

Testing the Standard Model with the lepton $g-2$

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Outline

- 1. μ : The muon g-2: a quick theory update
- 2. e : Testing the Standard Model with the electron g-2
- 3. τ : The tau g-2: opportunities or fantasies?

1. The muon $g-2$: a quick theory update

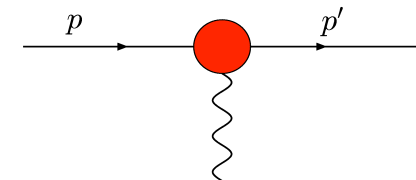
The anomalous magnetic moment: the basics

- The Dirac theory predicts for a lepton $l=e,\mu,\tau$:

$$\vec{\mu}_l = g_l \left(\frac{e}{2m_l c} \right) \vec{s} \quad g_l = 2$$

- QFT predicts deviations from the Dirac value:

$$g_l = 2(1 + a_l)$$



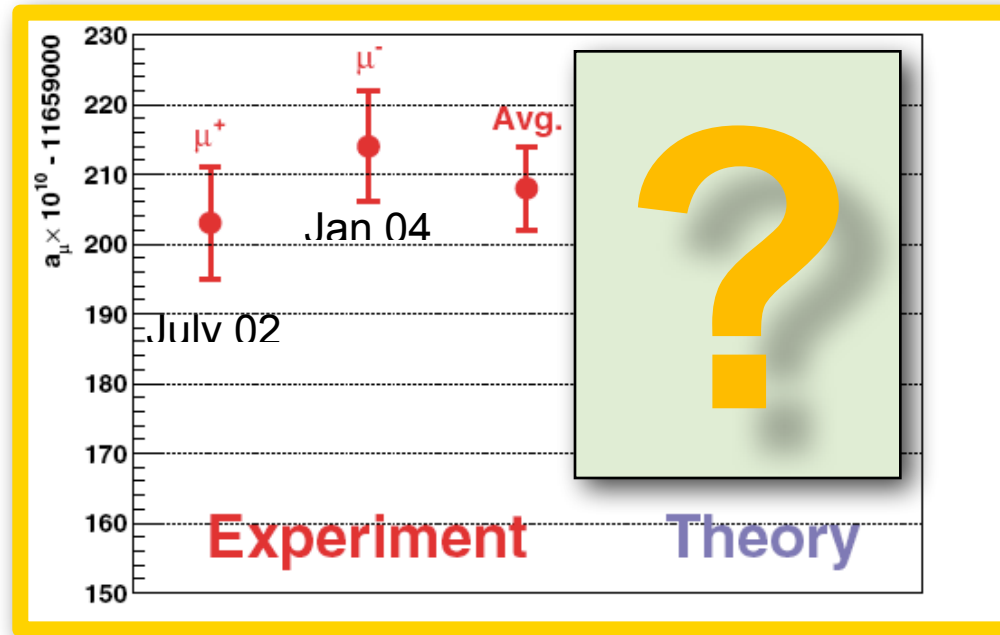
- Study the photon-lepton vertex:

$$\bar{u}(p') \Gamma_\mu u(p) = \bar{u}(p') \left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu} q^\nu}{2m} F_2(q^2) + \dots \right] u(p)$$

$$F_1(0) = 1 \quad F_2(0) = a_l$$

A pure “quantum correction” effect!

The muon g-2: the experimental result



- Today: $a_\mu^{\text{EXP}} = (116592089 \pm 54_{\text{stat}} \pm 33_{\text{sys}}) \times 10^{-11}$ [0.5ppm].
- Future: new muon g-2 experiments proposed at:
 - Fermilab E989, aiming at $\pm 16 \times 10^{-11}$, ie 0.14ppm
 - J-PARC aiming at 0.1 ppmSee G. Grange & K. Ishida's talks.
- Are theorists ready for this (amazing) precision? No(t yet)

The muon g-2: the QED contribution

$$a_{\mu}^{\text{QED}} = (1/2)(\alpha/\pi) \quad \text{Schwinger 1948}$$

$$+ 0.765857426 (16) (\alpha/\pi)^2$$

Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; MP '04

$$+ 24.05050988 (28) (\alpha/\pi)^3$$

Remiddi, Laporta, Barbieri ... ; Czarnecki, Skrzypek; MP '04;
Friot, Greynat & de Rafael '05, Mohr, Taylor & Newell 2012

$$+ 130.8796 (63) (\alpha/\pi)^4$$

Kinoshita & Lindquist '81, ... , Kinoshita & Nio '04, '05;
Aoyama, Hayakawa, Kinoshita & Nio, 2007, Kinoshita et al. 2012,
Steinhauser et al. 2013 (analytic, in progress).

$$+ 753.29 (1.04) (\alpha/\pi)^5 \quad \text{COMPLETED!}$$

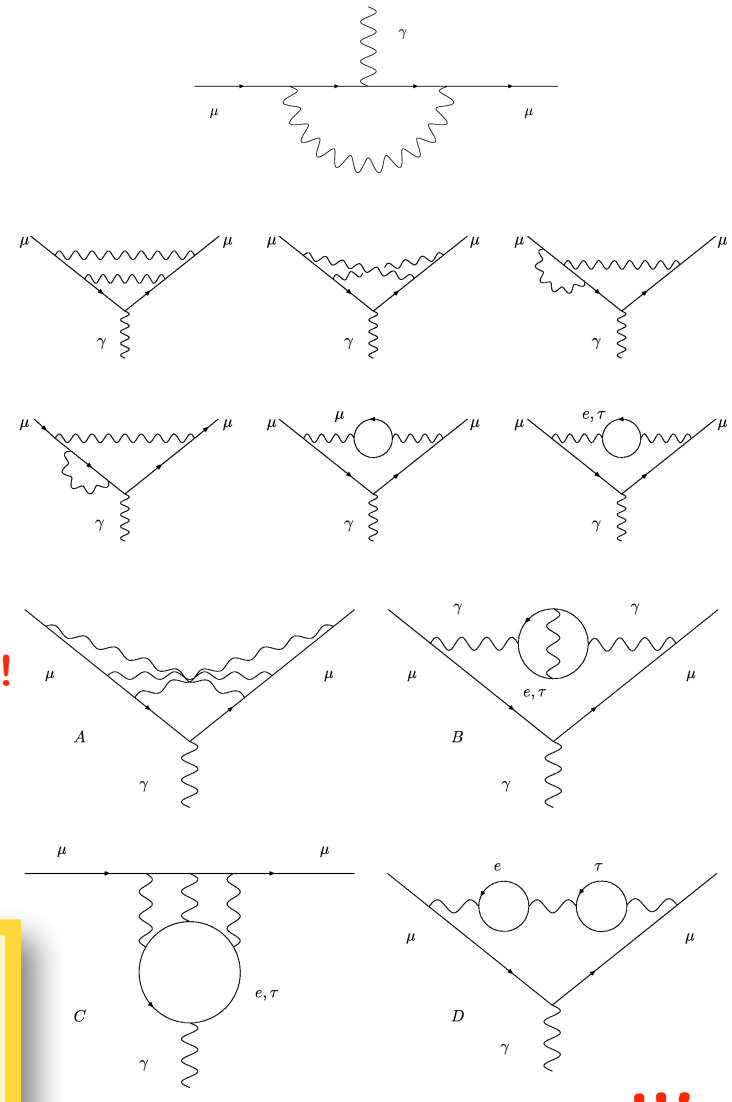
Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta,
Karshenboim, ..., Kataev, Kinoshita & Nio '06, Kinoshita et al. 2012

Adding up, we get:

$$a_{\mu}^{\text{QED}} = 116584718.951 (22)(77) \times 10^{-11}$$

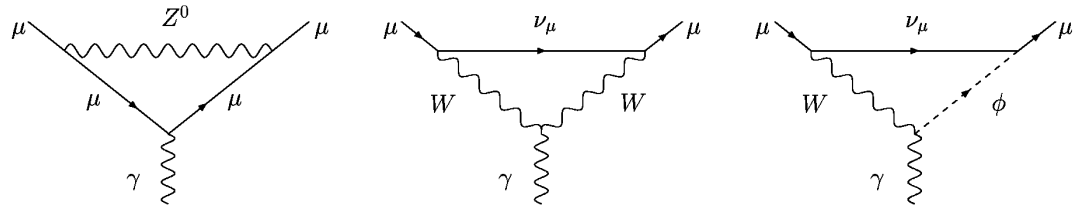
from coeffs, mainly from 4-loop unc \leftarrow \rightarrow from $\delta\alpha(\text{Rb})$

with $\alpha = 1/137.035999049(90)$ [0.66 ppb]



The muon g-2: the electroweak contribution

● One-loop term:



$$a_{\mu}^{\text{EW}}(1\text{-loop}) = \frac{5G_{\mu}m_{\mu}^2}{24\sqrt{2}\pi^2} \left[1 + \frac{1}{5} (1 - 4\sin^2\theta_W)^2 + O\left(\frac{m_{\mu}^2}{M_{Z,W,H}^2}\right) \right] \approx 195 \times 10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda; Studenikin et al. '80s

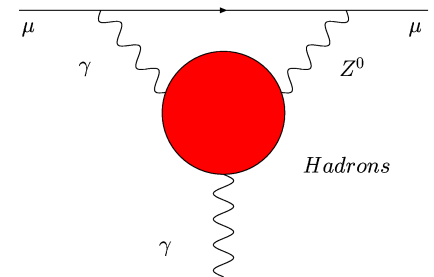
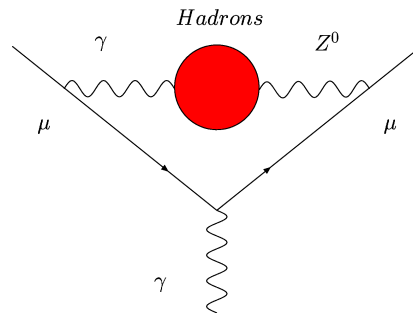
● One-loop plus higher-order terms:

$$a_{\mu}^{\text{EW}} = 153.6 (1) \times 10^{-11}$$

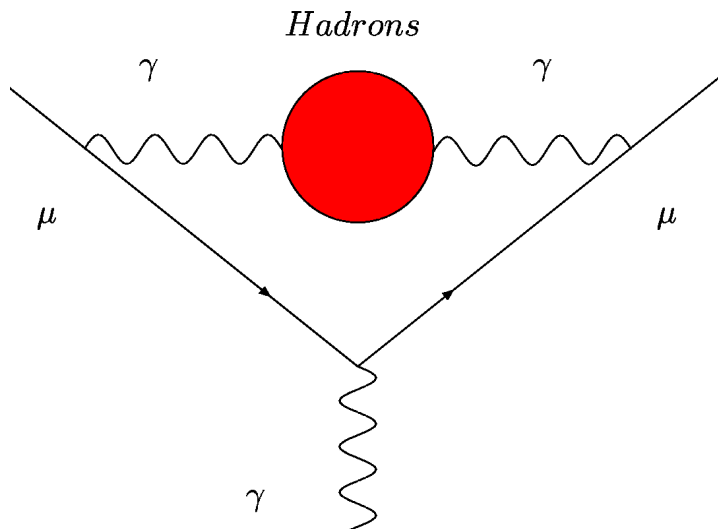
with $M_{\text{Higgs}} = 125.6 (1.5) \text{ GeV}$

Hadronic loop uncertainties and 3-loop nonleading logs.

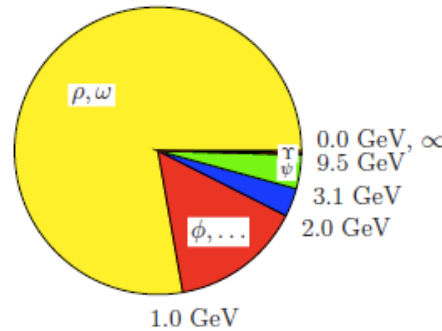
Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrossi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013.



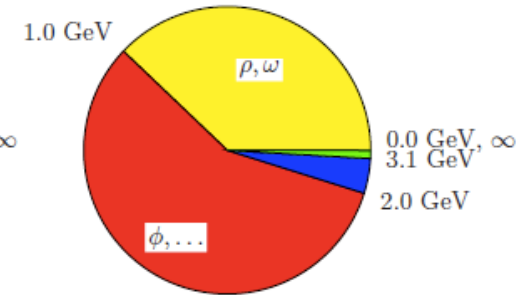
The muon g-2: the hadronic LO contribution (HLO)



Central values



Errors²



F. Jegerlehner and A. Nyffeler, Phys. Rept. 477 (2009) 1

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)(s/m^2)}$$

$$a_\mu^{\text{HLO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^\infty ds K(s) \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^\infty \frac{ds}{s} K(s) R(s)$$

$$a_\mu^{\text{HLO}} = 6903 (53)_{\text{tot}} \times 10^{-11}$$

F. Jegerlehner, A. Nyffeler, Phys. Rept. 477 (2009) 1

$$= 6923 (42)_{\text{tot}} \times 10^{-11}$$

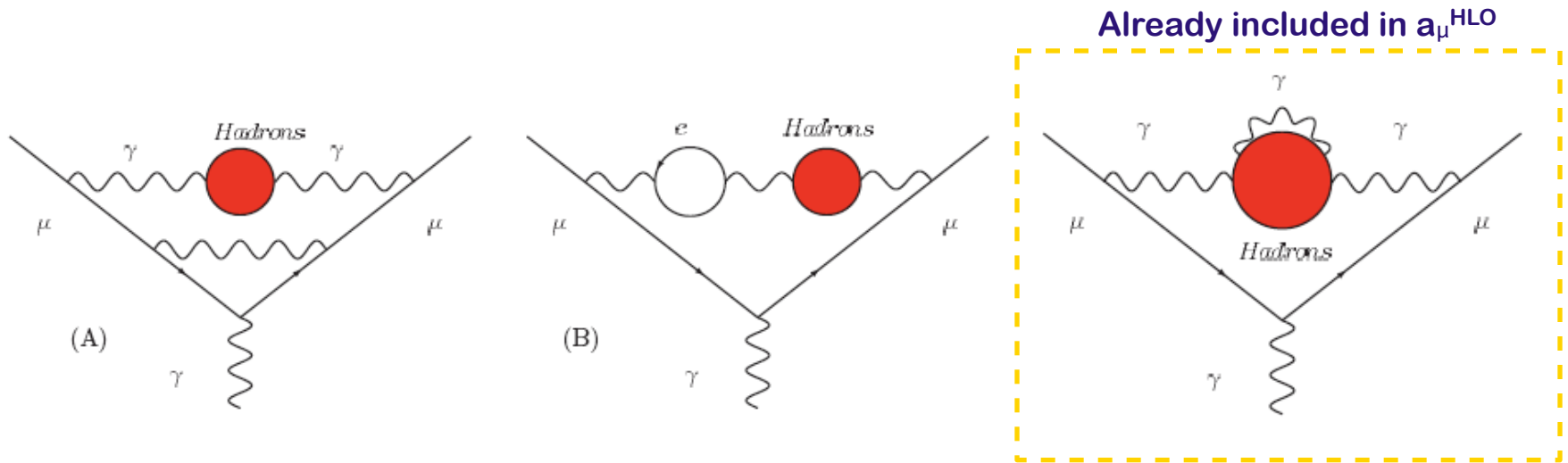
Davier et al, EPJ C71 (2011) 1515 (incl. BaBar & KLOE10 2 π)

$$= 6949 (37)_{\text{exp}} (21)_{\text{rad}} \times 10^{-11}$$

Hagiwara et al, JPG 38 (2011) 085003



- HNLO: Vacuum Polarization**



$O(\alpha^3)$ contributions of diagrams containing hadronic vacuum polarization insertions:

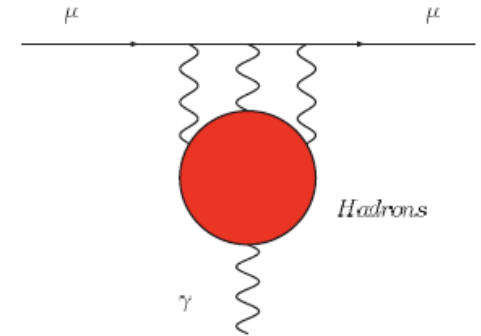
$$a_\mu^{\text{HNLO}}(\text{vp}) = -98 (1) \times 10^{-11}$$

Krause '96, Alemany et al. '98, Hagiwara et al. 2011

● HNLO: Light-by-light contribution

🔗 Unlike the HLO term, the hadronic l-b-l term relies at present on theoretical approaches.

🔗 This term had a troubled life! Latest values:



$a_{\mu}^{\text{HNLO}}(b) = +80 (40) \times 10^{-11}$	Knecht & Nyffeler '02
$a_{\mu}^{\text{HNLO}}(b) = +136 (25) \times 10^{-11}$	Melnikov & Vainshtein '03
$a_{\mu}^{\text{HNLO}}(b) = +105 (26) \times 10^{-11}$	Prades, de Rafael, Vainshtein '09
$a_{\mu}^{\text{HHO}}(b) = +116 (39) \times 10^{-11}$	Jegerlehner & Nyffeler '09

Results based also on Hayakawa, Kinoshita '98 & '02; Bijmens, Pallante, Prades '96 & '02

🔗 **“Bound”** $a_{\mu}^{\text{HNLO}}(|b|) < \sim 160 \times 10^{-11}$ Erler, Sanchez '06, Pivovarov '02; also Boughezal, Melnikov '11

🔗 **Lattice? Very hard... but in progress.** Blum, Chowdhury, Hayakawa, Izubuchi, 1407.2923

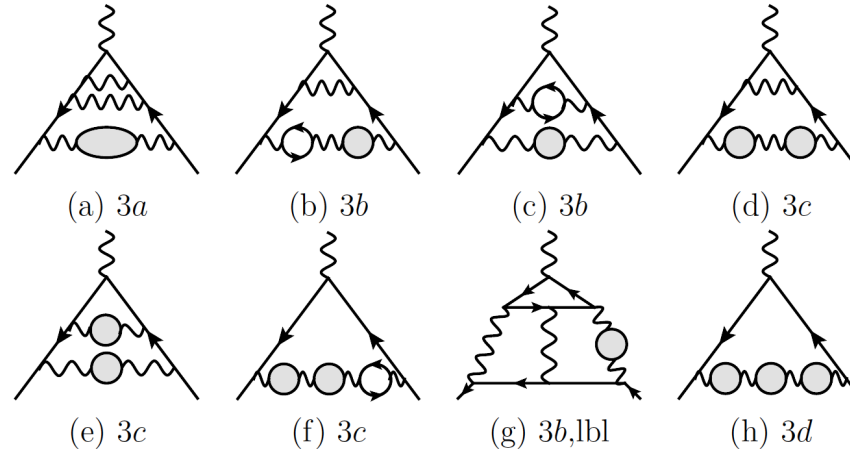
🔗 **Pion exch. in holographic QCD agrees.** D.K.Hong, D.Kim '09; Capiello, Catà, D'Ambrosio '11

🔗 **“By far not complete” calculation: 188×10^{-11}** Fischer et al, PRD87(2013)034013

🔗 **Dispersive approach recently proposed** Colangelo, Hoferichter, Procura, Stoffer 1402.7081

The muon g-2: the hadronic NNLO contributions (HNNLO)

● HNNLO: Vacuum Polarization



$O(\alpha^4)$ contributions of diagrams containing hadronic vacuum polarization insertions:

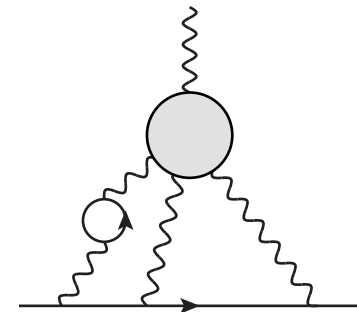
$$a_\mu^{\text{HNNLO}}(\text{vp}) = 12.4 (1) \times 10^{-11}$$

Kurz, Liu, Marquard, Steinhauser 2014

● HNNLO: Light-by-light

$$a_\mu^{\text{HNNLO}}(|b|) = 3 (2) \times 10^{-11}$$

Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014



The muon g-2: SM vs. Experiment

Adding up all contributions, we get the following SM predictions and comparisons with the measured value:

$$a_{\mu}^{\text{EXP}} = 116592091 (63) \times 10^{-11}$$

E821 – Final Report: PRD73 (2006) 072 with latest value of $\lambda=\mu_{\mu}/\mu_{\text{p}}$ from CODATA'10

$a_{\mu}^{\text{SM}} \times 10^{11}$	$\Delta a_{\mu} = a_{\mu}^{\text{EXP}} - a_{\mu}^{\text{SM}}$	σ
116 591 809 (66)	282 (91) $\times 10^{-11}$	3.1 [1]
116 591 829 (57)	262 (85) $\times 10^{-11}$	3.1 [2]
116 591 855 (58)	236 (86) $\times 10^{-11}$	2.8 [3]

with the “conservative” $a_{\mu}^{\text{HNLO}}(|b|) = 116 (39) \times 10^{-11}$ and the LO hadronic from:

- [1] Jegerlehner & Nyffeler, Phys. Rept. 477 (2009) 1
- [2] Davier et al, EPJ C71 (2011) 1515 (includes BaBar & KLOE10 2π)
- [3] Hagiwara et al, JPG38 (2011) 085003 (includes BaBar & KLOE10 2π)

Note that the th. error is now about the same as the exp. one

The muon g-2: connection with the SM Higgs mass

- Δa_μ can be explained by errors in QED, EW, NNLO, g-2 EXP, HLO, or, we hope, by **New Physics!** (See Paradisi's talk)
- Can Δa_μ be due to **hypothetical mistakes in the hadronic $\sigma(s)$** ?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$.
- **Consider:**

$$\begin{aligned} a_\mu^{\text{HLO}} &\rightarrow a = \int_{4m_\pi^2}^{s_u} ds f(s) \sigma(s), & f(s) &= \frac{K(s)}{4\pi^3}, \quad s_u < M_Z^2, \\ \Delta\alpha_{\text{had}}^{(5)} &\rightarrow b = \int_{4m_\pi^2}^{s_u} ds g(s) \sigma(s), & g(s) &= \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)}, \end{aligned}$$

and the increase

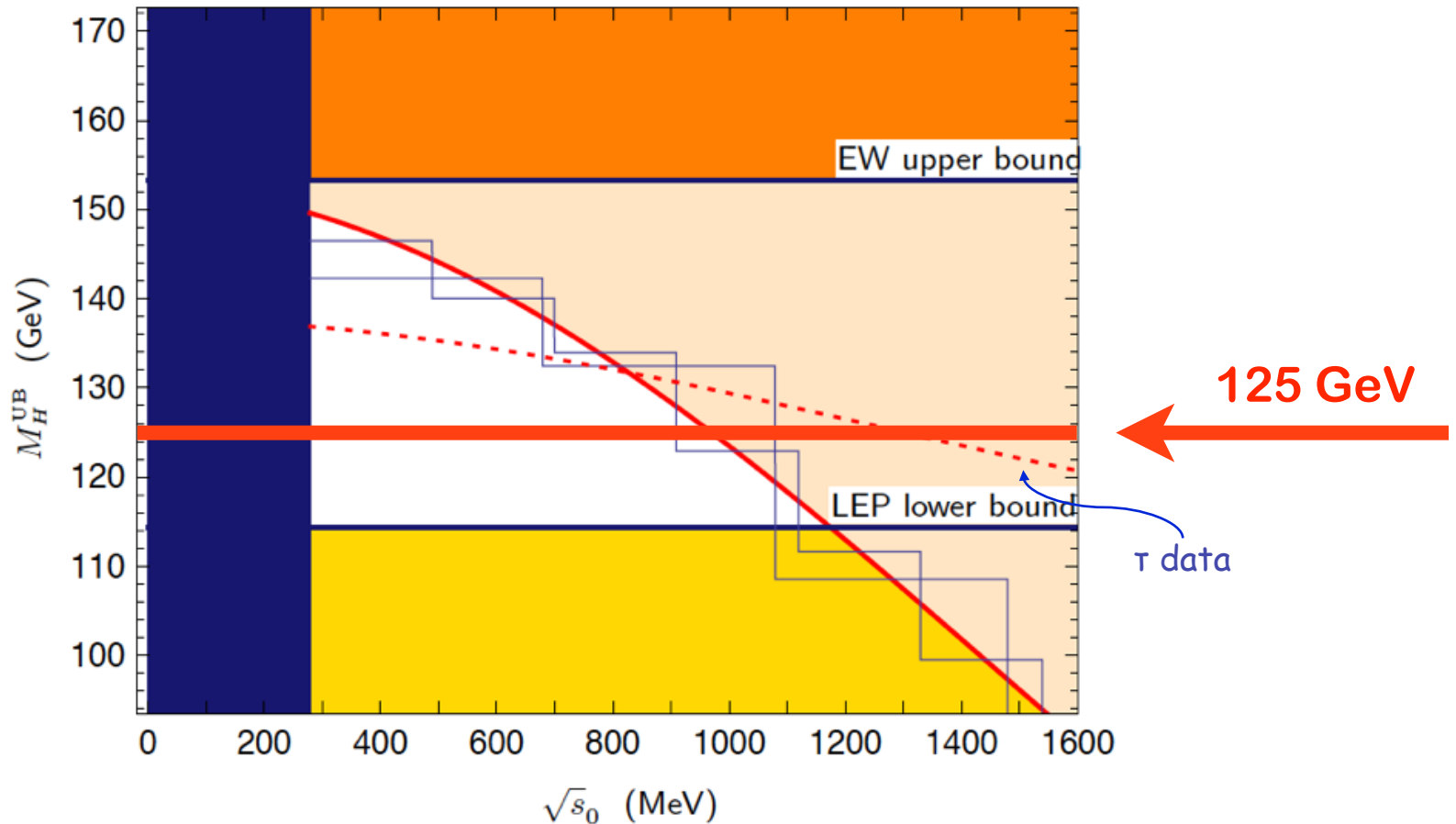
$$\Delta\sigma(s) = \epsilon\sigma(s)$$

($\epsilon > 0$), in the range:

$$\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2] \quad \longrightarrow$$



The muon g-2: connection with the SM Higgs mass (2)

- How much does the M_H upper bound from the EW fit change when we shift $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$] to accommodate Δa_μ ?



W.J. Marciano, A. Sirlin, MP, 2008 & 2010

The muon g-2: connection with the SM Higgs mass (3)

-  Given the quoted exp. uncertainty of $\sigma(s)$, the possibility to explain the muon g-2 with these very large shifts $\Delta\sigma(s)$ appears to be **very unlikely**.
-  Also, given a **125 GeV** SM Higgs, these hypothetical shifts $\Delta\sigma(s)$ could only occur at very low energy (below ~ 1 GeV).

W.J. Marciano, A. Sirlin, MP, 2008 & 2010 (and work in progress)

2. Testing the SM with the electron $g-2$

G.F. Giudice, P. Paradisi & MP, arXiv:1208.6583

M. Fael & MP, arXiv:1402.1575

The QED prediction of the electron g-2

$$a_e^{\text{QED}} = + (1/2)(\alpha/\pi) - 0.328\,478\,444\,002\,55(33)(\alpha/\pi)^2$$

Schwinger 1948 Sommerfeld; Petermann; Suura&Wichmann '57; Elend '66; CODATA Mar '12

$$A_1^{(4)} = -0.328\,478\,965\,579\,193\,78\dots$$

$O(10^{-18})$ in a_e

$$A_2^{(4)}(m_e/m_\mu) = 5.197\,386\,68(26) \times 10^{-7}$$

$$A_2^{(4)}(m_e/m_\tau) = 1.837\,98(33) \times 10^{-9}$$

$$+ 1.181\,234\,016\,816(11)(\alpha/\pi)^3$$

$O(10^{-19})$ in a_e

Kinoshita; Barbieri; Laporta, Remiddi, ... , Li, Samuel; MP '06; Giudice, Paradisi, MP 2012

$$A_1^{(6)} = 1.181\,241\,456\,587\dots$$

$$A_2^{(6)}(m_e/m_\mu) = -7.373\,941\,62(27) \times 10^{-6}$$

$$A_2^{(6)}(m_e/m_\tau) = -6.5830(11) \times 10^{-8}$$

$$A_3^{(6)}(m_e/m_\mu, m_e/m_\tau) = 1.909\,82(34) \times 10^{-13}$$

$$- 1.9097(20)(\alpha/\pi)^4$$

$0.6 \cdot 10^{-13}$ in a_e

Kinoshita & Lindquist '81, ... , Kinoshita & Nio '05; Aoyama, Hayakawa, Kinoshita & Nio 2012; Kurz, Liu, Marquard & Steinhauser 2014: analytic mass dependent part.

$$+ 9.16(58)(\alpha/\pi)^5 \quad \text{Complete Result! (12672 mass indep. diagrams!)}$$

Aoyama, Hayakawa, Kinoshita, Nio, PRL 109 (2012) 111807 (and work in progress to reduce error).

$0.4 \cdot 10^{-13}$ in a_e NB: $(\alpha/\pi)^6 \sim O(10^{-16})$

The SM prediction of the electron g-2

The SM prediction is:

$$a_e^{\text{SM}}(\alpha) = a_e^{\text{QED}}(\alpha) + a_e^{\text{EW}} + a_e^{\text{HAD}}$$

The EW (1&2 loop) term is: Czarnecki, Krause, Marciano '96 [value from CODATA10]

$$a_e^{\text{EW}} = 0.2973(52) \times 10^{-13}$$

The Hadronic contribution (LO+NLO+NNLO) is:

Nomura & Teubner '12, Jegerlehner & Nyffeler '09; Krause'97; Kurz, Liu, Marquard & Steinhauser 2014

$$a_e^{\text{HAD}} = 17.10(17) \times 10^{-13}$$

$$a_e^{\text{HLO}} = +18.66(11) \times 10^{-13}$$

$$a_e^{\text{HNLO}} = [-2.234(14)_{\text{vac}} + 0.39(13)_{\text{lbl}}] \times 10^{-13}$$

$$a_e^{\text{HNNLO}} = +0.28(1) \times 10^{-13}$$

Which value of α should we use to compute a_e^{SM} ?

The electron g-2 gives the best determination of alpha

- The 2008 measurement of the electron g-2 is:

$$a_e^{\text{EXP}} = 11596521807.3 (2.8) \times 10^{-13} \quad \text{Hanneke et al, PRL100 (2008) 120801}$$

vs. old (factor of 15 improvement, 1.8σ difference):

$$a_e^{\text{EXP}} = 11596521883 (42) \times 10^{-13} \quad \text{Van Dyck et al, PRL59 (1987) 26}$$

- Equate $a_e^{\text{SM}}(\alpha) = a_e^{\text{EXP}}$ → best determination of alpha:

$$\alpha^{-1} = 137.035\,999\,177 (34) \quad [0.25 \text{ ppb}]$$

- Compare it with other determinations (independent of a_e):

$$\alpha^{-1} = 137.036\,000\,0 (11) \quad [7.7 \text{ ppb}] \quad \text{PRA73 (2006) 032504 (Cs)}$$

$$\alpha^{-1} = 137.035\,999\,049 (90) \quad [0.66 \text{ ppb}] \quad \text{PRL106 (2011) 080801 (Rb)}$$

Excellent agreement → beautiful test of QED at 4-loop level!

The electron g-2: SM vs. Experiment

- Using $\alpha = 1/137.035\,999\,049\,(90)$ [^{87}Rb , 2011], the SM prediction for the electron g-2 is

$$a_e^{\text{SM}} = 115\,965\,218\,18.1\,(0.6)\,(0.4)\,(0.2)\,(7.6) \times 10^{-13}$$

δC_4^{qed} δC_5^{qed} δa_e^{had} from $\delta\alpha$

- The EXP-SM difference is:

$$\Delta a_e = a_e^{\text{EXP}} - a_e^{\text{SM}} = -10.8\,(8.1) \times 10^{-13}$$

The SM is in very good agreement with experiment (1.3σ).

NB: The 4-loop contrib. to a_e^{QED} is $-556 \times 10^{-13} \sim 70 \delta\Delta a_e!$

(the 5-loop one is 6.2×10^{-13})

The electron g-2 sensitivity and NP tests

- The present sensitivity is $\delta\Delta a_e = 8.1 \times 10^{-13}$, ie (10^{-13} units):

$$(0.6)_{\text{QED4}}, \quad (0.4)_{\text{QED5}}, \quad (0.2)_{\text{HAD}}, \quad (7.6)_{\delta\alpha}, \quad (2.8)_{\delta a_e^{\text{EXP}}}$$

$$(0.7)_{\text{TH}} \quad \leftarrow \text{may drop to 0.2 or 0.3}$$

- The $(g-2)_e$ exp. error may soon drop below 10^{-13} and work is in progress for a significant reduction of that induced by $\delta\alpha$.

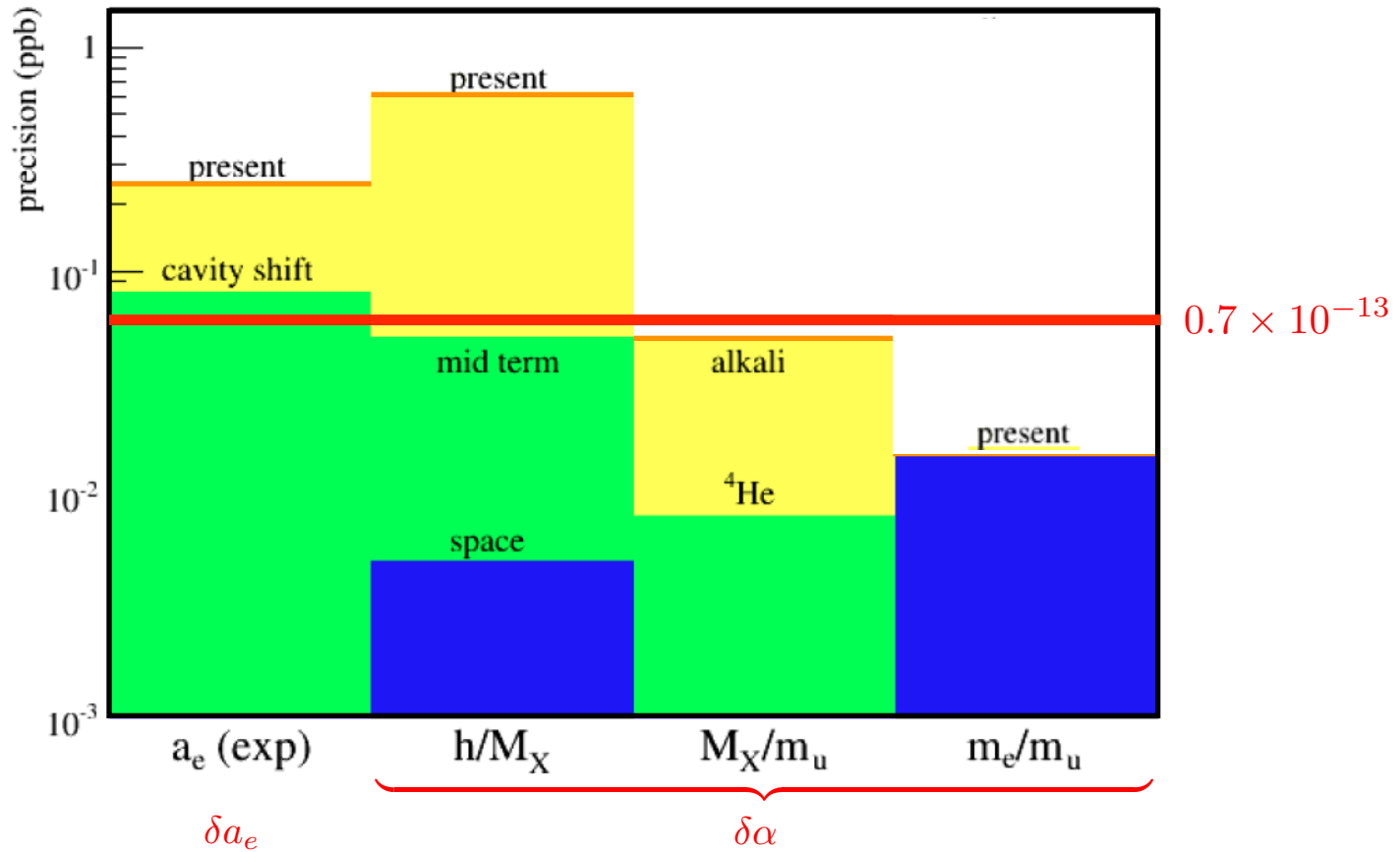
→ sensitivity of 10^{-13} may be reached with ongoing exp. work

- In a broad class of BSM theories, contributions to a_l scale as

$$\frac{\Delta a_{\ell_i}}{\Delta a_{\ell_j}} = \left(\frac{m_{\ell_i}}{m_{\ell_j}} \right)^2 \quad \text{This Naive Scaling leads to:}$$

$$\Delta a_e = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 0.7 \times 10^{-13}; \quad \Delta a_\tau = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 0.8 \times 10^{-6}$$

The electron g-2 sensitivity and NP tests (2)



Summary of the contributions to the relative precision in a_e (in ppb)

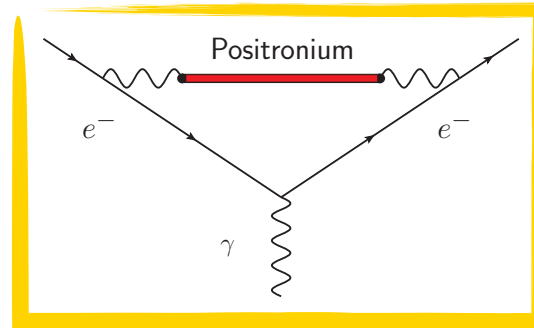
F. Terranova & G.M. Tino, arXiv:1312.2346

- The experimental sensitivity in Δa_e is not far from what is needed to **test if the discrepancy in $(g-2)_\mu$ also manifests itself in $(g-2)_e$** under the naive scaling hypothesis.
- NP scenarios exist which **violate Naive Scaling**. They can lead to larger effects in Δa_e (& Δa_τ) and contributions to EDMs, LFV or lepton universality breaking observables. See **Calibbi and Paradisi's talk**.

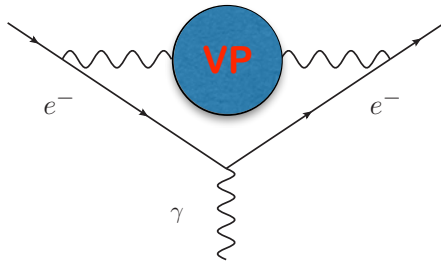


● The leading contribution of positronium to a_e comes from:

Mishima 1311.7109; Fael & MP 1402.1575; Melnikov et al. 1402.5690; Eides 1402.5860; Hayakawa 1403.0416



● The e^+e^- bound states appear as poles in the vac. pol. $\Pi(q^2)$ right below the branch-point $q^2 = (2m)^2$. Their contribution is:



$$\Rightarrow a_e(\text{vp}) = \frac{\alpha}{\pi^2} \int_0^\infty \frac{ds}{s} \text{Im} \Pi(s + i\epsilon) K(s)$$

$$a_e^{\text{P}} = \frac{\alpha^5}{4\pi} \zeta(3) \left(8 \ln 2 - \frac{11}{2} \right) = 0.9 \times 10^{-13} = 1.3 \left(\frac{\alpha}{\pi} \right)^5$$

Mishima 1311.7109

● Of the same magnitude of the exp. unc. of a_e & the naively rescaled muon Δa_μ . Of the same order of α as the 5-loop term!



- Melnikov, Vainshtein & Voloshin (MVV) 1402.5690 determined a nonpert. contrib. of the e^+e^- continuum right above threshold that cancels one-half of a_e^P :

$$a_e(\text{vp})^{\text{cont,np}} = -\frac{|\alpha|^5}{8\pi} \zeta(3) \left(8 \ln 2 - \frac{11}{2} \right)$$

- In fact the **total positronium poles + continuum** nonperturbative contribution to a_e arising from the threshold region at LO in α is:

$$a_e^{\text{thr}}(\text{vp}) = -\frac{\alpha}{\pi} K(4m^2) \text{Re} A(1)$$

with

$$A(\beta) = -\frac{\alpha^2}{2} \left[\gamma + \psi \left(1 - \frac{i\alpha}{2\beta} \right) \right] = \frac{\alpha^2}{2} \sum_{k=1}^{\infty} \zeta(k+1) \left(\frac{i\alpha}{2\beta} \right)^k$$

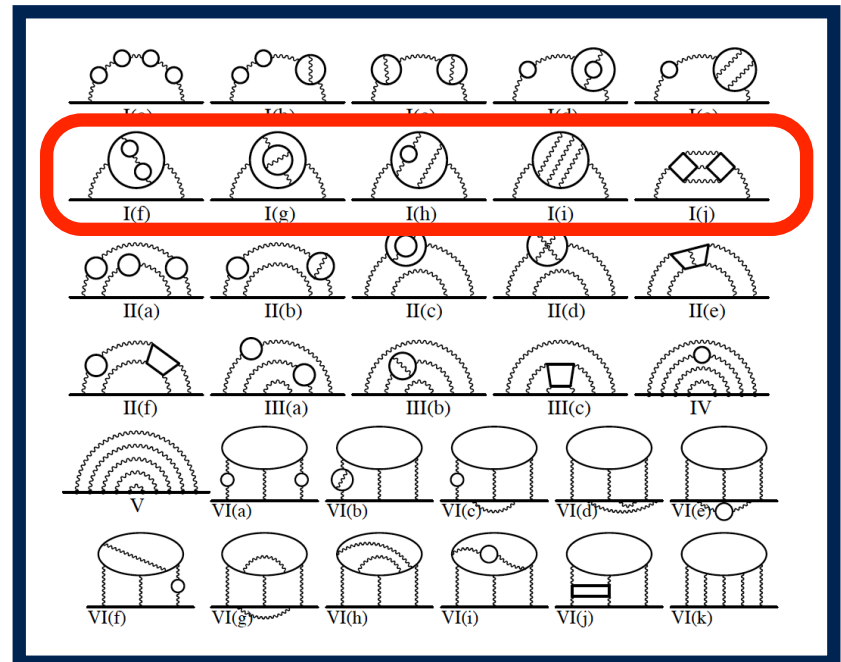
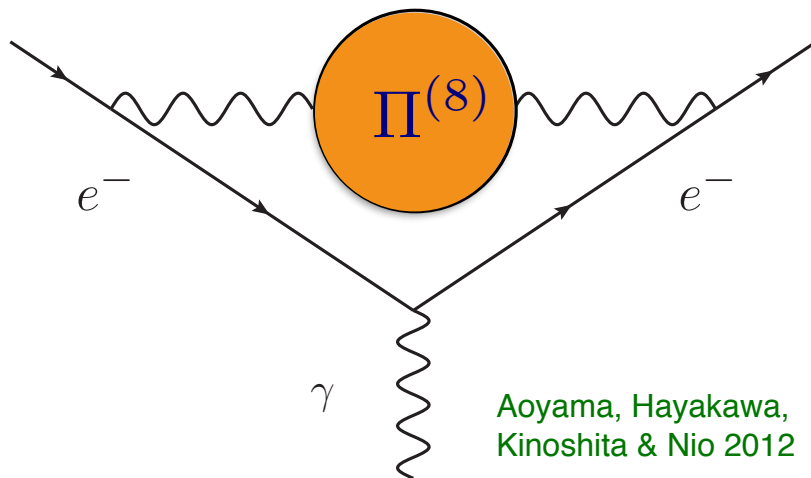
so that

$$a_e^{\text{thr}}(\text{vp}) = \frac{\alpha^5}{8\pi} \zeta(3) K(4m^2) = \frac{a_e^P}{2}$$

Is there a positronium contribution to the electron $g-2$? (III)



- So, should we add this total threshold contribution $a_e^P/2$ to the perturbative QED 5-loop result of Kinoshita and collaborators?
- Using the Coulomb Green's function, MVV 1402.5690 argued that it is already contained in the contribution of $O(\alpha^5)$.
- Hayakawa 1403.0416 claimed that positronium contributes to a_e only through a specific class of diagrams of $O(\alpha^7)$.
- The 5-loop QED contribution to a_e arising from the insertion of the 4-loop VP in the photon line has been computed via:

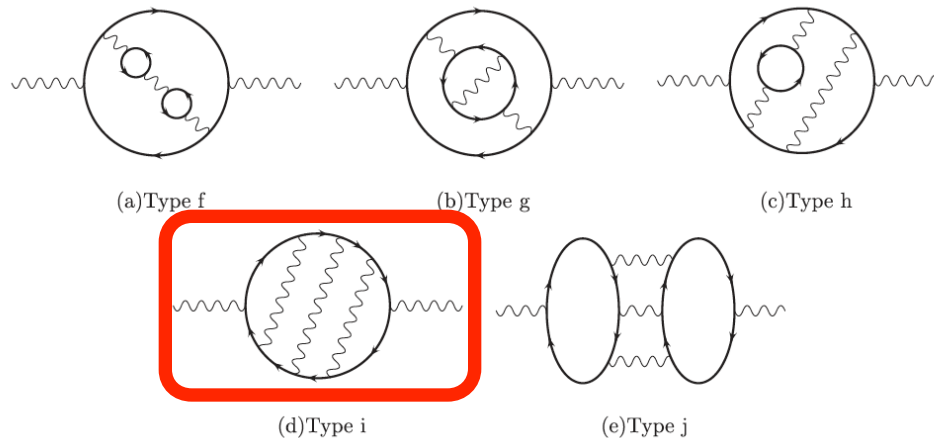




- Using explicit expressions for $\Pi^{(8)}(q^2)$ (Baikov, Maier, Marquard '13) we obtain:

$$a_e^{(10)}(\text{vp}) = -\frac{\alpha}{\pi} \int_0^1 dx (1-x) \Pi^{(8)}\left(-\frac{m^2 x^2}{1-x}\right)$$

$$a_e^{(10)}(\text{vp}) = n_e \frac{\alpha^5}{8\pi} \zeta(3) K(4m^2) + \dots = \frac{a_e^{\text{P}}}{2} + \dots$$



- $a_e^{\text{P}}/2$ is already included in the 5-loop contrib. of class I(i).
- There is no additional contrib of QED bound states beyond PT!

M.A. Braun 1968; Barbieri, Christillin, Remiddi 1973

3. The tau $g-2$: opportunities or fantasies?

Work in progress in collaboration with
S. Eidelman, D. Epifanov, M. Fael, L. Mercolli

arXiv:1301.5302

arXiv:1310.1081

The SM prediction of the tau g-2

The Standard Model prediction of the tau g-2 is:

$$\begin{aligned} a_{\tau}^{\text{SM}} &= 117324 \quad (2) && \times 10^{-8} && \text{QED} \\ &+ 47.4 \quad (0.5) && \times 10^{-8} && \text{EW} \\ &+ 337.5 \quad (3.7) && \times 10^{-8} && \text{HLO} \\ &+ 7.6 \quad (0.2) && \times 10^{-8} && \text{HHO (vac)} \\ &+ 5 \quad (3) && \times 10^{-8} && \text{HHO (lbl)} \end{aligned}$$

$$a_{\tau}^{\text{SM}} = 117721 \quad (5) \times 10^{-8}$$

Eidelman & MP
2007

$(m_{\tau}/m_{\mu})^2 \sim 280$: great opportunity to look for New Physics, and a “clean” NP test too...

	Muon	Tau
$a_{\text{EW}}/a_{\text{H}}$	1/45	1/7
$a_{\text{EW}}/\delta a_{\text{H}}$	3	10

... if only we could measure it!!

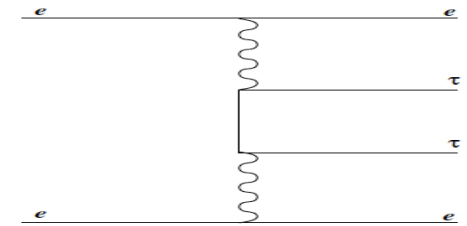
The tau g-2: experimental bounds

- The very short lifetime of the tau makes it very difficult to determine a_τ measuring its spin precession in a magnetic field.

- DELPHI's result, from $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$ total cross-section measurements at LEP 2 (the PDG value):

$$a_\tau = -0.018 (17)$$

PDG 2012



- With an effective Lagrangian approach, using data on tau lepton production at LEP1, SLC, and LEP2:

$$-0.004 < a_\tau^{\text{NP}} < 0.006 \quad (95\% \text{ CL})$$

Escribano & Massó 1997

$$-0.007 < a_\tau^{\text{NP}} < 0.005 \quad (95\% \text{ CL})$$

González-Sprinberg et al 2000

- Bernabéu et al, propose the measurement of $F_2(q^2=M_\gamma^2)$ from $e^+e^- \rightarrow \tau^+\tau^-$ production at B factories. NPB 790 (2008) 160

The tau g-2 via its radiative leptonic decays: a proposal

- **Tau radiative leptonic decays at LO:**

$$\frac{d^3\Gamma}{dx dy d\cos\theta} = \frac{\alpha M_\tau^5 G_F^2 y \sqrt{x^2 - 4r^2}}{2\pi(4\pi)^6} G_0(x, y, \cos\theta, r)$$

Kinoshita & Sirlin PRL2(1959)177; Kuno & Okada, RMP73(2001)151

$$\left. \frac{\Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau \gamma)}{\Gamma_{\text{total}}} \right|_{E_\gamma > 10\text{MeV}} = 1.836\% \quad \text{vs} \quad 1.75(18)\%$$

$$\left. \frac{\Gamma(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau \gamma)}{\Gamma_{\text{total}}} \right|_{E_\gamma > 10\text{MeV}} = 0.367\% \quad \text{vs} \quad 0.361(38)\%$$

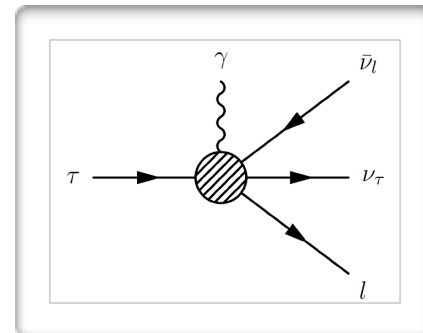
CLEO 2000

- **Add the contribution of the effective coupling and the QED corrections:**

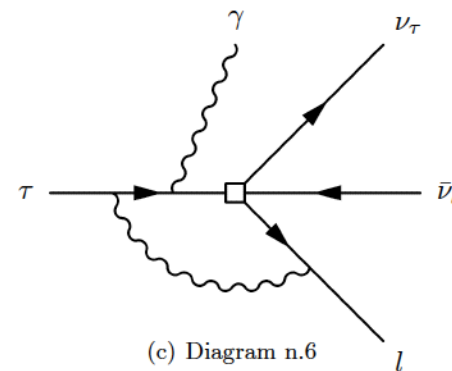
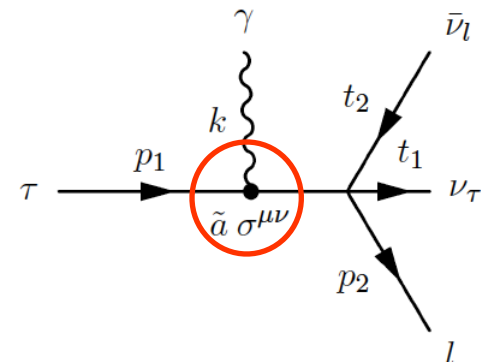
$$G_0 \rightarrow G_0 + \tilde{a}_\tau G_a + \frac{\alpha}{\pi} G_{\text{RC}}$$

- **Measure $d^3\Gamma$ precisely and get \tilde{a}_τ !**

[see also Laursen, Samuel, Sen, PRD29 (1984) 2652]



$$x = \frac{2E_l}{M_\tau}, \quad y = \frac{2E_\gamma}{M_\tau}, \quad r = \frac{m_l}{M_\tau}$$



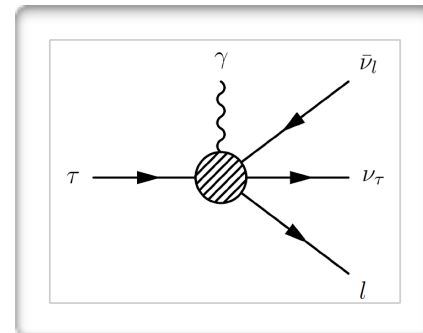
(c) Diagram n.6

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Kinoshita & Sirlin PRL2(1959)177; Kuno & Okada, RMP73(2001)151



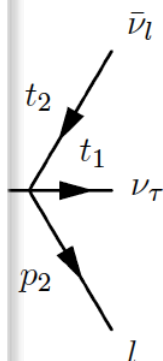
$\Gamma(\tau^- \rightarrow \nu_\tau^- \gamma)$

$\Gamma(\tau^- \rightarrow \nu_\tau^- l \bar{\nu}_l)$

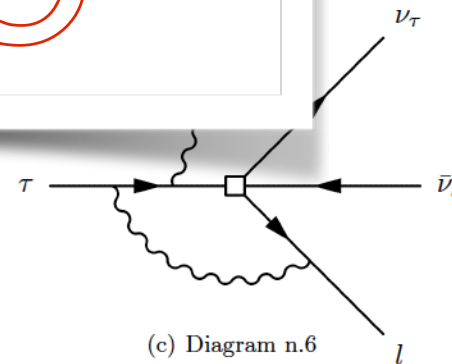
WORK IN PROGRESS

- **Add couplings**

$$r = \frac{m_l}{M_\tau}$$



- **Measure $d^3\Gamma$ precisely and get \tilde{a}_τ !**
[see also Laursen, Samuel, Sen, PRD29 (1984) 2652]



(c) Diagram n.6

Conclusions

- The lepton $g-2$ provide beautiful examples of interplay between theory and experiment.
- The muon discrepancy is $\Delta a_\mu \sim 3\div 3.5 \sigma$. Is it NP? New upcoming $g-2$ experiment: QED & EW terms ready for the challenge; how about the hadronic one? Future of LBL??
- Could Δa_μ be due to **mistakes in the hadronic $\sigma(s)$** ? Very unlikely. Also, given a 125 GeV SM Higgs, these hypothetical shifts $\Delta\sigma(s)$ could only occur at very low energies (below $\sim 1\text{GeV}$).
- The sensitivity of the electron $g-2$ has improved. It may soon be possible to **test if Δa_μ manifests itself also in the electron $g-2$** ! A robust and ambitious experimental program is under way to improve α & a_e . The **positronium** contribution shouldn't be added.
- The tau $g-2$ is essentially unknown: we propose to measure it at Belle II via its radiative leptonic decays.

The End