NUFACT14

Working group 1: Neutrino oscillation physics

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Neutrino masses & mixings from discrete symmetries

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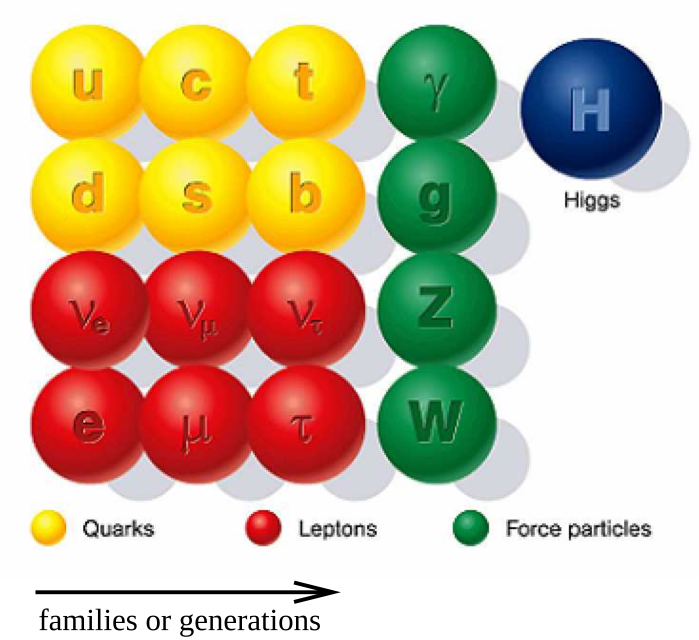
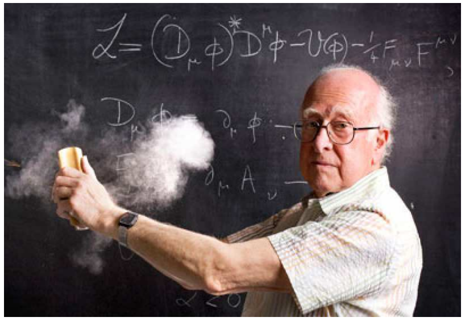
Outline

- triplication of chiral families
  - observation of mixing
  - simple (approximate) patterns
- non-Abelian discrete family symmetries
  - finite groups
  - direct implementation
  - indirect implementation
- accommodating large θ_{13}
  - new family symmetries with more structure
  - perturbation of simple mixing patterns
  - benchmark model based on S_4
  - new vacuum alignments for indirect models

Standard Model (of particle physics)



- highly successful theory
- based on gauge symmetry
  $SU(3)_C \times SU(2)_W \times U(1)_Y$
- broken by Higgs vacuum



some nagging questions

- "Who ordered that?" (I. I. Rabi on the discovery of the muon)
- origin of three families of quarks & leptons
- neutrino masses and mixing
- hierarchy problem, baryogenesis, dark matter, dark energy . . .

Possible hints from mixing

Fermion mixings

- mismatch of flavour (weak) and mass eigenstates

$$\Psi_{\text{flavour}} = V^\dagger \, \Psi_{\text{mass}}$$

- quark sector: V_L^u and V_L^d

$$U_{\text{CKM}} = V_L^u \, V_L^{d\dagger} \approx \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \qquad \lambda \sim 0.22$$

- lepton sector: V_L^e and V_L^ν

$$U_{\text{PMNS}} = V_L^e \, V_L^{\nu\dagger} \approx \begin{pmatrix} 0.82 & 0.55 & 0.15 \\ 0.37 & 0.57 & 0.70 \\ 0.39 & 0.59 & 0.68 \end{pmatrix}$$

www.nu-fit.org (2014)

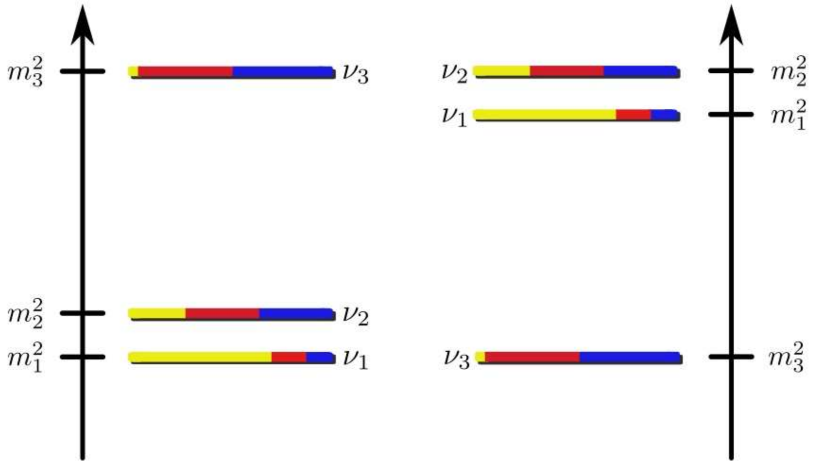
mixing $\Longleftrightarrow$ each family knows of the existence of the others!

Three neutrino flavour mixing

(in diagonal charged lepton basis)

flavour | PMNS mixing | mass

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



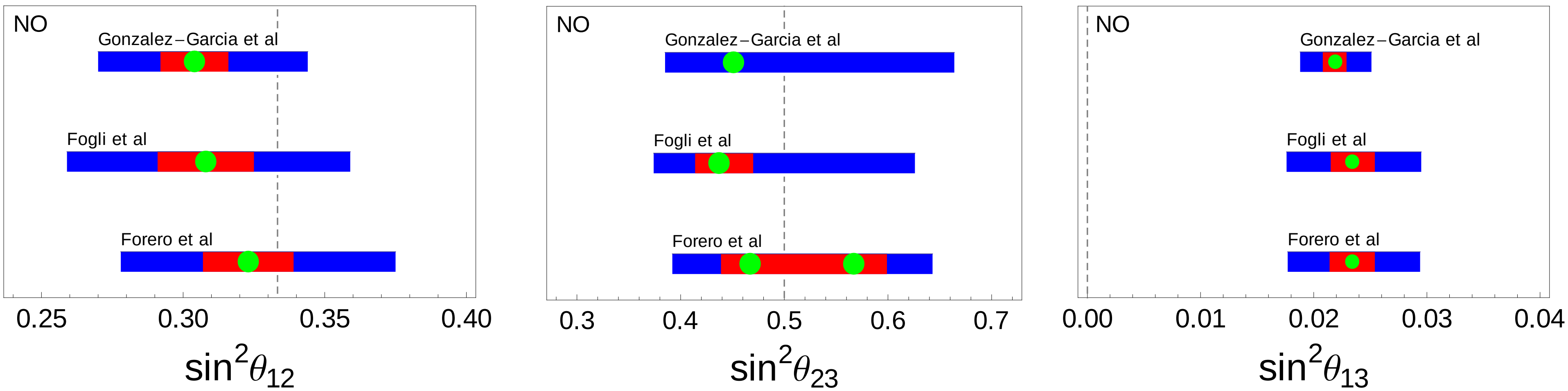
atmospheric | reactor + Dirac | solar | Majorana

$$U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_2}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_3}{2}} \end{pmatrix}$$

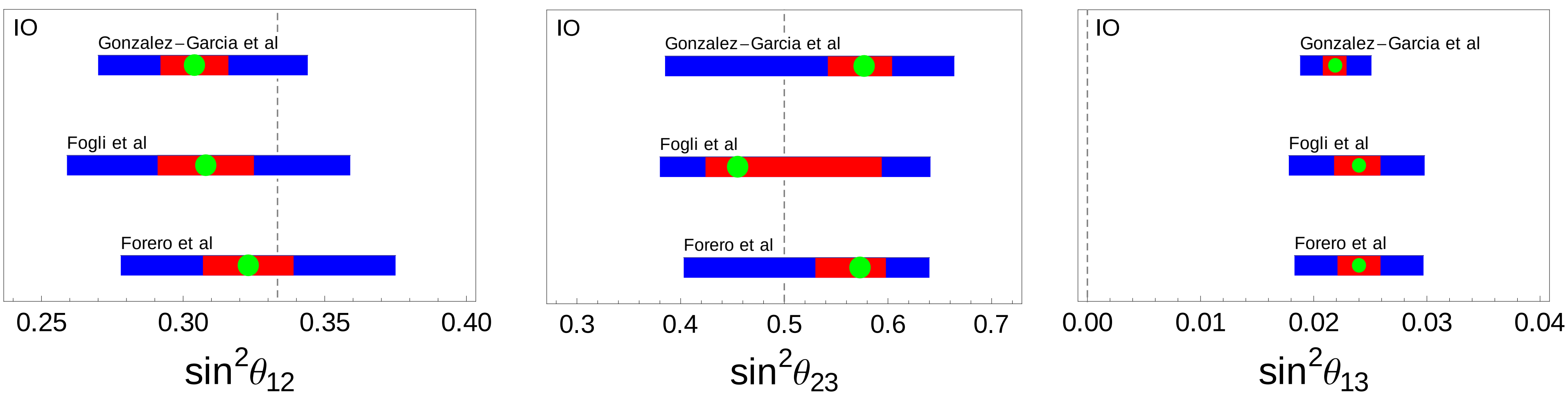
$\theta_{23} \approx 45^\circ$ | $\theta_{13} \approx 9^\circ$ | $\theta_{12} \approx 33^\circ$

Global neutrino fits

normal mass ordering



inverted mass ordering



Simple mixing patterns – tri-bimaximal

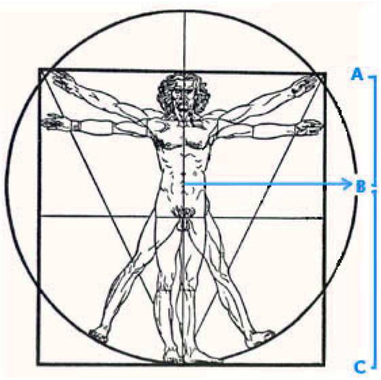


Harrison Perkins Scott

$$U_{\mathrm{PMNS}} \quad \approx \quad U_{\mathrm{TB}} \quad \equiv \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\theta_{12} \approx 35.3^\circ \qquad \theta_{23} = 45^\circ \qquad \theta_{13} = 0^\circ$$

Simple mixing patterns – golden ratio



$$\varphi = \frac{1+\sqrt{5}}{2} \approx 1.618$$

$$\tan\theta_{12} = \frac{1}{\varphi}$$

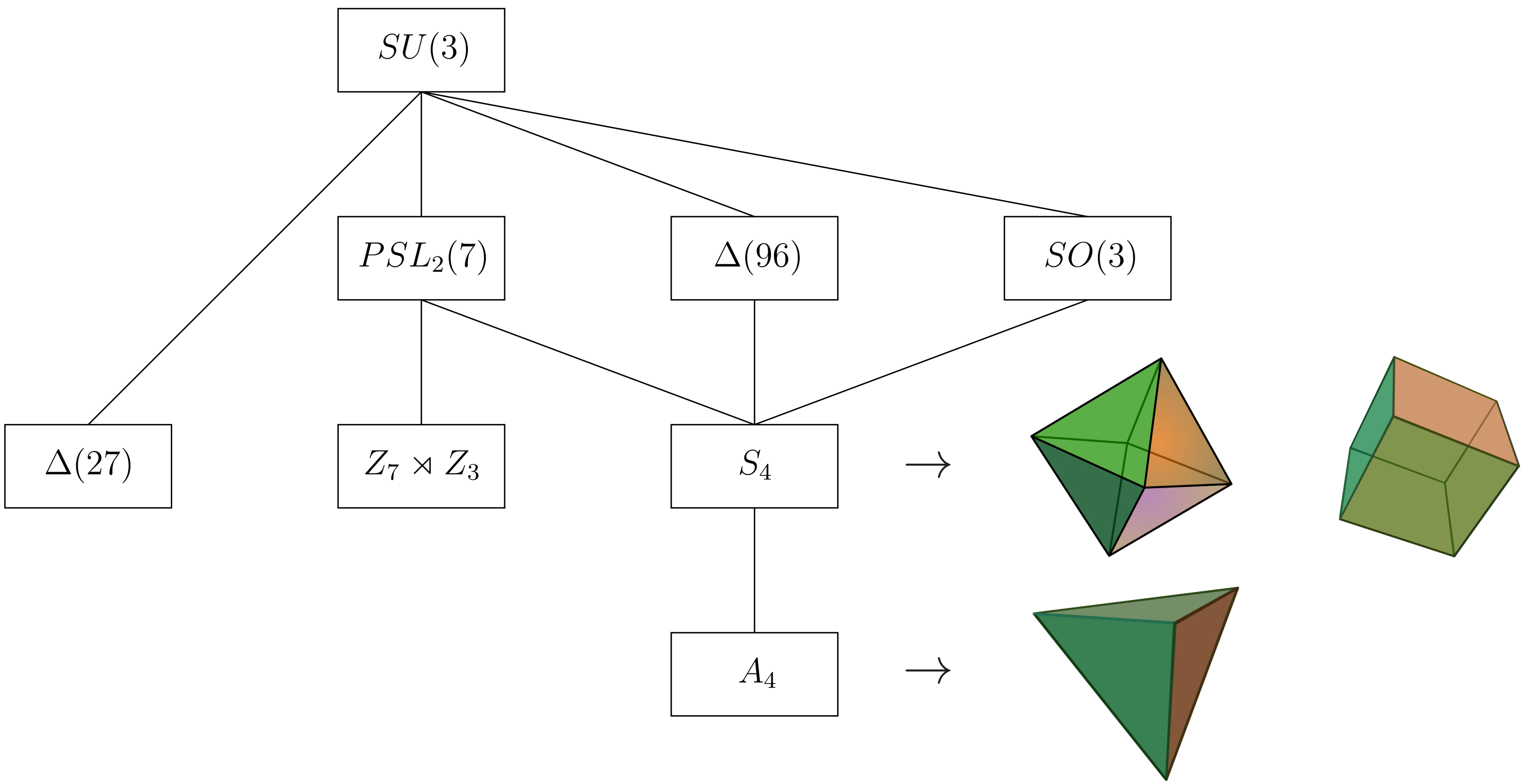
$$U_{\rm PMNS} \quad \approx \quad U_{\rm GR} \quad \equiv \quad \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\frac{\sin\theta_{12}}{\sqrt{2}} & \frac{\cos\theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{\sin\theta_{12}}{\sqrt{2}} & \frac{\cos\theta_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\theta_{12} \approx 31.7^\circ \qquad \theta_{23} = 45^\circ \qquad \theta_{13} = 0^\circ$$

Family symmetries (aka horizontal symmetries)

Candidate symmetry groups G

- non-Abelian to unify families (G should have triplet representations)
- discrete to facilitate obtaining simple mixing patterns   Merlo's talk



Essentials of finite group theory – S_4 example

e.g. Ramond, Group theory: a physicist's survey (2010)

- finite number of group elements g $\rightarrow$ each g has finite order ($g^n = 1$)
- construct all elements from a small number of generators
- presentation of S_4: generators S, T, U which satisfy

$$S^2 = T^3 = U^2 = (ST)^3 = (SU)^2 = (TU)^2 = (STU)^4 = 1$$

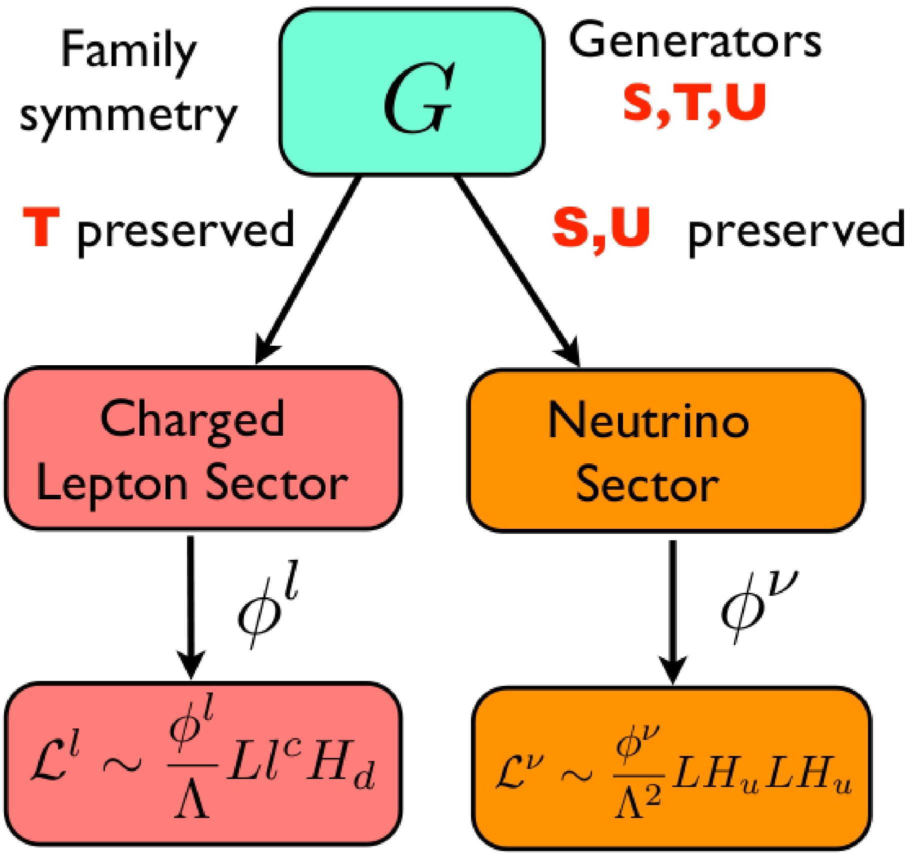
- matrix representations for S, T, U $\rightarrow$ irreps $\mathbf{1}$ $\mathbf{1}'$ $\mathbf{2}$ $\mathbf{3}$ $\mathbf{3}'$
- in physics we are mainly interested in multiplication rules
- Kronecker products: e.g. $\mathbf{3} \otimes \mathbf{3} = \mathbf{1} + \mathbf{2} + \mathbf{3} + \mathbf{3}'$
- Clebsch-Gordan coefficients: e.g. $\mathbf{3} \otimes \mathbf{3} \rightarrow \mathbf{1}$

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} \otimes \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} \rightarrow \alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2$$

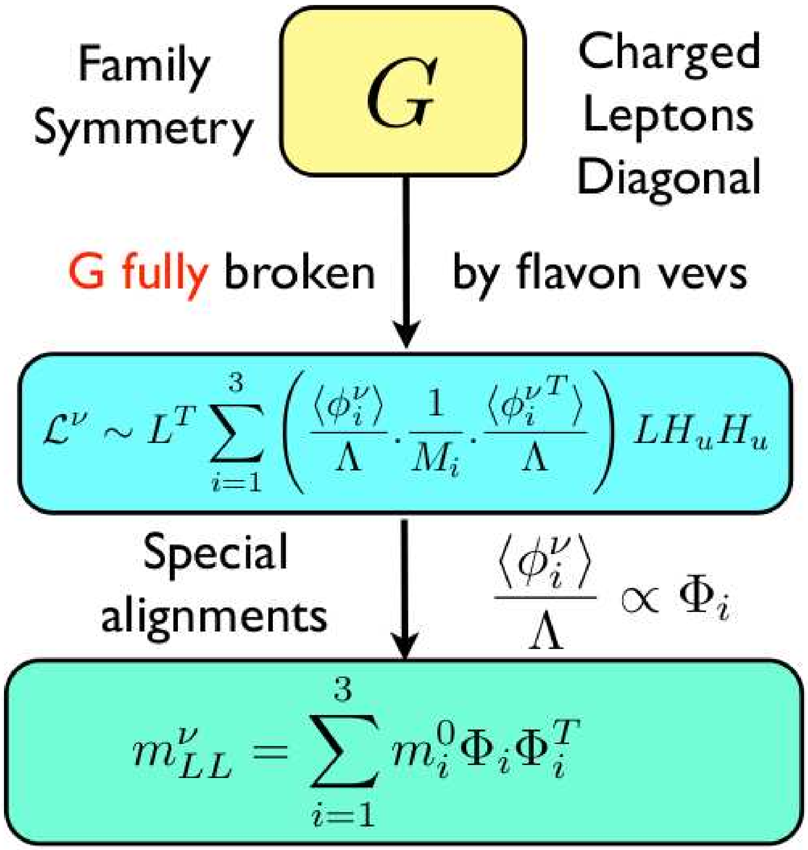
basis dependent !!

Two model building strategies

direct models
(mixing from residual symmetries)



indirect models
(mixing from flavon alignments Φ)



King, Luhn (JHEP 0910, 2009)

Building a direct model with tri-bimaximal mixing

- choose family symmetry group – S_4
- identify VEV configurations for family symmetry breaking fields ϕ

$$S\langle\phi^\nu\rangle = U\langle\phi^\nu\rangle = \langle\phi^\nu\rangle \qquad T\langle\phi^\ell\rangle = \langle\phi^\ell\rangle \qquad \text{flavon VEVs}$$

S_4	S	U	T	$\langle\phi^\nu\rangle$	$\langle\phi^\ell\rangle$
$\mathbf{1}, \mathbf{1}'$	1	± 1	1	$\mathbf{1}$	$\mathbf{1}, \mathbf{1}'$
$\mathbf{2}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}$	$\mathbf{2} \propto \begin{pmatrix} 1 \\ 1 \end{pmatrix}$	–
$\mathbf{3}, \mathbf{3}'$	$\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$	$\mp\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$	$\mathbf{3}' \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$	$\mathbf{3}, \mathbf{3}' \propto \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

- control coupling of flavons to fermions by extra Z_N or $U(1)$ symmetry

$$\frac{\phi^\nu}{\Lambda^2} L H_u L H_u \qquad \frac{\phi^\ell}{\Lambda} L\, \ell^c H_d$$

- with type I seesaw

$$\mathcal{L}_\nu \;\sim\; L\, H_u\, \nu^c \;+\; \phi^\nu \nu^c \nu^c$$

Building an indirect model with tri-bimaximal mixing

- family symmetry $G \subset SU(3)$
- diagonal charged leptons
- type I seesaw with 2 or 3 ν_a^c in singlet representation of G
- diagonal right-handed neutrino mass matrix (e.g. due to Z_2 symmetry)

$$\mathcal{L}_\nu \sim \sum_a \frac{\phi_a^\nu}{\Lambda} L\, H_u\, \nu_a^c + M_a \nu_a^c \nu_a^c$$

- $\phi_a^\nu \sim \overline{\mathbf{3}}$ and $L \sim \mathbf{3}$ of G
- G or $SU(3)$ invariant $\rightarrow$ $\phi^\nu_{a\,1} L_1 + \phi^\nu_{a\,2} L_2 + \phi^\nu_{a\,3} L_3 = {\phi_a^\nu}^T L$
- integrate out ν_a^c (seesaw formula)

$$\mathcal{L}_\nu \sim L^T \sum_a \left(\frac{\langle \phi_a^\nu \rangle}{\Lambda} \cdot \frac{1}{M_a} \cdot \frac{\langle \phi_a^\nu \rangle^T}{\Lambda} \right) L H_u H_u$$

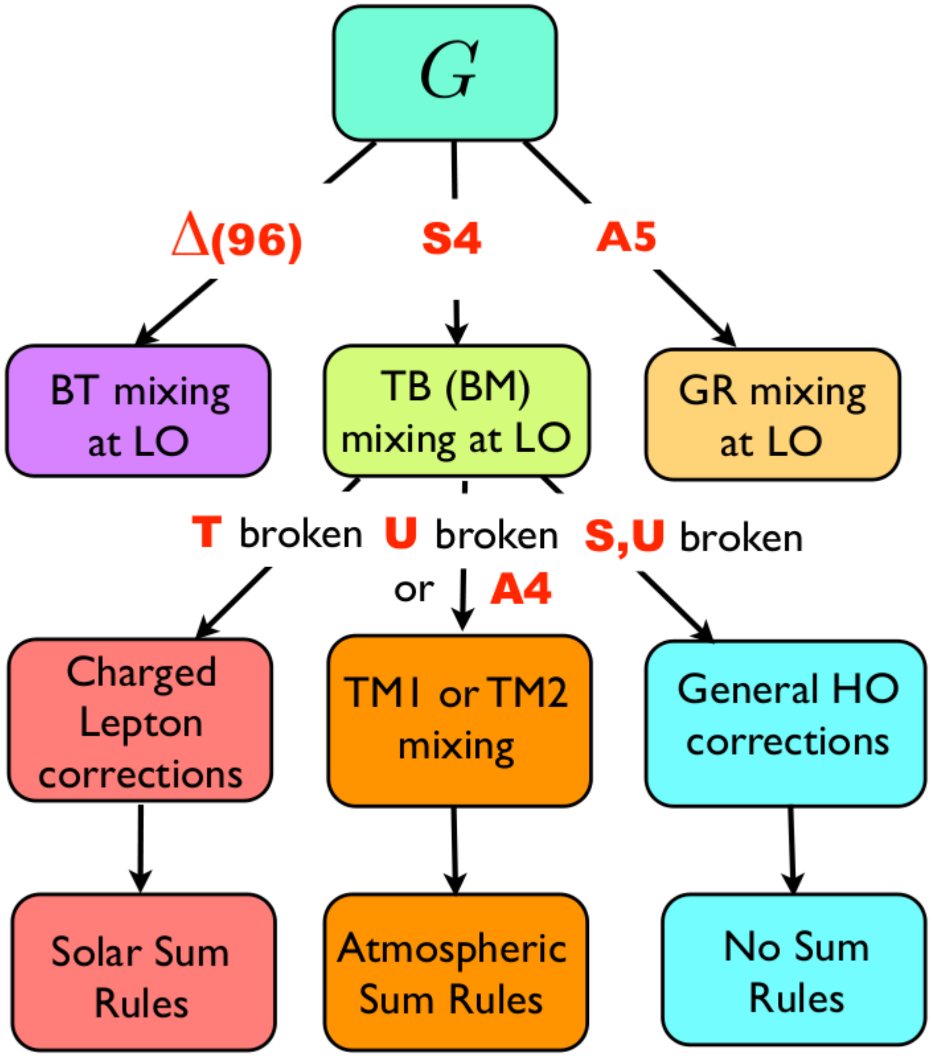
- tri-bimaximal if $\left[\langle \phi_1^\nu \rangle \propto \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \right] \quad \langle \phi_2^\nu \rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \langle \phi_3^\nu \rangle \propto \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

How to accommodate $\theta_{13} \sim 9^\circ$

King, Luhn (Rept. Prog. Phys. 76, 2013)

King et al. (New J. Phys. 16, 2014)

Direct models after 2012



mixing patterns:

	θ_{13}	θ_{23}	θ_{12}
TB	0°	45°	35.3°
BM	0°	45°	45°
GR	0°	45°	31.7°
BT	12.2°	36.2°	36.2°
TM	$\neq 0^\circ$	$\neq 45^\circ$	35.3°

TB = tri-bimaximal
BM = bimaximal
GR = golden ratio
BT = bi-trimaximal
TM = trimaximal

New family symmetries

- scans of "small" finite groups   Holthausen, Lim, Lindner (Phys. Lett. B721, 2013)

$$\Delta(6\cdot 10^2) \qquad \Delta(6\cdot 16^2) \qquad (Z_{18}\times Z_6)\rtimes S_3 \subset \Delta(6\cdot 18^2)$$

- mixing patterns derived from $\Delta(6n^2)$ King, Neder (Phys. Lett. B726, 2013)

$$U_{\Delta(6n^2)} = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\vartheta & \sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}}\sin\vartheta \\ -\sqrt{\frac{2}{3}}\sin(\frac{\pi}{6}+\vartheta) & \sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}}\cos(\frac{\pi}{6}+\vartheta) \\ \sqrt{\frac{2}{3}}\sin(\frac{\pi}{6}-\vartheta) & -\sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}}\cos(\frac{\pi}{6}-\vartheta) \end{pmatrix} \quad \text{with} \quad \vartheta = \pi\,\frac{p}{n}$$

- all possible mixing patterns from finite groups   Fonseca, Grimus (1405.3678)

$\rightarrow$ 17 sporadic cases, but all incompatible with observed mixing

$\rightarrow$ one series of mixing patterns related to $\Delta(6n^2)$

limited possibilities for lepton mixing from residual symmetries alone

Perturbation I – solar mixing sum rule

- T symmetry of charged lepton sector "slightly" broken (e.g. GUTs)
- $U_{\rm PMNS} = V_{\ell_L} V_{\nu_L}^\dagger$ and $V_{\nu_L}^\dagger = U_{\rm TB}$

$$U_{\rm PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & \hat{s}_{23} \\ 0 & -\hat{s}_{23}^* & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & \hat{s}_{13} \\ 0 & 1 & 0 \\ -\hat{s}_{13}^* & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & \hat{s}_{12} & 0 \\ -\hat{s}_{12}^* & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$s_{12}\, e^{i\delta_{12}} \approx \frac{1}{\sqrt{3}} \left(e^{i\delta_{12}^\nu} - \theta_{12}^\ell\, e^{i\delta_{12}^\ell} + \theta_{13}^\ell\, e^{i(\delta_{13}^\ell - \delta_{23}^\nu)} \right)$$

$$s_{23}\, e^{i\delta_{23}} \approx \frac{1}{\sqrt{2}} \left(e^{i\delta_{23}^\nu} - \theta_{23}^\ell\, e^{i\delta_{23}^\ell} \right)$$

$$s_{13}\, e^{i\delta_{13}} \approx \frac{1}{\sqrt{2}} \left(-\theta_{12}^\ell\, e^{i(\delta_{12}^\ell + \delta_{23}^\nu)} - \theta_{13}^\ell\, e^{i\delta_{13}^\ell} \right)$$

$$c_{ij} = \cos\theta_{ij}$$
$$\hat{s}_{ij} = \sin\theta_{ij}\, e^{-i\delta_{ij}}$$

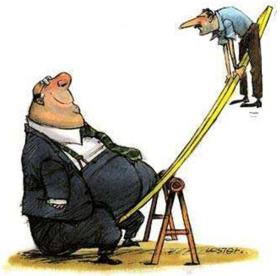
- $\theta_{12}^\ell \sim \theta_C \sim 0.22 \quad \to \quad \theta_{13} \sim 9^\circ$
- first order relation $\boxed{\theta_{12} \approx 35.3^\circ + \theta_{13} \cos\delta}$

Perturbation II – atmospheric mixing sum rule

- U symmetry of neutrino sector "slightly" broken $\rightarrow$ $U^{13}_{\text{PMNS}} \neq 0$
- conserve one Z_2 symmetry of Klein symmetry $Z_2^S \times Z_2^U$

	trimaximal 1 (TM_1)	trimaximal 2 (TM_2)
unbroken Z_2	$SU = -\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & 2 & -1 \\ 2 & -1 & 2 \end{pmatrix}$	$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$
PMNS mixing	$\frac{1}{\sqrt{6}}\begin{pmatrix} 2 & \cdot & \cdot \\ -1 & \cdot & \cdot \\ -1 & \cdot & \cdot \end{pmatrix}$	$\frac{1}{\sqrt{3}}\begin{pmatrix} \cdot & 1 & \cdot \\ \cdot & 1 & \cdot \\ \cdot & 1 & \cdot \end{pmatrix}$
solar angle	$\theta_{12} \approx 34.2^\circ$	$\theta_{12} \approx 35.8^\circ$
first order relation	$\theta_{23} \approx 45^\circ + \sqrt{2}\,\theta_{13}\cos\delta$	$\theta_{23} \approx 45^\circ - \frac{1}{\sqrt{2}}\,\theta_{13}\cos\delta$

Direct model of leptons based on S_4 and CP



- add CP symmetry to King, Luhn (JHEP 1109, 2011)
- diagonal charged leptons (enforced by T symmetry)
- in neutrino sector $(L \sim \nu^c \sim \mathbf{3} \qquad H_u \sim \mathbf{1})$

$$W_\nu \ \sim \ y_D L \nu^c H_u + (y_{\mathbf{3'}}\phi_{\mathbf{3'}} + y_{\mathbf{2}}\phi_{\mathbf{2}} + y_{\mathbf{1}}\phi_{\mathbf{1}})\nu^c\nu^c + \frac{y_{\mathbf{1'}}}{M}\widetilde{\phi}_{\mathbf{1'}}\phi_{\mathbf{2}}\nu^c\nu^c$$

- tri-bimaximal mixing at leading order (without flavon $\widetilde{\phi}_{\mathbf{1'}}$)
- $\langle\widetilde{\phi}_{\mathbf{1'}}\rangle$ breaks $Z_2^S \times Z_2^U$ down to $Z_2^S \quad \longrightarrow \quad$ TM$_2$ mixing
- resulting PMNS parameters

	θ_{13}	θ_{23}	θ_{12}	δ	(α_1, α_2)
(i)	free	$45^\circ \mp \frac{1}{\sqrt{2}}\theta_{13}$	35.3°	0 or π	0 or π
(ii)	free	45°	35.3°	$\pm\frac{\pi}{2}$	0 or π

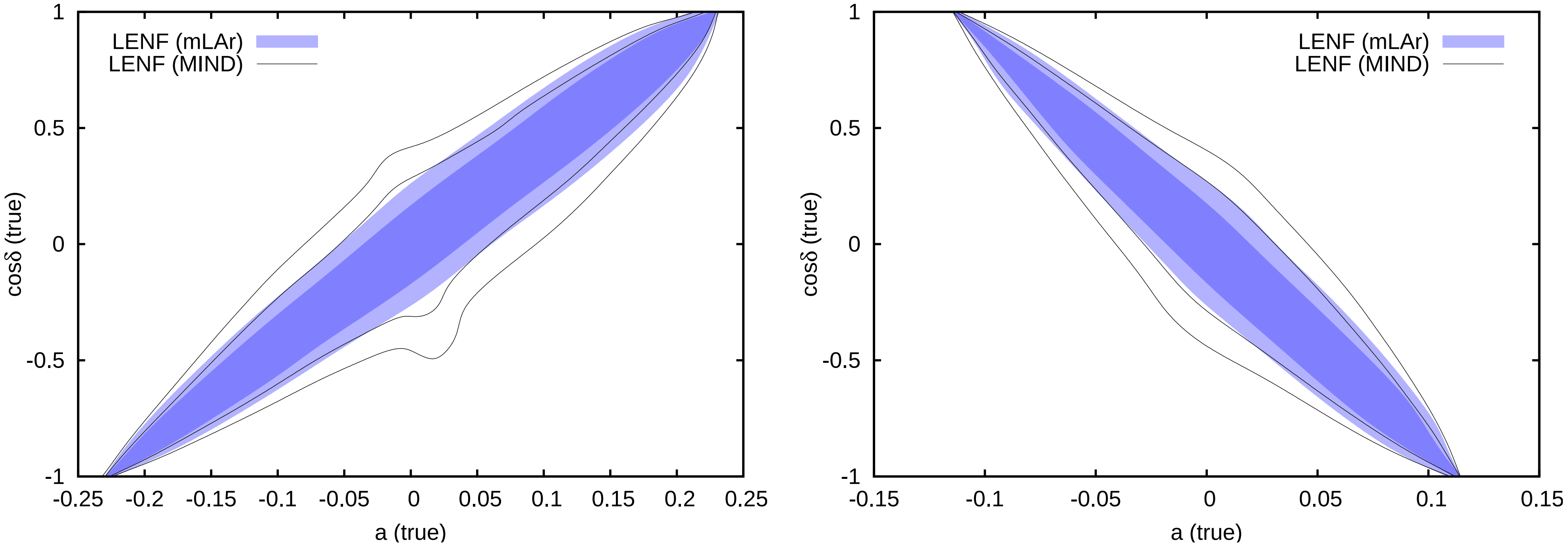
$\Longrightarrow$ five low-energy predictions with imposed CP symmetry

Testing the atmospheric sum rule

Meloni's talk

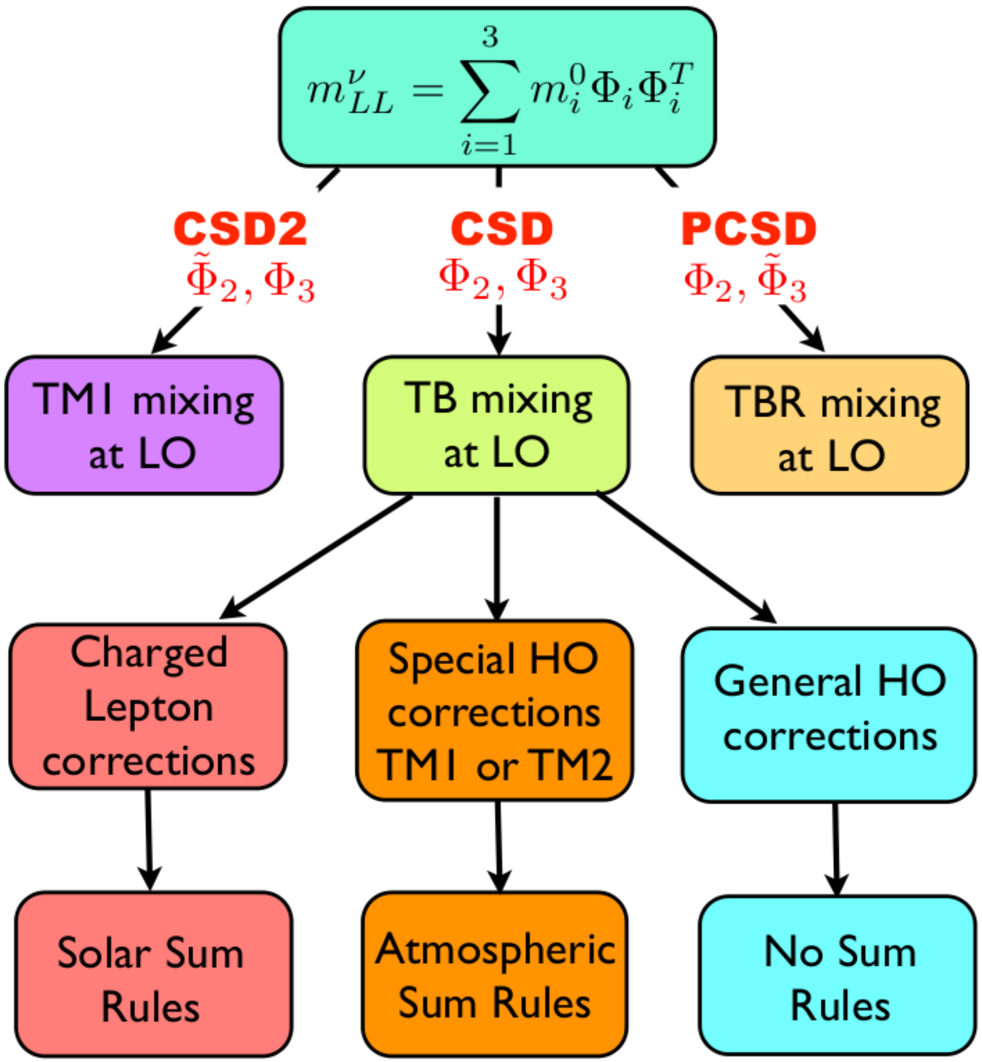
- low energy neutrino factory could measure θ_{23} and δ to high precision
- expected sensitivity for ruling out atmospheric sum rule

trimaximal 1 trimaximal 2



Ballett et al. (Phys. Rev. D89, 2014)

Indirect models after 2012



flavon alignments:

	$\langle \Phi_2 \rangle$	$\langle \Phi_3 \rangle$
CSD	$\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}$	$\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-1\end{pmatrix}$
CSD2	$\frac{1}{\sqrt{5}}\begin{pmatrix}1\\2\\0\end{pmatrix}$	$\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-1\end{pmatrix}$
PCSD	$\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}$	$\frac{1}{\sqrt{2}}\begin{pmatrix}\epsilon\\1\\-1\end{pmatrix}$

other alignments possible as well
(CSD3, CSD4, ...)

Variations of constrained sequential dominance (CSD)

$$\boxed{m_\nu \;=\; m_2^0\,\Phi_2\Phi_2^T \;+\; m_3^0\,\Phi_3\Phi_3^T} \qquad\qquad m_2^0 \ll m_3^0$$

- CSD

tri-bimaximal

$$\frac{m_2^0}{3}\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix} + \frac{m_3^0}{2}\begin{pmatrix}0 & 0 & 0\\ 0 & 1 & -1\\ 0 & -1 & 1\end{pmatrix} \quad\Longrightarrow\quad \boxed{\begin{array}{ll}\theta_{13} = 0 & m_1^\nu = 0\\ \theta_{23} = 45^\circ & m_2^\nu = m_2^0\\ \theta_{12} = 35.3^\circ & m_3^\nu = m_3^0\end{array}}$$

- CSD2

trimaximal 1 (TM_1)

$$\frac{m_2^0}{5}\begin{pmatrix}1 & 2 & 0\\ 2 & 4 & 0\\ 0 & 0 & 0\end{pmatrix} + \frac{m_3^0}{2}\begin{pmatrix}0 & 0 & 0\\ 0 & 1 & -1\\ 0 & -1 & 1\end{pmatrix} \qquad \boxed{\begin{array}{ll}\theta_{13} \approx \frac{\sqrt{2}}{3}\frac{m_2^\nu}{m_3^\nu} & m_1^\nu = 0\\ \theta_{23} \approx 45^\circ + \sqrt{2}\theta_{13}\cos\delta & m_2^\nu \approx \frac{3}{5}m_2^0\\ \theta_{12} \approx 35.3^\circ & m_3^\nu \approx m_3^0\end{array}}$$

- PCSD (partially CSD)

tri-bimaximal-reactor

$$\frac{m_2^0}{3}\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix} + \frac{m_3^0}{2}\begin{pmatrix}\epsilon^2 & \epsilon & -\epsilon\\ \epsilon & 1 & -1\\ -\epsilon & -1 & 1\end{pmatrix} \quad\Longrightarrow\quad \boxed{\begin{array}{ll}\theta_{13} \approx \frac{\epsilon}{\sqrt{2}} & m_1^\nu = 0\\ \theta_{23} \approx 45^\circ & m_2^\nu \approx m_2^0\\ \theta_{12} \approx 35.3^\circ & m_3^\nu \approx m_3^0\end{array}}$$

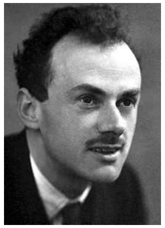
Summary

- non-Abelian (discrete) symmetries
  - unify three families of chiral fermions
  - still attractive despite $\theta_{13} \sim 9^\circ$
- direct models with realistic θ_{13}
  - large family symmetries [e.g. $\Delta(600)$]
  - small family symmetries [e.g. S_4] plus perturbations
  - testable mixing sum rules
- indirect models with realistic θ_{13}
  - requires more complicated flavon alignments Φ
  - relation between mixing angles and masses possible [e.g. in CSD2]
- gain predictivity by imposing CP symmetry
- crucial to measure mixing parameters to a high precision

Thank you

Symmetries of the mass matrices (in flavour basis)

charged leptons



Dirac

$M_\ell = \text{diag}(m_e, m_\mu, m_\tau)$

symmetric under diagonal phase transformation h

$$\boxed{M_\ell = h^T M_\ell h^*}$$

e.g. $h = \text{diag}(1, e^{\frac{4\pi i}{3}}, e^{\frac{2\pi i}{3}})$

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neutrinos

Majorana

$M_\nu = U_{\text{PMNS}} \, \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \, U_{\text{PMNS}}^T$

symmetry of M_ν depends on U_{PMNS}

$$\boxed{M_\nu = k^T M_\nu k}$$

$k = U_{\text{PMNS}}^* \, \text{diag}(+1, -1, -1) \, U_{\text{PMNS}}^T$

four different k $\rightarrow$ generate $Z_2 \times Z_2$ symmetry group

<u>Klein symmetry:</u> $K = \{1, k_1, k_2, k_3\}$

for $U_{\text{PMNS}} = U_{\text{TB}}$:

$$k_1 = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad k_2 = -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad k_3 = k_1 k_2$$

Origin of the Klein symmetry K

- direct models
  - $\boxed{\text{Klein symmetry } K \ \subset \ \text{family symmetry } G}$
  - flavons ϕ are multiplets of G
  - their VEVs $\langle\phi\rangle$ break G down to K in neutrino sector
  - for TB mixing (k_1, k_2, h) generate permutation group S_4
- indirect models
  - $\boxed{\text{Klein symmetry } K \ \not\subset \ \text{family symmetry } G}$
  - G responsible for generating particular flavon VEV configurations $\langle\phi\rangle$
  - for TB mixing – from e.g. $\Delta(27)$, $Z_7 \rtimes Z_3$

$$\langle\phi_1\rangle \propto \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \qquad \langle\phi_2\rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \langle\phi_3\rangle \propto \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$\Rightarrow \quad \mathcal{L}_\nu \quad \sim \quad \nu\,(\phi_1\phi_1^T + \phi_2\phi_2^T + \phi_3\phi_3^T)\,\nu \ H\,H$$

Aligning triplet flavons in $\Delta(27)\,,\ Z_7 \rtimes Z_3\,,\ A_4$

$$\boxed{V(\phi) \;=\; -\,m^2 \sum_i {\phi_i}^\dagger \phi_i \;+\; \lambda \left(\sum_i {\phi_i}^\dagger \phi_i\right)^2 \;+\; \Delta V}$$

central terms in ΔV

$(i)\quad \kappa \sum_i \phi_i^\dagger \phi_i \phi_i^\dagger \phi_i$

$$\kappa > 0 \quad \rightarrow \quad \langle \phi \rangle \;\propto\; \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\kappa < 0 \quad \rightarrow \quad \langle \phi \rangle \;\propto\; \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$(ii)\quad \tilde{\kappa} \sum_{i,j} (\phi_i^\dagger \tilde{\phi}_i)(\tilde{\phi}_j^\dagger \phi_j)$

$\tilde{\kappa} > 0 \quad \rightarrow \quad$ orthogonality condition $\langle \phi \rangle \perp \langle \tilde{\phi} \rangle$

e.g. $\langle \tilde{\phi} \rangle \;\propto\; \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

... $\langle \widetilde{\phi} \rangle \;\propto\; \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$

Flavon alignment in supersymmetry

- SUSY unbroken at scale of family symmetry breaking
- introduce so-called driving fields X which couple to flavons
- flavon superpotential W^{flavon} linear in X due to $U(1)_R$ symmetry
- F-terms of driving fields need to vanish

$$\boxed{F^*_{X_i} = -\frac{\partial W^{\text{flavon}}}{\partial X_i} = 0}$$

- two examples in S_4

$$W^{\text{flavon}} \sim X_{\mathbf{1}}\phi_{\mathbf{2}}\phi_{\mathbf{2}} = X_{\mathbf{1}}(\phi_{\mathbf{2},1}\phi_{\mathbf{2},2} + \phi_{\mathbf{2},2}\phi_{\mathbf{2},1}) = 2X_{\mathbf{1}}\phi_{\mathbf{2},1}\phi_{\mathbf{2},2}$$

$$\longrightarrow \quad \langle\phi_{\mathbf{2}}\rangle \propto \begin{pmatrix}1\\0\end{pmatrix} \quad \text{or} \quad \langle\phi_{\mathbf{2}}\rangle \propto \begin{pmatrix}0\\1\end{pmatrix}$$

$$W^{\text{flavon}} = g_0 X_{\mathbf{3}}\phi_{\mathbf{3'}}\phi_{\mathbf{2}} + X_{\mathbf{3'}}\left(g_1\phi_{\mathbf{3'}}\phi_{\mathbf{3'}} + g_2\phi_{\mathbf{3'}}\phi_{\mathbf{2}} + g_3\phi_{\mathbf{3'}}\phi_{\mathbf{1}}\right)$$

$$\longrightarrow \quad \langle\phi_{\mathbf{3'}}\rangle = \varphi_{\mathbf{3'}}\begin{pmatrix}1\\1\\1\end{pmatrix} \qquad \langle\phi_{\mathbf{2}}\rangle = \varphi_{\mathbf{2}}\begin{pmatrix}1\\1\end{pmatrix} \qquad \varphi_{\mathbf{2}} = -\tfrac{g_3}{2g_2}\varphi_{\mathbf{1}}$$

- flavon alignments independent of g_i

Details of the direct S_4 model

matter	L	τ^c	μ^c	e^c	ν^c	H_u	H_d
S_4	$\mathbf{3}$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$
Z_3^ν	1	2	2	2	2	0	0
Z_3^ℓ	0	2	1	0	0	0	0

King, Luhn
(JHEP 1109, 2011)

$$\langle \varphi_\ell \rangle = \begin{pmatrix} 0 \\ v_\ell \\ 0 \end{pmatrix} \qquad \langle \eta_\mu \rangle = \begin{pmatrix} 0 \\ w_\mu \end{pmatrix} \qquad \langle \eta_e \rangle = \begin{pmatrix} w_e \\ 0 \end{pmatrix}$$

flavons	φ_ℓ	η_μ	η_e	$\phi_{\mathbf{3}'}$	$\phi_{\mathbf{2}}$	$\phi_{\mathbf{1}}$	$\widetilde{\phi}_{\mathbf{1}'}$
S_4	$\mathbf{3}'$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{3}'$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}'$
Z_3^ν	0	0	0	2	2	2	0
Z_3^ℓ	1	1	2	0	0	0	0

$$\langle \phi_{\mathbf{3}'} \rangle = v_{\mathbf{3}'} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \langle \phi_{\mathbf{2}} \rangle = v_{\mathbf{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \langle \phi_{\mathbf{1}} \rangle = v_{\mathbf{1}} \qquad \langle \widetilde{\phi}_{\mathbf{1}'} \rangle = \widetilde{v}_{\mathbf{1}'}$$

Charged lepton sector

$$\boxed{W_\ell \;\sim\; \left[\tfrac{1}{M}(L\varphi_\ell)_{1'}\,\tau^c + \tfrac{1}{M^2}(L\varphi_\ell)_2\,\eta_\mu\,\mu^c + \tfrac{1}{M^2}(L\varphi_\ell)_2\,\eta_e\,e^c\right]H_d}$$

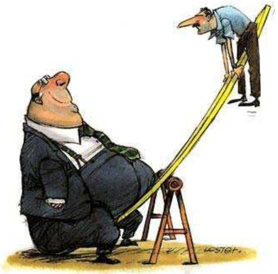
- Z_3^ℓ controls pairing of flavons with right-handed charged fermions
- different S_4 contractions of $(L\varphi_\ell)$ pick out different L_i components

$$(L\varphi_\ell)_{1'} \;=\; L_1\varphi_{\ell 1} + L_2\varphi_{\ell 3} + L_3\varphi_{\ell 2} \;\rightarrow\; L_3$$

$$(L\varphi_\ell)_2 \;=\; \begin{pmatrix} L_1\varphi_{\ell 3} + L_2\varphi_{\ell 2} + L_3\varphi_{\ell 1} \\ L_1\varphi_{\ell 2} + L_2\varphi_{\ell 1} + L_3\varphi_{\ell 3} \end{pmatrix} \;\rightarrow\; \begin{pmatrix} L_2 \\ L_1 \end{pmatrix}$$

- mass matrix diagonal by construction
- m_τ heavier than m_μ and m_e
- hierarchy between m_μ and m_e due to hierarchy of VEVs w_μ and w_e
- just a toy model of charged lepton sector (with GUTs off-diagonals)

Neutrino sector



- type I seesaw mechanism
- Dirac coupling without flavon field
- breaking of S_4 via mass matrix of right-handed neutrinos
- use assignments $\quad L \sim \nu^c \sim \mathbf{3} \qquad H_u \sim \mathbf{1}$

$$\boxed{W_\nu \;\sim\; y_D L \nu^c H_u + (y_{\mathbf{3}'}\phi_{\mathbf{3}'} + y_{\mathbf{2}}\phi_{\mathbf{2}} + y_{\mathbf{1}}\phi_{\mathbf{1}})\nu^c\nu^c + \frac{y_{\mathbf{1}'}}{M}\widetilde{\phi}_{\mathbf{1}'}\phi_{\mathbf{2}}\nu^c\nu^c}$$

S_4 irrep	S	U	VEV alignment
$\mathbf{3}'$	$\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$	$\langle \phi_{\mathbf{3}'} \rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$
$\mathbf{2}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\langle \phi_{\mathbf{2}} \rangle \propto \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
$\mathbf{1}$	1	1	$\langle \phi_{\mathbf{1}} \rangle \propto 1$
$\mathbf{1}'$	1	-1	$\langle \widetilde{\phi}_{\mathbf{1}'} \rangle \propto 1$

U broken $\;\&\;$ S conserved $\;\longrightarrow\;$ TM_2 mixing

Flavon alignment

$$\langle\phi_{\mathbf{3}'}\rangle = v_{\mathbf{3}'}\begin{pmatrix}1\\1\\1\end{pmatrix} \qquad \langle\phi_{\mathbf{2}}\rangle = v_{\mathbf{2}}\begin{pmatrix}1\\1\end{pmatrix} \qquad \langle\phi_{\mathbf{1}}\rangle = v_{\mathbf{1}} \qquad \langle\widetilde{\phi}_{\mathbf{1}'}\rangle = \widetilde{v}_{\mathbf{1}'}$$

- SUSY unbroken at scale of family symmetry breaking
- F-terms of driving fields $\phi^0_{\mathbf{r}}$ need to vanish

$$\begin{aligned} W_\nu^{\text{flavon}} \;=\;& \phi^0_{\mathbf{3}'}(g_1\,\phi_{\mathbf{3}'}\phi_{\mathbf{3}'} + g_2\,\phi_{\mathbf{3}'}\phi_{\mathbf{2}} + g_3\,\phi_{\mathbf{3}'}\phi_{\mathbf{1}}) \\ & + \phi^0_{\mathbf{3}}(g_4\,\phi_{\mathbf{3}'}\phi_{\mathbf{2}}) \\ & + \phi^0_{\mathbf{1}}(g_5\,\phi_{\mathbf{3}'}\phi_{\mathbf{3}'} + g_6\,\phi_{\mathbf{2}}\phi_{\mathbf{2}} + g_7\,\phi_{\mathbf{1}}\phi_{\mathbf{1}}) \\ & + \widetilde{\phi}^0_{\mathbf{1}}(g_8\,\widetilde{\phi}_{\mathbf{1}'}\widetilde{\phi}_{\mathbf{1}'} + M^2) \end{aligned}$$

- previously assumed flavon alignments independent of g_i with

$$v_{\mathbf{2}} = -\frac{g_3}{2g_2}v_{\mathbf{1}} \qquad v^2_{\mathbf{3}'} = -\frac{1}{3g_5}\left(g_7 + \frac{g_3^2 g_6}{2g_2^2}\right)v^2_{\mathbf{1}} \qquad \widetilde{v}^2_{\mathbf{1}'} = -\frac{1}{g_8}M^2$$

Imposing CP symmetry (straightforward in S_4)

$$M_R = y_{\mathbf{3'}} v_{\mathbf{3'}} \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + y_{\mathbf{2}} v_{\mathbf{2}} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} + y_{\mathbf{1}} v_{\mathbf{1}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$
$$+ \frac{y_{\mathbf{1'}}}{M} \tilde{v}_{\mathbf{1'}} v_{\mathbf{2}} \begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

$$v_{\mathbf{2}} = -\frac{g_3}{2g_2} v_{\mathbf{1}} \qquad v_{\mathbf{3'}}^2 = -\frac{1}{3g_5}\left(g_7 + \frac{g_3^2 g_6}{2g_2^2}\right) v_{\mathbf{1}}^2 \qquad \tilde{v}_{\mathbf{1'}}^2 = -\frac{1}{g_8} M^2$$

- CP symmetry $\rightarrow$ couplings y_i and g_i real
- phases of $v_{\mathbf{1}}$, $v_{\mathbf{2}}$, $v_{\mathbf{3'}}$ identical up to π or $\pm\pi/2$
- absorb phase of $v_{\mathbf{1}}$ into redefinition of ν^c

	$v_{\mathbf{1}}$	$v_{\mathbf{2}}$	$v_{\mathbf{3'}}$	$\tilde{v}_{\mathbf{1'}}$
(A)	real	real	real	real
(B)	real	real	real	imaginary
(C)	real	real	imaginary	real
(D)	real	real	imaginary	imaginary

Predictions with CP symmetry

- seesaw mechanism

$$M_\nu = m_D \, {M_R}^{-1} \, m_D^T \qquad \text{with} \qquad m_D \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\longrightarrow \quad \boxed{U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} U_R} \qquad \text{with} \qquad U_R^T \, M_R \, U_R = M_R^{\text{diag}}$$

- resulting PMNS parameters

	θ_{13}	θ_{23}	θ_{12}	δ	(α_1, α_2)
(A)	free	$45^\circ \mp \frac{1}{\sqrt{2}}\theta_{13}$	35.3°	0 or π	0 or π
(B)	free	45°	35.3°	$\pm\frac{\pi}{2}$	0 or π
(C)	unphysical: two degenerate neutrino masses				
(D)	unphysical: $\theta_{13} = 35.3^\circ$				

$\Longrightarrow$ five low-energy predictions with imposed CP symmetry
(up to a finite choice)