Variations on Final "Cooling"

David Neuffer
Fermilab
Don Summers, Terry Hart Ole Miss
R. Palmer, H. Sayed, BNL

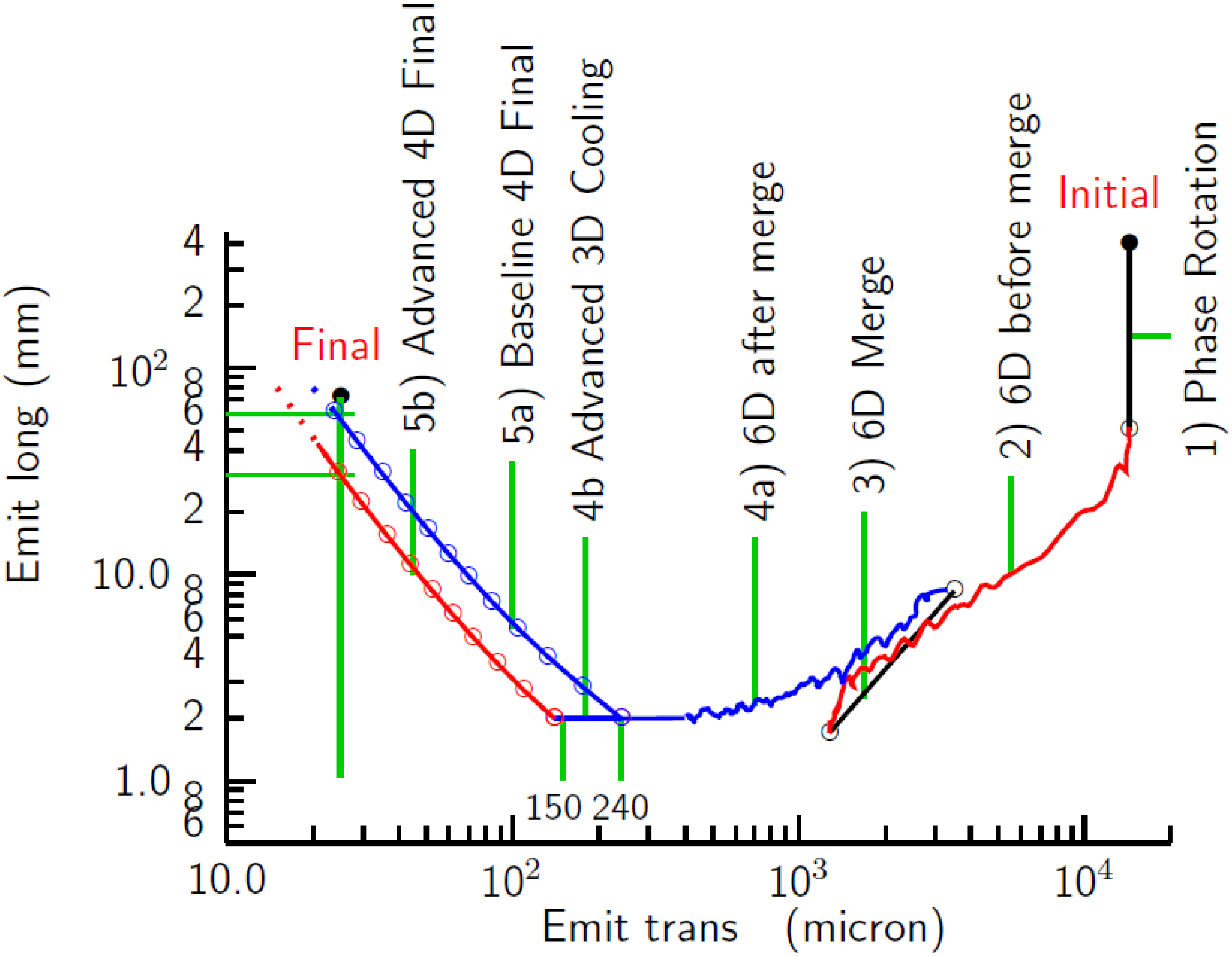
Outline

- Final Cooling for a Collider
  - – R. Palmer
- Final Cooling Simulation –
  - – H. Sayed
- Final scenario variation
  - – w /D. Summers & T. Hart
  - – round to flat and slicing ….
- Variations on Round to Flat
  - – 1-D cooling …

Final cooling

- Baseline High energy collider has final "cooling"
  - – ε_{x}, ε_{y}: 0.0003 → 0.00003m
  - – ε_{L} : 0.001 → 0.1m
    - Mostly emittance exchange…
- Outline
  - – Baseline scenario
  - – Simulation
  - – Variation
    - Can we use the round to flat beam "emittance exchange" ? –
      - – to change the rules
    - cool, rotate, slice (transverse) recombine (longitudinal)

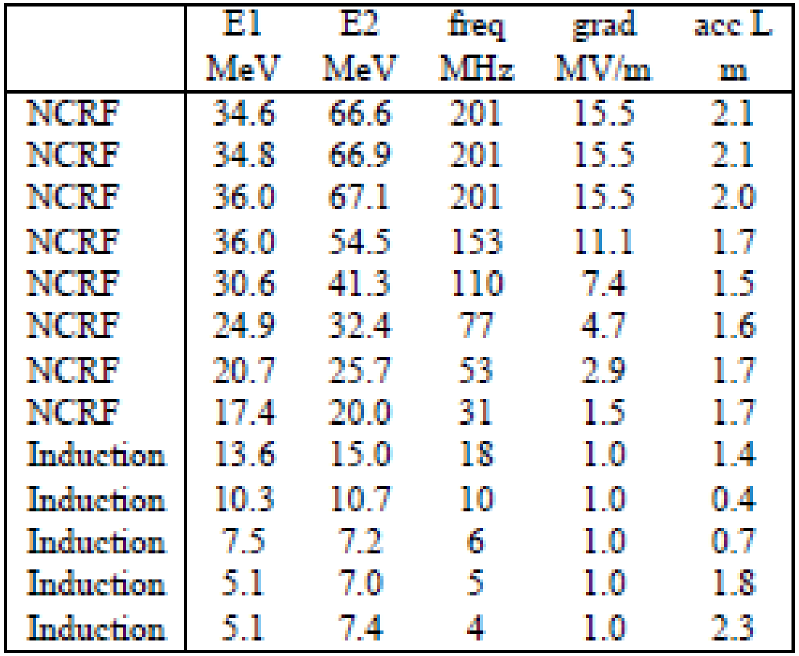
Set up for final cooling



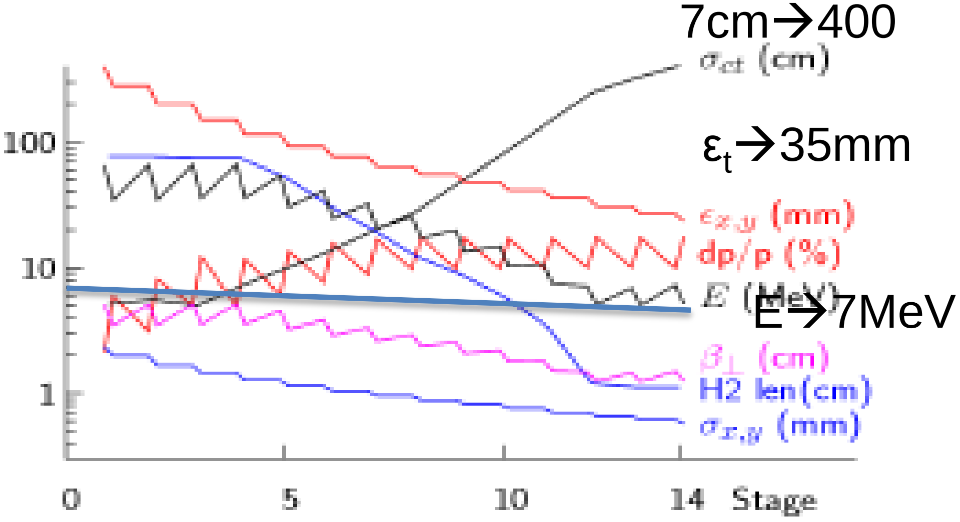
Baseline scenario (2011)

- “Baseline” Muon Collider final cooling stages
  - No actual cooling – emittance exchange
  - High magnetic fields
  - Impossible “rf”

Table 1: Rf Parameters of 40 T Example



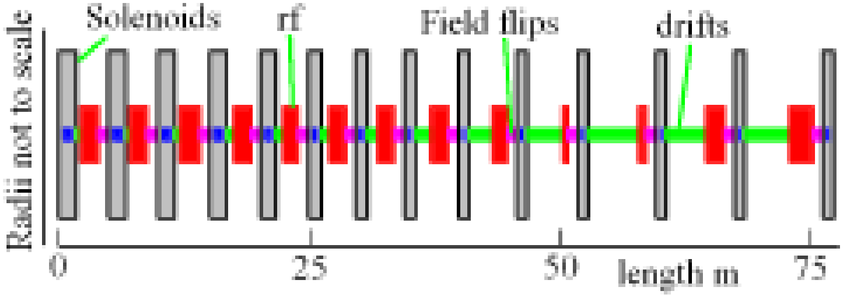
	E1 MeV	E2 MeV	freq MHz	grad MV/m	acc L m
NCRF	34.6	66.6	201	15.5	2.1
NCRF	34.8	66.9	201	15.5	2.1
NCRF	36.0	67.1	201	15.5	2.0
NCRF	36.0	54.5	153	11.1	1.7
NCRF	30.6	41.3	110	7.4	1.5
NCRF	24.9	32.4	77	4.7	1.6
NCRF	20.7	25.7	53	2.9	1.7
NCRF	17.4	20.0	31	1.5	1.7
Induction	13.6	15.0	18	1.0	1.4
Induction	10.3	10.7	10	1.0	0.4
Induction	7.5	7.2	6	1.0	0.7
Induction	5.1	7.0	5	1.0	1.8
Induction	5.1	7.4	4	1.0	2.3



7cm→400

ε_t→35mm

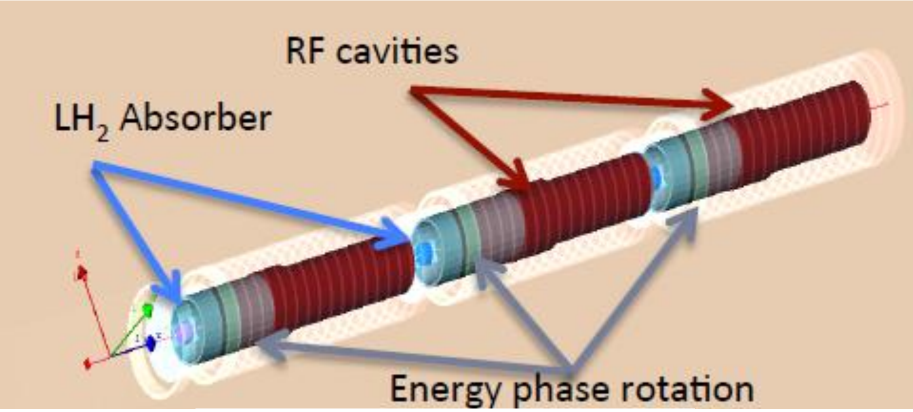
E→7MeV

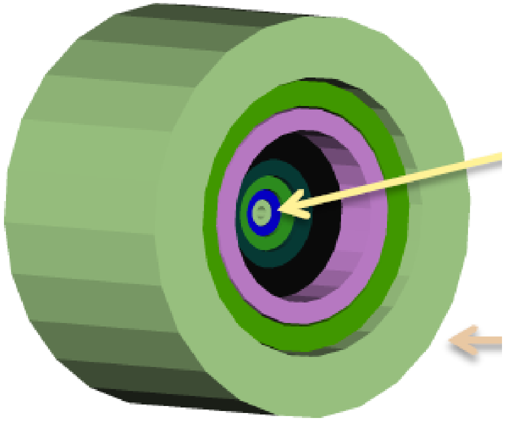


Detailed simulation of final cooling

(H. Sayed)

- **G4Beamline simulation of final cooling scenario**
  - absorbers, rf for bunching & reacceleration, magnets
  - 17 stages, 140m long
  - absorbers within strong magnetic fields
- **Get smaller ε_N by smaller P_μ, larger B**
  - P_μ :135 → 70 MeV/c
  - B: 25 →30 T
    - Palmer used 40 MeV/c, 40T





$$\beta_t \cong \frac{2P_\mu}{0.3B}$$

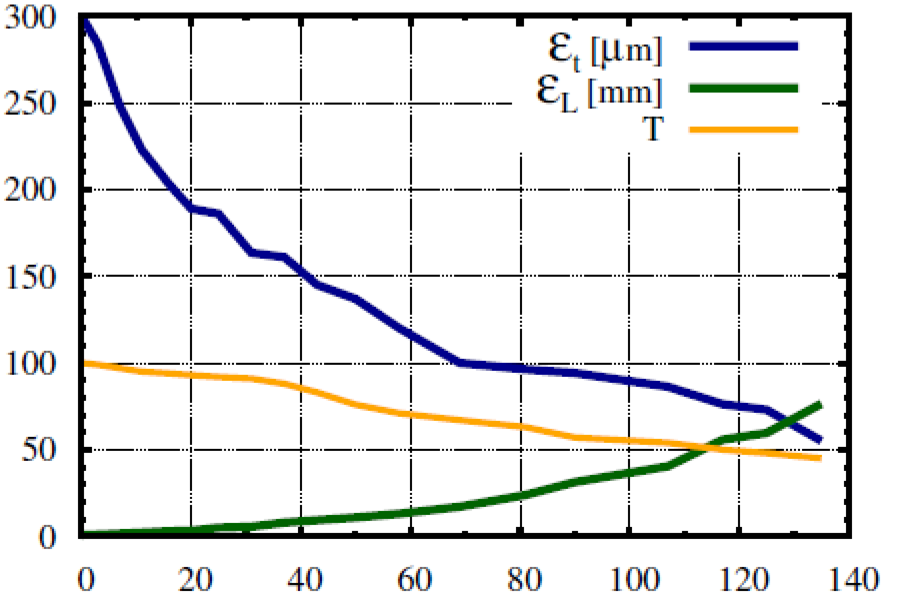
$$\varepsilon_{N,eq} \cong \frac{\beta_t E_s^2}{2\beta mc^2 L_R (dE/ds)}$$

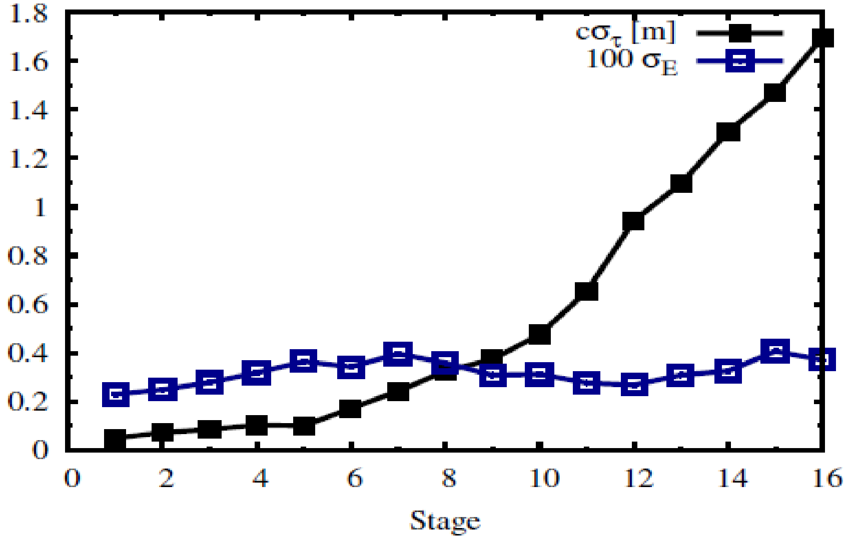
Simulation results

- **System is ~135m long**
  - $\varepsilon_{t,N}$: 300 →55 10^{-6} m
  - ε_L : 1.5→75mm
    - not quite specs
  - Transmission ~ 50%

- **Parameter changes**
  - rf 325 → 10 MHz
  - σ_z : 5 cm →180cm

- **Could be improved with iteration**
  - 40T, induction linac ?





Stages

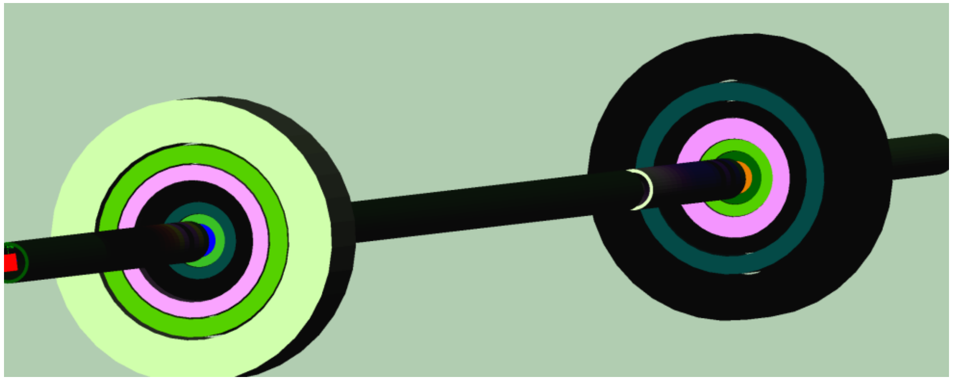
Status

- **Simulation status**
  - ~confirms baseline design
  - needs a bit further optimization / extension
  - uses somewhat extreme components
    - B → 40T; f_{rf} → < 10 MHz (induction linac)

Almost entirely emittance exchange

$\varepsilon_t - \varepsilon_L$ exchange

- **Obtain exchange without cooling hardware ??**



Variant approach: Cool, Round-to-flat, Slice, Recombine (w/ D. Summers, T. Hart)

1. **Cool**
    - Cool until system parameters are difficult
        - $\varepsilon_{x,y}$ (ε_t) →~10^{-4} m, ε_L → ~0.004 m ?
    - set up beam for round to flat transform
2. **Round to flat beam transform**
    - ε_t → ε_x =0.004; ε_y =0.00025 ?
        - method used in ILC source
3. **Slice transversely in large emittance**
    - using “slow extraction-like” septum to form 16 (?) bunches
        - ε_x =0.00025; ε_y =0.00025
4. **Recombine longitudinally at high energy**
    - bunch recombination in 10 GeV storage ring (C. Bhat)
        - ε_x =0.00025; ε_y =0.00025, ε_L=0.07m

1. Cool

- **Start with "final cooling" scenario**
- **Stop at ~step 5 – where parameters are still reasonable..**
  - ε_t ~ 0.0001m
  - ε_L~ ~0.003m
- **Beam is at ~100--135 MeV/c**
  - 66→ 40 MeV kinetic energy
- **No field flips to obtain high-canonical momentum**
  - Nonflip lattices have smaller β^*
  - Small cyclotron mode emittance; Large drift mode
- **No more cooling**

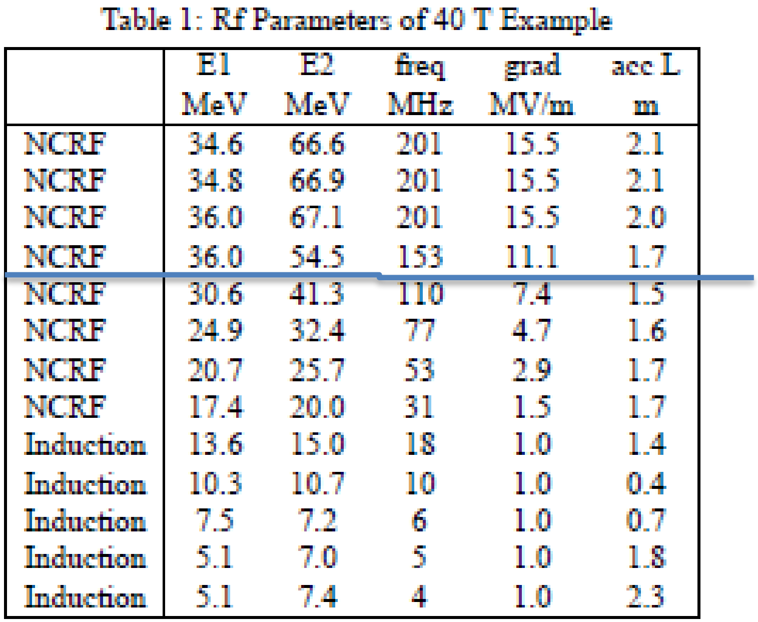
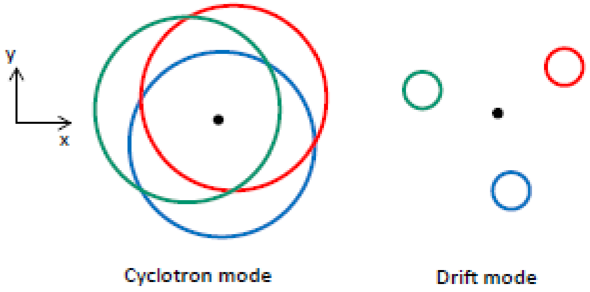


Table 1: Rf Parameters of 40 T Example

	E1 MeV	E2 MeV	freq MHz	grad MV/m	acc L m
NCRF	34.6	66.6	201	15.5	2.1
NCRF	34.8	66.9	201	15.5	2.1
NCRF	36.0	67.1	201	15.5	2.0
NCRF	36.0	54.5	153	11.1	1.7
NCRF	30.6	41.3	110	7.4	1.5
NCRF	24.9	32.4	77	4.7	1.6
NCRF	20.7	25.7	53	2.9	1.7
NCRF	17.4	20.0	31	1.5	1.7
Induction	13.6	15.0	18	1.0	1.4
Induction	10.3	10.7	10	1.0	0.4
Induction	7.5	7.2	6	1.0	0.7
Induction	5.1	7.0	5	1.0	1.8
Induction	5.1	7.4	4	1.0	2.3

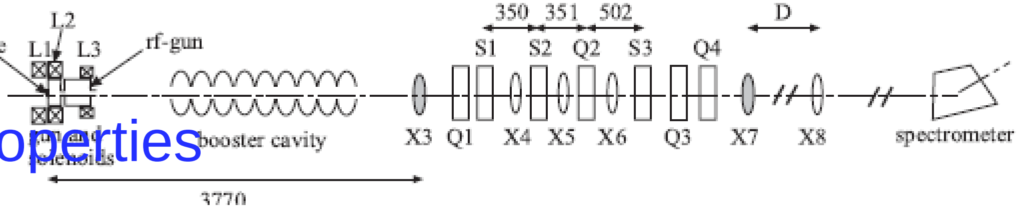


2. Round to Flat beam transform

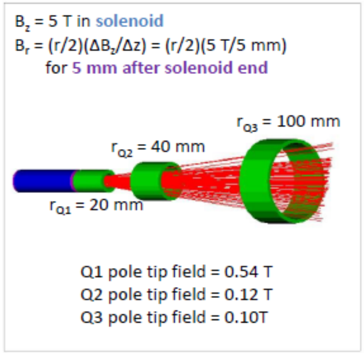
- **Beam has large angular momentum L from non-flip**
  - means beam internally has asymmetric emittance

$$\varepsilon_{4D} = \varepsilon_T^2 = \varepsilon_+ \varepsilon_- = (\varepsilon_P + L)(\varepsilon_P - L)$$

- **Beam is in same format as in electron source**
  - Beam cooled to thermal properties within large B



- **Round to Flat beam transform**
  - Demonstrated at FNAL (electron injector)
  - ~3 skew quads +
  - ε_+, ε_- → ε_x, ε_y

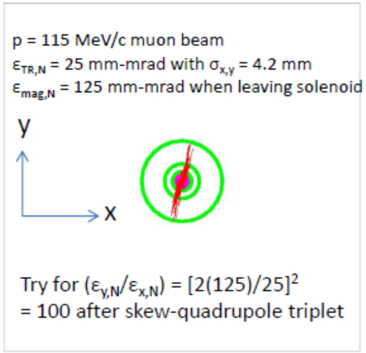


B_z = 5 T in solenoid

$B_r = (r/2)(\Delta B_z/\Delta z) = (r/2)(5\ T/5\ mm)$

for 5 mm after solenoid end

Q1 pole tip field = 0.54 T
Q2 pole tip field = 0.12 T
Q3 pole tip field = 0.10T



p = 115 MeV/c muon beam

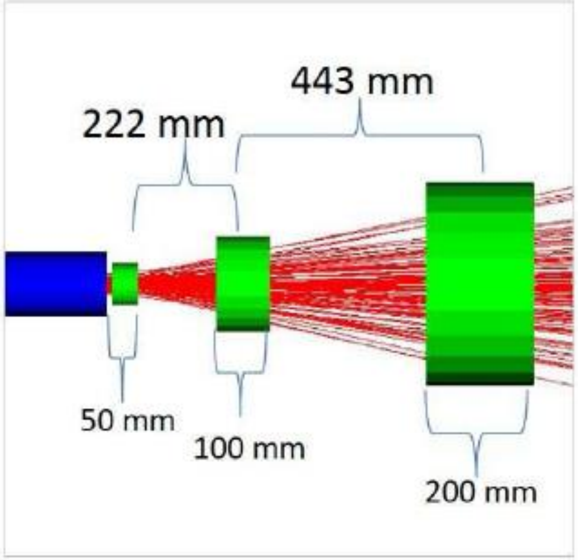
$\varepsilon_{TR,N}$ = 25 mm-mrad with $\sigma_{x,y}$ = 4.2 mm

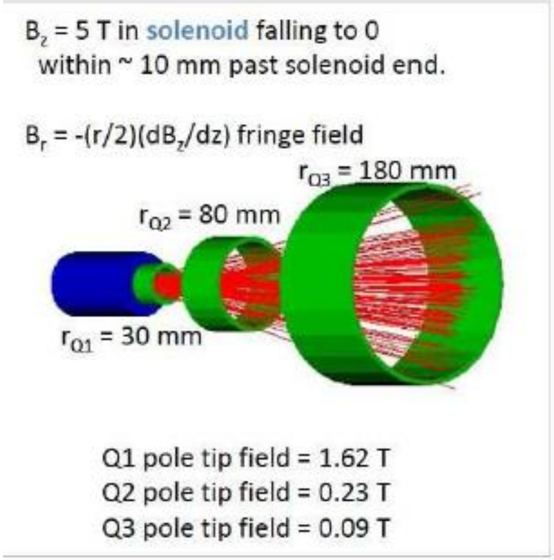
$\varepsilon_{mag,N}$ = 125 mm-mrad when leaving solenoid

Try for $(\varepsilon_{y,N}/\varepsilon_{x,N}) = [2(125)/25]^2$
= 100 after skew-quadrupole triplet

Simulation: Round to flat

- **Simulated at T. Hart at Muon final cooling parameters**
  - 115 MeV/c
  - symmetric emittance within B=5T solenoid

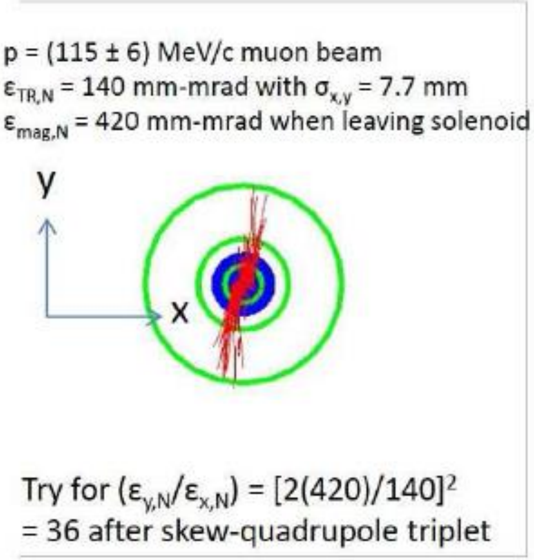




B_z = 5 T in solenoid falling to 0 within ~ 10 mm past solenoid end.

$B_r = -(r/2)(dB_z/dz)$ fringe field

Q1 pole tip field = 1.62 T
Q2 pole tip field = 0.23 T
Q3 pole tip field = 0.09 T



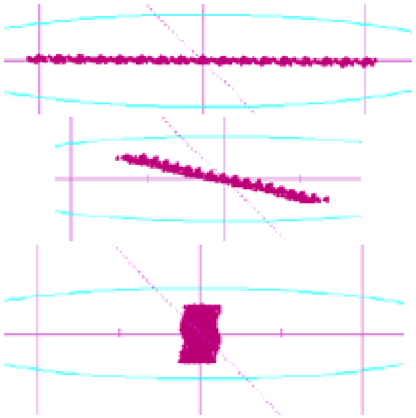
p = (115 ± 6) MeV/c muon beam
$\varepsilon_{TR,N}$ = 140 mm-mrad with $\sigma_{x,y}$ = 7.7 mm
$\varepsilon_{mag,N}$ = 420 mm-mrad when leaving solenoid

Try for $(\varepsilon_{y,N}/\varepsilon_{x,N}) = [2(420)/140]^2$
= 36 after skew-quadrupole triplet

- **Factor of 16 transform ratio:**
  - **$\varepsilon_x \sim 4 \cdot 10^{-3}$ m; $\varepsilon_y \sim 2.5 \cdot 10^{-5}$ m (($\varepsilon_x * \varepsilon_y$ constant)**
  - **$\varepsilon_L \sim 3 \cdot 10^{-2}$ m (unchanged)**

3. Slice transversely & 4. Recombine

- **Flat beam is accelerated to Slicer**
  - – match into slicer optics (~linear or ring)
  - – small storage ring (?) with slow extraction-like optics
    - slicer is electrostatic; slices in large emittance
    - N slices → string of N bunches
- **recombine Longitudinally**
  - – to High Energy Storage ring
    - snap coalescence
    - C Bhat (R. Johnson et al. PAC07)
      - – 21 GeV storage ring, 55μs, 19→1
  - – modeled on pbar coalescence



Variant without Round to Flat transform:

- **1. Cool bunch to ~10^{-4}m ε_T**
  - ~3×10^{-3} ε_L
- **2. Slow extraction slice to 10 bunches:**
  - $10^{-4}\varepsilon_x \times 10^{-5}$m ε_y
  - Separated longitudinally

- **3. Accelerate as bunch train; recombine longitudinally**
  - 10^{-4}m $\varepsilon_x \times 10^{-5}$m ε_y
  - ~3×10^{-2} ε_L
- **Collide as flat beams;**
  - luminosity ~ same as ε_t= ~3×10^{-5}

High Energy Collisions of flat beams

- **IF x-y emittance product same as for baseline (round) Collider scenario**
  - Can obtain ~ same luminosity

- **Flat beam lattice easier to design**
  - **Chromatic correction easier**
    - **10/1 emittance aspect ratio ?**

- **Some disadvantages**

- **Flat beam may be more natural result of cooling with round to flat transform as well**

Round to flat and beam eigenmodes

- Most ionization cooling scenarios use solenoidal focusing
- beam dynamics within solenoids is not x-y
  - more like r – θ (cyclotron – drift)

- Exploring eigenmodes to understand cooling and develop variations

Beam Dynamics: Eigenmodes in solenoid

- **Round to Flat transform requires round beam formation in a solenoid**
- **In solenoid:**
  - Coordinates are x, $p_{x,}$ y, p_y

$$p_x = k_x + \frac{eB}{2c} y \qquad p_y = k_y - \frac{eB}{2c} x$$

- $k_x = m\gamma v_x$

$$\begin{pmatrix} d_x \\ d_y \end{pmatrix} = \begin{pmatrix} x - \frac{c}{eB} k_y \\ y + \frac{c}{eB} k_x \end{pmatrix}$$

References

A. Burov, S. Nagaitsev, A. Shemyakin, PRSTAB 3 094002 (2000)

A. Burov, S. Nagaitsev, Y. Derbenev, Phys. Rev. E 66, 016503 (2002)

K.J. Kim, PRSTAB 6 104002 (2003)

- **Alternative canonical coordinates:**
  - Cyclotron mode

$$\begin{pmatrix} \kappa_1 \\ \kappa_2 \end{pmatrix} = \sqrt{\frac{c}{eB}} \begin{pmatrix} k_y \\ k_x \end{pmatrix} = \sqrt{\frac{c}{eB}} \begin{pmatrix} p_y + \frac{eB}{2c} x \\ p_x - \frac{eB}{2c} y \end{pmatrix}$$

  - Drift mode

$$\begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = \sqrt{\frac{eB}{c}} \begin{pmatrix} d_x \\ d_y \end{pmatrix} = \sqrt{\frac{eB}{c}} \begin{pmatrix} \frac{x}{2} - \frac{c}{eB} p_y \\ \frac{y}{2} + \frac{c}{eB} p_x \end{pmatrix}$$

  - Round to flat transforms
  - (k, d) to (x, y)

Cooling within solenoids

- **Ionization cooling**
  - Absorbers within solenoids
    - Cools k_1, k_2
  - Cyclotron mode is preferentially cooled
  - With

$$\varepsilon_x = \sqrt{\langle x^2 \rangle \langle p_x{}^2 \rangle - \langle x\, p_x \rangle^2}$$

    - and

$$\ell = \frac{1}{2}\langle x p_y - y p_x \rangle$$

then:

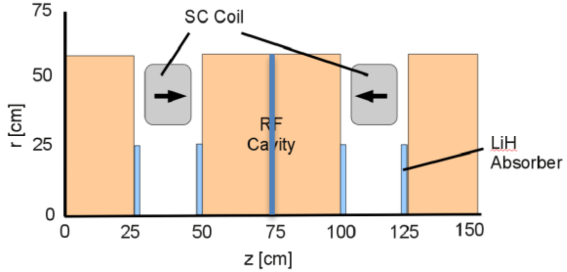
$$\varepsilon_1 \varepsilon_2 = \varepsilon_x \varepsilon_y - \ell^2$$

  - Typically (at $\varepsilon_x = \varepsilon_y = \varepsilon_t$)
    - $\varepsilon_1 \varepsilon_2 = \varepsilon_k\, \varepsilon_c = (\varepsilon_t - \ell)\,(\varepsilon_t + \ell)$

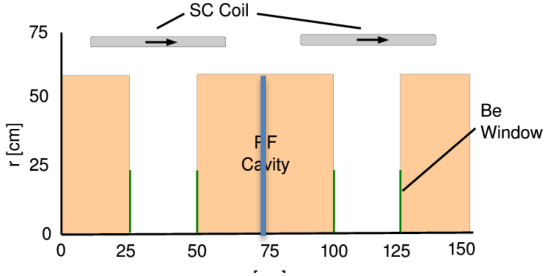
- **With field flips:**
  - k_1, k_2 and d_1, d_2 change identities with each flip
  - Both modes are equally damped
    - Angular momentum is damped
- **Without field flips**
  - One mode is preferentially cooled
  - Canonical angular momentum not damped

Example: Front End Cooling

- **With field flip**

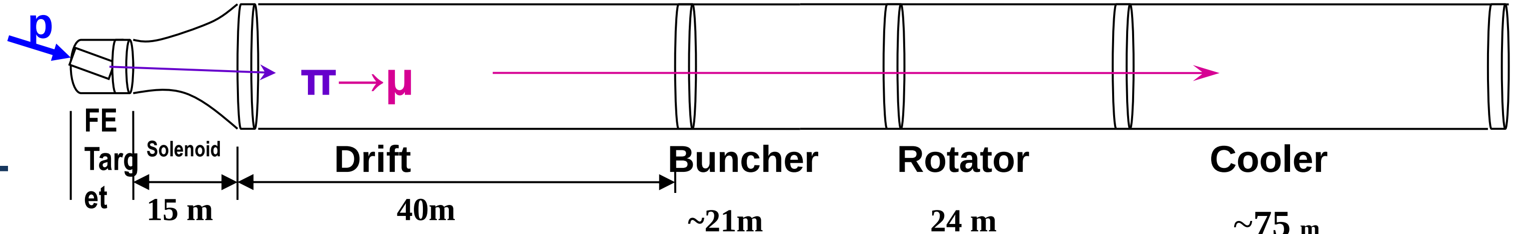


- **Without field flip**



- **75m of cooling:**
  - $\varepsilon_{\perp,N}$: 0.016→0.0064
  - ℓ damped:0.27→0.05
  - x_{rms}, y_{rms} p_{xrms}, p_{yrms} all damped

- **75m of cooling**
  - $\varepsilon_{\perp,N}=(\varepsilon_+ \varepsilon_-)^{1/2}$:0.016→0.0085
  - ℓ increases:0.27→1.44
    - $\varepsilon_+ / \varepsilon_- = \sim 9$
  - k_x, k_y are damped
    - d_x, d_y not damped



Comparison of flip and non-flip cooling

- **Buncher/Rotator ends and Cooling starts at L=102m**
  - obtain “1-D” cooling --- useful in other applications ?

flip mode (~NF) **non-flip mode (B=constant)**

L	ε_t cm	L/ε_t		ε_t	ε_c	ε_d	L/ε_t
102	1.61	0.27		1.61	1.44	2.1	0.27
120	1.21	0.21		1.21	1.08	2.1	0.67
135	1.01	0.18		1.02	0.85	2.2	0.96
150	0.82	0.15		0.89	0.69	2.3	1.21
165	0.71	0.11		0.85	0.58	2.4	1.36
180	0.64	0.08		0.84	0.50	2.4	1.44
195	0.57	0.045		0.81	0.44	2.5	1.54
210	0.54	0.035		0.78	0.38	2.6	1.60

Summary

- **Final Cooling for a high-energy collider**
  - G4Beam line simulation presented
  - Close to desired values
    - does have high-field magnets; very low frequency
    - mostly “emittance exchange”
- **Alternative to “baseline low energy” cooling**
  - emittance exchange from slicing
  - Could use round to flat transform
- **Solenoidal focusing → 1-D cooling**
  - 2-D from field flips; 3-D from emittance exchange

Final Cooling System variations considered …