^\tau(M_Z^2) - \alpha_s^Z(M_Z^2) = 0.0005 \pm 0.0015_\tau \pm 0.0028_Z$$

# $V_{us}$ Determination

Gámiz-Jamin-Pich-Prades-Schwab

$$|V_{us}|^2 = \frac{R^{00}_{\tau,S}}{\frac{R^{00}_{\tau,V+A}}{|V_{ud}|^2} - \delta R^{00}_{\tau,\text{th}}}$$

$$\delta R^{kl}_\tau \approx 24 \, \frac{m_s^2(m_\tau^2)}{m_\tau^2} \, \Delta_{kl}(\alpha_s)$$

$$\delta R^{00}_{\tau,\text{th}} \equiv \underbrace{0.1544\,(37)}_{J=0} + \underbrace{0.086\,(32)}_{m_s(2\text{ GeV}) = 94\,(6)\ \text{MeV}} = 0.240\,(32)$$

$R^{00}_{\tau,S} = 0.1614\ (28)$

$R^{00}_{\tau,V+A} = 3.4712\ (79)$ ⟹ $|V_{us}| = 0.2173 \pm 0.0020_{\text{exp}} \pm 0.0010_{\text{th}}$

$|V_{ud}| = 0.97425\ (22)$

$R^{00}_{\tau,S} = 0.1665\ (34)$ ⟹ $\mathbf{|V_{us}| = 0.2207 \pm 0.0023_{exp} \pm 0.0011_{th}}$

**$K_{l3}$:** $|V_{us}| = 0.2239 \pm 0.0009$ $\quad [f_+(0) = 0.9661 \pm 0.0032]$

**The τ could give the most precise $V_{us}$ determination**

# Cabibbo-suppressed $\tau$ Decays

**HFAG**

| $X_s^-$ | $\mathrm{Br}(\tau^- \to \nu_\tau X_s^-)$ (%) | $X_s^-$ | $\mathrm{Br}(\tau^- \to \nu_\tau X_s^-)$ (%) |
|---|---|---|---|
| $K^-$ | $(0.6955 \pm 0.0096)$ | $K^-\eta$ | $(0.0153 \pm 0.0008)$ |
| $K^-\pi^0$ | $(0.4322 \pm 0.0149)$ | $K^-\pi^0\eta$ | $(0.0048 \pm 0.0012)$ |
| $K^-2\pi^0$ (ex. $K^0$) | $(0.0630 \pm 0.0222)$ | $\pi^-\bar{K}^0\eta$ | $(0.0094 \pm 0.0015)$ |
| $K^-3\pi^0$ (ex. $K^0$, $\eta$) | $(0.0419 \pm 0.0218)$ | $K^-\omega$ | $(0.0410 \pm 0.0092)$ |
| $\pi^-\bar{K}^0$ | $(0.8206 \pm 0.0182)$ | $K^-\phi$ ($\phi \to K\bar{K}$) | $(0.0037 \pm 0.0014)$ |
| $\pi^-\bar{K}^0\pi^0$ | $(0.3649 \pm 0.0108)$ | $K^-\pi^-\pi^+$ (ex. $K^0$, $\omega$) | $(0.2923 \pm 0.0068)$ |
| $\pi^-\bar{K}^0 2\pi^0$ | $(0.0269 \pm 0.0230)$ | $K^-\pi^-\pi^+\pi^0$ (ex. $K^0$, $\omega$, $\eta$) | $(0.0411 \pm 0.0143)$ |
| $\bar{K}^0 h^- h^+ h^-$ | $(0.0222 \pm 0.0202)$ | | |

$\mathrm{Br}(\tau^- \to \nu_\tau X_s^-) = (2.875 \pm 0.050)\%$ $\rightarrow$ $(2.967 \pm 0.060)\%$

**Unaccounted modes:** $1 - \sum_i \mathrm{Br}(\tau^- \to \nu_\tau^- X_i) = (0.0704 \pm 0.1060)\%$

**Antonelli-Cirigliano-Lusiani-Passemar:**

$\Gamma(K\to\mu\nu_\mu)$

$K_{l3}$ + $\tau \to \nu_\tau(K\pi)^-$ spectra $\Rightarrow$

$\mathrm{Br}(\tau^- \to \nu_\tau K^-) = (0.713 \pm 0.003)\%$

$\mathrm{Br}(\tau^- \to \nu_\tau K^-\pi^0) = (0.471 \pm 0.018)\%$

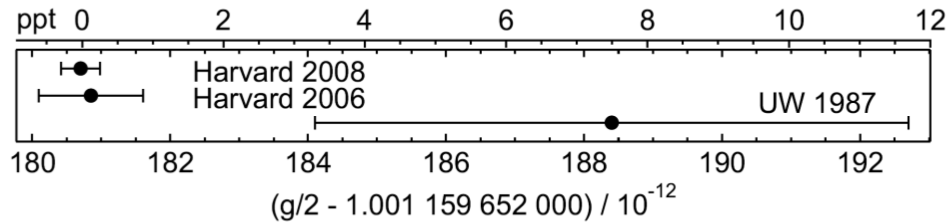
$\mathrm{Br}(\tau^- \to \nu_\tau \bar{K}^0\pi^-) = (0.857 \pm 0.030)\%$

$\Rightarrow$ $R_{\tau,S}^{00} = 0.1665\ (34)$

# Electron Anomalous Magnetic Moment

**Hanneke-Fogwell-Gabrielse '08**

$$a_e = 0.001\,159\,652\,180\,73\ (28)$$




$$a_e^{\rm QED} = \sum_{n=1} \left(\frac{\alpha}{\pi}\right)^n a_e^{(2n)} \qquad a_e^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_e/m_\mu) + A_2^{(2n)}(m_e/m_\tau) + A_3^{(2n)}(m_e/m_\mu, m_e/m_\tau)$$

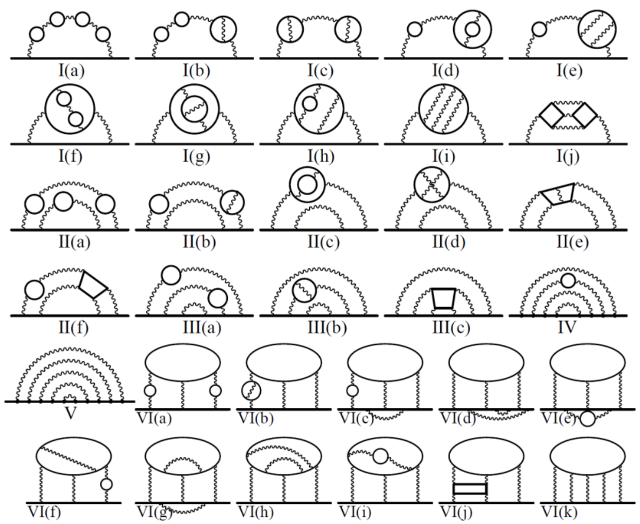
**Aoyama-Hayakawa-Kinoshita-Nio**

$$A_1^{(8)} = -1.9106\ (20) \quad , \quad A_2^{(8)}(m_e/m_\mu) = 9.222\ (66)\cdot 10^{-4}$$

$$A_2^{(8)}(m_e/m_\tau) = 8.24\ (12)\cdot 10^{-6} \quad , \quad A_3^{(8)}(m_e/m_\mu, m_e/m_\tau) = 7.465\ (18)\cdot 10^{-7}$$

$$A_1^{(10)} = 9.16\ (58) \quad , \quad A_2^{(10)}(m_e/m_\mu) = -3.82\ (39)\cdot 10^{-3}$$

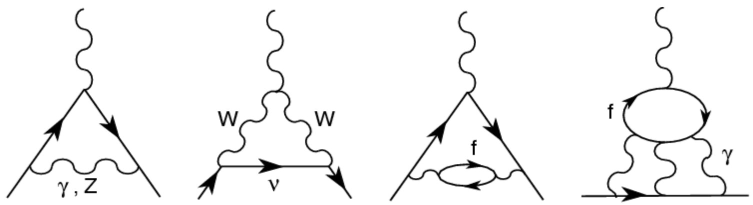
$$a_e^{\rm QCD} = 1.685\ (33)\times 10^{-12} \quad , \quad a_e^{\rm Weak} = 0.0297\ (5)\times 10^{-12}$$




$$\alpha^{-1}_{\rm Rb2010} = 137.035\,999\,049\ (90) \;\Rightarrow\; a_e^{\rm th} = 0.001\,159\,652\,181\,78\ (6)_{\rm 8th}(4)_{\rm 10th}(3)_{\rm had}(77)_\alpha$$

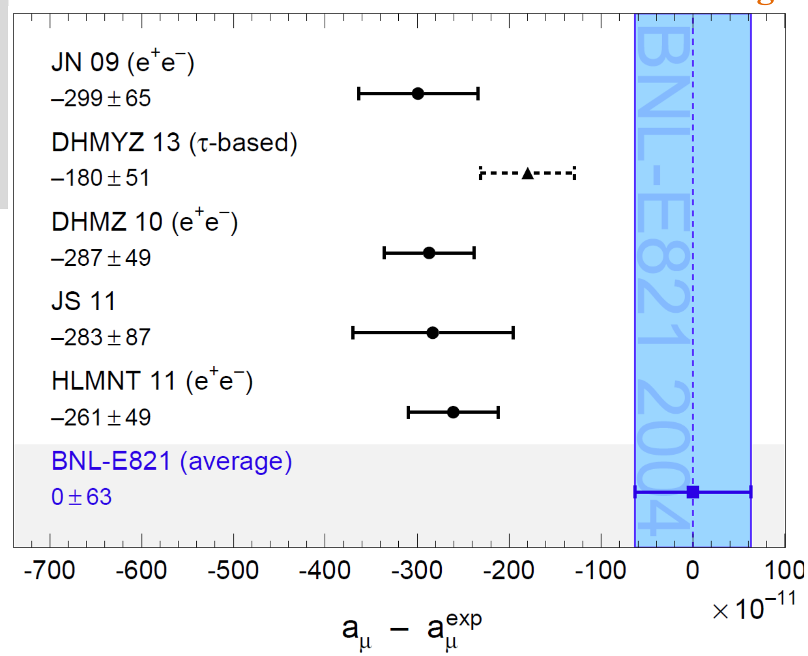
$$a_e \;\Rightarrow\; \alpha^{-1}_{a_e} = 137.035\,999\,173\,6\ (68)_{\rm 8th}(46)_{\rm 10th}(26)_{\rm had}(331)_{\rm exp} \qquad [0.25\ \rm ppb]$$

# $\mu$ Anomalous Magnetic Moment




$$a_\mu^{exp} = (11\,659\,208.9 \pm 6.3) \cdot 10^{-10}$$

**BNL-E821**




| | | | | |
|---|---|---|---|---|
| $10^{10} \cdot a_\mu^{th} =$ | $11\,658\,471.895 \pm 0.008$ | QED | | **Aoyama-Hayakawa-Kinoshita-Nio** |
| | $+ \; 15.4 \pm 0.1$ | EW | | **Gnendiger et al, Czarnecki et al**, Knecht et al |
| | $+ \; 697.4 \pm 5.3$ | hvp | $(703.0 \pm 4.4)_\tau$ , $(692.3 \pm 4.2)_{e^+e^-}$ | **Davier et al**, Hagiwara et al, Jegerlehner-Nyffeler |
| | $- \; 8.6 \pm 0.1$ | hvp NLO+NNLO | | **Kurz et al, Hagiwara et al**, Krause |
| | $+ \; 10.5 \pm 2.6$ | light-by-light | | **de Rafael-Prades-Vainshtein**, Knecht et al, Melnikov-Vainshtein, Nyffeler, Bijnens et al, Hayakawa et al, Goecke et al, Roig et al, Masjuan-Vanderhaeghen |

$$= 11\,659\,186.6 \pm 5.9 \qquad (11\,659\,192.2 \pm 5.1)_\tau \; , \; (11\,659\,181.5 \pm 4.9)_{e^+e^-}$$

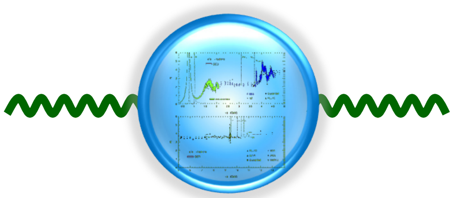
$$a_\mu^{exp} - a_\mu^{th} = 2.7\,\sigma \qquad 1.9\,\sigma \qquad 3.4\,\sigma$$

# Future Challenge

$\Delta a_\mu = 1.6\ 10^{-10}$ (0.14 ppm)

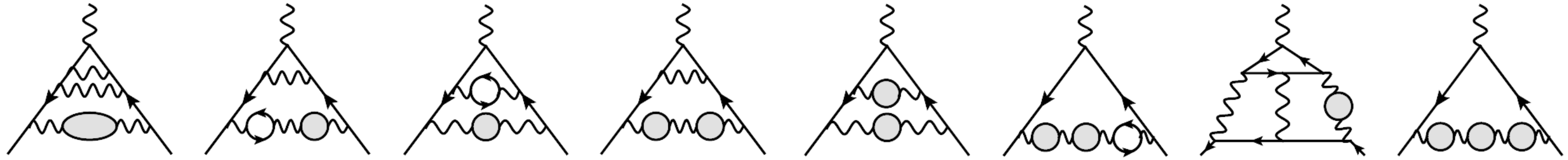
FNAL, J-PARC

- Hadronic Vacuum Polarization

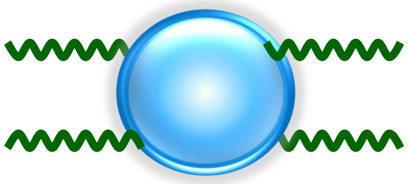


- Improved data
- Radiative return
- Isospin breaking
- Improved theoretical tools
- Lattice simulations

- NNLO HVP  Kurz et al



- Light-by-Light



- Lattice  Blum et al
- Dispersive approach  Colangelo et al, Pauk-Vanderhaeghen
- Analytical methods  de Rafael-Prades-Vainshtein, Knecht et al, Melnikov-Vainshtein, Nyffeler, Bijnens et al, Hayakawa et al, Goecke et al, Roig et al, Masjuan-Vanderhaeghen

# τ Anomalous Magnetic Moment

**Difficult to measure!** $a_\tau^{\exp} = (-0.018 \pm 0.017)$ **DELPHI**

$-0.007 < a_\tau^{\text{New Phys}} < 0.005$ **González-Springer, Santamaria, Vidal '00** **(LEP/SLD data)**

**Eidelman, Passera**

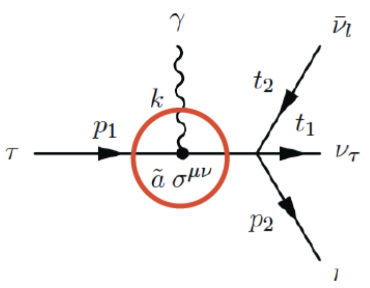
$$
\begin{aligned}
10^8 \cdot a_\tau^{\text{th}} = & \; 117\,324 \pm 2 && \text{QED} \\
& + 47.4 \pm 0.5 && \text{EW} \\
& + 337.5 \pm 3.7 && \text{hvp} \\
& + 7.6 \pm 0.2 && \text{hvp NLO} \\
& + 5 \pm 3 && \text{light-by-light} \\
= & \; \mathbf{117\,721 \pm 5}
\end{aligned}
$$

**Enhanced sensitivity to new physics:** $(m_\tau/m_\mu)^2 = 283$

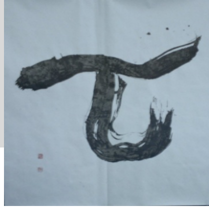
| | Electron | Muon | Tau |
|---|---|---|---|
| $a^{EW}/a^{HAD}$ | 1/56 | 1/45 | 1/7 |
| $a^{EW}/\delta a^{HAD}$ | 1.6 | 3 | 10 |

## Essentially unknown

May be accessible at BFs through radiative leptonic decays **(Fael et al)**



# SUMMARY



- Very precise determination of $\alpha_s$ from $\tau$ decays

$$\alpha_s(m_\tau^2) = 0.332 \pm 0.013 \quad \Longrightarrow \quad \alpha_s(M_Z^2) = 0.1202 \pm 0.0015$$

- The $\tau$ could give the most precise $V_{us}$ determination

$$|V_{us}| = 0.2207 \pm 0.0023_{\text{exp}} \pm 0.0011_{\text{th}} \qquad (\text{present } \tau \text{ data} + \text{K data})$$

$K_{l3}$: $$|V_{us}| = 0.2239 \pm 0.0009 \qquad [f_+(0) = 0.9661 \pm 0.0032]$$

- Many QCD tests with $\tau$ data: Chiral Dynamics

- Improved predictions for $a_\mu^{\text{QCD}}$ needed

**Theoretical and experimental challenge**

# Backup Slides

# Renormalons

$$D(s) \equiv -s \frac{d}{ds} \Pi^{(0+1)}(s) = \frac{1}{4\pi^2} \sum_{n=0} K_n \; a(-s)^n$$

**Asymptotic series**

**Borel Summation:**

$$B(t) \equiv \sum_{n=0} K_{n+1} \frac{t^n}{n!} \quad \Longrightarrow \quad D(s) = \frac{1}{4\pi^2} \left\{ 1 + \int_0^\infty dt\, e^{-t/a(-s)} \, B(t) \right\}$$

**However, $B(t)$ has pole singularities at**

- $u \equiv -\beta_1 t/2 = +n \qquad (n \geq 2)$ **Infrared Renormalons**
- $u \equiv -\beta_1 t/2 = -n \qquad (n \geq 1)$ **Ultraviolet Renormalons**

**IR - n Renormalon** $\Longrightarrow$ **Ambiguity:** $\delta D(s) \sim \left( \frac{\Lambda^2}{-s} \right)^n$

# Renormalon Hypothesis: Asymptotics already reached

## Modelling a better behaved FOPT

(Beneke – Jamin)

- **Large higher-order $K_n$ corrections could cancel the $g_n$ ones**

  **Happens in the "large-$\beta_0$" approximation (UV renormalon chain)**

- D = 4 corrections very suppressed in $R_\tau$

  ➡ **n = 2 IR renormalons can do the job** **($K_n \approx - g_n$)**

- No sign of renormalon behaviour in known coefficients

  ➡ **n = -1,2,3 renormalons + linear polynomial**

  5 unknown constants fitted to $K_n$ $(2 \le n \le 5)$. $K_5 = 283$ assumed

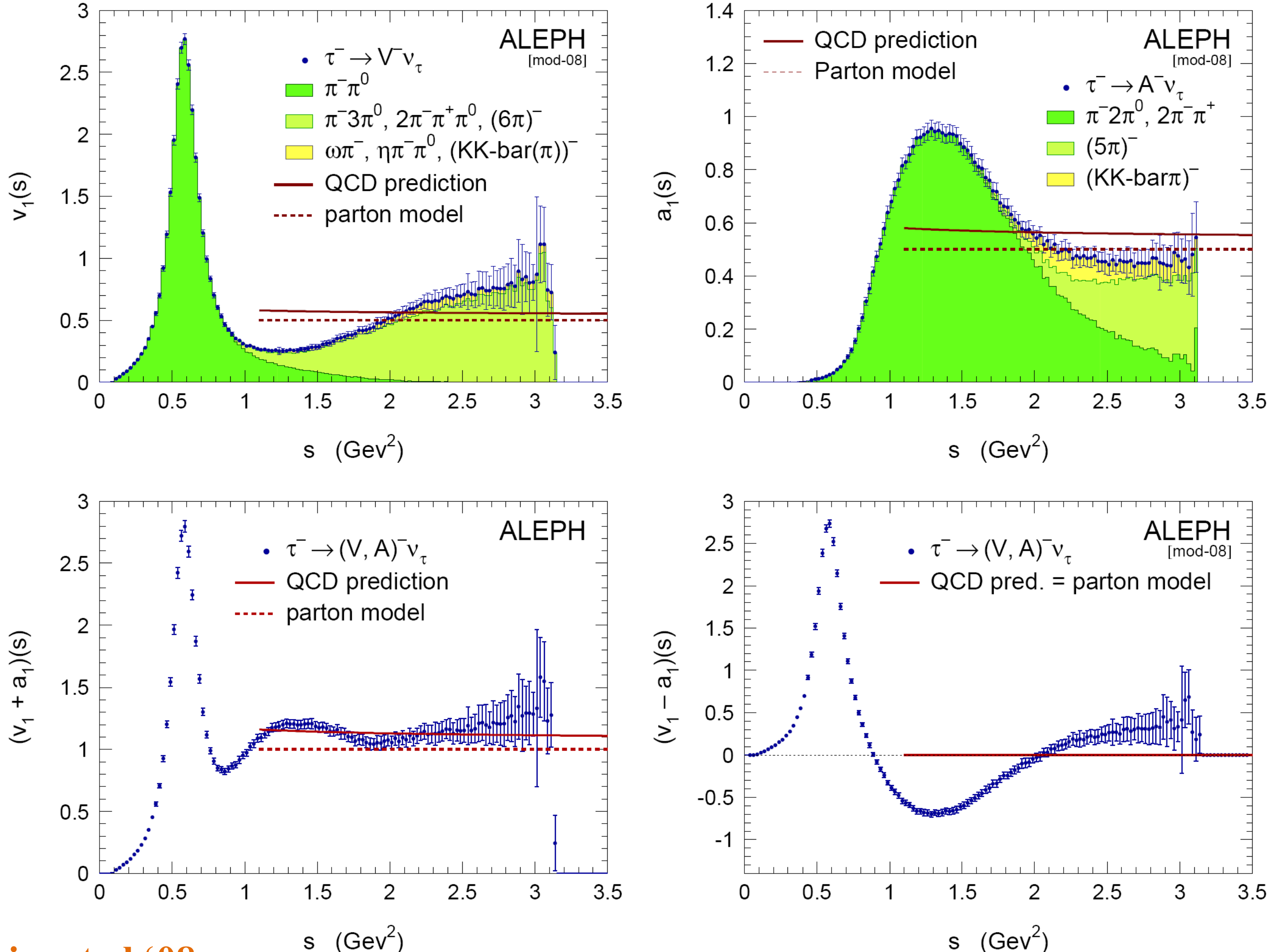
- **Borel summation:** large renormalon contributions. Smaller $\alpha$

**Nice model of higher orders. But too many different possibilities …**

(Descotes-Genon – Malaescu)

# SPECTRAL FUNCTIONS

$$v_1(s) = 2\pi \, \text{Im}\, \Pi^{(0+1)}_{ud,V}(s) \qquad a_1(s) = 2\pi \, \text{Im}\, \Pi^{(0+1)}_{ud,A}(s)$$




**BF data needed**

Davier et al '08