UNIVERSIDAD AUTONOMA DE MADRID

invisibles neutrinos, dark matter & dark energy physics

Instituto de Física Teórica

NEUTRINO MASSES AND MIXINGS FROM CONTINUOUS SYMMETRIES

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University of Glasgow **NUFACT 2014** SUPA

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Content

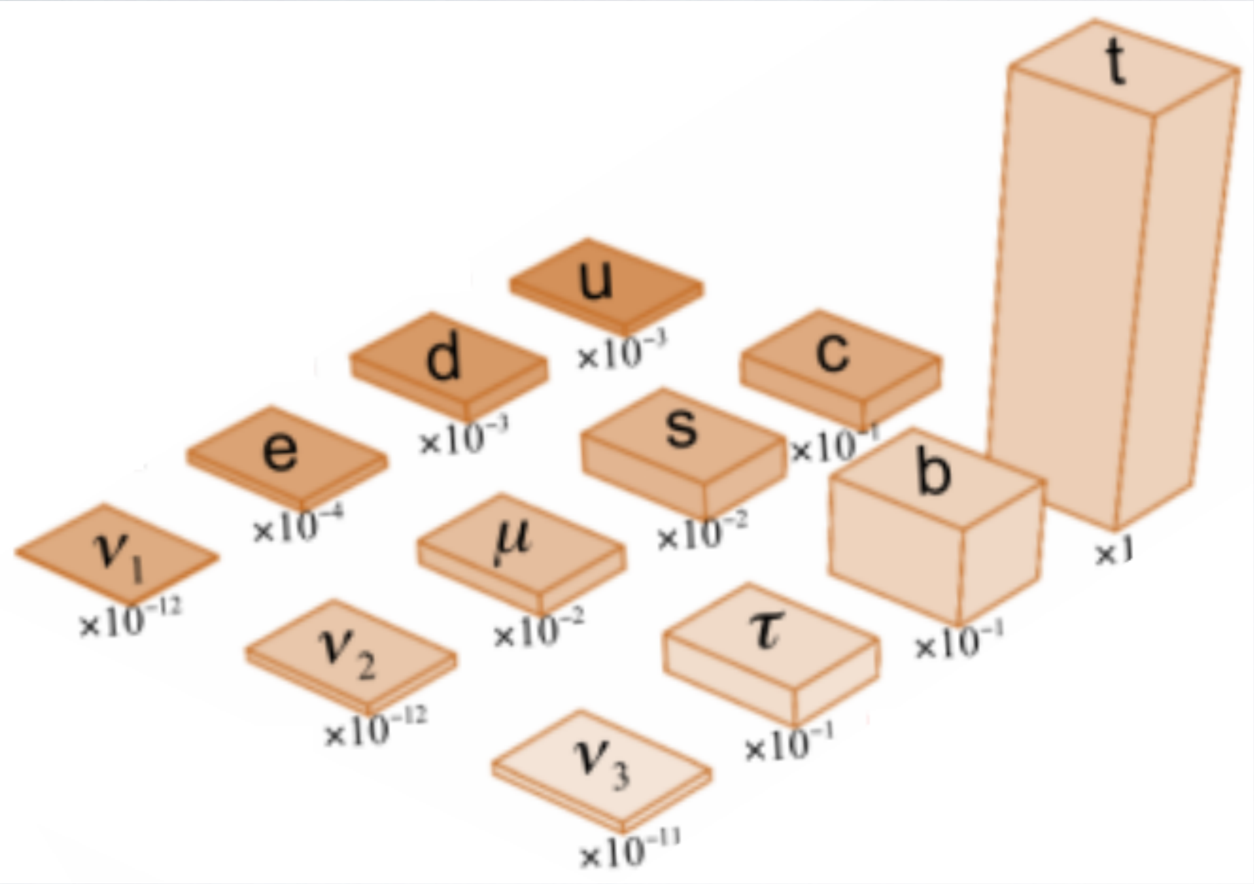
- Introduction
- Continuous Abelian U(1) Flavour Symmetry
  - The model building in the lepton sector
  - Comparison with Anarchy

[Altarelli, Feruglio, Masina & LM, JHEP **1211** (2012) 139
Bergstrom, Meloni and LM, Phys.Rev. **D89** (2014) 093021]

- Continuous non-Abelian Flavour Symmetries
  - The MLFV approach
  - Dynamical Yukawas

[Alonso, Isidori, LM, Munoz & Nardi, JHEP **1106** (2011) 037
Alonso, Gavela, Hernandez & LM, Phys.Lett. **B715** (2012) 194-198,
Alonso, Gavela, Hernandez, LM & Rigolin, JHEP **1308** (2013) 069,
Alonso, Gavela, Isidori, Maiani, JHEP **1311** (2013) 187]

The Flavour Puzzle



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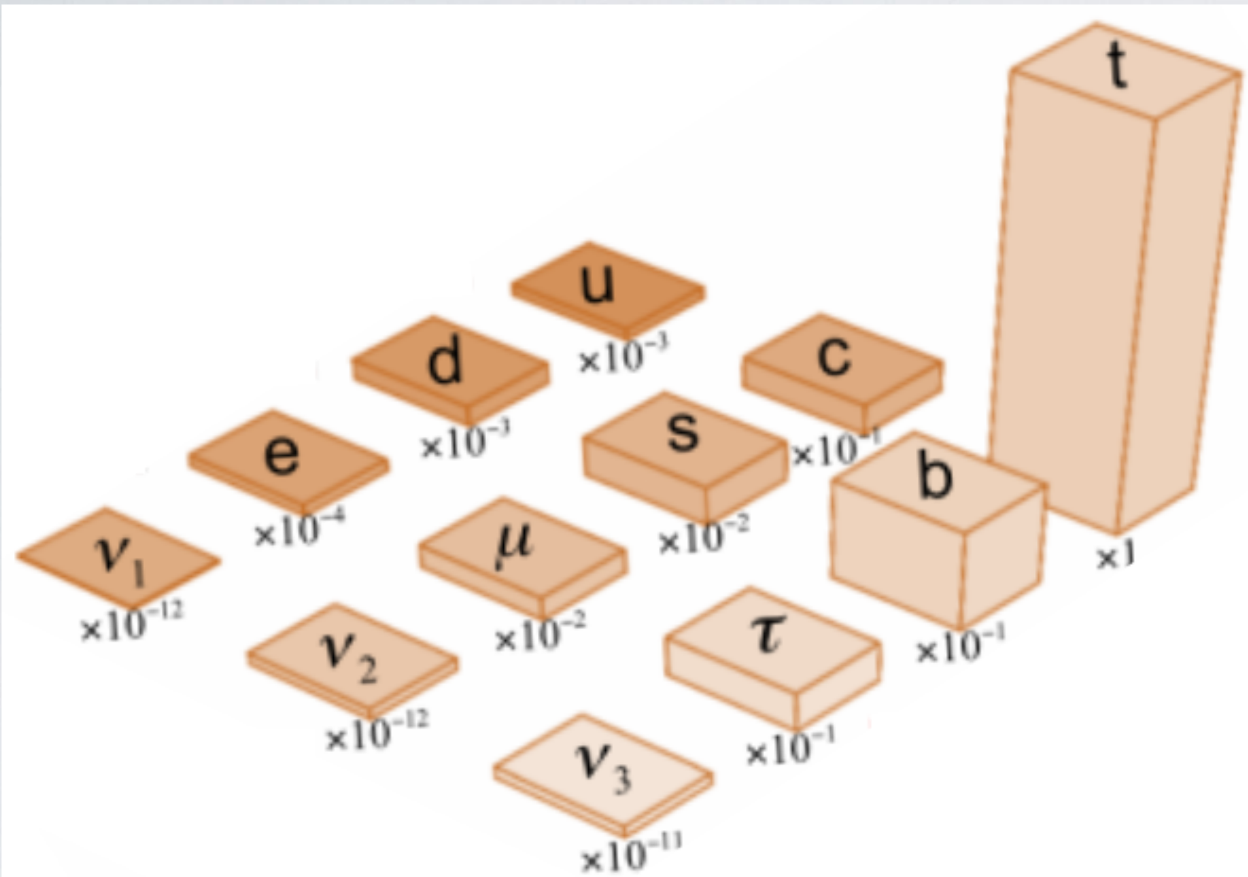

$$|V_{CKM}| \simeq \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$\Updownarrow$

$$|U_{PMNS}| \simeq \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.4 & 0.5 & 0.7 \\ 0.4 & 0.5 & 0.7 \end{pmatrix}$$

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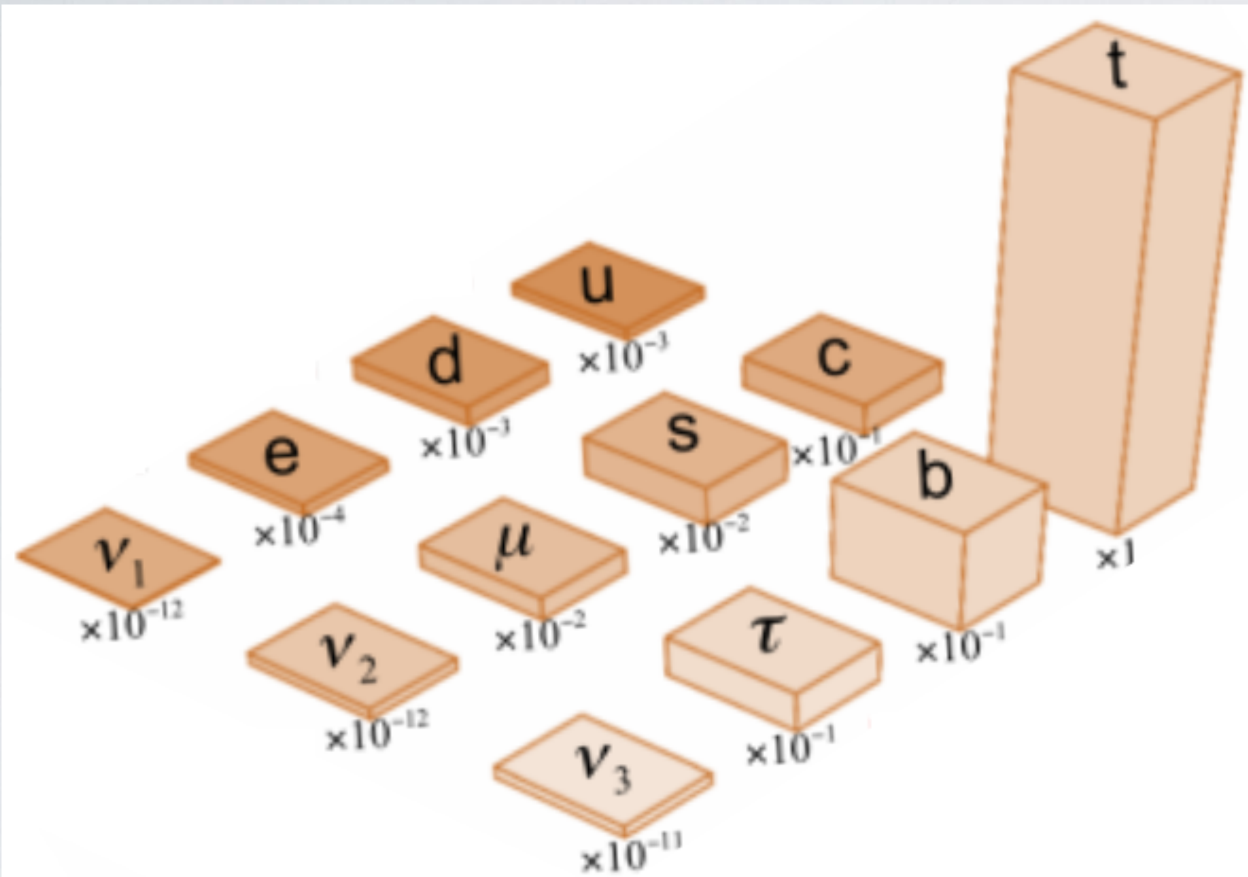

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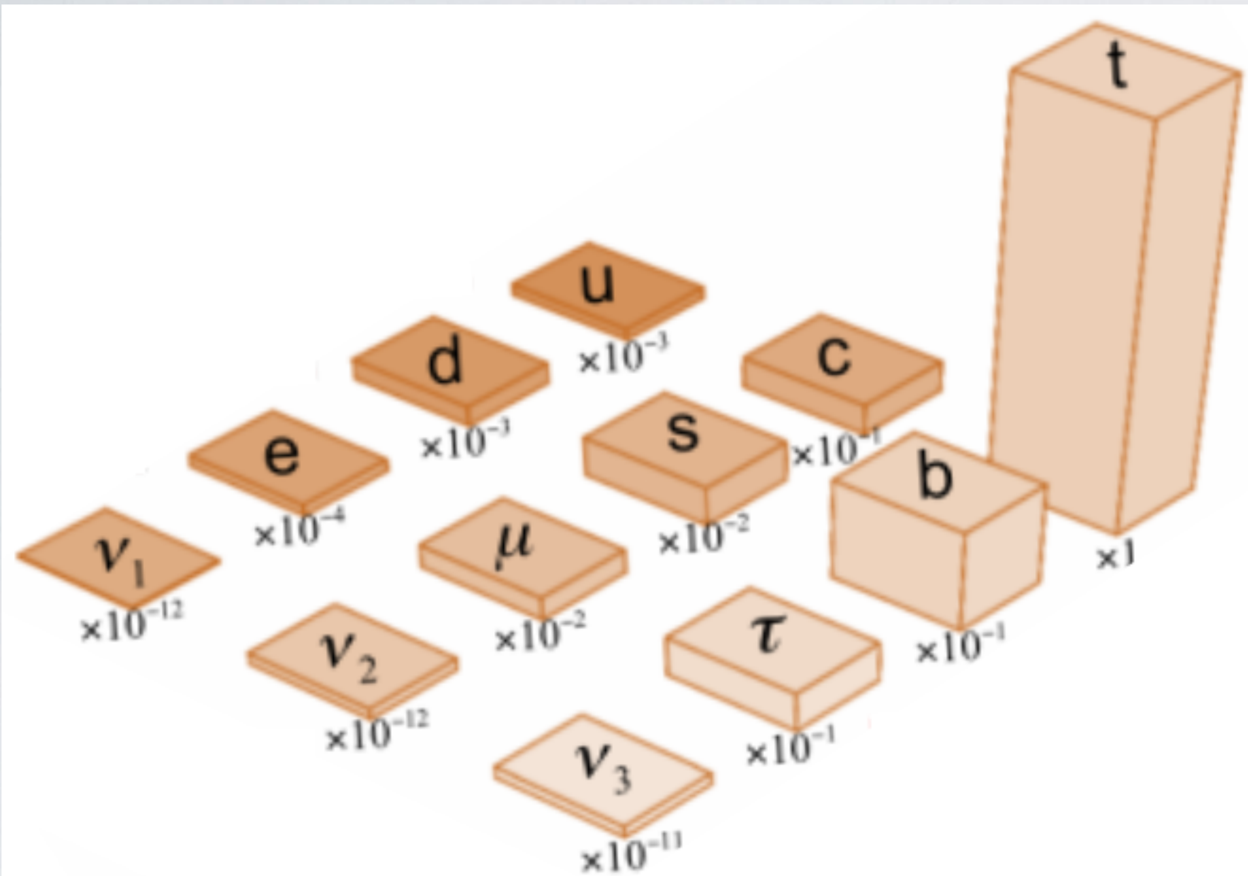

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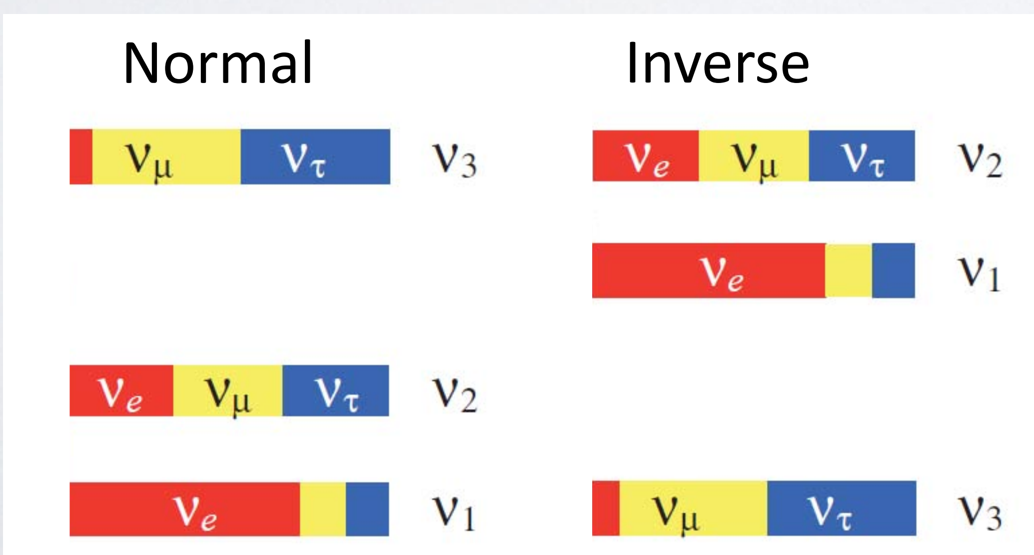

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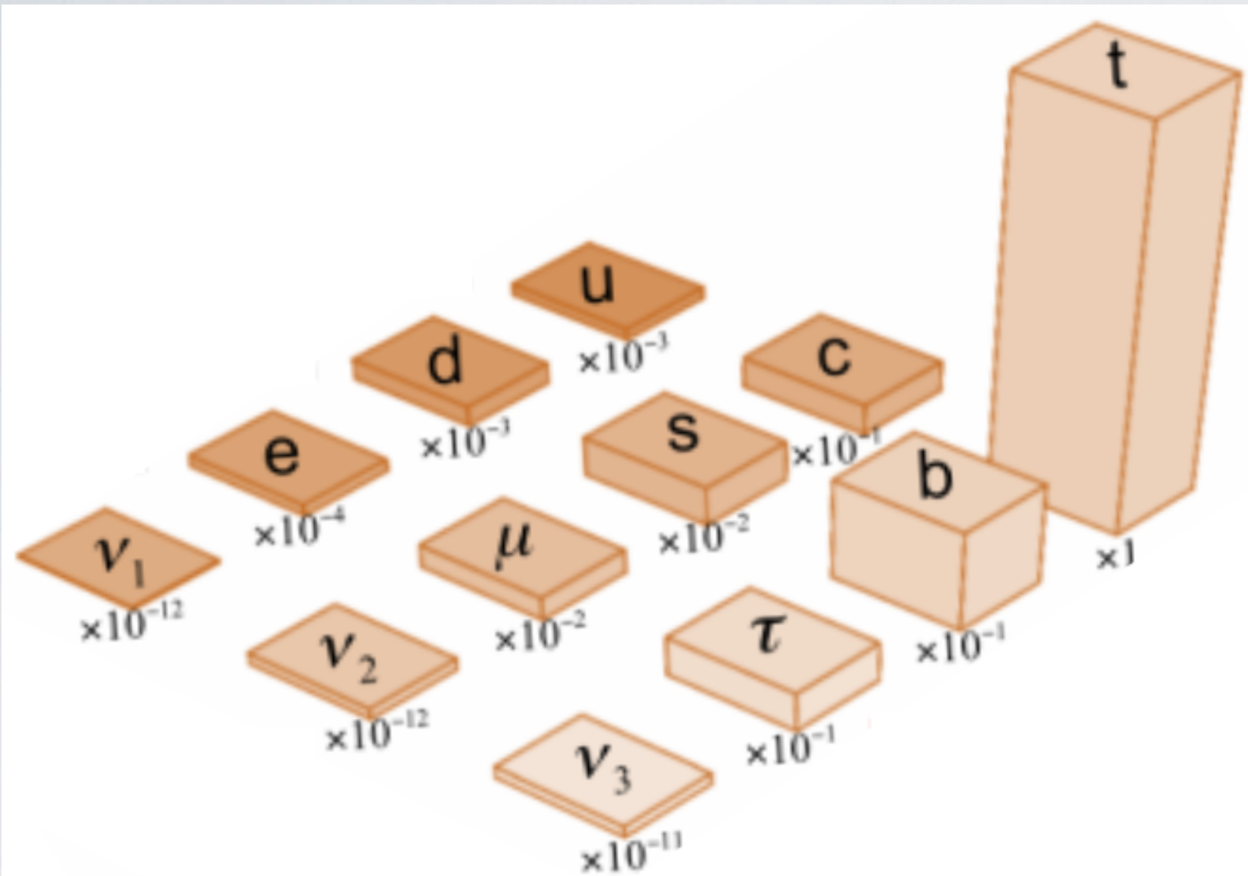

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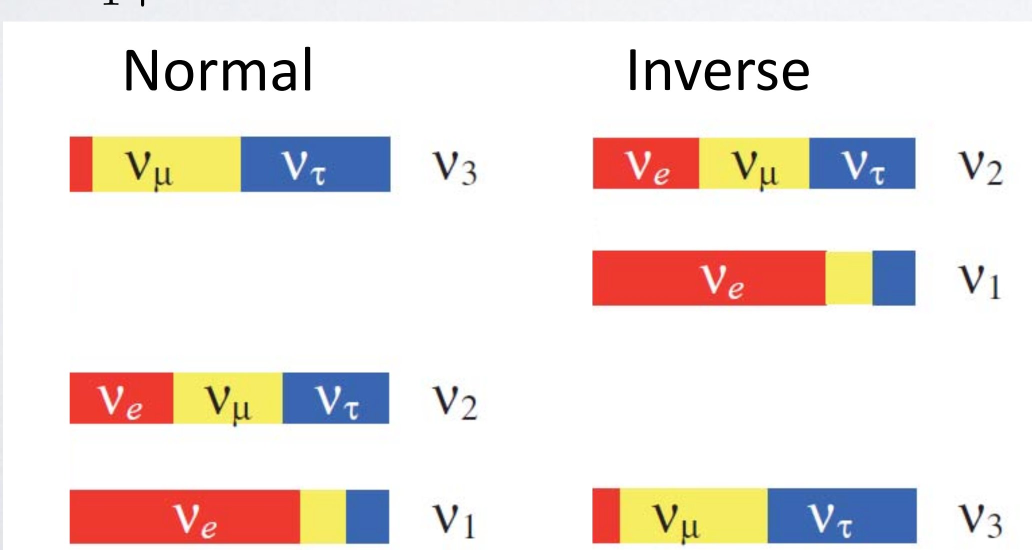

Nature of Neutrinos:

Majorana $\nu^C = \nu$

Dirac $\nu^C \neq \nu$

Pros and Cons of Discrete Syms

See talks by Luhn and Meloni

Pros:

- Great Predictivity: i.e. $$U_{TBM} = \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

  Not both the mass orderings are always allowed

- LO mixing angles determined by the CGs of the group: GEOMETRY

- Precise mass and angles sum rules: i.e. $\alpha\, m_1 + \beta\, m_2 = m_3$

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Cons:

- Largish θ_{13}: need of large perturbations/modifications $\Longrightarrow$ Naturalness
- different models give same/similar predictions
- why the specific discrete symmetry chosen
- involved high-energy scalar sector
- not so easy to describe at the same time quarks and leptons

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- the SM quarks are charged under $U(1)_{FN}$ as $FN(f) = n_f \geq 0$,

  while the flavon as a negative charge $FN(\theta) = -1$
- the corresponding non-renormalisable Lagrangian reads:

$$\mathcal{L}_Y = \left(\frac{\theta}{\Lambda}\right)^{n_{d_i^c}} \left(\frac{\theta}{\Lambda}\right)^{n_{q_j}} (Y_d)_{ij}\, d_i^c H^\dagger q_j + \\ + \left(\frac{\theta}{\Lambda}\right)^{n_{u_i^c}} \left(\frac{\theta}{\Lambda}\right)^{n_{q_j}} (Y_u)_{ij}\, u_i^c \widetilde{H}^\dagger q_j + \text{h.c.}$$

where $Y_{u,d} \approx \mathcal{O}(1)$ are free parameters

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when the symmetry is spontaneously broken by the VEV of θ, fermions receive different contributions in terms of ϵ. The Yukawa matrices are then given by:

$$y_u = F_{u^c} Y_u F_q \qquad\qquad y_d = F_{d^c} Y_d F_q$$

$$F_f = \begin{pmatrix} \epsilon^{n_{f1}} & 0 & 0 \\ 0 & \epsilon^{n_{f2}} & 0 \\ 0 & 0 & \epsilon^{n_{f3}} \end{pmatrix} \qquad (f = q, u^c, d^c)$$

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Assuming $n_{f1} > n_{f2} > n_{f3} \geq 0$, we move to the physical basis:

$$(V_{u,d})_{ii} \approx 1 \qquad\qquad (V_{u,d})_{ij} \approx \frac{n_{q_i}}{n_{q_j}} < 1 \qquad (i < j)$$

→

$$\left.\begin{array}{l} V_{ud} \approx V_{cs} \approx V_{tb} \approx \mathcal{O}(1) \\ V_{ub} \approx V_{td} \approx V_{us} \times V_{cb} \end{array}\right\}$$ independently of the particular charge choice

correct CKM with: $\begin{cases} n_q = (3,2,0) \\ \epsilon \approx 0.2 \end{cases}$ → $V = \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}$ (neglecting O(1) parameters)

$$\left[V \approx \begin{pmatrix} 1 & \lambda & \lambda^{3\div 4} \\ \lambda & 1 & \lambda^2 \\ \lambda^{3\div 4} & \lambda^2 & 1 \end{pmatrix} \right]$$

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correct quark masses with: $\begin{cases} n_{u^c} = (4,1,0) \\ n_{d^c} = (1,0,0) \end{cases}$

$\longrightarrow$ $M_u = \begin{pmatrix} \epsilon^7 & 0 & 0 \\ 0 & \epsilon^3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $M_d = \begin{pmatrix} \epsilon^4 & 0 & 0 \\ 0 & \epsilon^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (neglecting the mass and the O(1) parameters)

$$\left[M_u^{exp} = \begin{pmatrix} \epsilon^7 & 0 & 0 \\ 0 & \epsilon^3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad M_d^{exp} = \begin{pmatrix} \epsilon^4 & 0 & 0 \\ 0 & \epsilon^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

Lepton Sector

$$\mathcal{L}_Y = \left(\frac{\theta}{\Lambda}\right)^{n_{e^c_i}} \left(\frac{\theta}{\Lambda}\right)^{n_{\ell_j}} (Y_e)_{ij}\, e^c_i H^\dagger \ell_j$$

$$+ \left(\frac{\theta}{\Lambda}\right)^{n_{\ell_i}} \left(\frac{\theta}{\Lambda}\right)^{n_{\ell_j}} (Y_\nu)_{ij} \frac{\left(\overline{\ell^c}_i \widetilde{H}^*\right)\left(\widetilde{H}^\dagger \ell_j\right)}{\Lambda_L} + h.c.$$

(Similarly for See-Saw)

- charged leptons similar to quarks
- different choices are possible for neutrinos

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(Similarly for See-Saw)

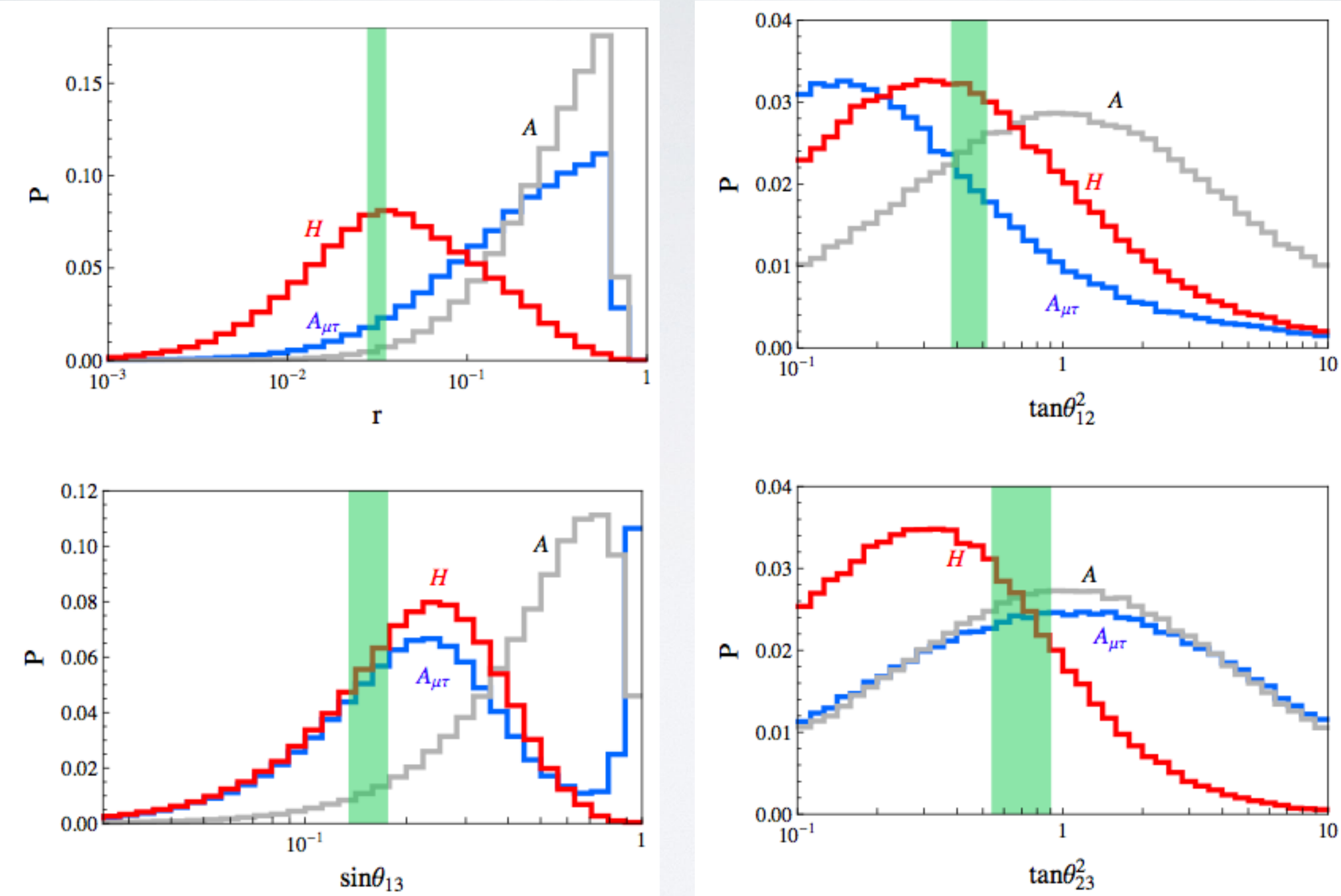
- charged leptons similar to quarks
- different choices are possible for neutrinos
- statistical approach to choose the best description

Frequentist approach

[Altarelli, Feruglio, Masina & LM, JHEP **1211** (2012) 139
Bergstrom, Meloni and LM, Phys.Rev. **D89** (2014) 093021]

Bayesian approach

[Altarelli, Feruglio, Masina & LM, JHEP **1211** (2012) 139



Anarchy vs. Hierarchy

Bergstrom, Meloni and LM, Phys.Rev. **D89** (2014) 093021]

Anarchy (A)

Hall, Murayama & Weiner PRL 84 (2000)
de Gouvea & Murayama, Phys. Rett. B573 (2003)

Based on the idea that structureless mass matrix can describe neutrino data.

Structureless = Random Entries

$$n_{e^c} = (3,1,0) \qquad n_\ell = (0,0,0)$$

$$Y_e = \begin{pmatrix} \epsilon^3 & \epsilon & 1 \\ \epsilon^3 & \epsilon & 1 \\ \epsilon^3 & \epsilon & 1 \end{pmatrix}$$

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Hierarchy (H)

All small parameters are naturally explained in terms of suitable suppression factors fixed by the charges.

$$n_{e^c} = (8,3,0) \qquad n_\ell = (2,1,0)$$

$$Y_e = \begin{pmatrix} \epsilon^{10} & \epsilon^6 & \epsilon^2 \\ \epsilon^9 & \epsilon^5 & \epsilon \\ \epsilon^8 & \epsilon^4 & 1 \end{pmatrix}$$

$$Y_\nu = \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}$$

Evidence of H with respect to A: depending on the prior on ϵ,

$$\log B \simeq 3 \div 4.5$$

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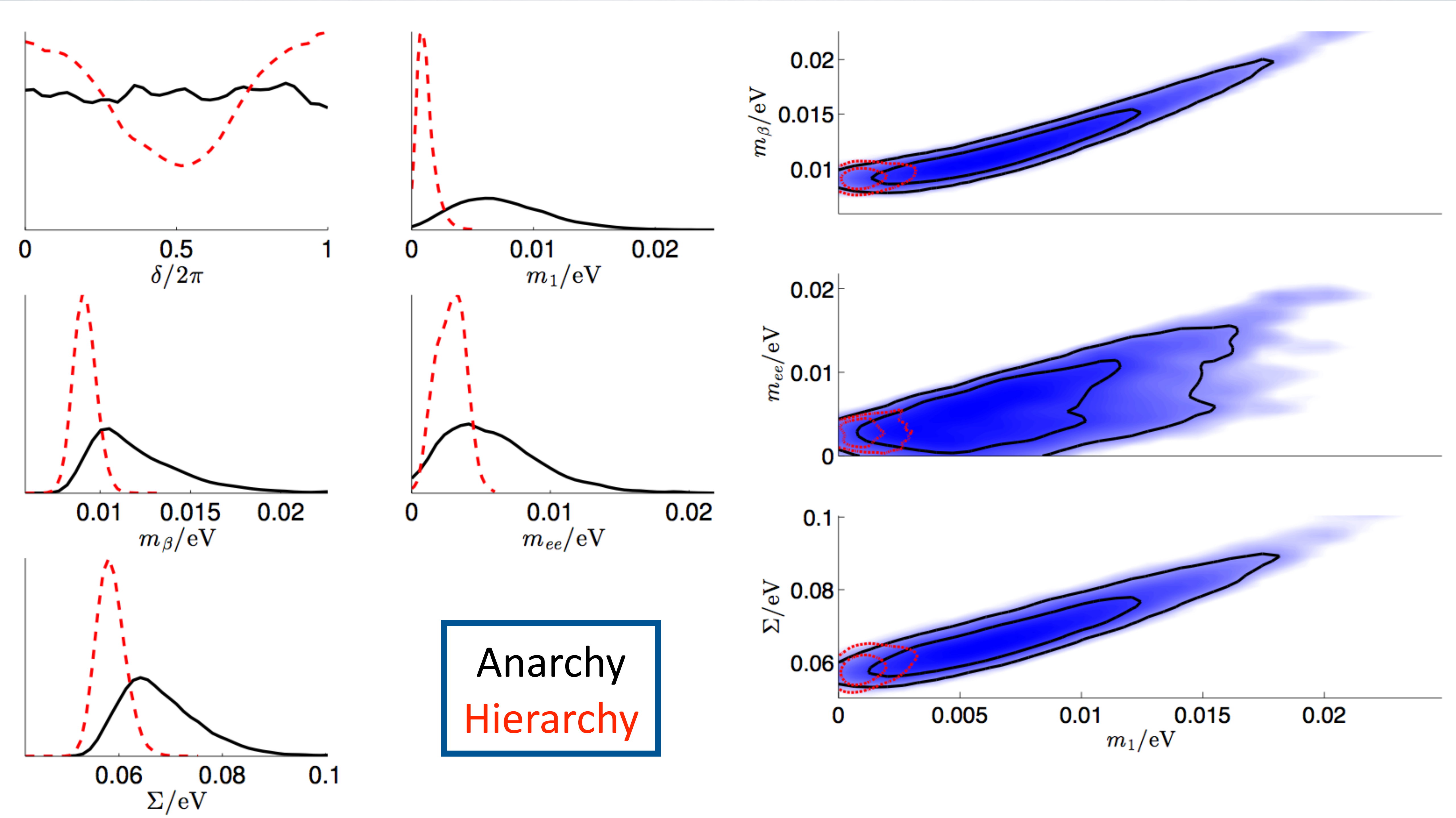

The area is normalised to 1

Pros and Cons of U(1) Sym

Pros:

- elegant approach in term of field content and symmetry
- all the observables can always be fitted
- U(1) symmetries are already in the SM and are predicted in many BSM theories
- both quarks and leptons can be described consistently

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Cons:

- enormous number of parameters: a parameter for each entry of the Y's
- all the predictions cannot be precise, due to O(1) parameters
- why a specific choice for the charges?
  (the same could be said for the SM: i.e. Higgs in doublet is not a prediction)

Continuous non-Abelian Syms

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The flavour symmetry of the SM gauge interactions is

$$\mathcal{L}_K \supset i\ell_L^\dagger \gamma^\mu D_\mu \ell_L + i e_R^\dagger \gamma^\mu D_\mu e_R + i Q_L^\dagger \gamma^\mu D_\mu Q_L + i u_R^\dagger \gamma^\mu D_\mu u_R + i d_R^\dagger \gamma^\mu D_\mu d_R$$

$$G_f = U(3)_{\ell_L} + U(3)_{e_R} + U(3)_{Q_L} + U(3)_{u_R} + U(3)_{d_R}$$

$$\begin{cases} \ell_L \to U_{\ell_L} \ell_L \\ e_R \to U_{e_R} e_R \end{cases} \qquad \begin{matrix} \ell_L \sim (3,1) \\ e_R \sim (1,3) \end{matrix}$$

$$\begin{cases} Q_L \to U_{Q_L} Q_L \\ u_R \to U_{u_R} u_R \\ d_R \to U_{d_R} d_R \end{cases} \qquad \begin{matrix} Q_L \sim (3,1,1) \\ u_R \sim (1,3,1) \\ d_R \sim (1,1,3) \end{matrix}$$

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This flavour symmetry is not respected by the Yukawa interactions:

$$\mathcal{L}_Y = \overline{\ell_L} H Y_e e_R + \overline{Q_L} H Y_d d_R + \overline{Q_L} \tilde{H} Y_u u_R + \text{h.c.}$$

$$\to \overline{\ell_L} H U_{\ell_L}^\dagger Y_e U_{e_R} e_R + \overline{Q_L} H U_{Q_L}^\dagger Y_d U_{d_R} d_R + \overline{Q_L} \tilde{H} U_{Q_L}^\dagger Y_u U_{u_R} u_R + \text{h.c.}$$

Minimal Flavour Violation

The formal invariance is recovered if the Yukawa matrices are promoted to auxiliary fields, called spurions, which transform as:

$$\begin{cases} Y_e \to U_{\ell_L} Y_e U^\dagger_{e_R} \end{cases} \qquad Y_e \sim (3, \bar{3})$$

$$\begin{cases} Y_u \to U_{Q_L} Y_u U^\dagger_{u_R} & Y_u \sim (3, \bar{3}, 1) \\ Y_d \to U_{Q_L} Y_d U^\dagger_{d_R} & Y_d \sim (3, 1, \bar{3}) \end{cases}$$

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Once the spurions acquire *ad hoc* background values, than quark masses and mixing, and charged lepton masses can be described in agreement with data:

$$Y_{e,d} = \begin{pmatrix} m_{e,d}/v & & \\ & m_{\mu,s}/v & \\ & & m_{\tau,b}/v \end{pmatrix} \quad Y_u = V^\dagger_{CKM} \begin{pmatrix} m_u/v & & \\ & m_c/v & \\ & & m_t/v \end{pmatrix}$$

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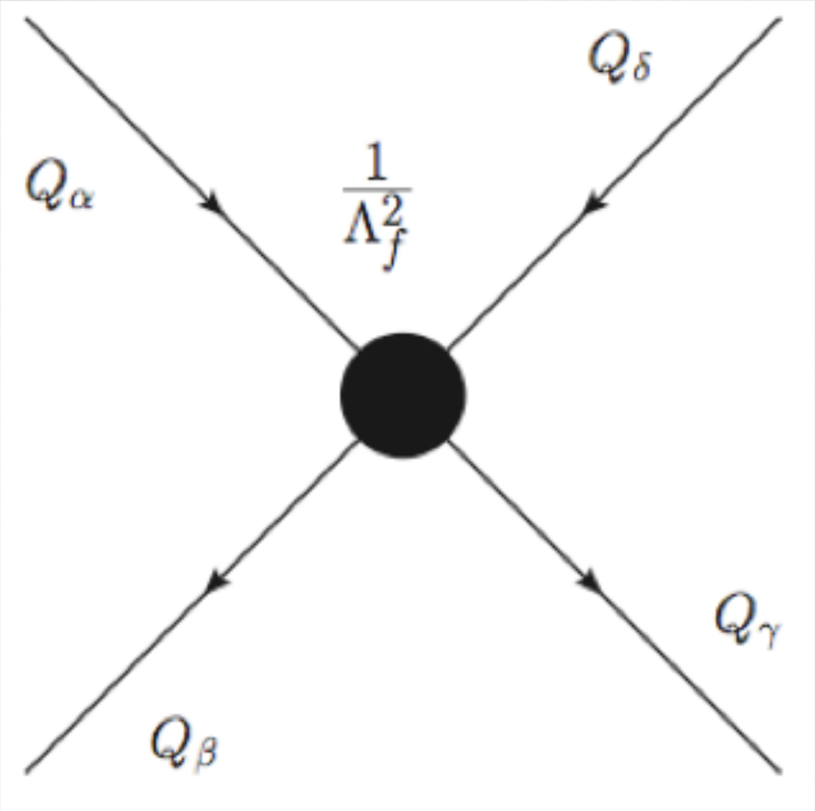
Assuming that any BSM flavour structure is governed by the spurions:

Minimal Flavour Violation

Chivukula & Georgi, Phys. Lett. B188 (1987) 99

D'Ambrosio *et al.*, Nucl. Phys. B645 (2002) 155

MFV helps solving the flavour problem in BSM theories:



$$\frac{c^{\alpha\beta\gamma\delta}}{\Lambda_f^2}\left(\overline{Q}_\alpha\gamma_\mu Q_\beta\right)\left(\overline{Q}_\gamma\gamma^\mu Q_\delta\right)$$

In general: $\Lambda_f > 10^2 \div 10^3\ \mathrm{TeV}$

G. Isidori, Y. Nir & G. Perez 2010

MFV: $c_{\alpha\beta\gamma\delta} = \left(Y_U Y_U^\dagger\right)_{\alpha\beta}\left(Y_U Y_U^\dagger\right)_{\gamma\delta} \Longrightarrow \Lambda_f \sim \mathcal{O}(1)\mathrm{TeV}$

Minimal Lepton Flavour Violation

Type I See-Saw mechanism to explain the smallness of neutrino masses:

i.e. with 3 RH neutrinos in the spectrum

$$G_f = SU(3)_{\ell_L} \times SU(3)_{e_R} \times SU(3)_{N_R} \times U(1)^3$$

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$$\mathcal{L}_Y = \overline{\ell_L} H Y_e e_R + \overline{\ell_L} \tilde{H} Y_\nu N_R + \overline{N_R^c} \frac{M_N}{2} N_R + \text{h.c.}$$

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- Most general case: no predictive, no protection from FCNC

$$Y_e \sim (3,\bar{3},1) \qquad Y_\nu \sim (3,1,\bar{3}) \qquad M_N \sim (1,1,\bar{6})$$

Minimal Lepton Flavour Violation

Type I See-Saw mechanism to explain the smallness of neutrino masses:

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$$m_\nu \propto Y_\nu \frac{1}{M_N} Y_\nu^T$$

No predictivity!!

$$BR(\mu \to e\gamma) \to \bar{\ell}_L H (Y_\nu Y_\nu^\dagger) Y_e \sigma_{\mu\nu} F^{\mu\nu} e_R$$

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Cirigliano, Grinstein, Isidori & Wise, NPB 728 (2005)

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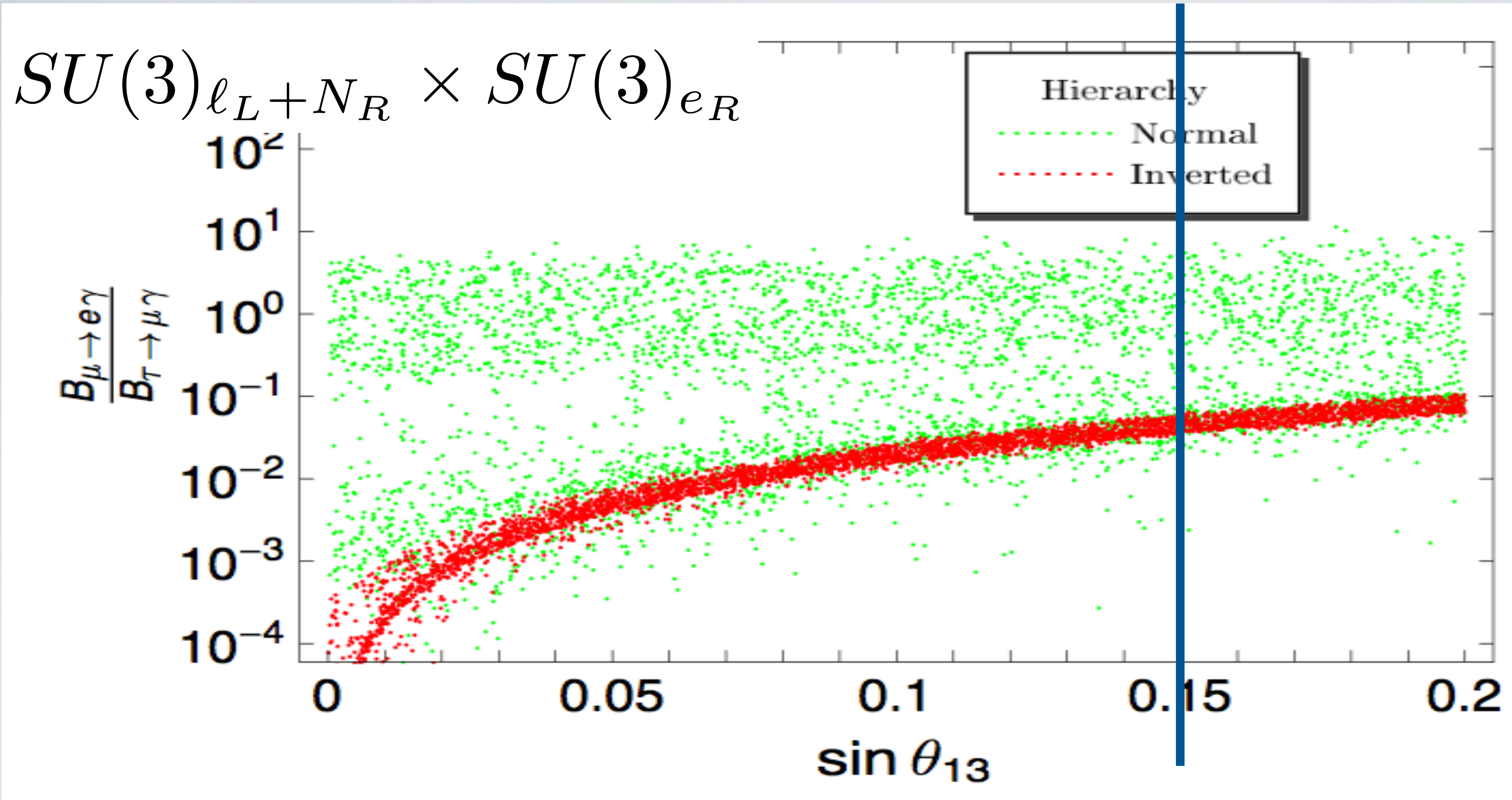
$$Y_e \sim (3,\bar{3},1) \qquad Y_\nu \sim (3,1,\bar{3})$$

- $SU(3)_{\ell_L + N_R} \times SU(3)_{e_R}$: then Y_ν is a unitary matrix and only two spurions
  Alonso, Isidori, LM, Munoz & Nardi, JHEP **1106** (2011)

$$Y_e \sim (3,\bar{3}) \qquad M_N \sim (\bar{6},1)$$



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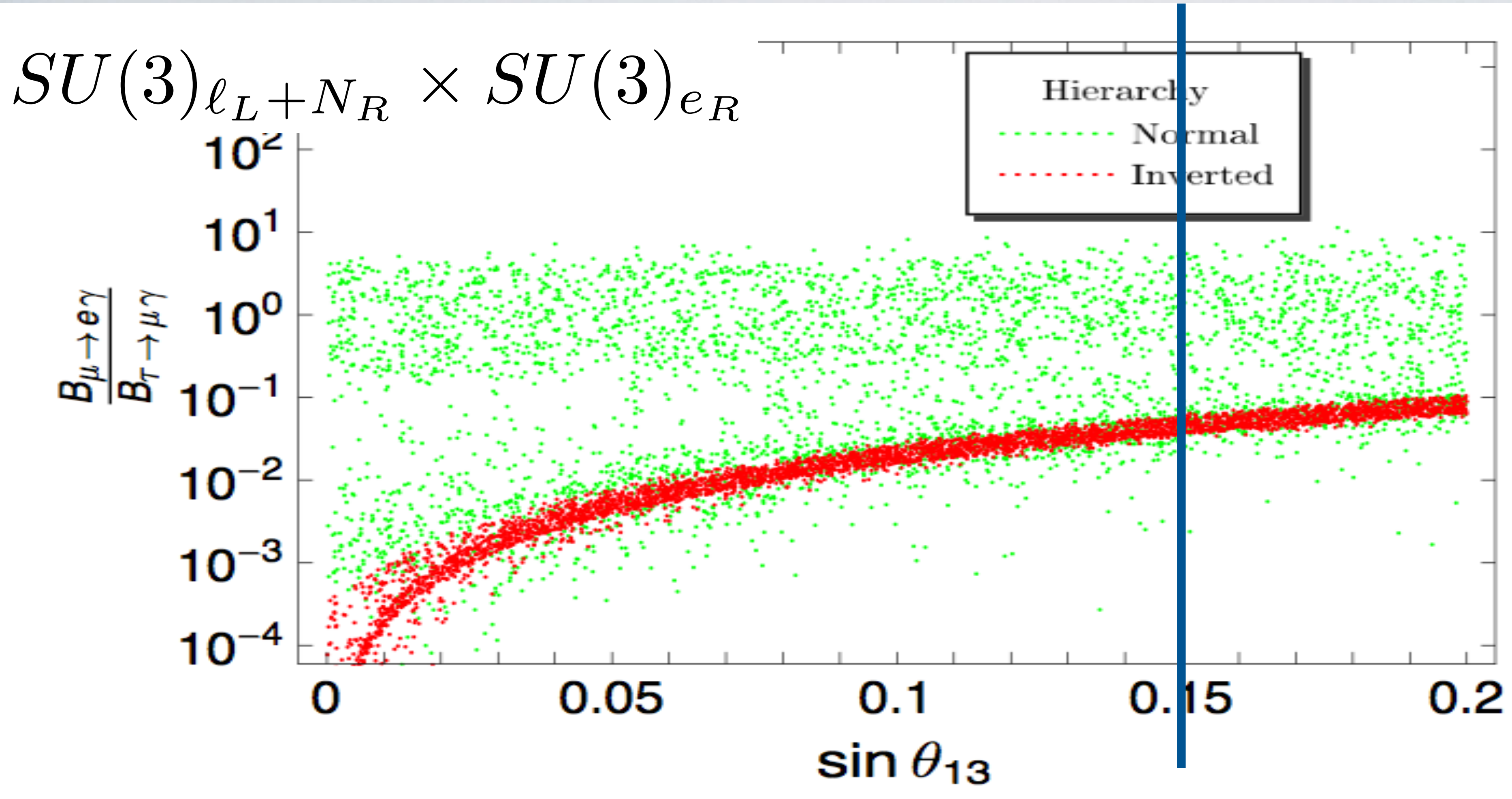

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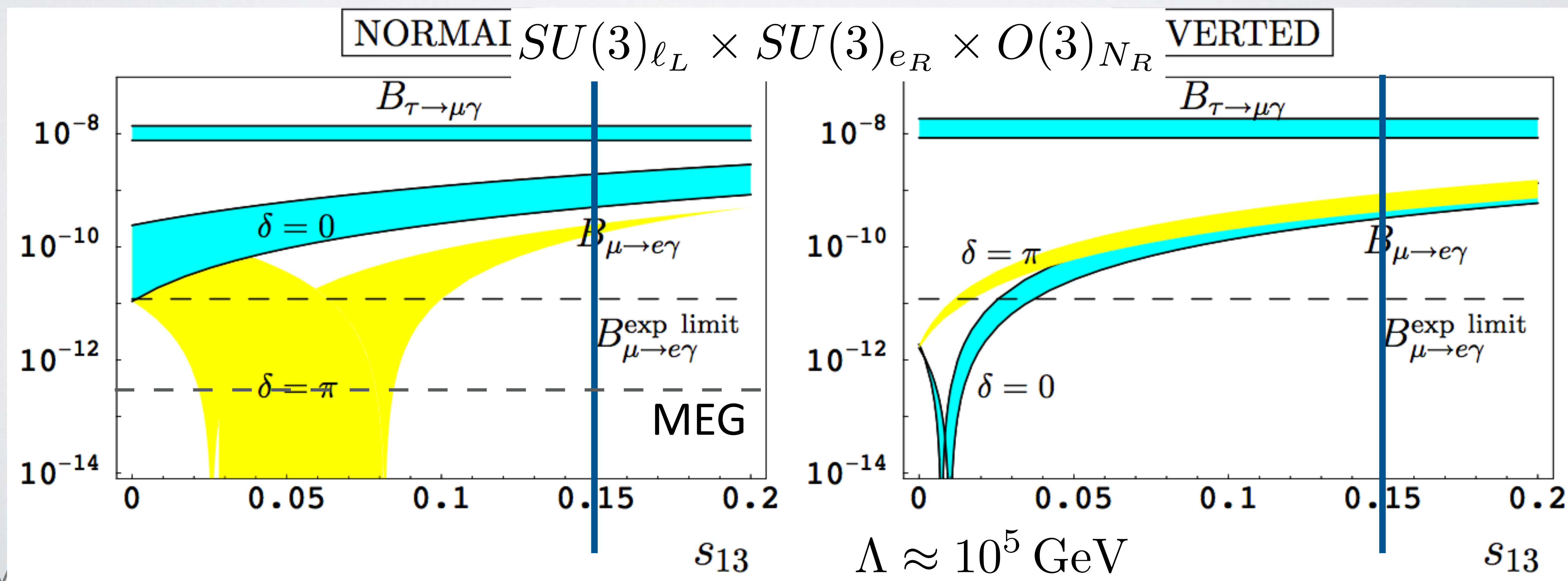

Minimal Flavour Violation

This flavour symmetry is not respected by the Yukawa interactions:

$$\mathcal{L}_Y = \overline{\ell_L} H Y_e e_R + \overline{Q_L} H Y_d d_R + \overline{Q_L} \tilde{H} Y_u u_R + \text{h.c.}$$
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The formal invariance is recovered if the Yukawa matrices are promoted to auxiliary fields, called spurions, which transform as:

$$\begin{cases} Y_e \to U_{\ell_L} Y_e U^\dagger_{e_R} \qquad Y_e \sim (3, \bar{3}) \end{cases} \qquad \begin{cases} Y_u \to U_{Q_L} Y_u U^\dagger_{u_R} \qquad Y_u \sim (3, \bar{3}, 1) \\ Y_d \to U_{Q_L} Y_d U^\dagger_{d_R} \qquad Y_d \sim (3, 1, \bar{3}) \end{cases}$$

Once the spurions acquire *ad hoc* background values, than quark masses and mixing, and charged lepton masses can be described in agreement with data.

Assuming that any BSM flavour structure is governed by the spurions:

Minimal Flavour Violation

Chivukula & Georgi, Phys. Lett. B188 (1987) 99
D'Ambrosio *et al.*, Nucl. Phys. B645 (2002) 155

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Is there a rational for this choice?

$$\begin{cases} Y_e \to U_{\ell_L} Y_e U^\dagger_{e_R} & Y_e \sim (3, \bar{3}) \end{cases}$$

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Once the spurions acquire *ad hoc* background values, than quark masses and mixing, and charged lepton masses can be described in agreement with data.

Assuming that ... ...purions:

The fermion masses and the mixing matrices are ONLY DESCRIBED but NOT EXPLAINED!!

...Lett. B188 (1987) 99
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Dynamical Fields

Alonso, Gavela, LM & Rigolin, JHEP **1107** (2011)

We can promote the spurions to be dynamical scalar fields, called flavons:

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In the following, we will consider only cases where $\mathcal{M}_N$ is not a flavon, but has specific structures (justified by lepton number approximate conservation in some cases).

Simplified Scenario: 2 Families

[Alonso, Gavela, Hernandez & LM, Phys.Lett. **B715** (2012) 194-198]

Consider a MLFV model with 2 RH neutrinos:

[Gavela, Hambye, D.Hernandez & P.Hernandez 2009]

$$\mathcal{L}_Y = \overline{\ell_L} H Y_E E_R + \overline{\ell_L} \tilde{H} \left(Y N + Y' N' \right) + \Lambda \overline{N'} N^c + \text{h.c.}$$

Degenerate RH neutrinos

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$$\mathcal{L}_Y = \overline{\ell_L} H Y_E E_R + \overline{\ell_L} \tilde{H} Y_\nu (N_1, N_2)^T + \Lambda (\overline{N}_1 N_1^c + \overline{N}_1 N_1^c) + \text{h.c.}$$

$$\mathcal{G}_f = SU(2)_{\ell_L} \times SU(2)_{E_R} \times O(2)_N$$

$$\mathcal{Y}_E \sim (2, \bar{2}, 1) \qquad \mathcal{Y}_\nu \sim (2, 1, 2)$$

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The light neutrino mass matrix is given by:

$$m_\nu = \frac{v^2}{\Lambda} \left(Y Y'^T + Y' Y^T \right) = \frac{v^2}{\Lambda} Y_\nu Y_\nu^T$$

The following invariants can be constructed:

$$\mathrm{Tr}\left(\mathcal{Y}_E\mathcal{Y}_E^\dagger\right) \qquad \mathrm{Tr}\left(\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\right) \qquad \det\left(\mathcal{Y}_E\right)$$

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The minimisation of the scalar potential leads to: (Casas-Ibarra par. for Y_ν)

$$(y^2-y'^2)\sqrt{m_{\nu_2}m_{\nu_1}}\sin 2\theta\cos 2\alpha = 0$$

$$\mathrm{tg}2\theta = \sin 2\alpha\frac{y^2-y'^2}{y^2+y'^2}\frac{2\sqrt{m_{\nu_2}m_{\nu_1}}}{m_{\nu_2}-m_{\nu_1}}$$

⟹

$$\alpha = \pm\pi/4$$

$m_{\nu_2}\approx m_{\nu_1}$ → θ_{12} large

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→

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Link between the spectrum, the mixing angles and the Majorana phase: this is due to the Majorana nature of neutrinos

3 Families: $O(2)_N$ Sym

Alonso, Gavela, Hernandez, LM & Rigolin, JHEP **1308** (2013)

Simple extension of the previous case:

$$\mathcal{G}_f = SU(3)_{\ell_L} \times SU(3)_{E_R} \times O(2)_N$$

$$\mathcal{L}_Y = \overline{\ell}_L H Y_E E_R + \overline{\ell}_L \tilde{H} Y'_\nu N'_R + \overline{\ell}_L \tilde{H} Y_\nu N_R + \frac{M'}{2} \overline{N}'^c_R N'_R + \frac{M}{2} \overline{N}^c_R \mathbb{1} N_R + \text{h.c.}$$

N_R doublet of $O(2)_N$ $\quad$ N'_R singlet of $O(2)_N$

$\mathcal{Y}_E \sim (3, \bar{3}, 1)$ $\quad$ $\mathcal{Y}_\nu \sim (3, 1, 2)$ $\quad$ $\mathcal{Y}'_\nu \sim (3, 1, 1)$

The light neutrino mass matrix is given by:

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The following invariants can be constructed:

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$$\mathrm{Tr}\left[\left(\mathcal{Y}_E\mathcal{Y}_E^\dagger\right)^2\right] \qquad \mathrm{Tr}\left[\left(\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\right)^2\right]$$

} masses

$$\mathrm{Tr}\left[\mathcal{Y}_E\mathcal{Y}_E^\dagger\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\right] \qquad \mathrm{Tr}\left[\mathcal{Y}_\nu\mathcal{Y}_\nu^T\mathcal{Y}_\nu^*\mathcal{Y}_\nu^\dagger\right]$$

$$\mathcal{Y}_\nu'^\dagger\mathcal{Y}_E\mathcal{Y}_E^\dagger\mathcal{Y}_\nu' \qquad \mathcal{Y}_\nu'^\dagger\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\mathcal{Y}_\nu'$$

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The minimisation of the scalar potential leads to the following exact solutions:

$$1)\begin{cases}\tan 2\theta_{12} = z/z' \\ m_{\nu_1} \neq m_{\nu_2} \\ m_{\nu_3} = 0\end{cases} \qquad 2)\begin{cases}\theta_{12} = \pi/4 \\ m_{\nu_1} = m_{\nu_2} \neq m_{\nu_3} \\ \alpha = \pi/4\end{cases}$$

$$3)\begin{cases}\theta_{23} = \pi/4 \\ m_{\nu_1} \neq m_{\nu_2} = m_{\nu_3} \\ \alpha = \pi/4\end{cases} \qquad 4)\begin{cases}\tan 2\theta_{23} = z/z' \\ m_{\nu_2} \neq m_{\nu_3} \\ m_{\nu_1} = 0\end{cases}$$

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Moreover, non-exact solutions are also present, interpolating these ones:

3 angles and both mass ordering possible; dependent on only 6 parameters.

3 Families: $O(3)_N$ Sym

Alonso, Gavela, Hernandez, LM & Rigolin, JHEP **1308** (2013)
Alonso, Gavela, Isidori, Maiani, JHEP **1311** (2013)

We can extend the RH neutrino symmetry, with three degenerate RH neutrinos:

$$\mathcal{G}_f = SU(3)_{\ell_L} \times SU(3)_{E_R} \times O(3)_N$$

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$$\text{Tr}\left[\mathcal{Y}_E \mathcal{Y}_E^\dagger \mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right] \qquad \text{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger\right] \rightarrow \text{mixing}$$

The minimisation of the scalar potential leads to this interesting solution:

$$\begin{cases} \theta_{23} = \pi/4 \\ m_{\nu_1} \neq m_{\nu_2} = m_{\nu_3} \\ \alpha = \pi/4 \end{cases}$$

In the almost degenerate spectrum, then even small perturbations can split the spectrum and a second sizable angle can arise in these scenario

Pros and Cons of non-Abelian Syms

Pros:

- elegant approach in term of field content and symmetry
- the M(L)FV symmetry is already encoded in the SM
- M(L)FV helps protecting from large FCNC in BSM
- both quarks and leptons can be described consistently
- small number of parameters: precise predictions
- link between spectrum and angles and Majorana phases

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Cons:

- very strongly constraint model building
- not yet successful model, but interesting and promising results

Final Remarks

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Thanks