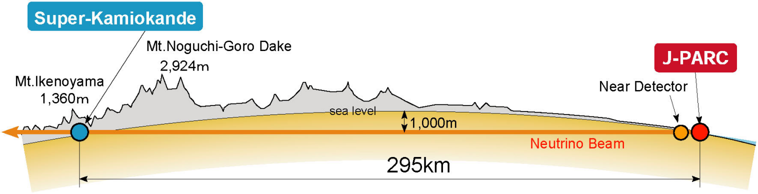
NEUT development for T2K and relevance of updated 2p2h models

Callum Wilkinson for the T2K collaboration

University of Sheffield

26/08/14

- New CCQE models in NEUT
- T2K analysis structure
- CCQE fits to external data
- Energy reconstruction
- Summary



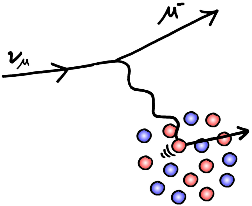
- T2K primary generator: **NEUT**[1].
- **ND280 targets:** *Carbon*, *Oxygen*, *Hydrogen*, Iron, Lead, Brass, and Argon.
- **SK targets:** *Oxygen*, *Hydrogen*.

Italicised targets are used for the T2K oscillation analyses.

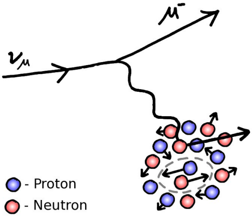
---

[1] Y. Hayato, *Acta Physica Polonica B* **40**, 2477 (2009)

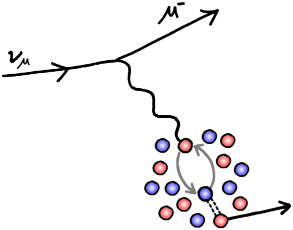
NEUT model updates



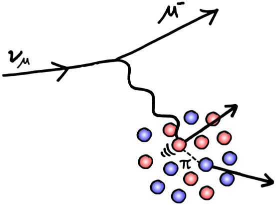
Global Fermi gas



Spectral Function

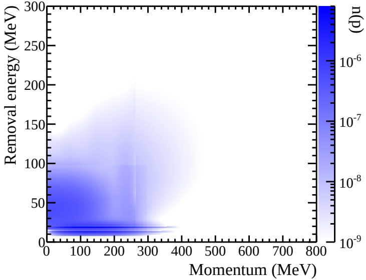


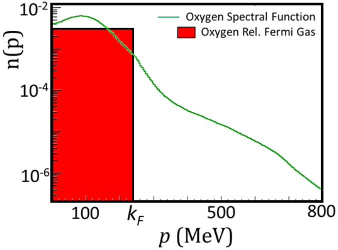
RPA



MEC

Base nuclear model





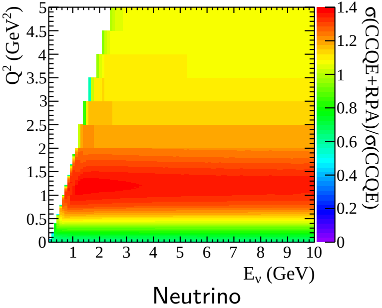
- Relativistic Fermi Gas (RFG), used for a long time in generators due to its simplicity (NEUT <v5.3.1).
- Omar Benhar's 2D Spectral Function[2] in momentum and removal energy has been implemented in NEUT (v5.3.1).
- The effective SF from Bodek *et al.*[3] has also been implemented in NEUT, but is not ready to be a candidate default model for T2K. A brief description of this model is available from the backup slide 37.

---

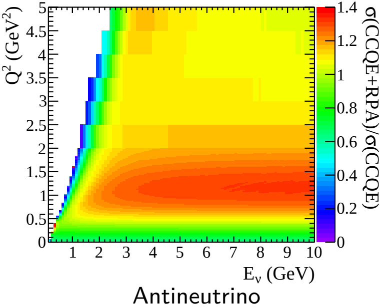
[2] O. Benhar, A. Fabrocini, *Phys. Rev.* **C62**, 034304 (2000)

[3] A. Bodek, E. Christy, B. Coopersmith (2014)

Random phase approximation (2D)



Neutrino



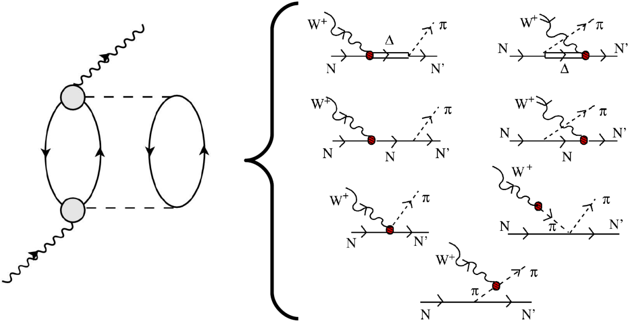
Antineutrino

- Random Phase Approximation (RPA), nuclear screening effect due to long range nucleon-nucleon correlations[4].
- NEUT implementation is dependent on Q^2 and E_ν.

---

[4] J. Nieves, I. R. Simo, M. J. V. Vacas, *Phys. Rev. C* **83**, 045501 (2011)

Nieves multi-nucleon interaction model

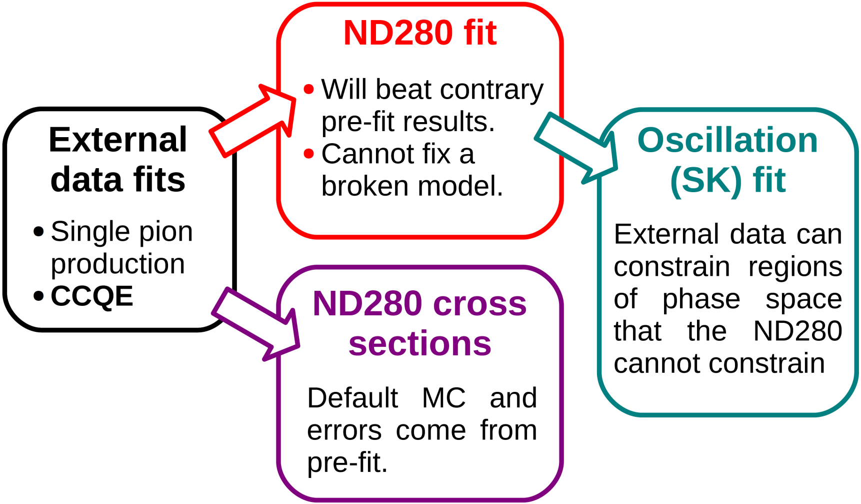


- Multi-nucleon interactions (MEC) from Nieves *et al.*[5], see Peter Sinclair's NuInt2014 talk for full implementation details (NEUT v5.3.2).
- Includes the high E_ν extension[6]. The low q3 part of the cross-section is accurate up to high energies.

---

[5] J. Nieves, I. R. Simo, M. J. V. Vacas, *Phys. Rev. C* **83**, 045501 (2011)

[6] R. Gran, J. Nieves, F. Sanchez, M. Vicente Vacas, *Phys. Rev.* **D88**, 113007 (2013)



External data fits

- Single pion production
- **CCQE**

ND280 fit

- Will beat contrary pre-fit results.
- Cannot fix a broken model.

ND280 cross sections

Default MC and errors come from pre-fit.

Oscillation (SK) fit

External data can constrain regions of phase space that the ND280 cannot constrain

CCQE fits

- **Aim:** constrain model parameters using all available CCQE data.
- Two candidate model combinations:
  - **SF+MEC**
  - **RFG+RPA+MEC**
- **Note:** we use BBBA05[7] vector form factors consistently for both models.
- Parameters which can be reweighted in NEUT:
  - MEC normalisation (as a percentage of the Nieves model)
  - Axial mass, M_{A}
  - Fermi momentum, pF (different for SF and RFG!)
  - Overall CCQE normalisation
  - RPA shape, accounting for different RPA models (affects the RPA enhancement at high Q^2)

---

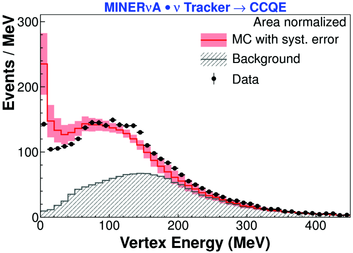
[7] R. Bradford, A. Bodek, H. Budd, J. Arrington, *Nuclear Physics B - Proceedings Supplements* **159**, 127 (2006)

Fit procedure

1. Select datasets for fit
2. Float model parameters in a χ^2 fit within each model.
3. Use Parameter Goodness of Fit (PGoF) test for consistency of datasets within each model.
4. Parameter error estimation. Rescaling procedure based on PGoF test.

Conclusions

1. Reasonable consistency between the datasets for the RFG+RPA+MEC model.
    - Not for the SF+MEC model, therefore we select RFG+RPA+MEC
2. Use external datasets to constrain the RFG+RPA+MEC parameters - the main result of this work.



- CCQE fits are to **lepton kinematics** only, cannot currently include hadronic data.
- A currently available example is the MINERvA vertex energy.

- We do not have a consistent description of the hadronic system for the new models.
- Nieves nucleon prediction not available to T2K, use the effective model from Jan Sobczyk for multi-nucleon ejection[8].
- NEUT FSI cascade model is applied to all outgoing nucleons.

---

[8] J. T. Sobczyk, *Phys. Rev. C* **86**, 015504 (2012)

Combined fit

- **Combined fit:**
  - MiniBooNE double-differential results (ν and $\bar{\nu}$)[9,10]
  - MINERvA results (ν and $\bar{\nu}$)[11,12] with restricted phase space $\theta_\mu \leq 20^\circ$ (slide 30), including cross-correlations (slide 31)
- Normalisation parameters for each MiniBooNE dataset are varied in all of the fits:

Fit type	χ^2/DOF	$\mathrm{M_A}$ (GeV)	MEC (%)	pF (MeV)
SF+MEC	161.3/197	1.33±0.03	0 (at limit)	209 (at limit)
RFG+RPA+MEC	109.6/195	1.02±0.03	58±10	239±7

- The relativistic RPA model was favoured in the fit.
- CCQE normalisation showed no tendency to be pulled from 1 for the RFG+RPA+MEC fit.

---

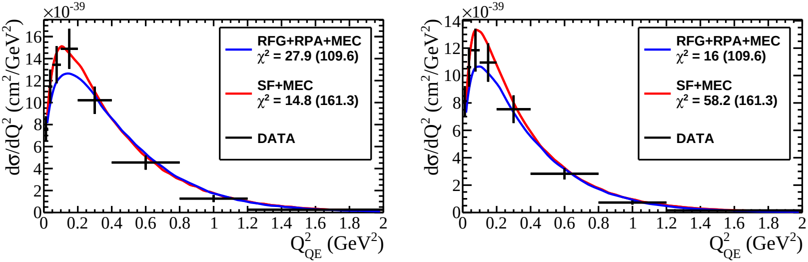
[9] A. A. Aguilar-Arevalo, *et al.*, *Phys. Rev. D* **81**, 092005 (2010)

[10] A. Aguilar-Arevalo, *et al.*, *Phys. Rev.* **D88**, 032001 (2013)

[11] G. Fiorentini, *et al.*, *Phys. Rev. Lett.* **111**, 022502 (2013)

[12] L. Fields, *et al.*, *Phys. Rev. Lett.* **111**, 022501 (2013)

MINERvA plots from combined fit results



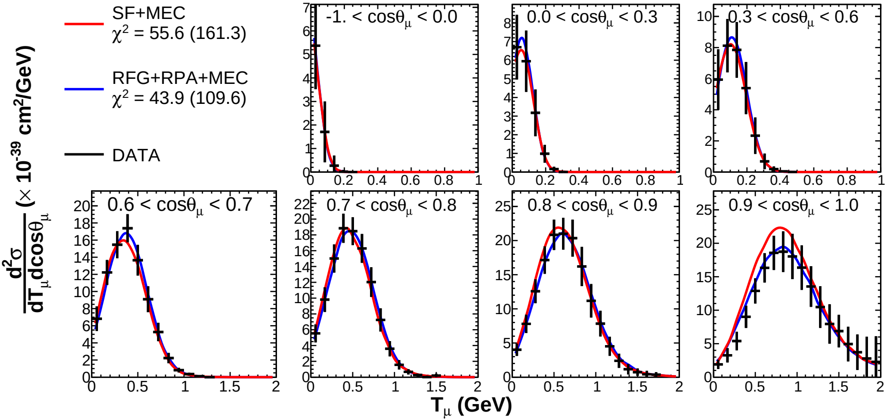
Neutrino

Antineutrino

- The χ^2 contributions indicate fairly strong tensions between MINERνA and the best fit values to all datasets for both models.
- Clear that MiniBooNE is not completely dominating the fits. We exploit the fact that, without correlations:

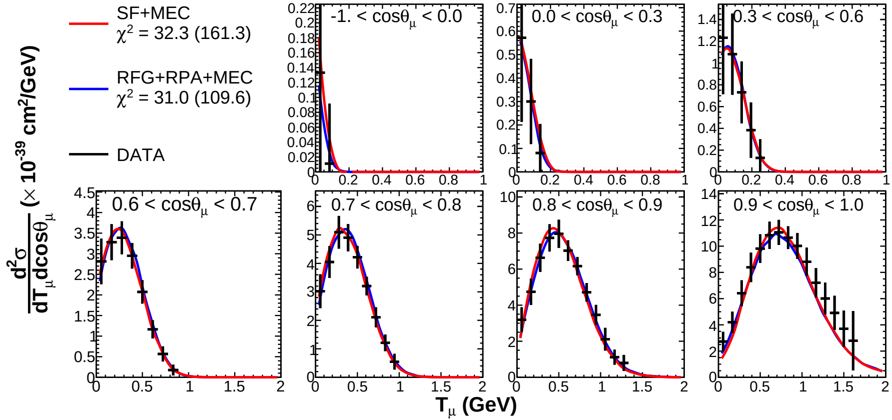
$$\chi^2_{\mathrm{MB}} \approx \chi^2_{\mathrm{MIN}}$$

MiniBooNE neutrino



- Worse agreement for SF+MEC because of the low momentum transfer bins.
- When fit individually, MiniBooNE neutrino would prefer to increase pF_{SF} to fit this region.

MiniBooNE antineutrino



- MiniBooNE antineutrino doesn't have the same tension in the low momentum transfer bins for SF+MEC.
- This probably stops MiniBooNE neutrino from pulling this region in the fits.

Are these results trustworthy?

- The best fit χ^2_{min} values seem to be very good for the both joint fits shown here:
  - **SF+MEC:** $\chi^2_{min} = 161.3/197$
  - **RFG+RPA+MEC:** $\chi^2_{min} = 109.6/195$
- **However**, MiniBooNE data lack correlations, so Gaussian statistics no longer work:
  - Standard goodness of fit tests are unreliable.
  - $\Delta\chi^2 = 1$ is no longer appropriate for calculating parameter errors.
- Need a better test statistic to properly assess agreement. Using the **Parameter Goodness of Fit** test[13], details are on slide 33.
  - Compares the best fit parameters for the complete dataset and for subsets of the data. Assesses the compatibility between datasets.

---

[13] M. Maltoni, T. Schwetz, *Phys. Rev.* **D68**, 033020 (2003)

- **RFG+RPA+MEC** (see slide 34 for a more detailed breakdown)

	χ^2_{min}/DOF	p-value (%)	χ^2_{PGoF}/DOF	PGoF (%)
All	109.61/195	100.00	27.14/9	0.13
MINERvA vs MiniBooNE			17.85/3	0.05
ν vs $\bar{\nu}$			5.14/3	16.18

- **SF+MEC** (see slide 35 for a more detailed breakdown)

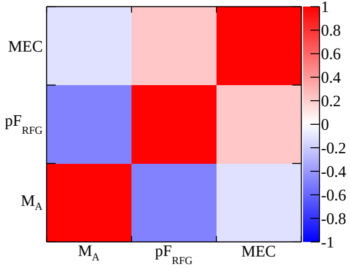
	χ^2_{min}/DOF	p-value (%)	χ^2_{PGoF}/DOF	PGoF (%)
All	161.34/197	97.02	83.42/9	0.00
MINERvA vs MiniBooNE			44.21/3	0.00
ν vs $\bar{\nu}$			54.38/3	0.00

- There is much less tension in the PGoF results for RFG+RPA+MEC than SF+MEC.
- Because of this lack of consistency between datasets, SF+MEC is a poor choice for the default T2K MC model.

- $\Delta\chi^2 \neq 1$ for 1σ errors.
- Parton density distribution fitters face similar challenges with the data they have available, with many papers discussing these issues[14].
  - **Solution:** inflate the $\Delta\chi^2$ value that defines the error.
- The PGoF gives a value for the incompatibility between the datasets: how much the χ^2 increases between the best fit point of each experiment, and the best fit for the combined dataset.
- We can use the reduced χ^2 from the PGoF test as the figure of merit, and scale the errors such that $\Delta\chi^2 = \chi^2_{PGoF}/DOF$. The rescaled error then spans the differences between the datasets.

---

[14] J. Pumplin, D. Stump, W. Tung, *Phys. Rev.* **D65**, 014011 (2001)

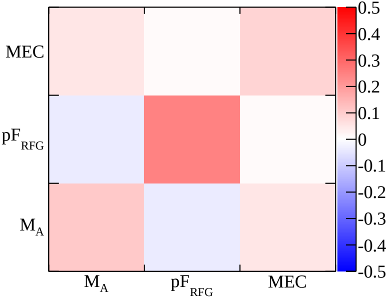


- Table of parameter errors, with and without scaling:

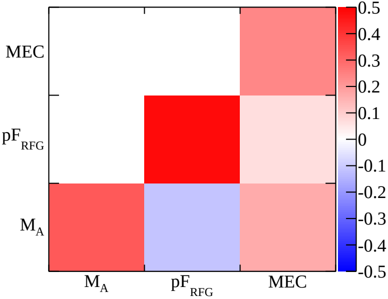
Fit type	χ^2/DOF	M_A (GeV)	MEC (%)	pF (GeV)
Unscaled	109.6/195	1.02±0.03	58±10	239±7
Scaled		1.02±0.08	58±25	239±16

- Fit favours relativistic RPA calculation.
- BBBA05 vector form factors are used.

Throwing the covariance matrix



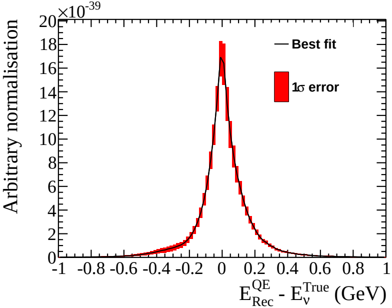
Covariance matrix



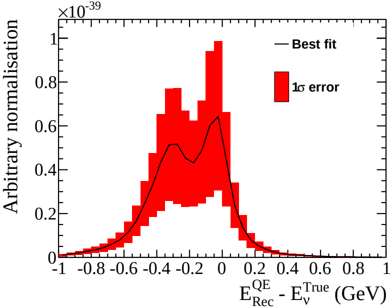
Cholesky decomposition

- Obtain a lower triangular matrix A from the covariance matrix Σ using a Cholesky decomposition: $AA' = \Sigma$
- A vector of randomly drawn parameter values, x, can be obtained:
  $x = \mu + Az$,
  where μ is the vector of best fit values for the parameters, and z is a vector of random draws from a 1-dimensional Gaussian centred on 0 and standard deviation of 1.

ND280 flux-averaged cross-section predictions



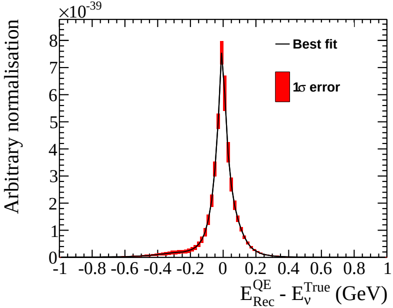
CCQE+MEC $\nu_\mu - {}^{12}$C (ν-mode)



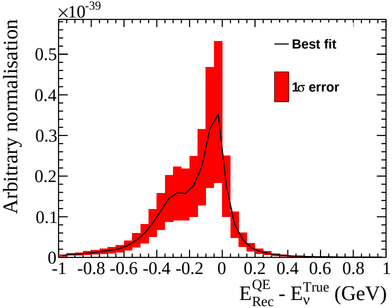
MEC $\nu_\mu - {}^{12}$C (ν-mode)

- Energy resolution plotted for the entire ND280 flux for neutrino running. The MEC component has been separated in the right plot.
- The 1σ error band is produced with 1000 throws of the scaled covariance matrix.

ND280 flux-averaged cross-section predictions



CCQE+MEC $\bar{\nu}_{\mu}-{}^{12}$C ($\bar{\nu}$-mode)



MEC $\bar{\nu}_{\mu}-{}^{12}$C ($\bar{\nu}$-mode)

- Energy resolution plotted for the entire ND280 flux for antineutrino running. The MEC component has been separated in the right plot.
- The 1σ error band is produced with 1000 throws of the scaled covariance matrix.

- A number of new models have been implemented in NEUT, and will be included in future T2K analyses:
    - Spectral Function (NEUT v5.3.1)
    - RPA (NEUT v5.3.2) - **note** we only have a calculation appropriate for RFG, **not** SF.
    - MEC (NEUT v5.3.2)
- Fits have been to all of the available CCQE data to form the nominal T2K MC.
    - Selected the RFG+RPA+MEC model as the default CCQE model. Obtained a set of parameters which describe all of the available CCQE data, and are consistent with expectation.
    - We can revisit this conclusion when we have implemented an RPA calculation appropriate for the SF.
- Investigating the effect that these model developments will have on energy reconstruction at ND280 and SK.

Backup

Fit details

- Generate 1 million events (CCQE + MEC) on the detector target material with NEUT
- Fit uses χ^2 minimisation through MINUIT
- Event by event reweighting through interface with T2KReWeight to build up model prediction at that iteration

$$
\begin{aligned}
\chi^2(\theta) = & \left[\sum_{k=0}^{N} \left(\frac{\nu_k^{DATA} - \lambda_\alpha^{-1} \nu_k^{MC}(\theta)}{\sigma_k} \right)^2 + \left(\frac{\lambda_\alpha - 1}{\varepsilon_\alpha} \right)^2 \right] \rightarrow \text{MiniBooNE } \nu \\
& + \left[\sum_{l=0}^{M} \left(\frac{\nu_l^{DATA} - \lambda_\beta^{-1} \nu_l^{MC}(\theta)}{\sigma_l} \right)^2 + \left(\frac{\lambda_\beta - 1}{\varepsilon_\beta} \right)^2 \right] \rightarrow \text{MiniBooNE } \bar{\nu} \\
& + \left[\sum_{i=0}^{16} \sum_{j=0}^{16} \left(\nu_i^{DATA} - \nu_i^{MC}(\theta) \right) V_{ij}^{-1} \left(\nu_j^{DATA} - \nu_j^{MC}(\theta) \right) \right] \rightarrow \text{MINER}\nu\text{A}
\end{aligned}
$$

- Where $\lambda_{\alpha,\beta}$ are normalisation parameters, and $\varepsilon_{\alpha,\beta}$ are the published normalisation uncertainties. θ are the fit parameters.

Reconstructed kinematics

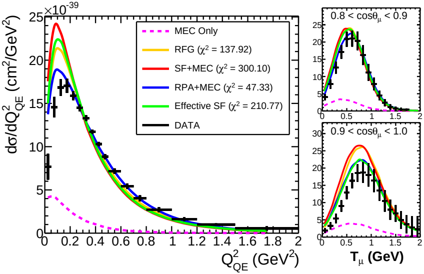
- Reconstructed neutrino energy E_{ν}^{QE} and reconstructed four-momentum transfer Q^2_{QE} are given by the equations:

$$E_{\nu}^{QE} = \frac{2M'_n E_{\mu} - (M'^2_n + m^2_{\mu} - M^2_p)}{2(M'_n - E_{\mu} + \sqrt{E^2_{\mu} - m^2_{\mu}})\cos\theta_{\mu}}$$

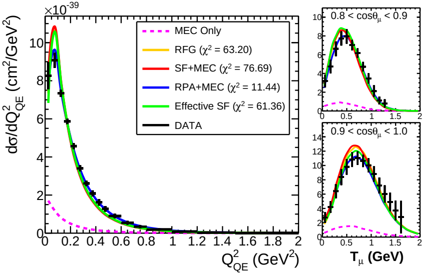
$$Q^2_{QE} = -m_{\mu} + 2E_{\nu}^{QE}(E_{\mu} - \sqrt{E^2_{\mu} - m^2_{\mu}}\cos\theta_{\mu})$$

- Where $E_{\mu} = T_{\mu} + m_{\mu}$ and $M'_n = M_n - E_B$
- $E_B = 34$ MeV for all datasets except MINERvA antineutrino, where $E_B = 30$ MeV is assumed.

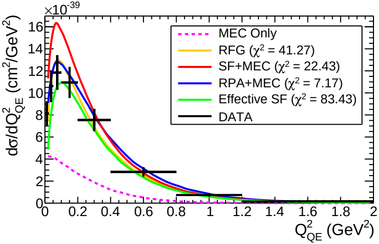
NEUT Nominal distributions ($M_A = 1.01$ GeV)



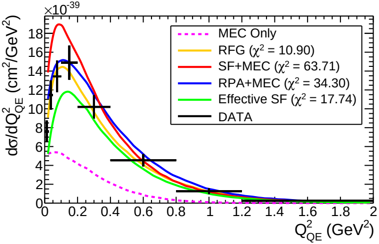
MiniBooNE neutrino



MiniBooNE antineutrino

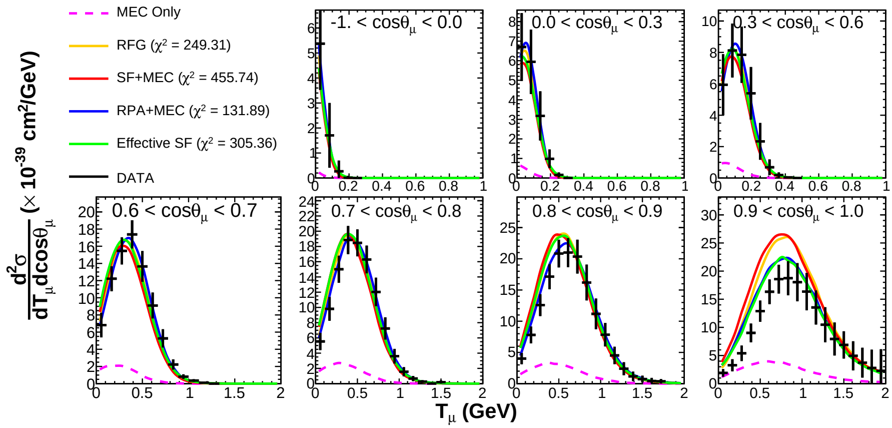


MINERvA neutrino

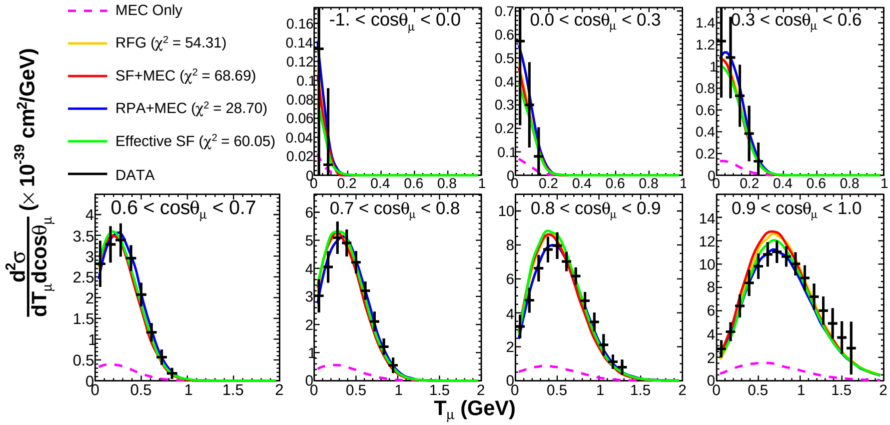


MINERvA antineutrino

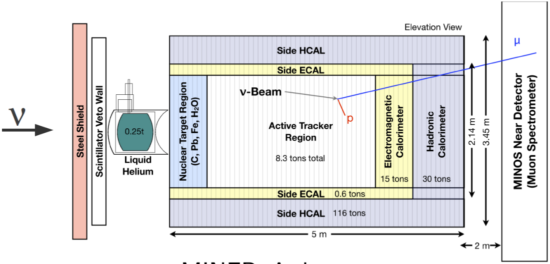
MiniBooNE ν 2D nominal



MiniBooNE $\bar{\nu}$ 2D nominal



MINERνA angular acceptance[15]



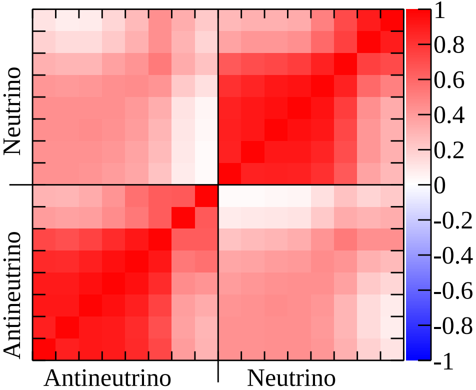
MINERνA detector

- Two presentations of MINERνA CCQE results are currently available:
    - $\theta_\mu < 20^\circ$
    - FULL (previous slide)
- The $\theta_\mu < 20^\circ$ sample excludes the unsampled regions of phase space (where MINOS can't tag muons), so is less model dependent
- Both samples are official MINERνA results.

---

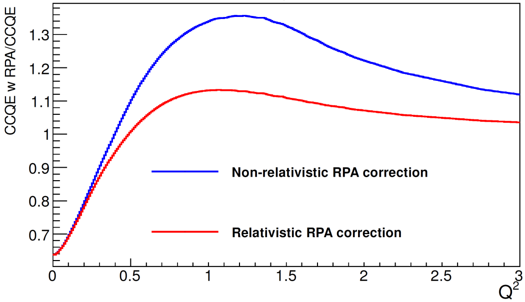
[15] M. Kordosky, Fermilab Wine and Cheese, June 2012

MINERvA cross-correlations



- MINERvA have kindly released the cross correlations between their neutrino and anti-neutrino samples.
- These are included in the CCQE fits

RPA shape



- This projection of the relativistic and non-relativistic RPA corrections onto the Q^2 axis shows the difference between the models. Both are implemented as functions of E_ν and Q^2.
- A reweighting dial has been implemented to reweight between the non-relativistic and relativistic models.

Parameter Goodness of Fit (PGoF) test[16]

- PGoF test statistic is defined:

$$\bar{\chi}^2(\theta) = \chi^2_{tot} - \sum_{r=1}^{D} \chi^2_{r,min}$$

  where D are the number of datasets and χ^2_{tot} is the minimum χ^2 in a fit to all D datasets.
- The number of degrees of freedom is given by:

$$P_{PGoF} = \sum_{r=1}^{D} P_r - P_{tot}$$

  where P_r and P_{tot} are the number of free parameters varied in each fit.
- Tests the compatibility of the different datasets in the framework of the model - basically, how badly are the best fit parameters pulled in the joint fit compared to those found in the independent fits.
- PGoF used extensively in sterile neutrino literature.

---

[16] M. Maltoni, T. Schwetz, *Phys. Rev.* **D68**, 033020 (2003)

PGoF RFG+RPA+MEC (relativistic RPA)

- In each fit, $\mathrm{M_A}$, $\mathrm{pF_{RFG}}$, MEC normalisation and MiniBooNE normalisation parameters are allowed to vary.
- RPA shape is fixed at the relativistic correction for all fits.

	χ^2_{min}/DOF	SGoF (%)	χ^2_{PGoF}/DOF	PGoF (%)
All	109.61/195	100.00	**27.14/9**	**0.13**
MINERvA	21.22/13	6.87	4.57/3	20.61
MiniBooNE	70.54/179	100.00	9.44/3	2.40
ν	69.10/127	100.00	12.70/3	0.53
$\bar{\nu}$	35.37/65	99.90	9.29/3	2.56
MINERvA vs MiniBooNE	109.61/195	100.00	**17.85/3**	**0.05**
ν vs $\bar{\nu}$	109.61/195	100.00	**5.14/3**	**16.18**

- There is a reasonable amount of tension, but in general the agreement is not bad.
- In particular, the agreement between ν and $\bar{\nu}$ is good, this is very important as we want to use the same systematics for antineutrino running.

PGoF SF+MEC

- In each fit, $\mathrm{M_A}$, $\mathrm{pF_{SF}}$, MEC normalisation and MiniBooNE normalisation parameters are allowed to vary

	χ^2_{min}/DOF	SGoF (%)	χ^2_{PGoF}/DOF	PGoF (%)
All	161.34/197	97.02	**83.42/9**	**0.00**
MINERvA	33.73/13	0.13	5.23/2	7.32
MiniBooNE	83.40/180	100.00	33.98/3	0.00
ν	54.77/128	100.00	24.96/4	0.01
$\bar{\nu}$	52.19/66	89.22	4.07/2	13.07
MINERvA vs MiniBooNE	161.34/197	97.02	**44.21/4**	**0.00**
ν vs $\bar{\nu}$	161.34/197	97.02	**54.38/3**	**0.00**

- Generally poor agreement between the datasets. In particular, nothing agrees well with MiniBooNE neutrino.
- Because of the lack of consistency, SF+MEC is a poor choice for the default T2K MC model.

PGoF RFG+RPA+MEC (non-relativistic RPA)

- In each fit, $\mathrm{M_A}$, $\mathrm{pF_{RFG}}$, MEC normalisation and MiniBooNE normalisation parameters are allowed to vary.
- RPA shape at the non-relativistic limit:

	χ^2_{min}/DOF	SGoF (%)	χ^2_{PGoF}/DOF	PGoF (%)
All	110.09/195	100.00	38.00/9	0.00
MINERvA	18.49/13	13.98	2.54/3	46.81
MiniBooNE	74.41/179	100.00	18.26/3	0.04
ν	62.65/127	100.00	16.15/3	0.11
$\bar{\nu}$	32.45/65	99.98	6.85/3	7.69
MINERvA vs MiniBooNE	110.09/195	100.00	17.19/3	0.06
ν vs $\bar{\nu}$	110.09/195	100.00	15.00/3	0.18

- There is more tension between datasets when RPA is at the non-relativistic limit rather than the relativistic limit.
- Although overall decrease in χ^2 is small between non-relativisitic and relativisitc RPA, the improved consistency between datasets indicates better agreement between relativistic RPA and data.

The effective spectral function and superscaling[17]

---

[17] A. Bodek, E. Christy, B. Coopersmith (2014)

What is superscaling?

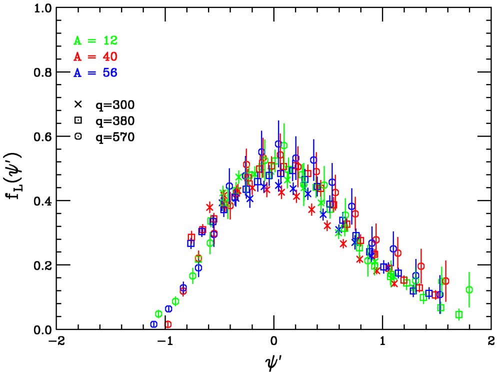
- Obtain a 'reduced' cross-section by dividing the experimental inclusive electron scattering data on a nucleus by the elastic cross section on a single nucleon smeared by the Fermi motion (as nucleons in the nuclear ground state are moving).
- If the reduced cross-section can be plotted against a variable, and it is shown that there is no dependence on that variable, the results scale.
- Superscaling is when two types of scaling occur[18,19]:
  1. No dependence on momentum transfer q.
  2. No dependence on the Fermi momentum of the nuclear species.
- Scaling occurs in the longitudinal, but not the transverse response. This lack of scaling in the transverse response is what motivates the transverse enhancement model.

---

[18] J. E. Amaro, *et al.*, *Phys. Rev.* **C71**, 015501 (2005)

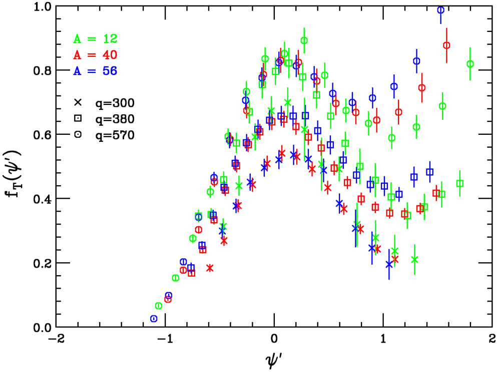
[19] P. Bosted, V. Mamyan (2012)

Electron scattering longitudinal response

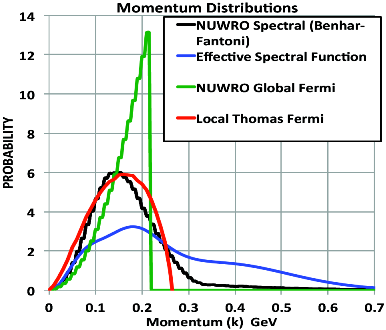


- Superscaling fit is to the longitudinal response for all available electron scattering data.
- By construction, the QE peak is at $\psi' = 0$, and $\psi' \propto \omega - Q^2/2M_P$.
- Helpful to just think of the plot as showing the shape of the QE peak.

Electron scattering transverse response

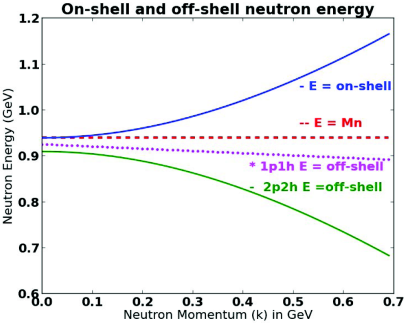


- Transverse response has QE peak + Δ peak and additional 2p2h effects modelled by the Transverse Enhancement Model (TEM).
- Note the TEM is **not** between QE peak and Δ response, it contributes across the **entire** QE peak. Without it, the entire peak would be under-estimated.

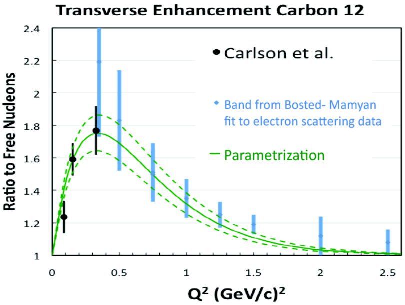


- **Basic idea:** model outgoing lepton kinematic distribution by changing initial state nucleon model. This effective modification is designed to cover a range of sins (additional nuclear effects), but in a way which is easy to implement in generators.
- Effective SF based on a parameterisation of the momentum distribution from Benhar's SF (from NOMAD collaboration), but parameters modified to fit superscaling function.
- Note that a significant high momentum component is required to fit electron scattering.

Effective spectral function



- Constant probability of being in a correlated state with another nucleon (2p2h), which affects how off-shell the interacting nucleon is.
- Difference is whether momentum and energy are being balanced by on-shell proton (2p2h), or on-shell A-1 nuclear remnant.
- On-shell proton in 2p2h events is also simulated (with equal and opposite momentum).



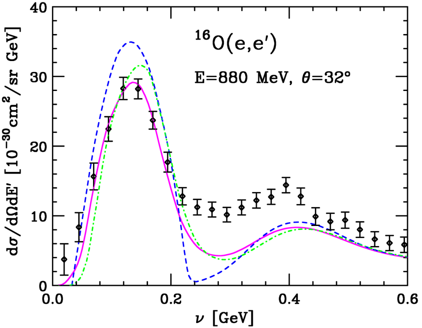
- Q^2 dependent excess in the transverse response compared with longitudinal response observed in electron scattering data.
- This excess is parameterised as a modification to the magnetic form factors for free nucleons[20]:

$$G_{M_n}^{nuclear} = G_{M_n} \times \sqrt{1 + AQ^2 \exp(-Q^2/B)}$$
$$G_{M_p}^{nuclear} = G_{M_p} \times \sqrt{1 + AQ^2 \exp(-Q^2/B)}$$

---

[20] A. Bodek, H. S. Budd, E. Christy, *Eur. Phys. J. C* **71**, 1726 (2011)

Differences in the approach



- The key argument here is that other predictions for the QE peak do not match the electron scattering data (longitudinal response).
- The superscaling function can be modelled by making changes to the initial state momentum distribution of the nucleons.
- Some of the apparent 'dip' region is due to not modelling the true CCQE response correctly (QE peak also contributes more in this region).