Extraction of Neutrino Flux from Inclusive Muon Cross Section

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Objective of Neutrino Experiments

One of the central interests of neutrino physics is to find out the mixing parameters of neutrino.

two flavor oscillation probability

$$\mathbf{P}(\nu_\mu \to \nu_\tau) = \mathbf{\sin^2 2}\theta_{\mathbf{23}}\mathbf{\sin^2}\left(\frac{\mathbf{1.27\Delta m^2_{23}[eV^2]L[km]}}{\mathbf{E}_\nu\mathbf{[GeV]}}\right)$$

Given L, Δm, θ can bee extracted from measured $\Phi(E_\nu)$.

The extraction of neutrino flux $\Phi(E_\nu)$ is a key to obtain the neutrino mixing parameters.

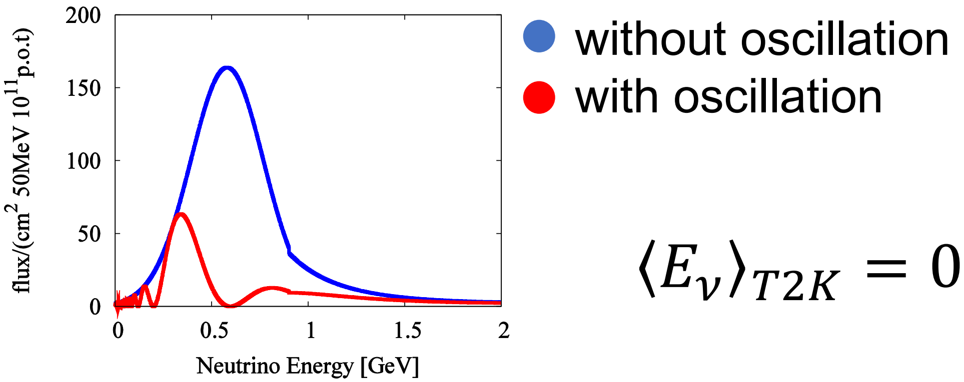
Long Base Line Experiments (example)

Neutrino-Nucleus (^{12}C, ^{16}O) reaction

$$\rightarrow \quad \Phi(E_\nu) \quad \rightarrow \quad \theta_{ij}\,,\delta_{CP}\,,\Delta m_{ij}^2 \quad \ldots$$

Neutrino flux — Mixing Parameters

T2K



$$\langle E_\nu \rangle_{T2K} = 0.6GeV$$

MiniBooNE

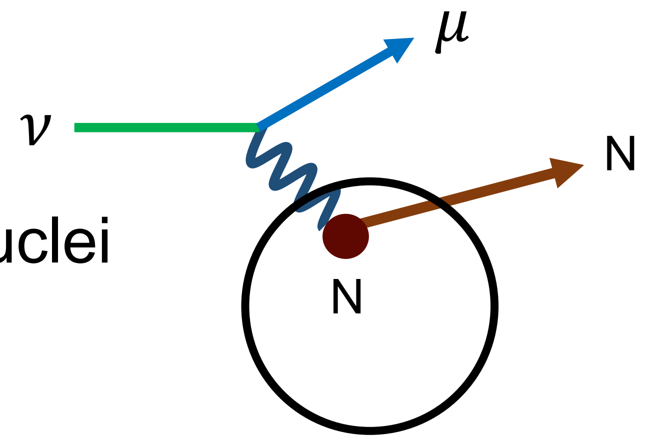
$$\langle E_\nu \rangle_{MiniBooNE} = 0.788GeV$$

Dominant mechanisms of neutrino-nucleus reactions

- Quasi-Elastic (QE) nucleon knock out
- Single pion production through Δ

Reconstruction of Neutrino Energy
(conventional method)

$$\mathbf{E}_\nu^{\mathbf{QF}}(\mathbf{E}_\mu, \theta_\mu) = \frac{\mathbf{E}_\mu - \mathbf{m}_\mu^2/(2\mathbf{M})}{1 - (\mathbf{E}_\mu - \mathbf{P}_\mu \cos\theta)/\mathbf{M}}$$



Assumption

- Initial nucleon is at rest inside nuclei
- No final state interaction

Reality

- Bound Nucleon
  (Fermi motion, correlation)
- Final state interaction
  (rescattering, π-absorption, MEC)

} QE-like

The contamination of QE-like events brings large systematic error in the extracted neutrino flux

Proposal for alternative method to reconstruct $\Phi(E_\nu)$

Extract $\Phi(E_\nu)$
from Inclusive muon production cross section

$$\left.\frac{d^2\sigma}{dE_\mu d\Omega_\mu}\right|_{exp} = \left\langle \frac{d^2\sigma}{d\Omega_\mu dE_\mu} \right\rangle = \int dE_\nu \frac{d^2\sigma}{d\Omega_\mu dE_\mu} \Phi(E_\nu)$$

observable

Flux averaged
cross section

Neutrino Flux

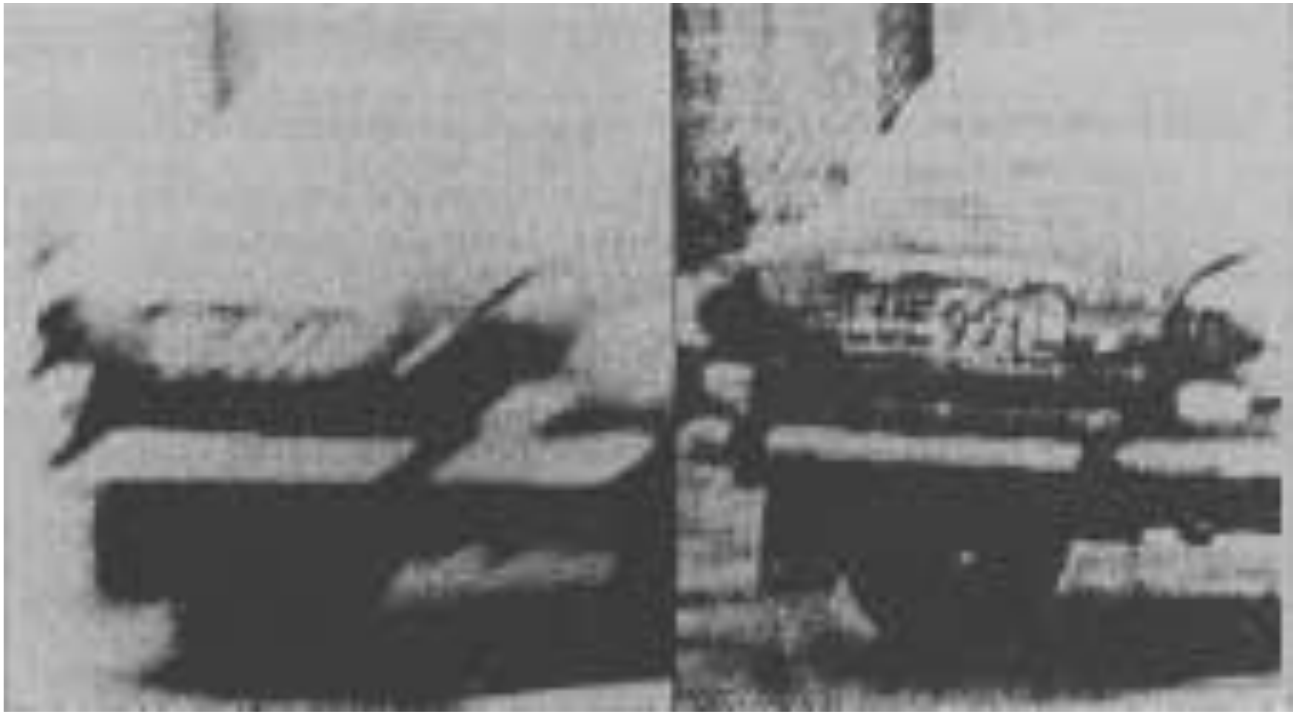
For solving this inversion problem, we use Maximum Entropy Method (MEM)

Contents

$$\left.\frac{d^2\sigma}{dE_\mu d\Omega_\mu}\right|_{exp} = \left\langle \frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega_\mu \mathrm{d}E_\mu} \right\rangle = \int \mathrm{d}E_\nu \frac{d^2\sigma}{dE_\mu d\Omega_\mu} \Phi(E_\nu)$$

1. Maximum Entropy Method (MEM)
2. Theoretical model of neutrino-nucleus cross section
3. Pseudo data
   3.1 Neutrino Flux
   3.2 Pseudo data of inclusive muon production
4. Result

1. Maximum Entropy Method



MEM

1. Maximum Entropy Method

Neutrino flux of MEM $\Phi_{MEM}(E_\nu)$ gives max of $P[\Phi(\mathrm{E}_\nu)|\bar{G}, I]$

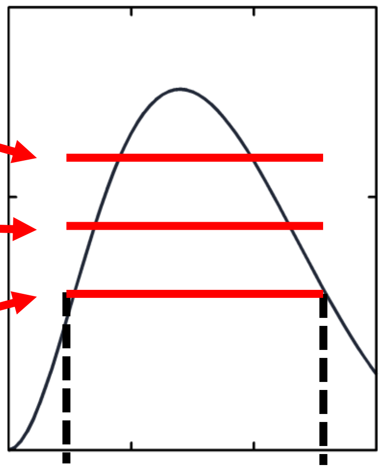
$P[\Phi(\mathrm{E}_\nu)|\bar{G}, I]$: conditional probability of having $\Phi(\mathrm{E}_\nu)$ given experimental data $\bar{G}$ and prior information of neutrino flux I

$$\sim \exp\left(-\frac{1}{2}\chi^2(\bar{G}, G) + \alpha S(G, I)\right)$$

- $\bar{G}$: $(\bar{G}_l , \sigma_l)$: experimental data $\left.\frac{d^2\sigma}{dE_\mu d\Omega_\mu}\right|_{exp}$
- G : $\left\langle\frac{d^2\sigma}{dE_\mu d\Omega_\mu}\right\rangle$: theoretical double differential cross section
- I : guess of neutrino flux $\Phi(\mathrm{E}_\nu)$ [default model of neutirno flux]

Error Estimation

Within MEM, we can estimate error of extracted flux.



$$\overline{\Phi}_{MEM} = \frac{\int_{E_{min}}^{E_{max}} dE_\nu \Phi_{MEM}(E_\nu)}{E_{max} - E_{min}}$$

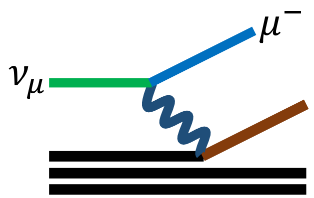
$$\langle \delta\overline{\Phi} \rangle^2 = \int [d\Phi(E_\nu)] \left(\delta\overline{\Phi}(\mathrm{E}_\nu)\right)^2 \mathrm{P}[\Phi(\mathrm{E}_\nu)|\bar{G}, I]$$

$$\left(\delta\overline{\Phi}(\mathrm{E}_\nu)\right)^2 = \frac{\left(\int_{E_{min}}^{E_{max}} dE_\nu \left(\Phi(E_\nu) - \Phi_{MEM}(E_\nu)\right)\right)^2}{(E_{max} - E_{min})^2}$$

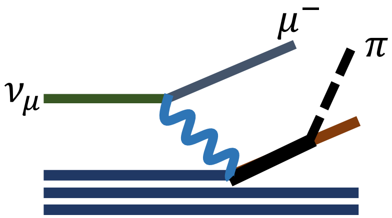
2. Theoretical double differential cross section

$\frac{d^2\sigma}{dE_\mu d\Omega_\mu}$: QE + single π production $\nu_\mu + {}^{12}C \rightarrow \mu^- + X$

B.Szczerbinska *et al* . Phys. Lett. B **649** 132 (2007)



Quasi-Elastic



Pion production

- Nuclear Spectral function

  O.Benhar *et al*. Phys. Rev. D **72** 053005 (2005)

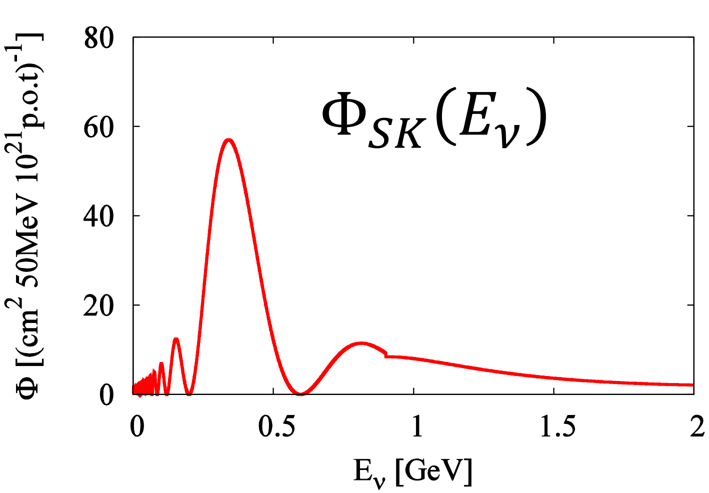
- Neutrino-induced pion production on nucleon

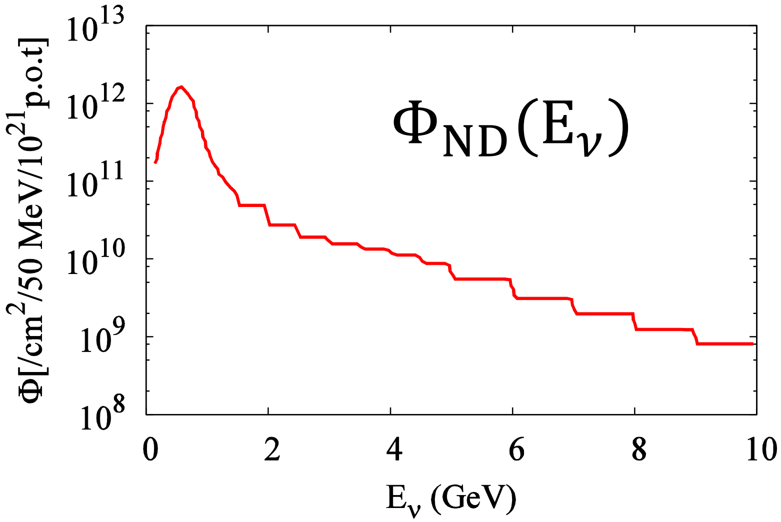
  T.Sato *et al*, Phys. Rev. C **67** 065201 (2003)

3. Pseudo data

3. 1 Neutrino Flux

$$\Phi_{SK}^{\nu_\mu}(E_\nu) = P(\nu_\mu \to \nu_\mu)\,\Phi_{\mathrm{ND}}^{\nu_\mu}(\mathrm{E}_\nu)$$





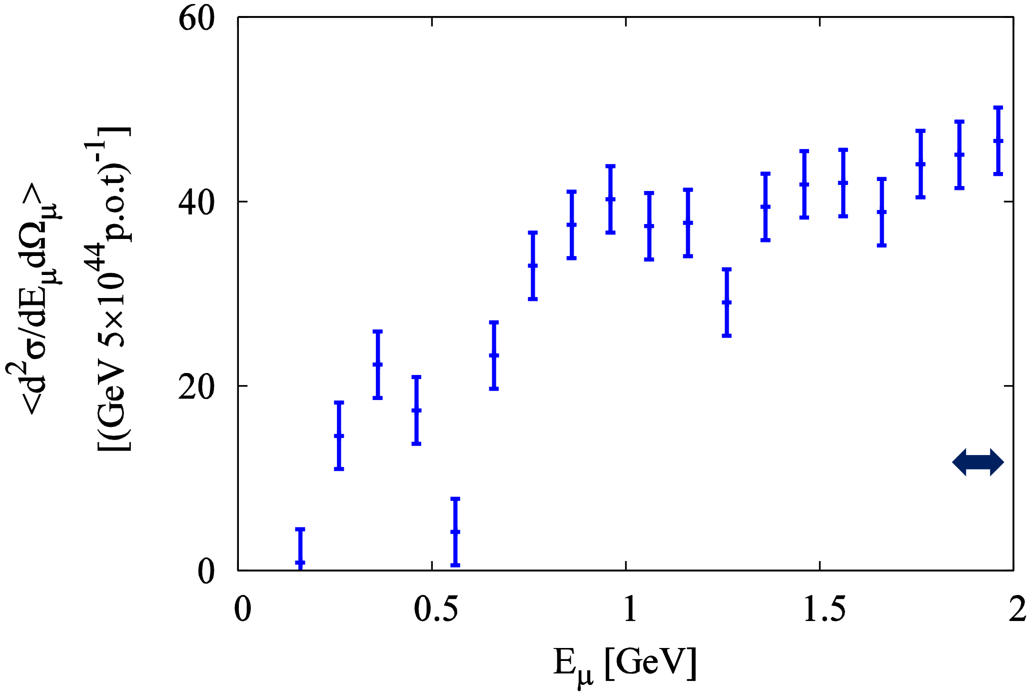
K. Abe *et al*. Phys. Lev. D **87** 012001 (2013)

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta_{23} \sin^2\left(\frac{1.27\Delta m_{23}^2[eV^2]L[km]}{E_\nu[GeV]}\right)$$

$$\Delta m^2 = 2.5 \times 10^{-3} eV^2$$
$$L = 295km$$
$$\sin^2 2\theta_{23} = 1.00$$

3.2 Pseudo Data of muon energy distribution

$$\left.\frac{d^2\sigma}{dE_\mu d\Omega_\mu}\right|_{exp} \Leftarrow \left\langle \frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega_\mu \mathrm{d}E_\mu} \right\rangle = \int \mathrm{d}E_\nu \frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega_\mu \mathrm{d}E_\mu} \Phi(E_\nu)$$



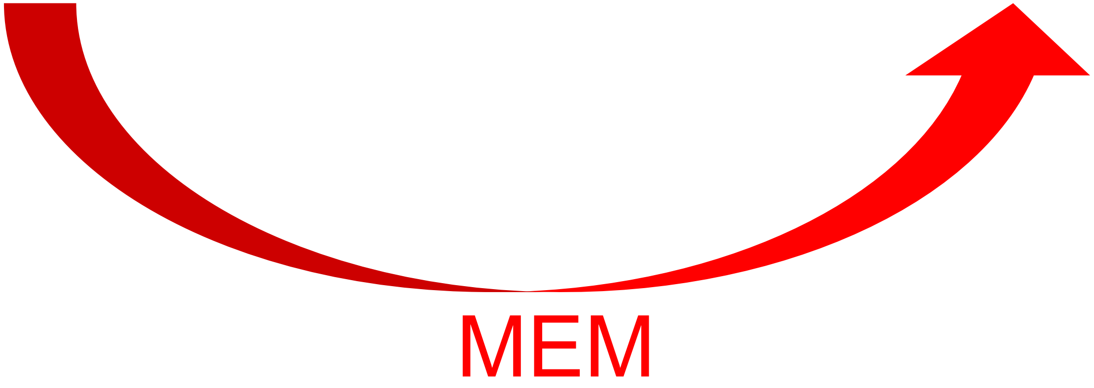
error
σ = 10%
Gaussian distribution

Resolution of muon energy
100MeV

4. Result

reconstruct the neutrino flux from 'data' by MEM

$$\left.\frac{d^2\sigma}{dE_\mu d\Omega_\mu}\right|_{exp} = \int \mathbf{dE}_\nu \frac{\mathbf{d^2}\sigma}{\mathbf{d\Omega}_\mu \mathbf{dE}_\mu} \mathbf{\Phi}(\mathbf{E}_\nu)$$

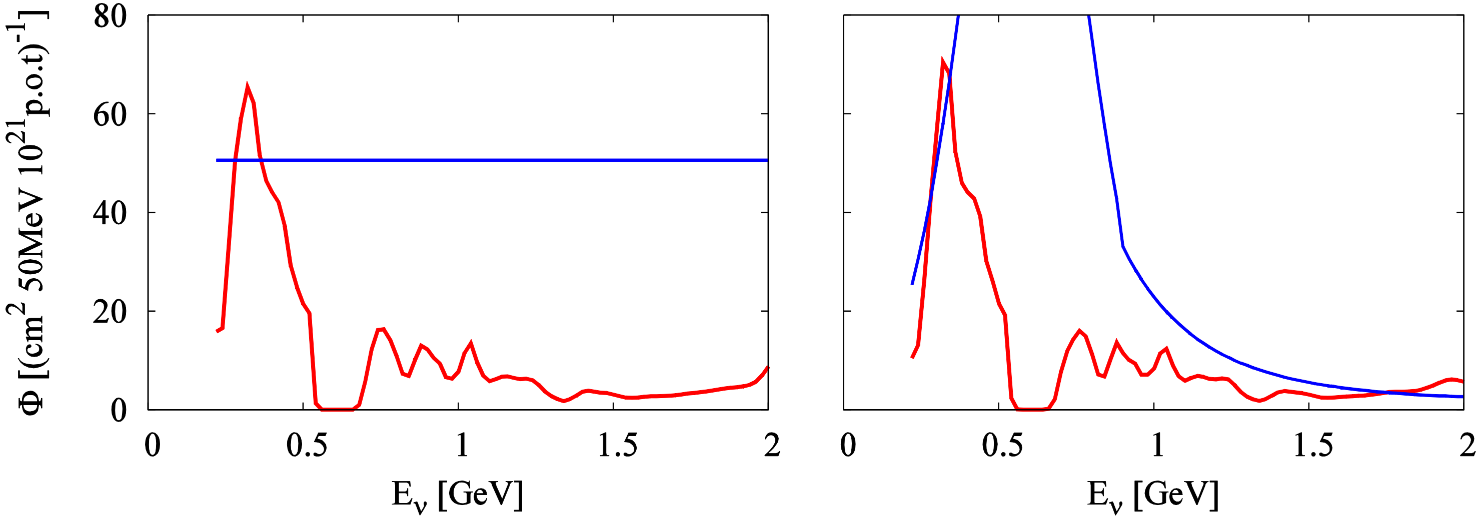


Dependence on Default Model (θ_μ=10°)

Reconstructed Flux by MEM

Default Model of Flux

$$\Phi(E_\nu)$$



Default Model =
(guesee of $\Phi(E_\nu)$)

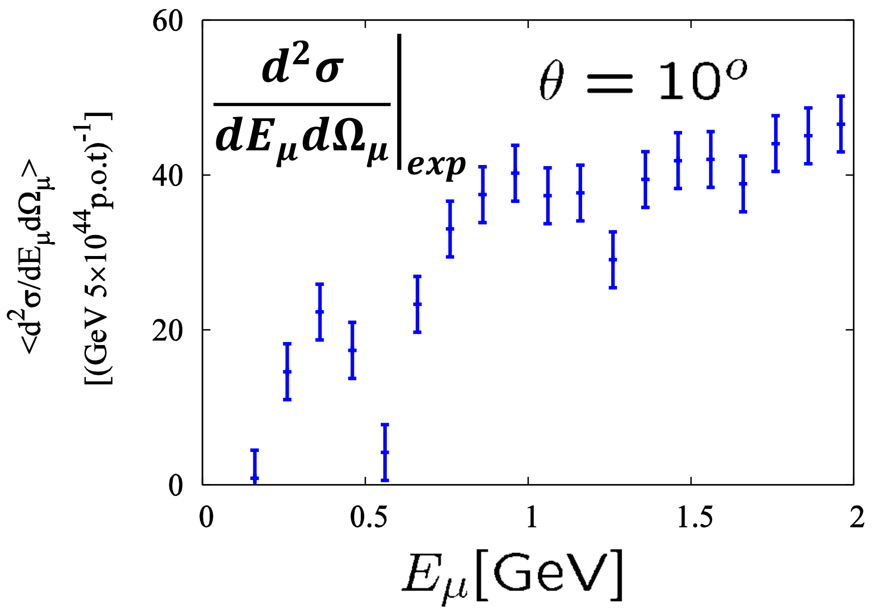
Constant

Without oscillation

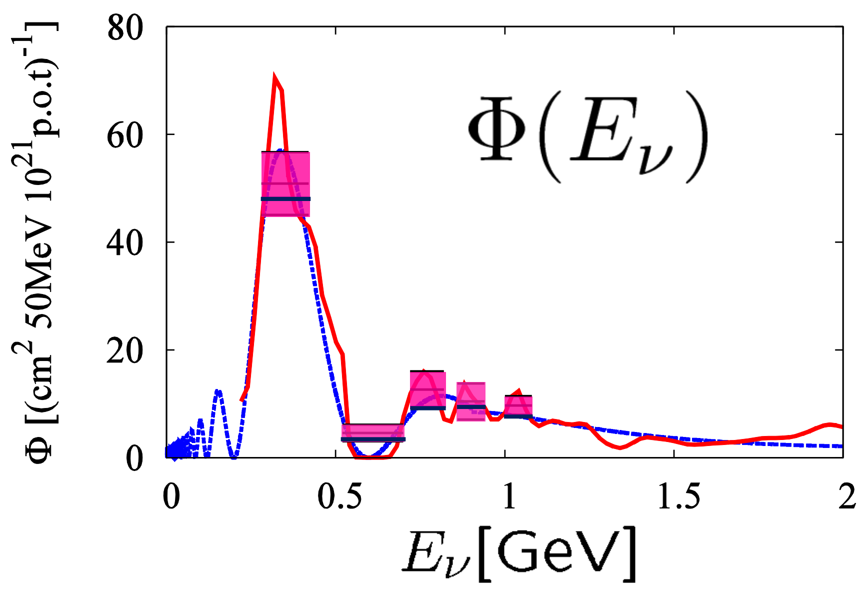
Dependence on default model is not so large.

Reconstructed Neutrino Flux (θ_μ=10°)

'pseudo data' of muon production



Reconstructed Neutrino Flux



Neutrino Flux is reconstructed well within error of MEM.

Summary

- We proposed a method for reconstructing the neutrino flux from muon distribution in inclusive neutrino nucleus reaction.

- We regarded the reconstruction problem as the Inversion problem and use maximum entropy method.

$$\left.\frac{d^2\sigma}{dE_\mu d\Omega_\mu}\right|_{exp} = \left\langle \frac{d^2\sigma}{d\Omega_\mu dE_\mu} \right\rangle = \int dE_\nu \frac{d^2\sigma}{d\Omega_\mu dE_\mu} \Phi(E_\nu)$$

MEM

- We have demonstrated that one can reconstruct neutrino flux from the inclusive cross section without using QE kinematics.

- The method of MEM can be useful alternative tool to extract neutrino flux provided accurate theoretical cross section is known.

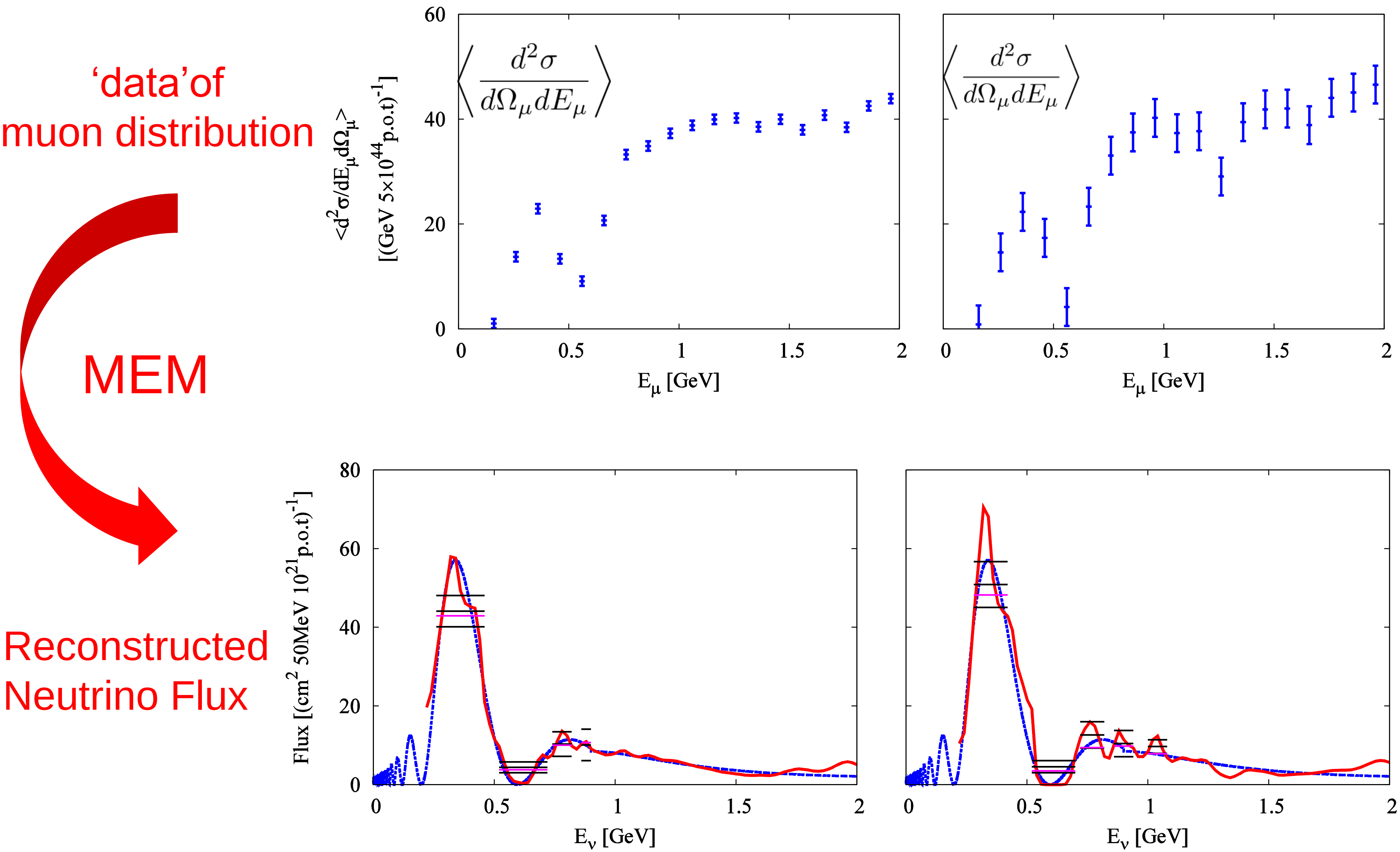
Back Up

Examine accuracy of data (θ_μ=10°)

‘data’of
muon distribution

MEM

Reconstructed
Neutrino Flux



Maximum Entropy Method (Principle)

$$\Phi_{MEM} = \int [d\Phi]\Phi P[\Phi|\bar{G}, I]$$

$$= \int [d\Phi]d\alpha\, \Phi\, P[\Phi, \alpha|\, \bar{G}, I]$$

$$= \int d\alpha\, \Phi_\alpha\, P[\alpha\, |\bar{G}, I]$$

$$\Phi_\alpha = \int [d\Phi]\Phi P[\Phi|\alpha, \bar{G}, I]$$

$$P[\Phi|\alpha, \bar{G}, I] \propto \exp\left[-\left(\frac{1}{2}\chi^2 - \alpha S\right)\right] = \exp Q$$

$$= \exp\left[-\frac{1}{2}\sum_l \left(\frac{\bar{G}_l - G_l}{\sigma_l}\right)^2 + \alpha \sum_i \left(\Phi_i - m_i - \Phi_i \ln\frac{\Phi_i}{m_i}\right)\right]$$

Maximum Entropy Method (Procedure)