Testing New Physics with the lepton $g-2$

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NP search strategies

- **High-energy frontier: A unique effort to determine the NP scale**

- **High-intensity frontier (flavor physics): A collective effort to determine the flavor structure of NP**

Where to look for New Physics at the low energy?

- **Processes very suppressed or even forbidden in the SM**
  - FCNC processes ($\mu \to e\gamma$, $\mu \to eee$, $\mu \to e$ in N, $\tau \to \mu\gamma$, $B^0_{s,d} \to \mu^+\mu^-$...)
  - CPV effects in the electron/neutron EDMs, $d_{e,n}$...
  - FCNC & CPV in $B_{s,d}$ & D decay/mixing amplitudes

- **Processes predicted with high precision in the SM**
  - EWPO as $(g-2)_{\mu,e}$: $a_\mu^{exp} - a_\mu^{SM} \approx (3 \pm 1) \times 10^{-9}$ [see Passera's talk]
  - LU in $R_M^{e/\mu} = \Gamma(M \to e\nu)/\Gamma(M \to \mu\nu)$ with $M = \pi, K$

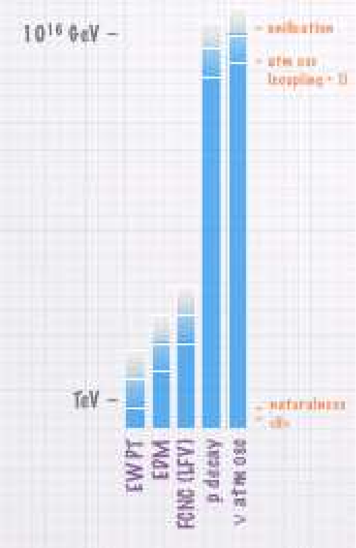
The NP "scale"

- **Gravity** $\Longrightarrow$ $\Lambda_{\text{Planck}} \sim 10^{18-19}$ GeV
- **Neutrino masses** $\Longrightarrow$ $\Lambda_{\text{see-saw}} \lesssim 10^{15}$ GeV
- **BAU**: evidence of CPV beyond SM
  - Electroweak Baryogenesis $\Longrightarrow$ $\Lambda_{\text{NP}} \lesssim$ TeV
  - Leptogenesis $\Longrightarrow$ $\Lambda_{\text{see-saw}} \lesssim 10^{15}$ GeV
- **Hierarchy problem**: $\Longrightarrow$ $\Lambda_{\text{NP}} \lesssim$ TeV
- **Dark Matter** $\Longrightarrow$ $\Lambda_{\text{NP}} \lesssim$ TeV

SM = effective theory at the EW scale

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_{d\geq 5} \frac{c_{ij}^{(d)}}{\Lambda_{NP}^{d-4}} O_{ij}^{(d)}$$

- $\mathcal{L}_{\text{eff}}^{d=5} = \frac{y_\nu^{ij}}{\Lambda_{\text{see-saw}}} L_i L_j \phi\phi$,
- $\mathcal{L}_{\text{eff}}^{d=6}$ generates FCNC operators



$a_\mu^{exp} - a_\mu^{SM} \sim 10^{-9} \rightarrow \Lambda_{NP} \lesssim 1\,\text{TeV}!!$

$\ell \to \ell'\gamma$: model-independent analysis

- **NP effects are encoded in the effective Lagrangian**

$$\mathcal{L} = e\frac{m_\ell}{2}\left(\bar{\ell}_R\sigma_{\mu\nu}A_{\ell\ell'}\ell'_L + \bar{\ell}'_L\sigma_{\mu\nu}A^\star_{\ell\ell'}\ell_R\right)F^{\mu\nu} \qquad \ell,\ell' = e,\mu,\tau\,,$$

$$A_{\ell\ell'} = \frac{1}{(4\pi\,\Lambda_{\rm NP})^2}\left[\left(g^L_{\ell k}\,g^{L*}_{\ell' k} + g^R_{\ell k}\,g^{R*}_{\ell' k}\right)f_1(x_k) + \frac{v}{m_\ell}\left(g^L_{\ell k}\,g^{R*}_{\ell' k}\right)f_2(x_k)\right]\,,$$

  - **Δa_ℓ and leptonic EDMs are given by**

$$\Delta a_\ell = 2m_\ell^2\,\mathrm{Re}(A_{\ell\ell}), \qquad \frac{d_\ell}{e} = m_\ell\,\mathrm{Im}(A_{\ell\ell})\,.$$

  - **The branching ratios of $\ell \to \ell'\gamma$ are given by**

$$\frac{\mathrm{BR}(\ell\to\ell'\gamma)}{\mathrm{BR}(\ell\to\ell'\nu_\ell\bar{\nu}_{\ell'})} = \frac{48\pi^3\alpha}{G_F^2}\left(|A_{\ell\ell'}|^2 + |A_{\ell'\ell}|^2\right)\,.$$

- **"Naive scaling":**

$$\Delta a_{\ell_i}/\Delta a_{\ell_j} = m^2_{\ell_i}/m^2_{\ell_j}, \qquad d_{\ell_i}/d_{\ell_j} = m_{\ell_i}/m_{\ell_j}\,.$$

(for instance, if the new particles have an underlying SU(3) flavor symmetry in their mass spectrum and in their couplings to leptons, which is the case for gauge interactions).

[Giudice, P.P., & Passera, '12]

Model-independent predictions

- $(g-2)_\ell$ **assuming "Naive scaling"** $\Delta a_{\ell_i}/\Delta a_{\ell_j} = m^2_{\ell_i}/m^2_{\ell_j}$

$$\Delta a_e = \left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right) 0.7\times 10^{-13}\,, \qquad \Delta a_\tau = \left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right) 0.8\times 10^{-6}.$$

- **EDMs assuming "Naive scaling"** $d_{\ell_i}/d_{\ell_j} = m_{\ell_i}/m_{\ell_j}$

$$d_e \simeq \left(\frac{\Delta a_e}{7\times 10^{-14}}\right) 10^{-24}\ \tan\phi_e\ \ e\,\mathrm{cm}\,,$$

$$d_\mu \simeq \left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right) 2\times 10^{-22}\ \tan\phi_\mu\ \ e\,\mathrm{cm}\,,$$

$$d_\tau \simeq \left(\frac{\Delta a_\tau}{8\times 10^{-7}}\right) 4\times 10^{-21}\ \tan\phi_\tau\ \ e\,\mathrm{cm}\,,$$

- $\mathrm{BR}(\ell_i \to \ell_j\gamma)$ **vs.** $(g-2)_\mu$

$$\mathrm{BR}(\mu\to e\gamma) \approx 3\times 10^{-13}\left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right)^2 \left(\frac{\theta_{e\mu}}{10^{-5}}\right)^2\,,$$

$$\mathrm{BR}(\tau\to \mu\gamma) \approx 4\times 10^{-8}\left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right)^2 \left(\frac{\theta_{\ell\tau}}{10^{-2}}\right)^2\,.$$

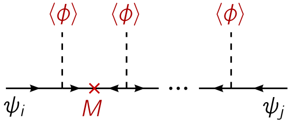
[Giudice, P.P., & Passera, '12]

SM vs. NP flavor problems

- Can the SM and NP flavour problems have a common explanation?

- **Froggat-Nielsen '79: Hierarchies from SSB of a Flavour Symmetry**

$$\epsilon = \frac{\langle \phi \rangle}{M} \ll 1 \Rightarrow Y_{ij} \propto \epsilon^{(a_i+b_j)}$$



- **Flavor protection from flavor models:** [Lalak, Pokorski & Ross '10]

Operator	$U(1)$	$U(1)^2$	$SU(3)$	MFV
$(\overline{Q}_L X^Q_{LL} Q_L)_{12}$	λ	λ^5	λ^3	λ^5
$(\overline{D}_R X^D_{RR} D_R)_{12}$	λ	λ^{11}	λ^3	$(y_d y_s) \times \lambda^5$
$(\overline{Q}_L X^D_{LR} D_R)_{12}$	λ^4	λ^9	λ^3	$y_s \times \lambda^5$

- Is the this flavor protection enough?

- Is it possible to disentangle among different flavour models by means of their predicted pattern of deviation w.r.t. the SM predictions in flavour physics?

The New Physics CP problem

- **Why CP violation? Motivation:**
  - Baryogenesis requires extra sources of CPV
  - The QCD $\overline{\theta}$-term $\mathcal{L}_{CP} = \overline{\theta}\frac{\alpha_s}{8\pi} G\tilde{G}$ is a CPV source beyond the CKM
  - Most UV completion of the SM, e.g. the MSSM, have many CPV sources
  - However, TeV scale NP with $\mathcal{O}(1)$ CPV phases generally leads to EDMs many orders of magnitude above the current limits $\Rightarrow$ the New Physics CP problem.

- **How to solve the New Physics CP problem?**
  - Decoupling some NP particles in the loop generating the EDMs (e.g. hierarchical sfermions, split SUSY, 2HDM limit...)
  - Generating CPV phases radiatively $\phi^f_{CP} \sim \alpha_w/4\pi \sim 10^{-3}$
  - Generating CPV phases via small flavour mixing angles $\phi^f_{CP} \sim \delta_{fj}\delta_{fj}$ with $f = e, u, d$: maybe the suppression of FCNC processes and EDMs have a common origin?

A concrete SUSY scenario: "Disoriented A-terms"

- **Challenge**: Large effects for $g-2$ keeping under control $\mu \to e\gamma$ and d_e
- **"Disoriented A-terms"** [Giudice, Isidori & P.P., '12]:

$$(\delta^{ij}_{LR})_f \sim \frac{A_f \theta^f_{ij} m_{f_j}}{m_{\tilde{f}}} \quad f = u, d, \ell \,,$$

  - Flavor and CP violation is restricted to the trilinear scalar terms.
  - Flavor bounds of the down-sector are naturally satisfied thanks to the smallness of down-type quark/lepton masses.
  - This ansatz arises in scenarios with partial compositeness where we a natural prediction is $\theta^{\ell}_{ij} \sim \sqrt{m_i/m_j}$ [Rattazzi et al.,'12].

- $\mu \to e\gamma$ and d_e are generated only by $U(1)$ interactions

$$A^{\mu e}_L \sim \frac{\alpha}{\cos^2\theta_W}\,\delta^{\mu e}_{LR}\,, \qquad \frac{d_e}{e} \sim \frac{\alpha}{\cos^2\theta_W}\,\mathrm{Im}\delta^{ee}_{LR}\,.$$

- $(g-2)_\mu$ is generated by $SU(2)$ interactions and is $\tan\beta$ enhanced therefore the relative enhancement w.r.t. $\mu \to e\gamma$ and d_e is $\tan\beta/\tan^2\theta_W \approx 100 \times (\tan\beta/30)$

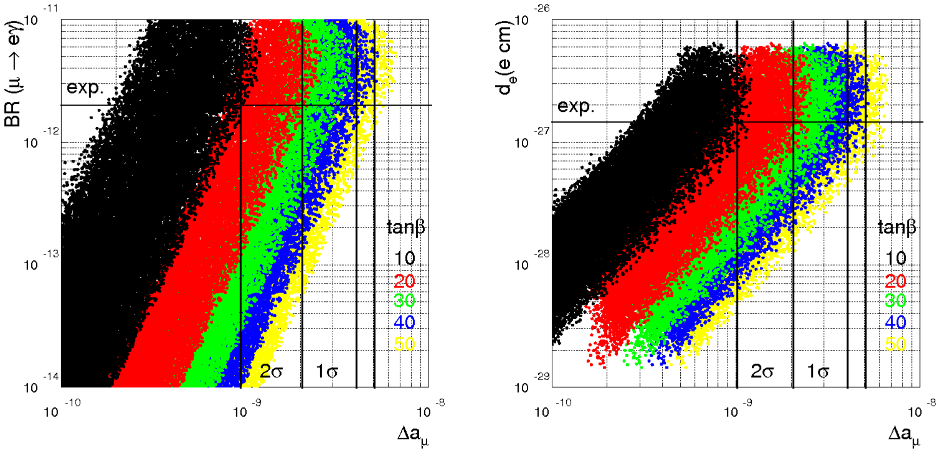
$$\Delta a_\ell \sim \frac{\alpha}{\sin^2\theta_W}\,\tan\beta$$

A concrete SUSY scenario: “Disoriented A-terms”

- **Numerical example**: $\tilde{m} = |A_e| = 1\ \mathrm{TeV}$, $\sin\phi_{A_e}$=1, $M_2 = \mu = 2M_1 = 0.2\ \mathrm{TeV}$, and $\tan\beta = 30$ [Giudice, P.P., & Passera, '12]

$$
\begin{aligned}
\mathrm{BR}(\mu \to e\gamma) &\approx 6 \times 10^{-13} \left| \frac{A_\ell}{\mathrm{TeV}} \frac{\theta^\ell_{12}}{\sqrt{m_e/m_\mu}} \right|^2 \left(\frac{\mathrm{TeV}}{m_{\tilde{\ell}}} \right)^4 , \\
d_e &\approx 4 \times 10^{-28}\, \mathrm{Im} \left(\frac{A_\ell\, \theta^\ell_{11}}{\mathrm{TeV}} \right) \left(\frac{\mathrm{TeV}}{m_{\tilde{\ell}}} \right)^2 e\, \mathrm{cm} , \\
\Delta a_\mu &\approx 1 \times 10^{-9} \left(\frac{\mathrm{TeV}}{m_{\tilde{\ell}}} \right)^2 \left(\frac{\tan\beta}{30} \right) .
\end{aligned}
$$

  - Disoriented A-terms can account for $(g-2)_\mu$, satisfy the bounds on $\mu \to e\gamma$ and d_e, while giving predictions for $\mu \to e\gamma$ and d_e within experimental reach.
  - The electron $(g-2)$ follows “naive scaling”.



Predictions for $\mu \to e\gamma$, Δa_μ and d_e in the disoriented A-term scenario with $\theta^\ell_{ij} = \sqrt{m_i/m_j}$. Left: $\mu \to e\gamma$ vs. Δa_μ. Right: d_e vs. Δa_μ [Giudice, P.P., & Passera, '12]

Testing new physics with the electron $g-2$

- **Longstanding muon $g-2$ anomaly**

$$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} = 2.90(90) \times 10^{-9}\,, \qquad \textbf{3.5}\boldsymbol{\sigma} \;\; \textbf{discrepancy}$$

- **NP effects are expected to be of order $a_\ell^{\rm NP} \sim a_\ell^{\rm EW}$**

$$a_\mu^{\rm EW} = \frac{m_\mu^2}{(4\pi v)^2}\left(1 - \frac{4}{3}\sin^2\theta_{\rm W} + \frac{8}{3}\sin^4\theta_{\rm W}\right) \approx 2 \times 10^{-9}.$$

- **Main question: how could we check if the a_μ discrepancy is due to NP?**
- **Answer: testing new-physics effects in a_e** [Giudice, P.P, & Passera, '12]
- **"Naive scaling":** $\Delta a_{\ell_i}/\Delta a_{\ell_j} = m_{\ell_i}^2/m_{\ell_j}^2$

$$\Delta a_e = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.7 \times 10^{-13}\,.$$

  - a_e has never played a role in testing beyond SM effects. From $a_e^{\rm SM}(\alpha) = a_e^{\rm EXP}$, we extract α which is is the most precise value of α available today!
  - The situation has now changed thanks to progresses both on the th. and exp. sides.

The Standard Model prediction of the electron $g-2$

- **Standard Model vs. measurement**

$$\Delta a_e = a_e^{\mathrm{EXP}} - a_e^{\mathrm{SM}} = -10.6\,(8.1) \times 10^{-13},$$

  - Beautiful test of QED at four-loop level!
  - $\delta\Delta a_e = 8.1 \times 10^{-13}$ is dominated by δa_e^{SM} through $\delta\alpha(^{87}\mathrm{Rb})$.

- **Future improvements in the determination of Δa_e**

$$\underbrace{(0.6)_{\mathrm{QED4}},\ \ (0.4)_{\mathrm{QED5}},\ \ (0.2)_{\mathrm{HAD}},}_{(0.7)_{\mathrm{TH}}}\ \ (7.6)_{\delta\alpha},\ \ (2.8)_{\delta a_e^{\mathrm{EXP}}}. \quad (1)$$

  - The first error, 0.6×10^{-13}, stems from numerical uncertainties in the four-loop QED. It can be reduced to 0.1×10^{-13} with a large scale numerical recalculation. [Kinoshita]
  - The second error, from five-loop QED term may soon drop to 0.1×10^{-13}.
  - Experimental uncertainties 2.8×10^{-13} ($\delta a_e^{\mathrm{EXP}}$) and 7.6×10^{-13} ($\delta\alpha$) dominate. We expect a reduction of the former error to a part in 10^{-13} (or better). [Gabrielse] Work is also in progress for a significant reduction of the latter error. [Nez]

- **Δa_e at the 10^{-13} (or below) is not too far! This will bring a_e to play a pivotal role in probing new physics in the leptonic sector.** [see Passera's talk]

Supersymmetry and a_e

- SUSY contributions to a_ℓ comes from loops with exchange of chargino/sneutrino or neutralino/charged slepton.

- **Violations of "naive scaling"** can arise through sources of non-universalities in the slepton mass matrices in two possible ways

  - **Lepton flavor conserving (LFC) case**. The charged slepton mass matrix violates the global non-abelian flavor symmetry, but preserves $\mathrm{U}(1)^3$. This case is characterized by non-degenerate sleptons ($m_{\tilde{e}} \neq m_{\tilde{\mu}} \neq m_{\tilde{\tau}}$) but vanishing mixing angles because of an exact alignment, which ensures that Yukawa couplings and the slepton mass matrix can be simultaneously diagonalized in the same basis.

  - **Lepton flavor violating (LFV) case**. The slepton mass matrix fully breaks flavor symmetry up to U(1) lepton number, generating mixing angles that allow for flavor transitions. Lepton flavor violating processes, such as $\mu \to e\gamma$, provide stringent constraints on this case. However, because of flavor transitions, a_e and a_μ can receive new large contributions proportional to m_τ (from a chiral flip in the internal line of the loop diagram), giving a new source of non-naive scaling.

Lepton flavor conserving case

- In the LFC case, we assume $m_{\tilde{e}} \neq m_{\tilde{\mu}} \neq m_{\tilde{\tau}}$ but flavor alignment between lepton and slepton mass matrices to avoid LFV. This is reminiscent of the alignment mechanism [Nir & Seiberg, '93], proposed to solve the supersymmetric flavor problem in the quark sector (which might arise naturally in the context of abelian flavor models).

$$\Delta a_\ell^{\rm LFC} \approx 3 \times 10^{-9} \left(\frac{m_\ell}{m_\mu}\right)^2 \left(\frac{100~{\rm GeV}}{m_{\tilde{\ell}}}\right)^2 \left(\frac{\tan\beta}{3}\right).$$

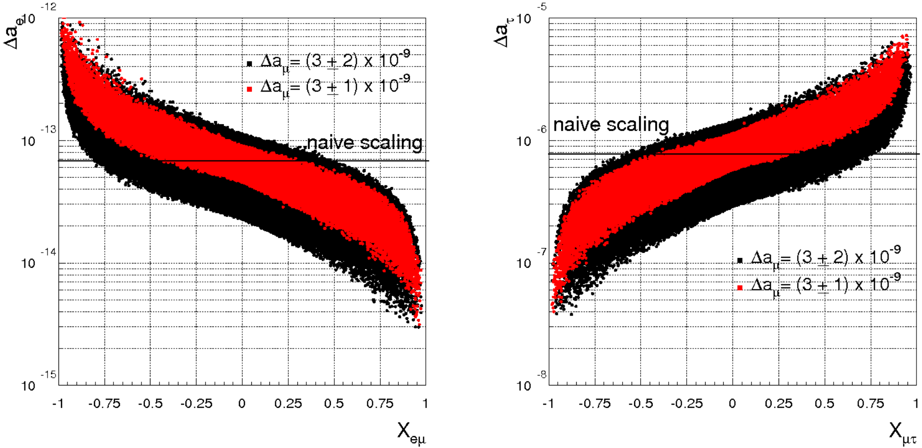
- Assuming that sleptons are the heaviest particles running in the loop

$$\Delta a_e \approx \Delta a_\mu \frac{m_e^2}{m_\mu^2} \frac{m_{\tilde{\mu}}^2}{m_{\tilde{e}}^2} \approx \frac{m_{\tilde{\mu}}^2}{m_{\tilde{e}}^2} \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 10^{-13},$$

$$\Delta a_\tau \approx \Delta a_\mu \frac{m_\tau^2}{m_\mu^2} \frac{m_{\tilde{\mu}}^2}{m_{\tilde{\tau}}^2} \approx \frac{m_{\tilde{\mu}}^2}{m_{\tilde{\tau}}^2} \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 10^{-6}.$$

- For values of Δa_μ explaining the muon $g-2$, non-degenerate sleptons at the level $m_{\tilde{\mu}} \approx 3\, m_{\tilde{e}}$ lead to $\Delta a_e \approx 10^{-12}$, which is at the limit of present experimental sensitivity.

Lepton flavor conserving case



Left: Δa_e as a function of $X_{e\mu} = (m^2_{\tilde{e}} - m^2_{\tilde{\mu}})/(m^2_{\tilde{e}} + m^2_{\tilde{\mu}})$. Right: Δa_τ as a function of $X_{\mu\tau} = (m^2_{\tilde{\mu}} - m^2_{\tilde{\tau}})/(m^2_{\tilde{\mu}} + m^2_{\tilde{\tau}})$. Black points satisfy the condition $1 \leq \Delta a_\mu \times 10^9 \leq 5$, while red points correspond to $2 \leq \Delta a_\mu \times 10^9 \leq 4$.

Correlation between a_e and violation of lepton universality in LFC

- In SUSY, "naive scaling" violations for $(g-2)_\ell$ can arise through sources of non-universalities in the slepton masses.
- In turn, these non-universalities will induce violations of lepton flavor universality in $P \to \ell\nu$, $\tau \to P\nu$ (where $P = \pi, K$), $\ell_i \to \ell_j \bar{\nu}\nu$, $Z \to \ell\ell$ and $W \to \ell\nu$ through loop effects.
- LFU has been tested at the 0.1% level so far.
- It is interesting to study the **correlation** between such **LFU** and departures from **"naive scaling"** for Δa_ℓ.
- Taking for example the process $P \to \ell\nu$, we can define the quantity

$$\frac{(R_P^{e/\mu})_{\rm EXP}}{(R_P^{e/\mu})_{\rm SM}} = 1 + \Delta r_P^{e/\mu} \, .$$

  - $R_P^{e/\mu} = \Gamma(P \to e\nu)/\Gamma(P \to \mu\nu)$
  - $\Delta r_P^{e/\mu} \neq 0$ signals the presence of new physics violating LFU.

Correlation between a_e and violation of lepton universality in LFC

- In SUSY, in the absence of LFV sources, $\Delta r_P^{e/\mu}$ is induced at the loop level through sparticle exchange. The parametrical structure of $\Delta r_{\mathrm{P}}^{e/\mu}$ is

$$\Delta r_{\mathrm{P}}^{e/\mu} \sim \frac{\alpha}{4\pi}\left(\frac{m_{\tilde{e}}^2 - m_{\tilde{\mu}}^2}{m_{\tilde{e}}^2 + m_{\tilde{\mu}}^2}\right)\frac{v^2}{\min(m_{\tilde{e},\tilde{\mu}}^2)},$$

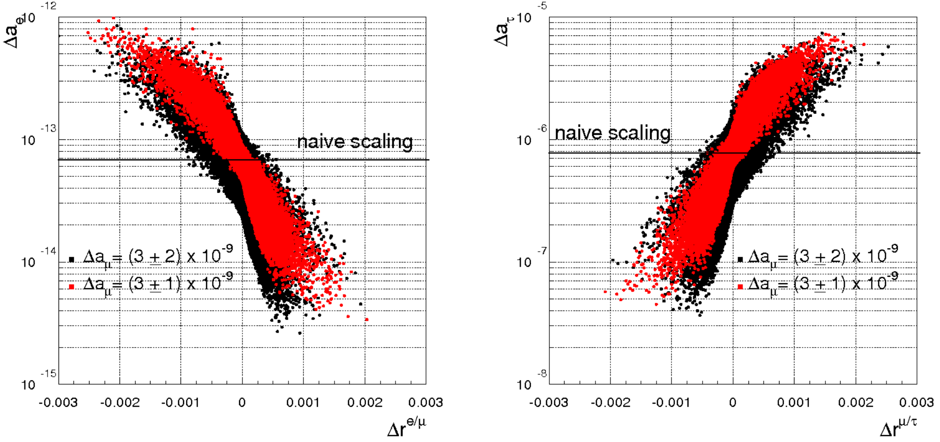
- The term $v^2/\min(m_{\tilde{e},\tilde{\mu}}^2)$ stems from SU(2) breaking effects which arise from 1) left-right soft breaking terms, 2) mixing terms in the chargino/neutralino mass matrices, or 3) D-terms.

- **"naive scaling"** violations for Δa_ℓ

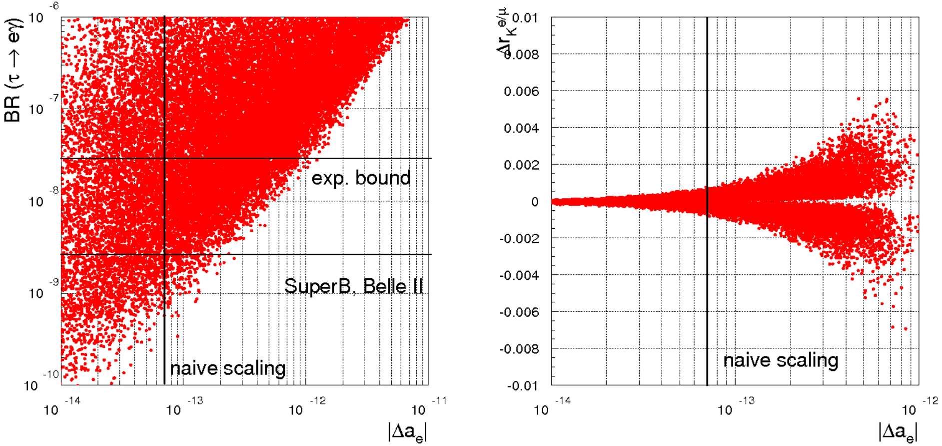
$$\Delta a_e \approx \Delta a_\mu \frac{m_e^2}{m_\mu^2}\frac{m_{\tilde{\mu}}^2}{m_{\tilde{e}}^2} \approx \frac{m_{\tilde{\mu}}^2}{m_{\tilde{e}}^2}\left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right)10^{-13},$$

- For values of $\Delta a_\mu \sim \textit{few} \times 10^{-9}$ (explaining the muon $g{-}2$ anomaly), non-degenerate sleptons at the level $m_{\tilde{\mu}} \approx 3\, m_{\tilde{e}}$ lead to $\Delta a_e \approx 10^{-12}$, ($\Delta a_e \approx 10^{-13}$ in "naive scaling") and $\Delta r_{\mathrm{P}}^{e/\mu} \approx 10^{-3}$.

Lepton flavor conserving case



Left: $\Delta r_{\rm P}^{e/\mu}$ vs. Δa_e, where $\Delta r_{\rm P}^{e/\mu}$ measures violations of lepton universality in $\Gamma(P \to e\nu)/\Gamma(P \to \mu\nu)$ with $P = K, \pi$. Right: $\Delta r_{\rm P}^{\mu/\tau}$ vs. Δa_τ where $\Delta r_{\rm P}^{\mu/\tau}$ measures violations of lepton universality in $\Gamma(P \to \mu\nu)/\Gamma(\tau \to P\nu)$.



Left: $\mathrm{BR}(\tau \to e\gamma)$ vs. $|\Delta a_e|$. Right: $\Delta r_K^{e/\mu}$ vs. $|\Delta a_e|$. The vertical line corresponds to the prediction for Δa_e assuming NS, setting Δa_μ equal to its central value $\Delta a_\mu = 3 \times 10^{-9}$.

Light (pseudo)scalars and a_e

- Yukawa interactions between a light scalar (pseudoscalar) ϕ (A) with leptons ℓ

$$\mathcal{L} = \left(\frac{g m_\ell}{2M_W}\right) C_\phi^\ell \, \bar{\ell}\ell\phi + i \left(\frac{g m_\ell}{2M_W}\right) C_A^\ell \, \bar{\ell}\gamma_5 \ell A$$

  - A could be a pseudo-Goldstone boson of an extended Higgs sector and ϕ a light gauge singlet coupled through a dimension-five interaction to the Yukawa terms.
  - Very light ϕ and A are constrained by low-energy data (meson decays) as well as reactor experiments (most of these bounds disappear for $M_A > 10$ GeV).

- One-loop effects to $(g-2)_\ell$

$$(\Delta a_\ell^{\phi A})_{1\mathrm{loop}} = \frac{g^2 m_\ell^4}{32\pi^2 M_W^2} \left(|C_\phi^\ell|^2 \frac{I_\phi^\ell}{M_\phi^2} - |C_A^\ell|^2 \frac{I_A^\ell}{M_A^2} \right) ,$$

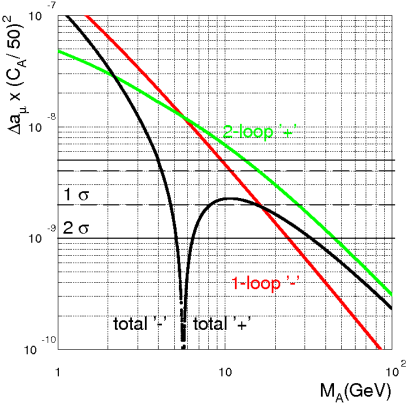
  - One-loop pseudoscalar (scalar) effect is unambiguously negative (positive).
  - "Naive scaling" $\Delta a_\ell \propto m_\ell^2$ for $m_\ell \gg M_{\phi,A}$, $\Delta a_\ell \propto m_\ell^4$ for $m_\ell \ll M_{\phi,A}$.

- Two-loop effects to $(g-2)_\ell$

$$(\Delta a_\ell^{\phi A})_{2\mathrm{loop}} = -\frac{\alpha^2}{8\pi^2 \sin^2\theta_W} \frac{m_\ell^2}{M_W^2} m_\tau^2 \left(\mathrm{Re}\left(C_\phi^\ell C_\phi^{\tau *}\right) \frac{L_\phi^\tau}{M_\phi^2} - \mathrm{Re}\left(C_A^\ell C_A^{\tau *}\right) \frac{L_A^\tau}{M_A^2} \right) ,$$

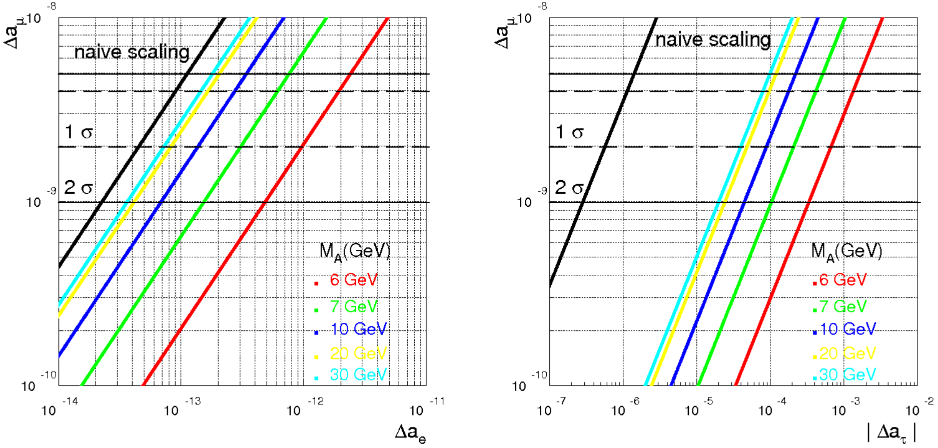
  - For $m_\ell \ll M_{\phi,A}$ we have the enhancement $m_\tau^2/m_{e,\mu}^2$ w.r.t. one-loop effects.
  - Two loop effects can be positive or negative depending on the sign of $\mathrm{Re}(C^\ell C^{\tau *})$.

Light (pseudo)scalars and a_e



- For $m_\ell \ll M_A$, where the Δa_μ anomaly can be explained, we have
  - Δa_e is always dominated by two-loop effects
  - Δa_μ receives comparable one- and two-loop contributions
  - Δa_τ is always dominated by one-loop effects.
  - As a result, we expect significant "naive scaling" violations

Light (pseudo)scalars and a_e



- In the regions where the Δa_μ anomaly is accommodated, Δa_e typically exceeds the 10^{-13} level, providing a splendid opportunity to test the $(g-2)_\mu$ anomaly.
- Δa_τ can reach values up to the level of 10^{-3}, well within the experimental resolutions expected at Belle II.

- An explanation of Δa_μ is given by a new hidden $U(1)_d$ symmetry which kinetically mixes with $U(1)_Y$ by [Marciano et al., '08]

$$\mathcal{L} = \frac{1}{2}\frac{\epsilon}{\cos\theta_W} B_{\mu\nu} Z_d^{\mu\nu} \,,$$

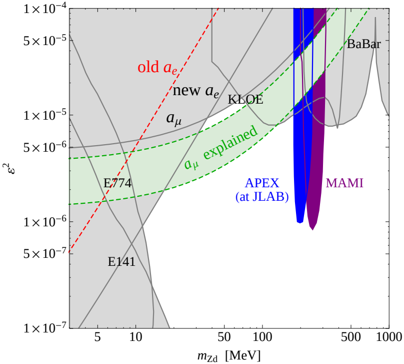
where ϵ parametrizes the mixing, $X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$, $X = B, Z_d$, is a $U(1)$ field strength tensor.

- Upon kinetic diagonalization, one obtains an induced coupling $e\,\epsilon Z_d^\mu J_\mu^{em}$, where J_μ^{em} is the the electromagnetic current.

- The 1-loop contribution of the Z_d to a_μ is given by

$$a_\mu^{Z_d} = \frac{\alpha}{2\pi}\epsilon^2 F_V\left(m_{Z_d}/m_\mu\right)$$

$$F_V(x) \equiv \int_0^1 dz \frac{2z(1-z)^2}{(1-z)^2 + x^2 z}$$



Exclusion region in $m_{Z_d} - \epsilon^2$ space.

Conclusions and future prospects

- **Important questions in view of ongoing/future experiments are**:
  - What are the expected deviations from the SM predictions induced by TeV NP?
  - Which observables are not limited by theoretical uncertainties?
  - In which case we can expect a substantial improvement on the experimental side?
  - What will the measurements teach us if deviations from the SM are [not] seen?

- **(Personal) answers**:
  - The expected deviations from the SM predictions induced by NP at the TeV scale with generic flavor structure are already ruled out by many orders of magnitudes.
  - On general grounds, we can expect any size of deviation below the current bounds.
  - cLFV processes, leptonic EDMs and LFU observables do not suffer from theoretical limitations (clean th. observables).
  - On the experimental side there are still excellent prospects of improvements in several clean channels especially in the leptonic sector: $\mu \to e\gamma$, $\mu N \to eN$, $\mu \to eee$, τ-LFV, EDMs and leptonic $(g-2)$.
  - The the origin of the $(g-2)_\mu$ discrepancy can be understood testing new-physics effects in the electron $(g-2)_e$. This would require improved measurements of $(g-2)_e$ and more refined determinations of α in atomic-physics experiments.

Conclusions

The origin of flavour is still, to a large extent, a mystery. The most important open questions can be summarized as follow:

- **Which is the organizing principle behind the observed pattern of fermion masses and mixing angles?**
- **Are there extra sources of flavour symmetry breaking beside the SM Yukawa couplings which are relevant at the TeV scale?**

Irrespectively of whether the LHC will discover or not new particles, leptonic dipoles (leptonic $g-2$, $\mu \to e\gamma$ and the electron EDM) will teach us a lot...