α_s & Tau Decays

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Outline

- Introduction
- Recent improvements/updates
- Results & comparison with others
- Summary

What Do We Measure?

Experimentally, we measure for many tau decay modes

Normalization: Branching fractions (BRs)
Shape: Invariant mass distributions (Spectral functions)

$$v_1/a_1\left[\tau^- \to V^-/A^-\nu_\tau\right] \propto \frac{\mathrm{BR}\left[\tau^- \to V^-/A^-\nu_\tau\right]}{\mathrm{BR}\left[\tau^- \to e^-\bar{\nu}_e\nu_\tau\right]} \frac{1}{N_{V/A}}\frac{dN_{V/A}}{ds} \frac{m_\tau^2}{\left(1-s/m_\tau^2\right)^2\left(1+2s/m_\tau^2\right)}$$

Vector/Axial-vector spectral functions — branching fractions — mass spectrum — kinematic factor

The total hadronic tau decay width can be determined from leptonic BRs

$$\begin{aligned} R_\tau &= \frac{\Gamma(\tau^- \to \text{hadrons } \nu_\tau)}{\Gamma(\tau^- \to e^-\bar{\nu}_e\nu_\tau)} \\ &= R_{\tau,V} + R_{\tau,A} + R_{\tau,S} \\ &= \frac{1 - B_e - B_\mu}{B_e} \end{aligned}$$

- V: even number of π's
- A: odd number of π's
- S: odd number of K's

Connection to Spectral Functions

Theoretically, R_τ can be expressed in terms of vacuum polarization functions as

$$R_{\tau,V+A} = 12\pi S_{\rm EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right){\rm Im}\Pi^{(1)}(s+i\varepsilon) + {\rm Im}\Pi^{(0)}(s+i\varepsilon)\right]$$

BNP, NPB373 (1992) 581

with $\Pi^{(J)} = |V_{ud}|^2 \left(\Pi^{(J)}_{\bar{u}d,V} + \Pi^{(J)}_{\bar{u}d,A}\right)$

$${\rm Im}\Pi^{(1)}_{\bar{u}d,V/A}(s) = \frac{1}{2\pi} v_1/a_1(s), \qquad {\rm Im}\Pi^{(0)}_{\bar{u}d,A} = \frac{1}{2\pi} a_0(s)$$

similar in e^+e^- annihilation into hadrons:

$$12\pi\, {\rm Im} \prod\nolimits_\gamma (s) = \frac{\sigma^{(0)}[e^+e^- \to \text{hadrons}]}{\sigma^{(0)}[e^+e^- \to \mu^+\mu^-]} \equiv R(s)$$

Therefore, R_τ is a weighted integral of spectral functions

➔ Basis for comparing measurements with theoretical predictions

Operator Product Expansion

Apply short distance Operator Product Expansion (OPE), one has

Small residual non-log EW correction & quark mass terms

SVZ, NPB147 (1979) 385, 448, 519;
BNP, NPB373 (1992) 581

$$R_{\tau,U}(s_0) \propto |V_{\rm CKM}|^2 S_{\rm EW}\left(1 + \delta^{(0)} + \delta'_{\rm EW} + \delta_U^{(2,m_q)} + \sum_{D=4,6,\ldots} \delta_U^{(D)}\right)$$

$U=V, A$

Dominant perturbative contribution (~20%) depends on α_s expansion schemes, one has e.g.

Le Diberder-Pich, PLB286 (1992) 147

- FOPT: Fixed-Order Pert. Theory expansion

$$1+\delta^{(0)} = 1 + a_s(s_0) + 5.2023a_s^2(s_0) + 26.366a_s^3(s_0) + (K_4 + 78.0)a_s^4(s_0)$$

$$\text{with} \quad a_s(s_0) = \frac{\alpha_s(s_0)}{\pi}$$

- CIPT: Contour-Improved Pert. Theory expansion

Nonperturbative contribution
Dimensions 4, 6 and 8 terms:

- $\langle \alpha_s/\pi\, GG\rangle$ (GeV^4)
- $\delta^{(6)}$
- $\delta^{(8)}$

➔ We have 4 parameters to determine:
α_s + 3 non-pert. ones

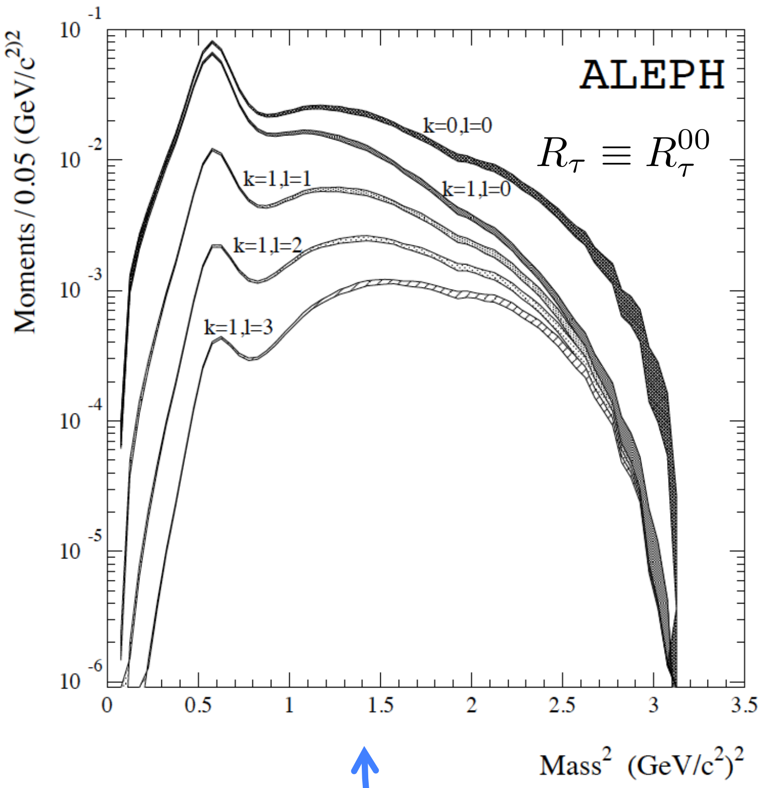
Spectral Moments

Exploit shape of SFs to obtain additional experimental information:

Le Diberder-Pich, PL B289, 165 (1992)

$$R_{\tau,U}^{k\ell}(s_0) = \int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right)^k \left(\frac{s}{s_0}\right)^\ell \frac{dR_{\tau,U}(s_0)}{ds}$$

Weighting factor suppresses the region where OPE fails and we have small statistics.



- Theory prediction very similar to R_τ:

$$R_{\tau,U}^{k\ell}(s_0) \propto |V_{\rm CKM}|^2 S_{\rm EW} \left(1 + \delta^{(0,k\ell)} + \delta'_{\rm EW} + \delta_U^{(2,m_q,k\ell)} + \sum_{D=4,6,\ldots} \delta_U^{(D,k\ell)}\right)$$

with corresponding perturbative and nonperturbative OPE terms

- Because of the strong correlations, only four moments are used.

➔ Five experimental inputs (R_τ + 4 moments) for four unknowns

ALEPH Measurements

1st ALEPH measurement:

1993, using ~8500 τ decays

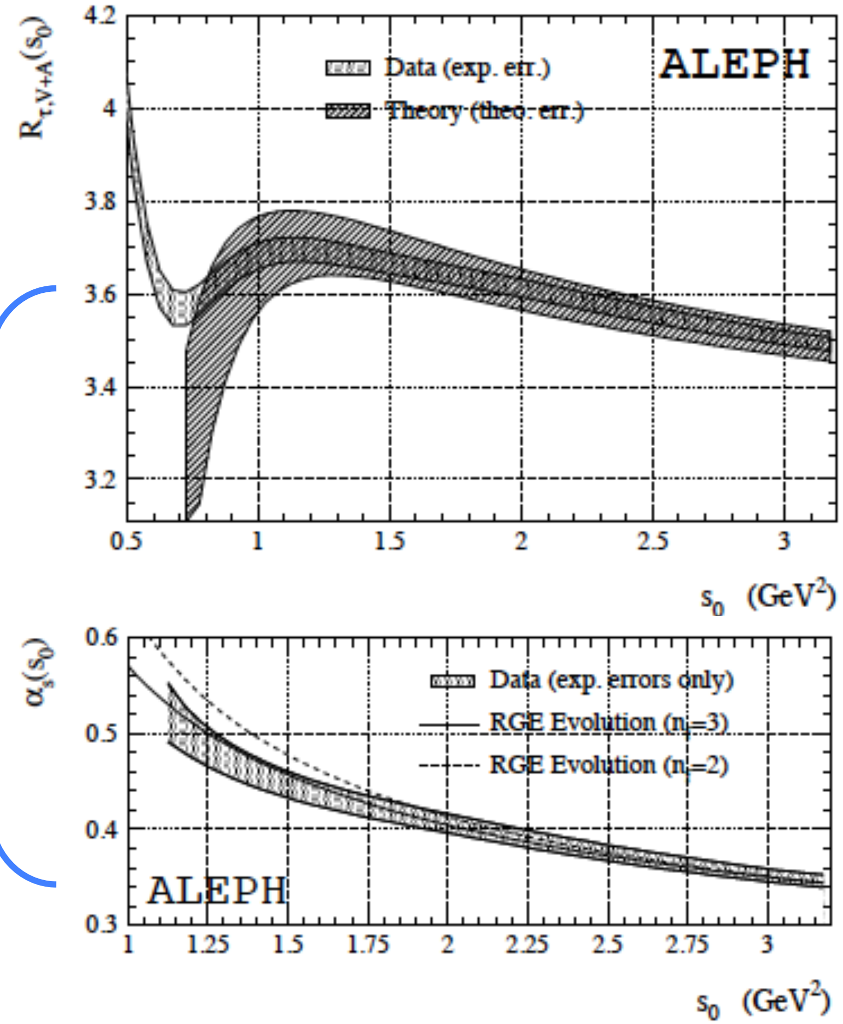
$\alpha_s(m^2_\tau)$=0.330±0.046$_{tot}$ [PLB 307 (1993) 209]

followed by CLEO (1995) [PLB 356 (1995) 580]
OPAL (1999) [EPJC 7 (1999) 571]

Updated ALEPH measurements:

- **1998, using ~30x more statistics**
  0.334±0.007$_{exp}$±0.021$_{th}$ (<CIPT+FOPT>)
  V/A separation, fits to V, A, V+A
  Provided a direct test of running $\alpha_s(s_0)$
  Checked the stability of OPE
  [EPJC 4 (1998) 409]

- **2005, using full LEP-1 data**
  Experimental improvement (γ, π^0)
  0.340±0.005$_{exp}$±0.014$_{th}$ (<CIPT+FOPT>)
  [Phys. Rep. 421 (2005) 191]



Recent Updates/Improvements

2008 version:

- K_4 became known & used
- Better V/A separation
- Improved strange BRs
- Quark-hadron duality violation study

BCK, PRL101 (2008) 012002 [0801.1821]

See also Boito et al., NPPS218 (2011) 104 [1011.4426]

$0.344 \pm 0.005_{exp} \pm 0.007_{th}$ (CIPT only)

[Davier, Descotes-Genon, Hoecker, Malaescu, Zhang EPJC56 (2008) 305]

2013 version:

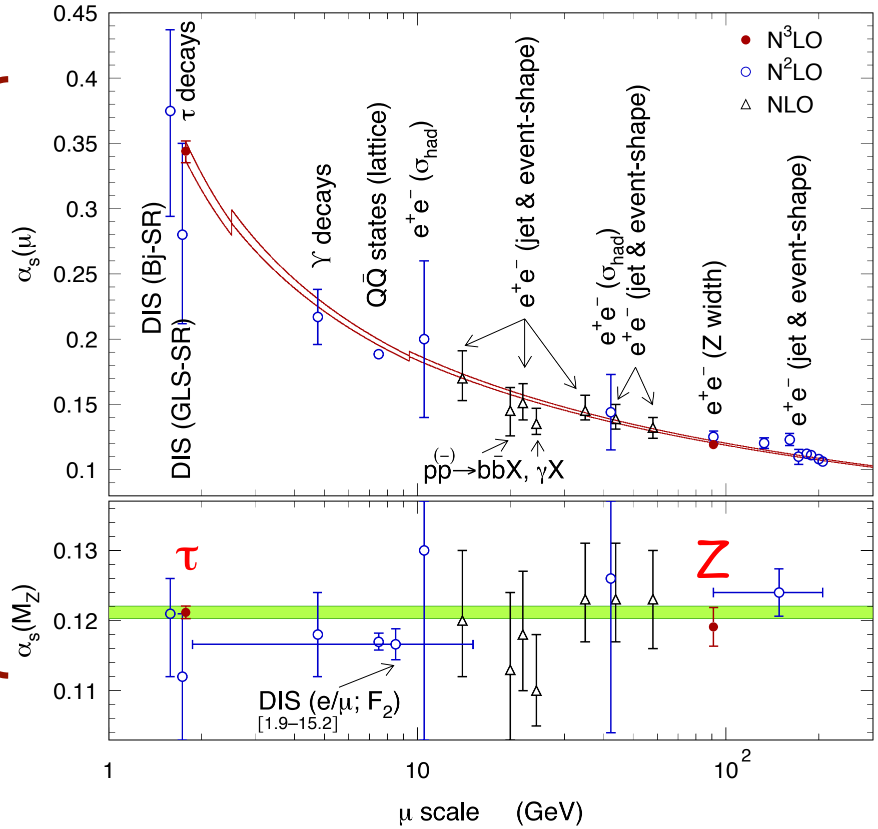
- Use a new unfolding method
- Fix a problem in correlation matrices

Thanks to D. Boito for bringing this issue to our attention

$0.332 \pm 0.005_{exp} \pm 0.011_{th}$ (<CIPT+FOPT>)

[Davier, Hoecker, Malaescu, Zhang for ALEPH, EPJC74 (2014) 2803]

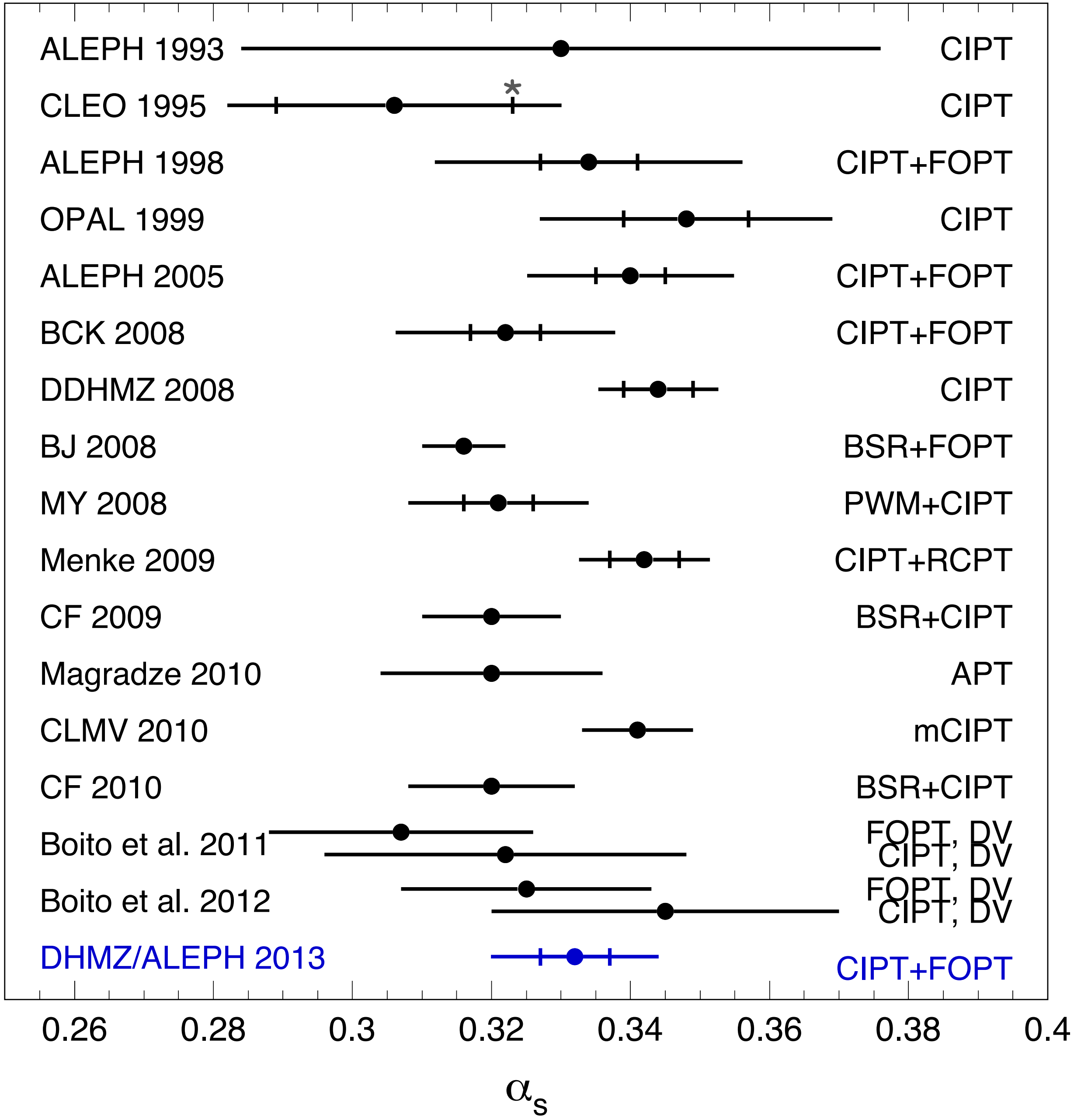
Updated version from Davier, Hoecker, Zhang, RMP 78 (2006) 1043



➔ Most precise test (2.4%) of asymptotic freedom in QCD

Comparison with Other Determinations

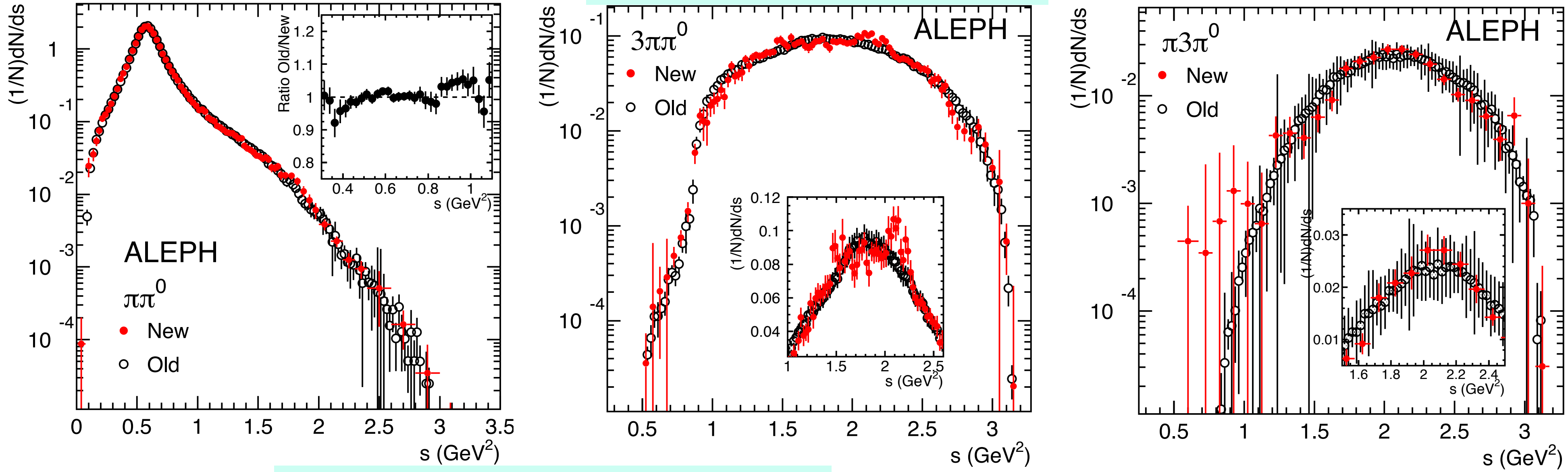
- Baikov, Chetyrkin, Köhn, PRL 101 (2008) 012002, [0801.1821]
- Beneke, Jamin, JHEP 0809 (2008) 044, [0806.3156]
- Maltman, Yavin, PRD78 (2008) 094020, [0807.0650]
- Menke, 0904.1796
- Caprini, Fischer, EPJC64 (2009) 35, [0906.5211]
- Magradze, Few Body Syst. 48 (2010) 143, Erratum-ibid. 53 (2012) 365, [1005.2674]
- Cvetic, Loewe, Martinez, Valenzuela, PRD82 (2010) 093007, [1005.4444]
- Caprini, Fischer, Rom.J.Phys. 55 (2010) 527, [1012.1132]
- Boito et al., PRD84 (2011) 113006, [1110.1127]; PRD85 (2012) 093015, [1203.3146]
- Beneke, Boito, Jamin, JHEP 1301 (2013) 125, [1210.8038]



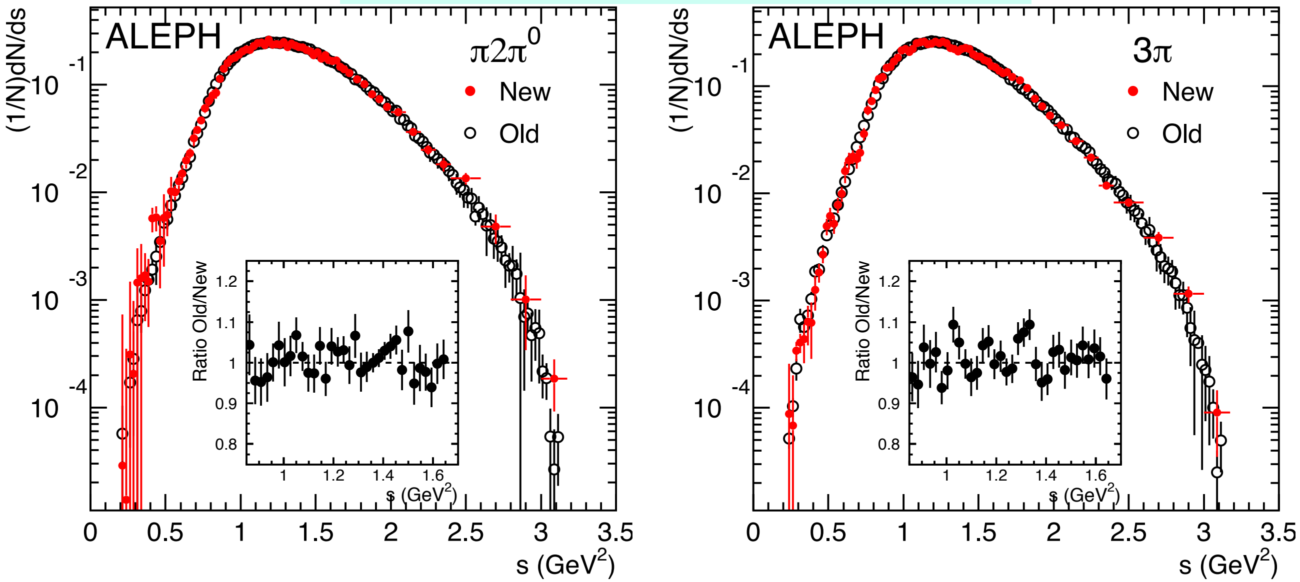
* experimental uncertainty when available is shown as inner error bar

New Spectral Functions

Vector spectral functions



Axial-vector spectral functions



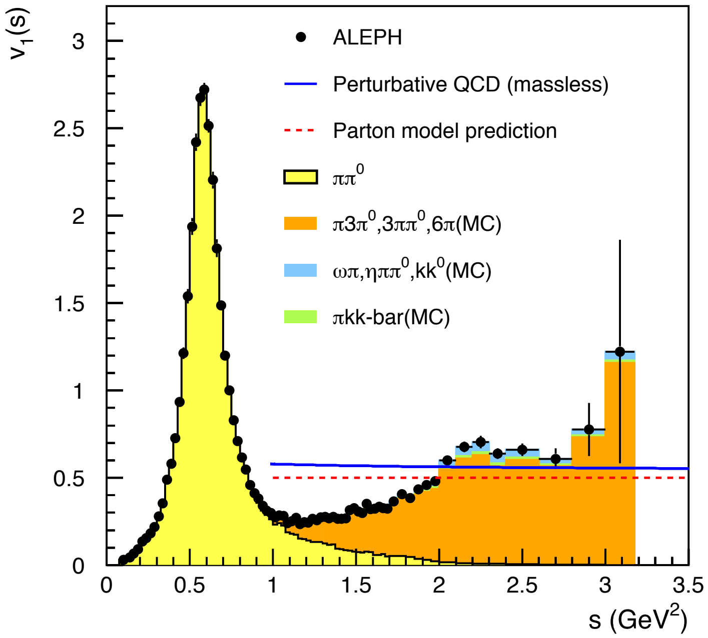
Reasonable good agreement observed in all distributions

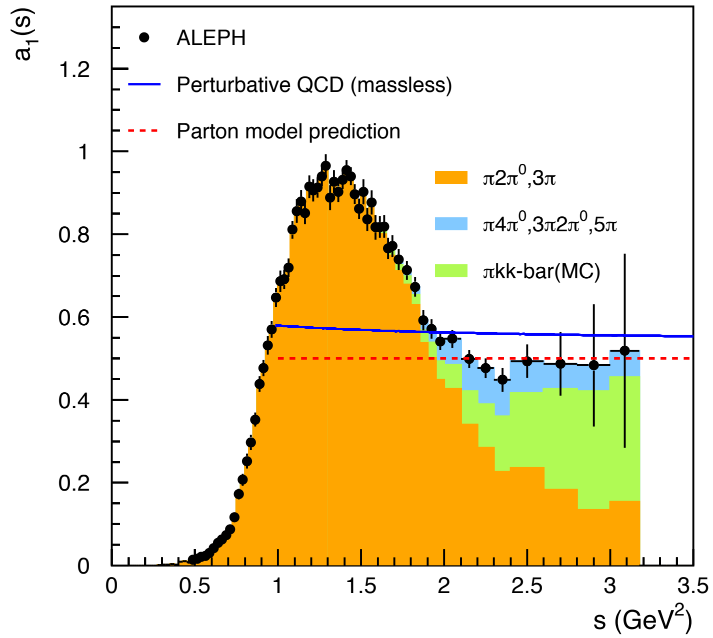
The new unfolding method uses a weaker regularization than the old SVD approach

➔ The new unfolding induces less smoothing/correlation between mass bins

New Spectral Functions V, A, V+A

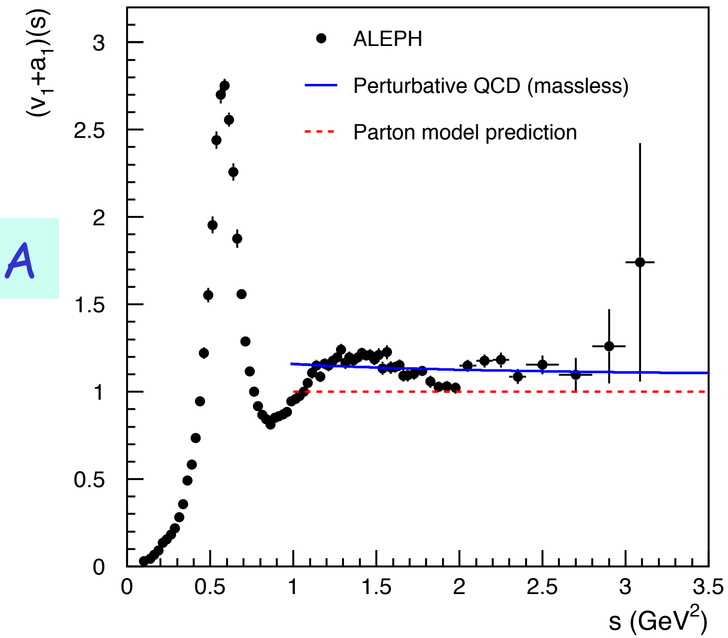
V





A

V+A



For V & A, the asymptotic region is not reached though statistical power is weak near the kinematic limit

V+A does approximately reach the asymptotic limit predicted by pQCD at the kinematic limit

Updated spectral functions and their correlation matrices are publicly available at http://aleph.web.lal.in2p3.fr/tau/specfun13.html

Correlation Between Inputs

Vector	D_V^{10}	D_V^{11}	D_V^{12}	D_V^{13}
$R_{\tau V}$	−0.377	0.215	0.365	0.389
D_V^{10}	1	−0.615	−0.929	−0.959
D_V^{11}	–	1	0.803	0.597
D_V^{12}	–	–	1	0.956

Axial-vector	D_A^{10}	D_A^{11}	D_A^{12}	D_A^{13}
$R_{\tau A}$	−0.659	0.420	0.589	0.594
D_A^{10}	1	−0.429	−0.899	−0.970
D_A^{11}	–	1	0.701	0.414
D_A^{12}	–	–	1	0.934

$V+A$	D_{V+A}^{10}	D_{V+A}^{11}	D_{V+A}^{12}	D_{V+A}^{13}
D_{V+A}^{10}	1	−0.483	−0.906	−0.969
D_{V+A}^{11}	–	1	0.743	0.508
D_{V+A}^{12}	–	–	1	0.949

In practice, we use normalized spectral moments

$$D_{\tau,V/A}^{kl} = \frac{R_{\tau,V/A}^{kl}}{R_{\tau,V/A}}$$

to decorrelate normalization and shape information between R_τ and spectral moments

Fit Results

CIPT

Fitted variable	Vector (V)	Axial-Vector (A)	$V+A$
$\alpha_s(m_\tau^2)$	$0.346 \pm 0.007 \pm 0.008$	$0.335 \pm 0.008 \pm 0.009$	$0.341 \pm 0.005 \pm 0.006$
$\langle \frac{\alpha_s}{\pi} GG \rangle (\mathrm{GeV}^4)$	$(-0.5 \pm 0.3) \cdot 10^{-2}$	$(-3.4 \pm 0.4) \cdot 10^{-2}$	$(-2.0 \pm 0.3) \cdot 10^{-2}$
$\delta^{(6)}$	$(2.8 \pm 0.2) \cdot 10^{-2}$	$(-3.7 \pm 0.2) \cdot 10^{-2}$	$(-4.6 \pm 1.5) \cdot 10^{-3}$
$\delta^{(8)}$	$(-8.2 \pm 0.5) \cdot 10^{-3}$	$(10.9 \pm 0.5) \cdot 10^{-3}$	$(1.3 \pm 0.3) \cdot 10^{-3}$
$\chi^2/1\mathrm{DF}$	0.43	3.4	1.1
$\delta^{(2)}$	$(-3.2 \pm 3.0) \cdot 10^{-4}$	$(-5.1 \pm 3.0) \cdot 10^{-4}$	$(-4.2 \pm 2.0) \cdot 10^{-4}$
$\delta^{(4)}$	$(1.0 \pm 1.6) \cdot 10^{-4}$	$(-6.3 \pm 0.1) \cdot 10^{-3}$	$(-3.1 \pm 0.1) \cdot 10^{-3}$
Total δ_{NP}	$(2.0 \pm 0.3) \cdot 10^{-2}$	$(-3.2 \pm 0.2) \cdot 10^{-2}$	$(-6.4 \pm 1.3) \cdot 10^{-3}$

FOPT central value for V+A: 0.324

➔ Average CIPT & FOPT (take half of the difference in theory uncertainty):

$$0.332 \pm 0.005_{exp} \pm 0.011_{th}$$

➔ Extrapolation to M_Z:

$$0.1199 \pm \underbrace{0.0006_{exp} \pm 0.0012_{th} \pm 0.0005_{evol}}_{0.0015_{tot}}$$

One of the most precise α_s measurements

Summary

Tau decays provide many interesting & sometimes unique possibilities to test the SM

- precise α_s measurement is a good example

Tau spectral functions have recently been updated & publicly available

- using a new unfolding method
- fixing a problem in the correlation matrices

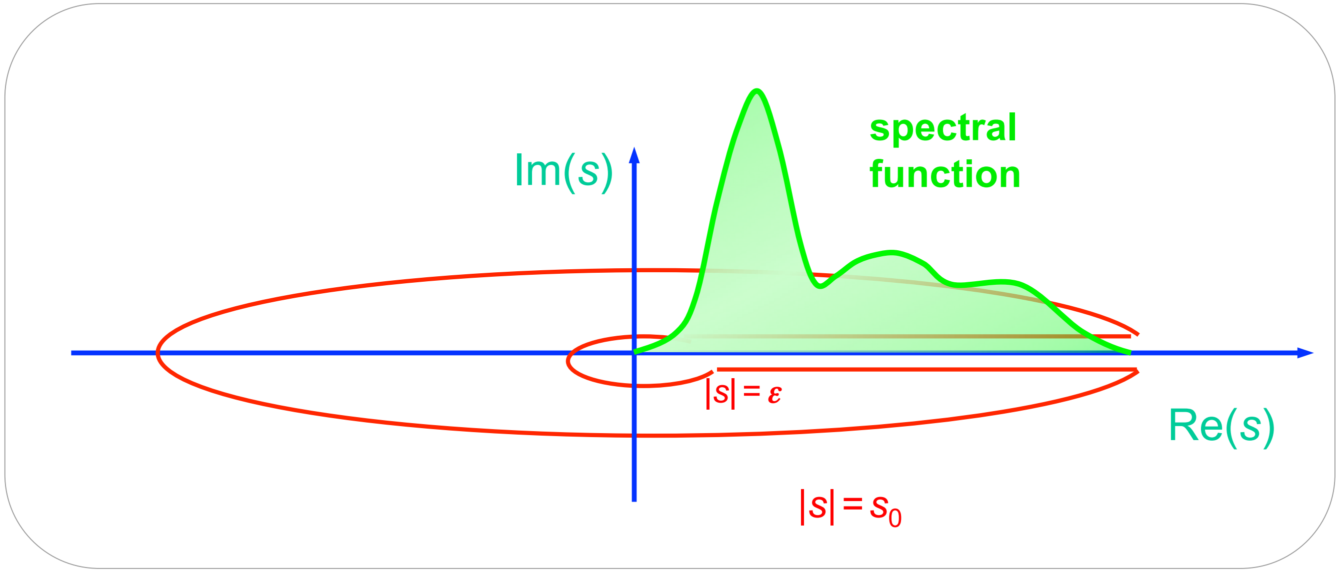
However, the impact of the changes on all results (including α_s) is small

Application of Cauchy's Theorem

$\text{Im}\,\Pi_{V/A}^{(J)}(s)$ contains hadronic physics that cannot be predicted in QCD in this region of the real axis

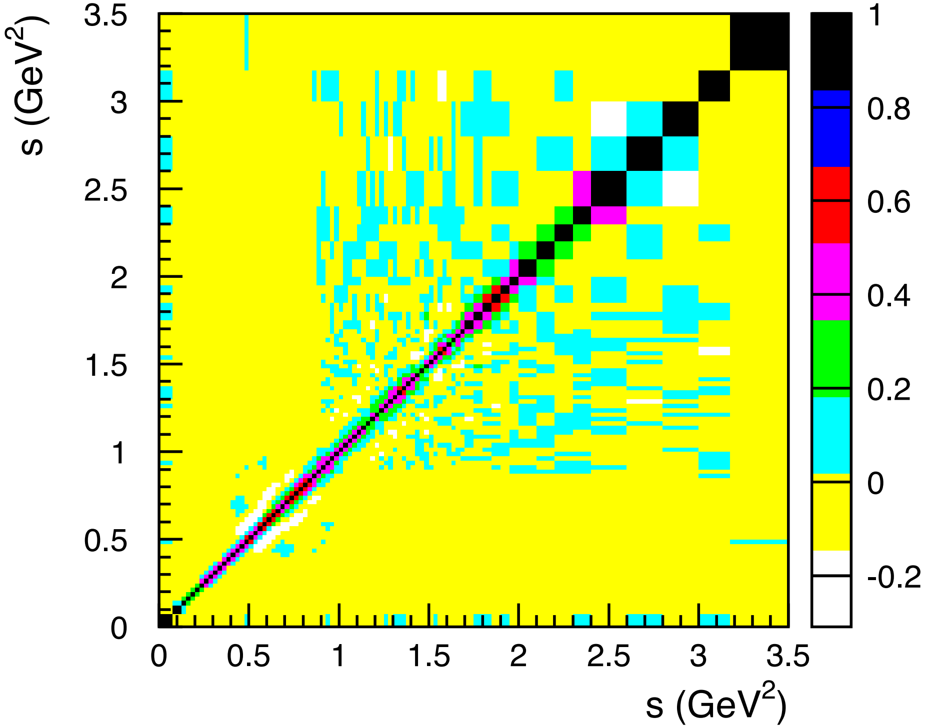
➔ Use Cauchy's theorem to perform a contour integral

$$R(s_0) \propto \int_0^{s_0} ds\; w(s)\; \text{Im}\Pi(s+i\varepsilon) \quad \Leftrightarrow \quad -\frac{1}{2i}\oint_{|s|=s_0} ds\; w(s)\Pi(s) \qquad \text{with } s_0 = m_\tau^2$$



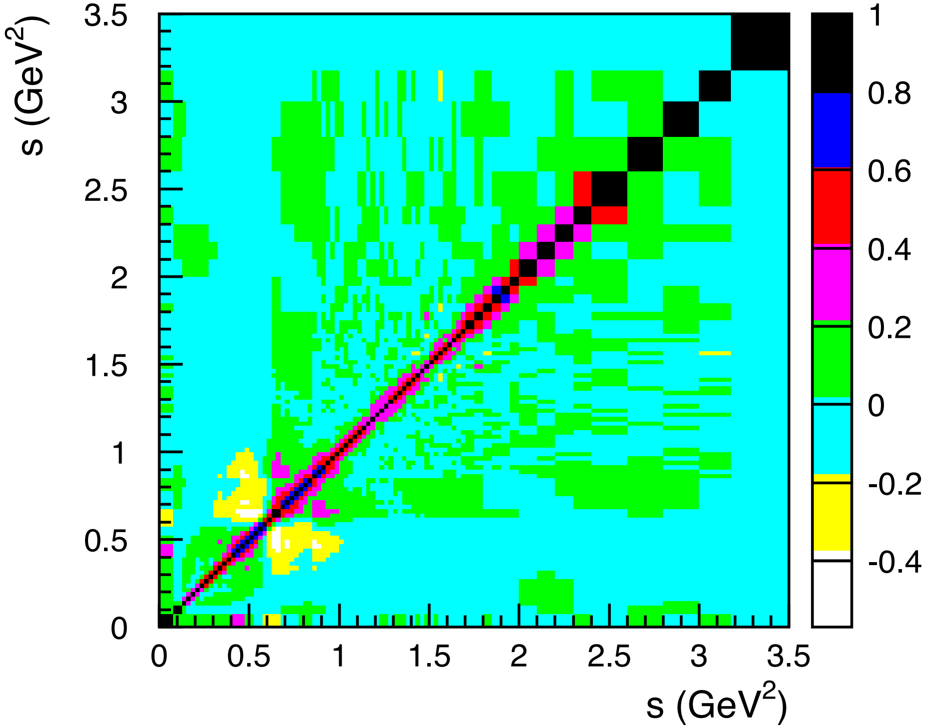
Correlation Matrices (e.g. V)

Statistical



Including correlation induced by unfolding procedure

Statistical + systematic



Larger correlation & anticorrelation in ρ region due to π^0 reconstruction, calibration and resolution effects

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Correlation Between Fitted Parameters

Moment	$\langle GG\rangle_V$	$\delta_V^{(6)}$	$\delta_V^{(8)}$	$\langle GG\rangle_A$	$\delta_A^{(6)}$	$\delta_A^{(8)}$	$\langle GG\rangle_{V+A}$	$\delta_{V+A}^{(6)}$	$\delta_{V+A}^{(8)}$
$\alpha_s(m_\tau^2)$	−0.50	−0.61	−0.71	−0.56	0.80	−0.76	−0.32	0.55	−0.64
$\langle GG\rangle_{V/A/V+A}$	1	0.44	0.71	1	−0.53	0.78	1	−0.10	0.54
$\delta_{V/A/V+A}^{(6)}$	–	1	0.92	–	1	−0.92	–	1	−0.87
$\delta_{V/A/V+A}^{(8)}$	–	–	1	–	–	1	–	–	1

➔ The correlation is reduced in the fit to V+A spectral function