α_s *from the (revised) Aleph data for* τ *decay*

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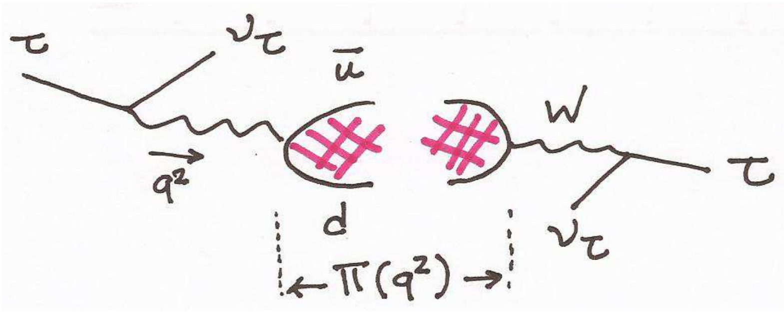
WORK IN PROGRESS

QCD in τ decay

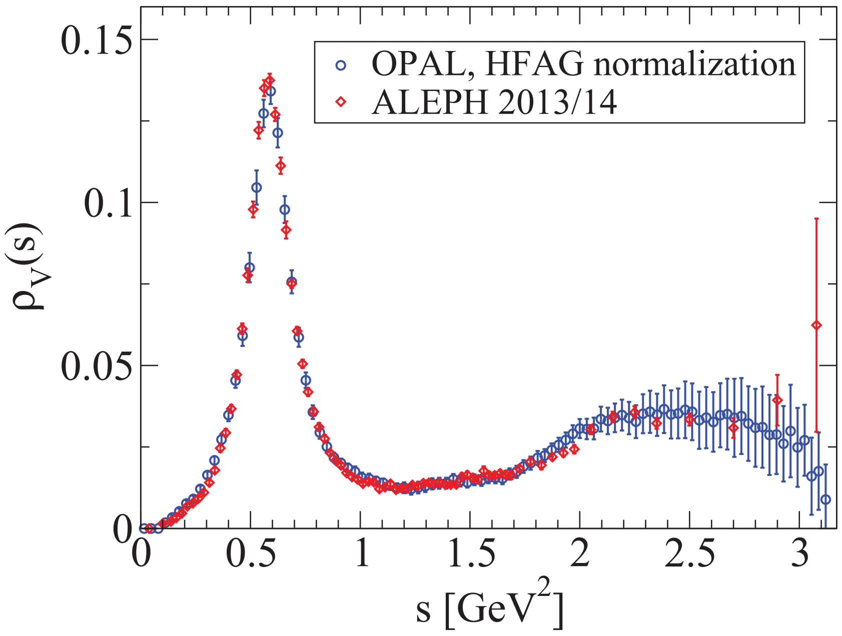


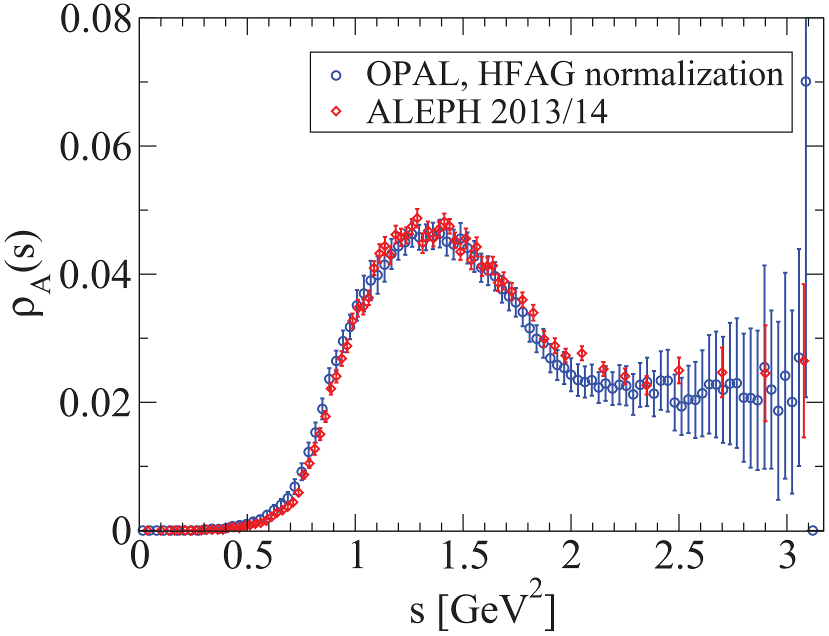
$$w_T(s;s_0) = (1+2\frac{s}{s_0})\ \underbrace{(1-\frac{s}{s_0})^2}_{\text{doubly pinched}}$$

$$w_L(s;s_0) = 2(\frac{s}{s_0})\ \underbrace{(1-\frac{s}{s_0})^2}_{\text{doubly pinched}}$$

$s_0 = m_\tau^2 \qquad \rho_{V,A} = \frac{1}{\pi}\mathrm{Im}\Pi_{V,A}$

$$\frac{\Gamma(\tau\to\nu_\tau \mathrm{H}_{ud}(\gamma))}{\Gamma[\tau\to\nu_\tau e\bar{\nu}_e(\gamma)]} = 12\pi^2|V_{ud}|^2 S_{EW} \int_0^{s_0} \frac{ds}{s_0} \left[w_T(s;s_0)\rho_{V+A}^{(1+0)}(s) - w_L(s;s_0)\rho_A^{(0)}(s) \right]$$

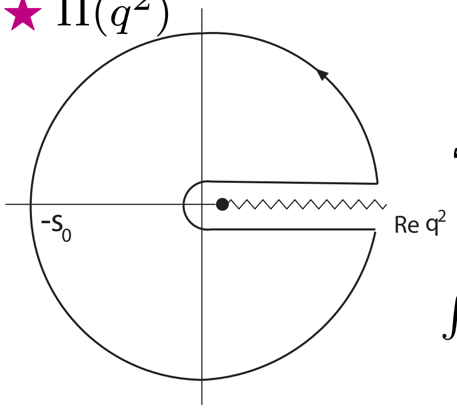




Theoretical Foundations

Shankar '77; Braaten-Narison-Pich '92

★ $\Pi(q^2)$



"Cauchy's Theorem" $(z = q^2\ ;\ \rho(t) = \frac{1}{\pi}\mathrm{Im}\Pi\ ;\ w_n = \text{polynomial})$:

$$\int_0^{s_0} dt\ w_n(t)\ \underbrace{\rho(t)}_{exp.} = \frac{1}{2\ i\pi}\oint_{|z|=s_0} dz\ w_n(z)\ \Pi(z)$$

$$= \frac{1}{2\ i\pi}\oint_{|z|=s_0} dz\ w_n(z)\ \Big[\underbrace{\Pi_{\mathrm{OPE}}(z)}_{\mathcal{O}(\alpha_s^4)} + \underbrace{\Pi(z) - \Pi_{\mathrm{OPE}}(z)}_{\Pi_{DV}(z)}\Big]$$

★ $\Pi_{DV} \to 0 \Longleftrightarrow \Pi_{\mathrm{OPE}} \to \Pi.$

(Cata-Golterman-S.P. '05)

However, Π_{OPE} expected asymptotic.

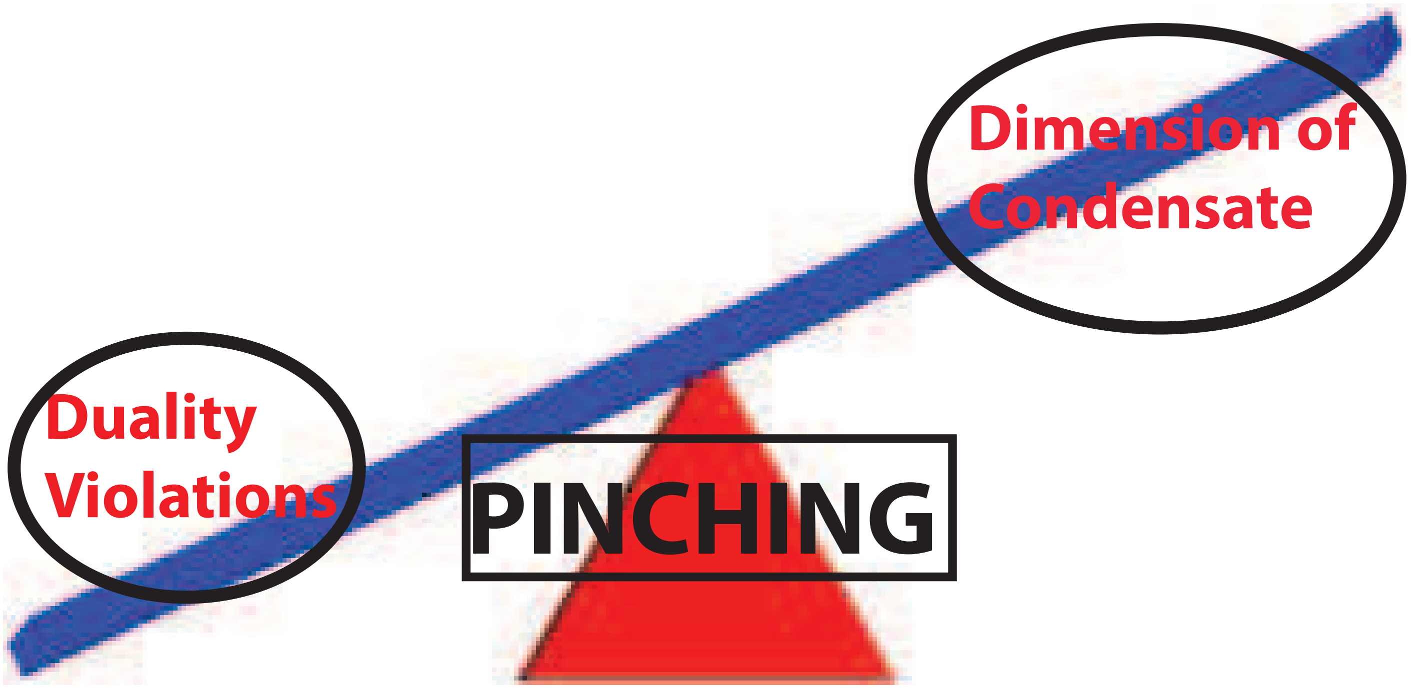
For s_0 large enough:

$$-\frac{1}{2\pi i}\oint_{|z|=s_0} dz\ w(z)\ \Pi_{DV}(z) = \quad - \underbrace{\int_{s_0}^{\infty} ds}_{\text{extrapolation!}}\quad w(s)\quad \underbrace{\frac{1}{\pi}\mathrm{Im}\Pi_{DV}(s)}_{\simeq\ \mathrm{e}^{-\delta-\gamma s}\ \sin(\alpha+\beta s)}$$

Main Theoretical Message

★ “Seesaw” mechanism at work:

Pinching: It is not possible to suppress simultaneously DVs and condensates.



New Analysis

New Ingredients: Experiment

- There was a problem in Aleph's data correlation matrices that has been corrected.
  (Davier et al. '14)

- We have used the SM value for $\pi \to \mu\nu$ (instead of the Aleph determination).
  For consistency this requires a (global) rescaling of all data by a factor 0.9987.

New Ingredients: Theory

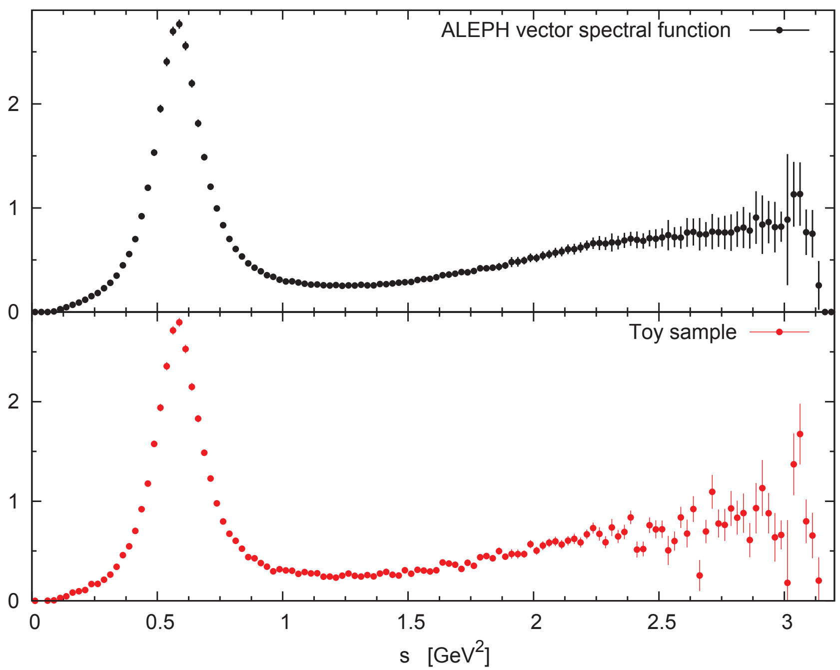
- New strategy.

New correlation matrices.

Boito et al. '11
Davier et al. '14

Take Aleph's correlation matrices, use MonteCarlo to recreate data.

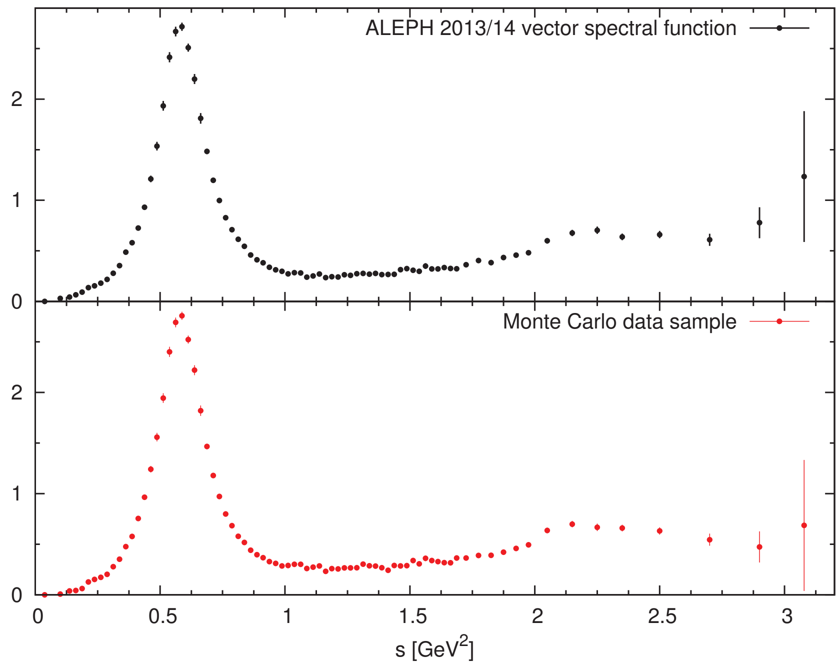
<u>2011</u>



WRONG!

Old Aleph correlations underestimated.

<u>2014</u>



OK!

A Change of Strategy

Old Strategy: (LeDiberder-Pich '92)

- Use 5 pinched weights

$$w_{kl}(y) = (1-y)^2(1+2y)(1-y)^k y^l \quad , \quad y = s/s_0$$

  with $(k,l) = \{(0,0),(1,0),(1,1),(1,2),(1,3)\}$.

- Set OPE condensates $C_{10,12,14,16} = 0$.
- Set Duality Violations =0. (Do not look at spectral functions.)
- Fit to extract 4 param. : α_s and $C_{4,6,8}$ only at $s = m_\tau^2$.
- May use V and A, but assume $V + A$ more reliable.

(Davier et al. '14)

$$\begin{aligned}
\langle \frac{\alpha_s}{\pi} GG \rangle \quad &= \quad (-0.5 \pm 0.3) \times 10^{-2}\ \text{GeV}^4 , \qquad \chi^2 = 0.43,\ p = 51\% \qquad V , \\
&\quad\ \ (-3.4 \pm 0.4) \times 10^{-2}\ \text{GeV}^4 , \qquad \chi^2 = 3.4,\ p = 7\% \qquad A , \\
&\quad\ \ (-2.0 \pm 0.3) \times 10^{-2}\ \text{GeV}^4 , \qquad \chi^2 = 1.1,\ p = 29\% \qquad V + A .
\end{aligned}$$

- Check Weinberg sum rules.

A Change of Strategy

New Strategy (Boito et al. '11 and '12):

- Do not use $w(y)$ with a term linear in y. (Beneke et al. '13)
- Do not assume any condensate is zero. (Let the data speak.)
- Do not assume that Duality Violations are zero. (Let the data speak.)

  For $s \geq s_{min}$,

$$\rho_{DV}^{V,A}(s) = \mathrm{e}^{-\delta_{V,A} - \gamma_{V,A}\, s} \sin\left(\alpha_{V,A} + \beta_{V,A}\, s\right)$$

  The old strategy assumes $\delta_{V,A} = \infty$.

- Fit to $\alpha_s, C_{6,8}$ and DV parameters with 3 weights:

$$w_0 = 1, w_2 = 1 - y^2 \quad \text{and} \quad w_3 = (1-y)^2(1+2y)$$

  Use all data for $s_0 \geq s_{min}$, to be determined by the fit as well.

- Use V and A. Check spectral functions.
- Check Weinberg sum rules.

Fitting Strategy

Master Equation:

(Cata-Golterman-S.P. '05)

$$\int_0^{s_0} dt\, w_n(t)\, \underbrace{\rho(t)}_{exp.} = \frac{1}{2\,i\pi} \oint_{|z|=s_0} dz\, w_n(z)\, \underbrace{\Pi_{\rm OPE}(z)}_{\mathcal{O}(\alpha_s^4)} - \int_{s_0}^{\infty} ds\, w(s)\, \underbrace{\frac{1}{\pi}{\rm Im}\Pi_{DV}(s)}_{{\rm e}^{-\delta-\gamma s}\ \sin(\alpha+\beta s)}$$

Fits :

- V channel, $w_0 = 1$.
- V and A channels, $w_0 = 1$.
- V channel, $w_0 = 1$ and $w_2 = 1 - y^2$.
- V and A channels, $w_0 = 1$ and $w_2 = 1 - y^2$.
- V channel, $w_0 = 1, w_2 = 1 - y^2$ and $w_3 = (1-y)^2(1+2y)$.
- V and A channels, $w_0 = 1, w_2 = 1 - y^2$ and $w_3 = (1-y)^2(1+2y)$.

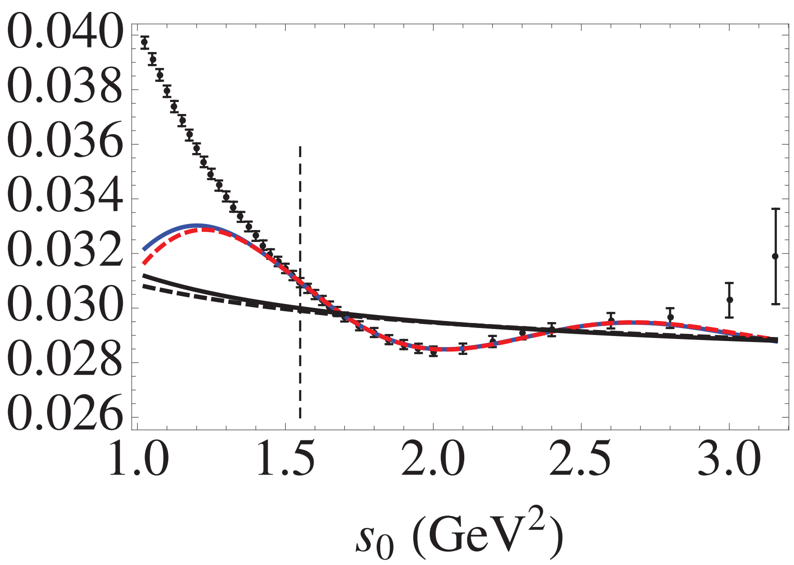
Consistent results in all cases.

Fit to $w_0 = 1$, Vector channel.

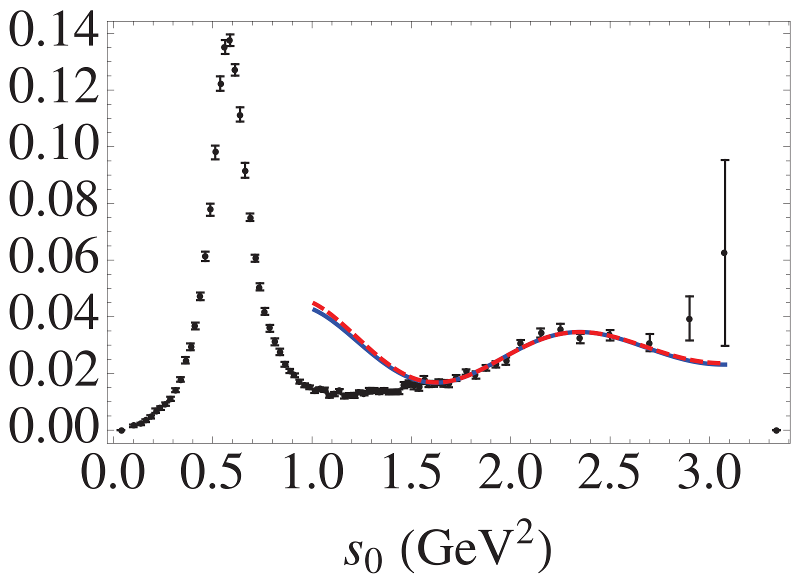
$s_{min} = 1.55\ \mathrm{GeV}^2$, $\chi^2/dof = 24.5/16$ ($p = 8\%$) (FOPT, CIPT similar)

<u>curves</u>: red=CIPT blue =FOPT black =no DV

$w_0 = 1$ spectral integral

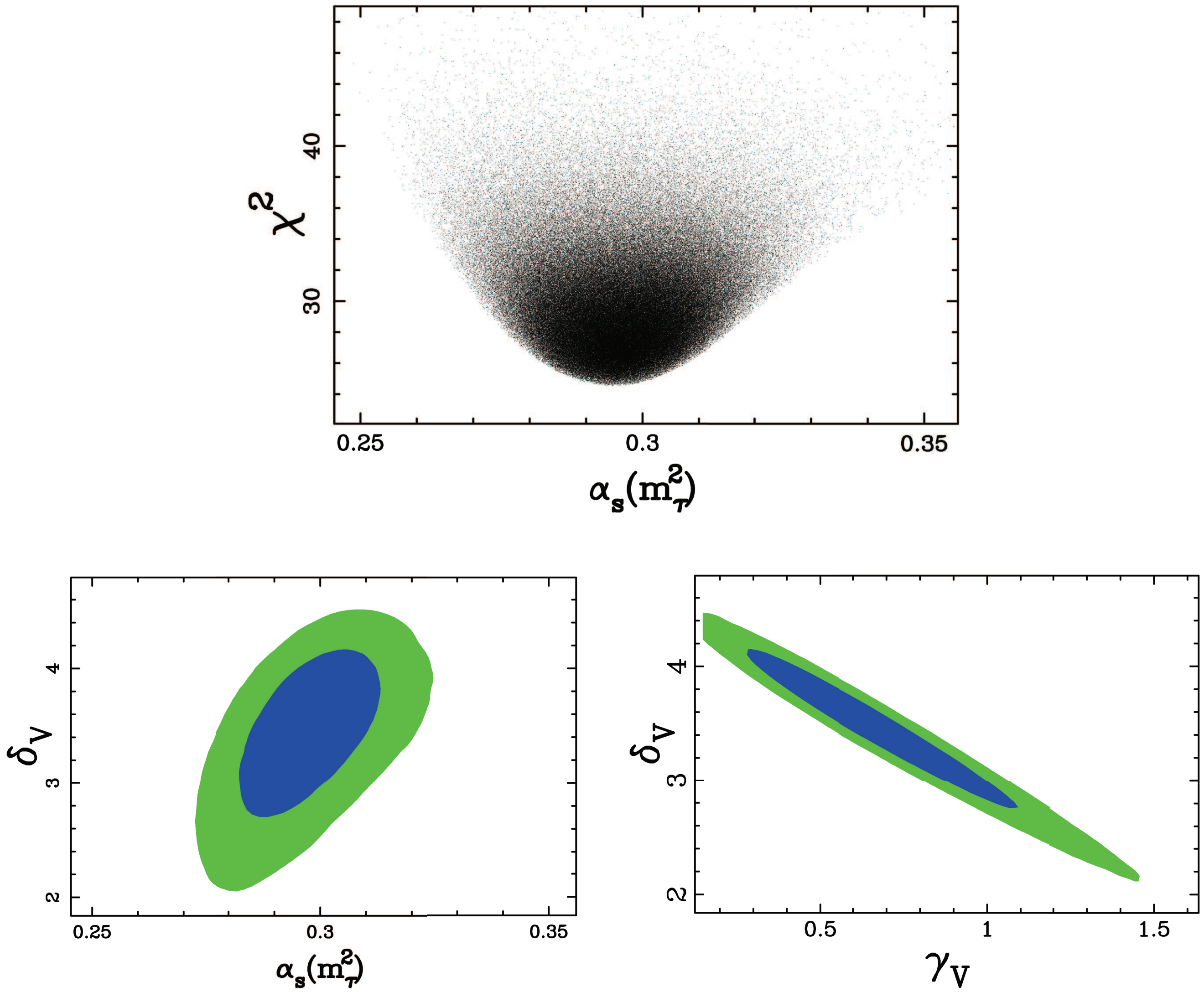


V spectrum



This curve is a prediction, not a fit !

Fit to $w_0 = 1$, *Vector channel.*



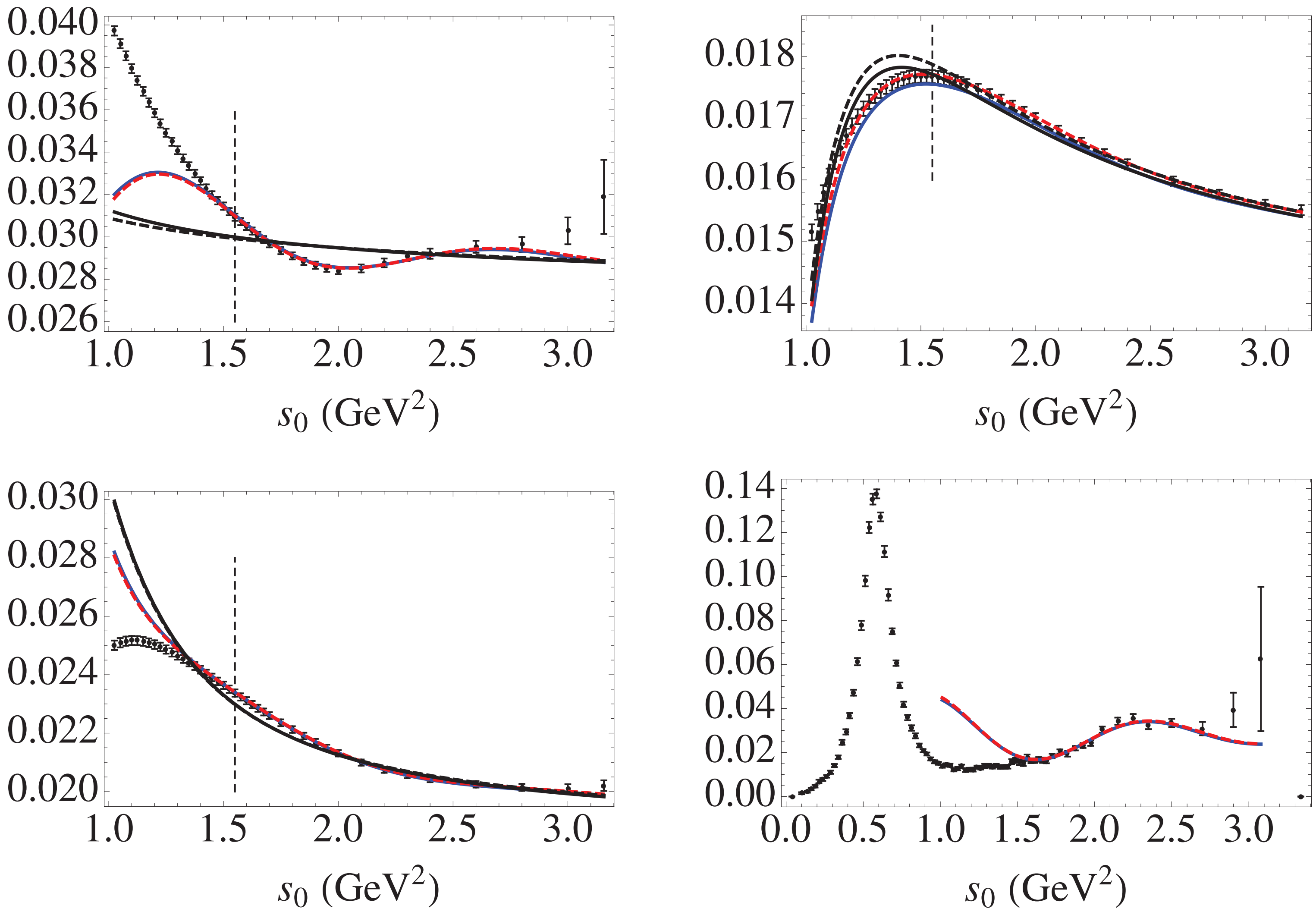
(68% and 95% contour plots), FOPT. Clearly $DVs \neq 0$.

Fit to $w_{0,2,3}$, Vector channel.

$s_{min} = 1.55\ \mathrm{GeV}^2$, w_n-block diagonal fit.

<u>curves</u>: red=CIPT blue =FOPT black =no DV

w_0, w_2 V spectral integral w_3, V spectrum

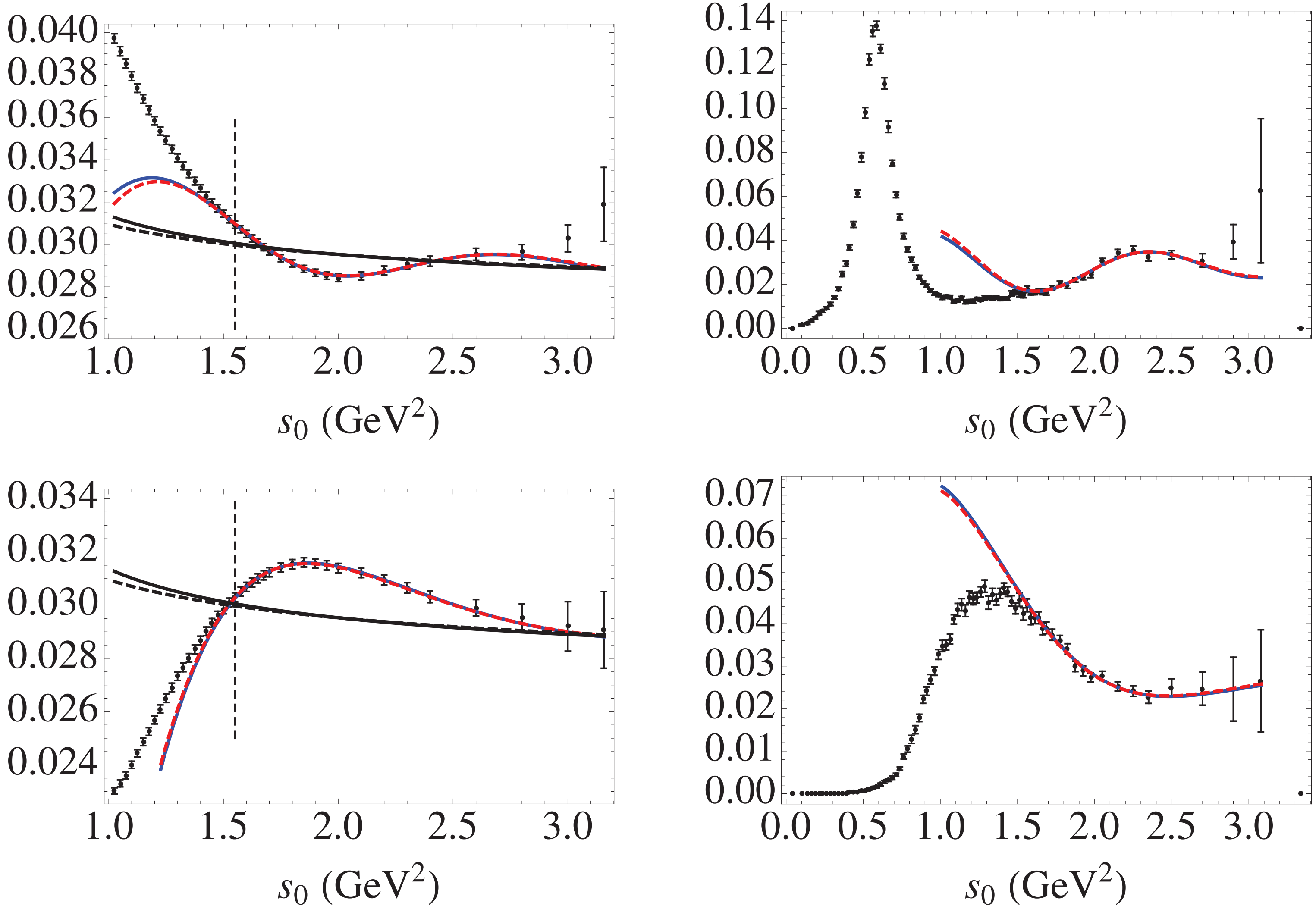


Fit to $w_0 = 1$, Vector and Axial combined.

$s_{min} = 1.55\ \mathrm{GeV}^2$, $\chi^2/dof = 40.0/33$ ($p = 19\%$).

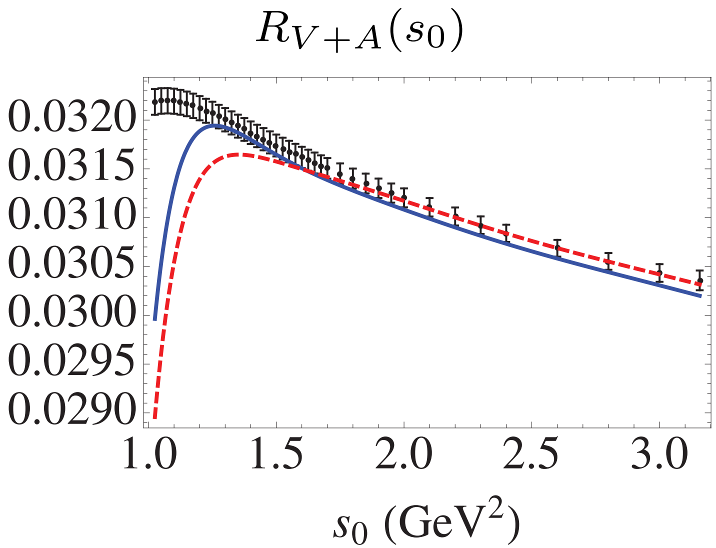
curves: red=CIPT blue =FOPT black =no DV

$w_0 = 1$ V,A spectral integral V,A spectrum

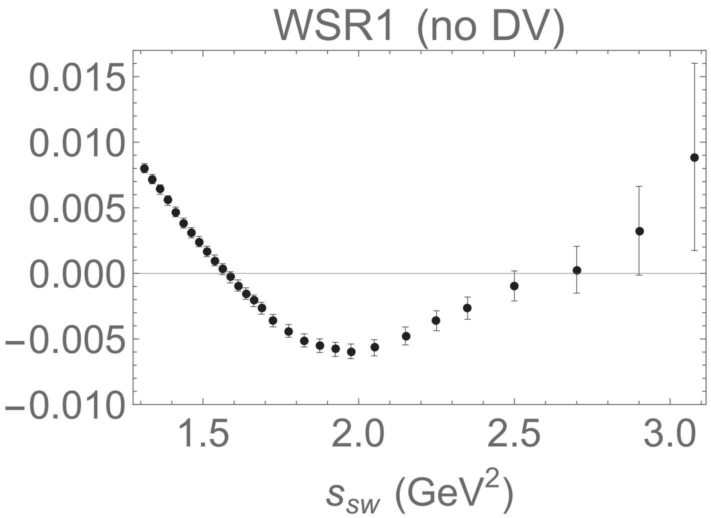


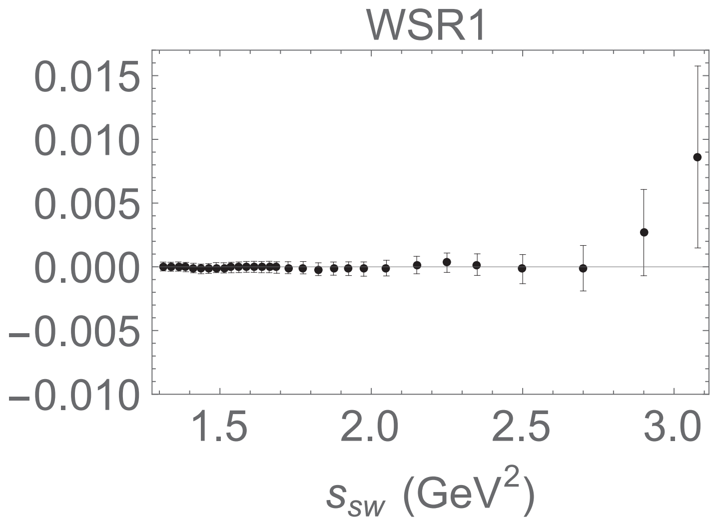
Classic Tests

red=CIPT blue =FOPT



Weinberg sum rule: $\int_0^\infty ds\, \left(\rho_V^{(1)}(s) - \rho_A^{(1)}(s)\right) - 2f_\pi^2 = 0$



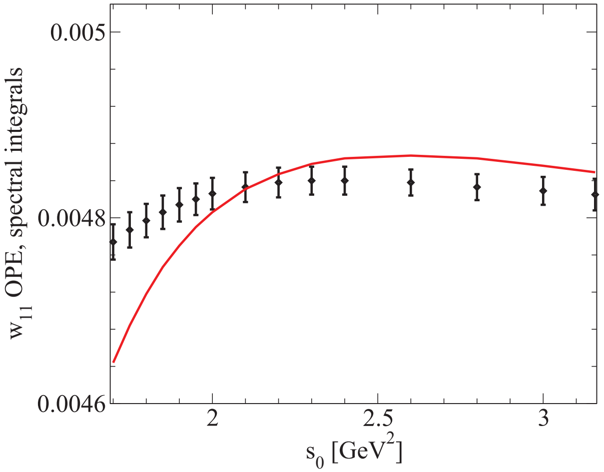


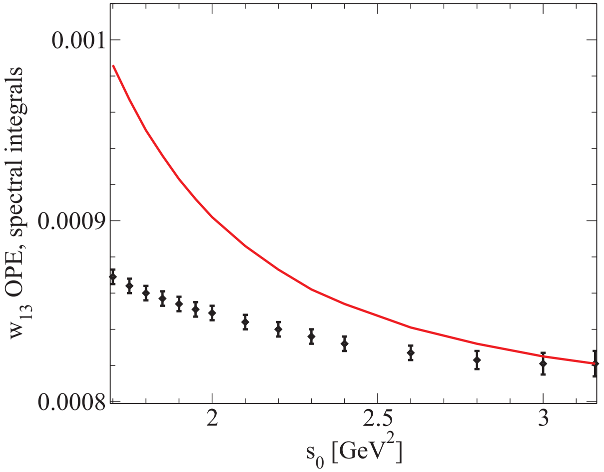
Further Tests

(Maltman-Yavin '08)

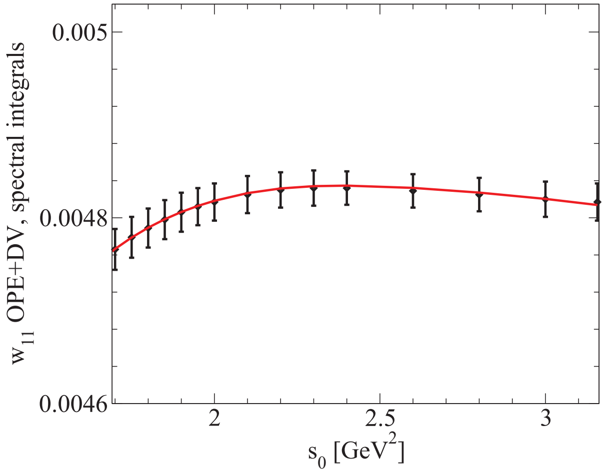
"Old vs. New" Strategy in other FESRs: w_{11}, w_{13}, etc...

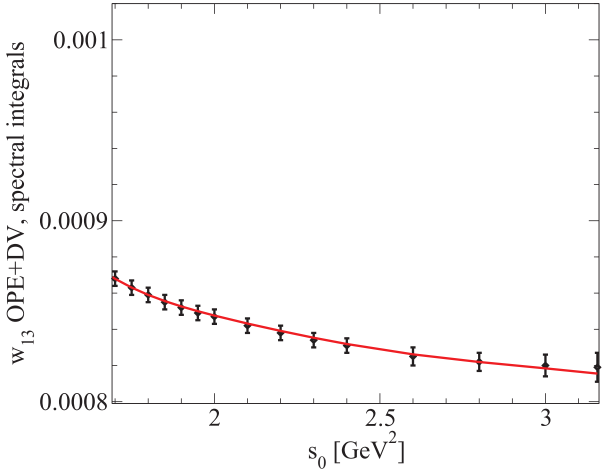
Old (fits)





New (predictions)





Results

(FOPT) $\alpha_s(m_\tau) = 0.296 \pm 0.010 \longrightarrow \alpha_s(m_Z) = 0.1155 \pm 0.0014$

(CIPT) $\alpha_s(m_\tau) = 0.310 \pm 0.014 \longrightarrow \alpha_s(m_Z) = 0.1174 \pm 0.0019$

Cfr. "Old Strategy" produces a shift: $\alpha_s(m_\tau) \sim +0.03$ higher, (and $\sim$ half errors)

(Davier et al. '14)

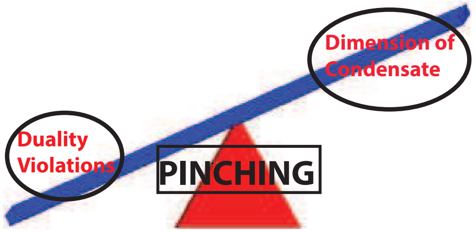
$$R_{V+A} = N_c \, S_{EW} |V_{ud}|^2 \Big(1 + \delta_P + \underbrace{\delta_6 + \delta_8 + \delta_{DV}}_{\delta_{NP}}\Big)$$

(FOPT) $\delta_{NP} = 0.020 \pm 0.009$

(CIPT) $\delta_{NP} = 0.016 \pm 0.010 \longleftrightarrow \delta_{NP}^{\text{“Old Strategy''}} = -0.0064 \pm 0.0013$ (Davier et al. '14)

Conclusions and Outlook

- DVs are clearly visible in the data.
- Pinching does not allow a simultaneous reduction of DVs and higher-dim condensates, unlike what has been assumed so far in the "standard method".



This introduced an unquantified systematic error.

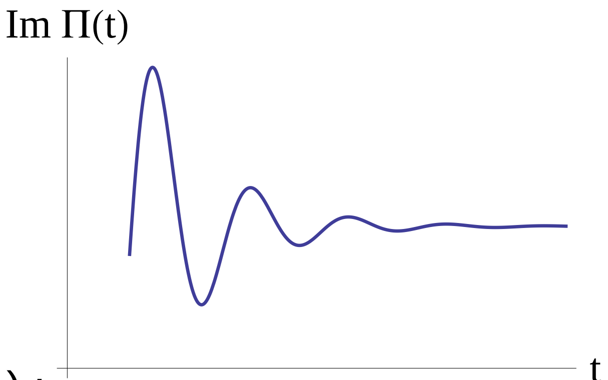
- We have introduced a new strategy which avoids this flaw and allows the data to determine both the contribution from DVs and condensates.
- The new strategy passes all tests, experimental and theoretical, performing better than the standard method.
- Better data (Babar and Belle ?) will tell whether the present errors can be reduced any further, or whether a more refined understanding of DVs is required.

$$\text{(FOPT)} \quad \alpha_s(m_\tau) = 0.296 \pm 0.010 \longrightarrow \alpha_s(m_Z) = 0.1155 \pm 0.0014$$
$$\text{(CIPT)} \quad \alpha_s(m_\tau) = 0.310 \pm 0.014 \longrightarrow \alpha_s(m_Z) = 0.1174 \pm 0.0019$$

BACK-UP SLIDES

Duality Violations

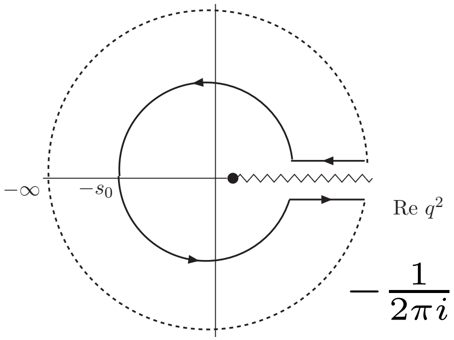
- OPE valid in euclidean, but not in minkowski. We know that spectrum $\neq$ OPE



- We expect (@ large t) :

$$\mathrm{Im}\Pi_{DV} \sim \mathrm{Im}(\Pi - \Pi_{OPE}) \sim \kappa \underbrace{e^{-\gamma t}}_{\text{OPE asympt.}} \underbrace{\sin(\alpha + \beta t)}_{\text{Regge}}$$

- $\Pi_{DV}(s) \to 0$ as $|s| \to \infty$. Then:



(Cata-Golterman-S.P. '05)

$$-\frac{1}{2\pi i}\oint_{|z|=s_0} dz\, w(z)\, \Pi_{DV}(z) = -\underbrace{\int_{s_0}^{\infty} ds}_{\text{extrapolation!}}\; w(s)\, \frac{1}{\pi}\mathrm{Im}\Pi_{DV}(s)$$

Duality Violations

Blok-Shifman-Zhang ’98; Cata-Golterman-SP ’05’08; Jamin ’11

Explicit realization only models, no theory. Take $\Lambda_{QCD} = 1$; $F \sim 0.1$, decay constant.

- 1 resonance ($M \to M + i\Gamma/2$):

$$\frac{F^2}{q^2 - n} \longrightarrow \frac{F^2}{q^2 - n - i\sqrt{n}\,\Gamma}$$

- Regge-like tower: $n = 1, 2, 3, \ldots$

$$\Pi(q^2) \sim \sum_n^\infty \frac{F^2}{z+n} \quad , \quad z = \underbrace{(-q^2)^\zeta}_{\text{cut, } q^2>0} \quad , \quad \zeta \simeq 1 - \mathcal{O}(\frac{1}{N_c})$$

$$\sim \quad \psi(z) = \frac{d \log \Gamma(z)}{dz}$$

- For $q^2 < 0 \longrightarrow \Pi(q^2) \sim \log z + \sum \frac{c_n}{z^n}$

- For $q^2 > 0 \longrightarrow \psi(z) = \psi(-z) - \frac{1}{z} - \pi \cot(\pi z)$,

$$\mathrm{Im}\Pi(q^2) \sim \mathrm{Im}(\log z) + \mathrm{Im}\sum \frac{c_n}{z^n} + \underline{\underline{F^2 \; e^{\frac{-q^2}{N_c}} \; \sin(\alpha + \beta\, q^2)}} \qquad F \sim 0.1 \quad ; \quad \alpha, \beta \sim 1$$