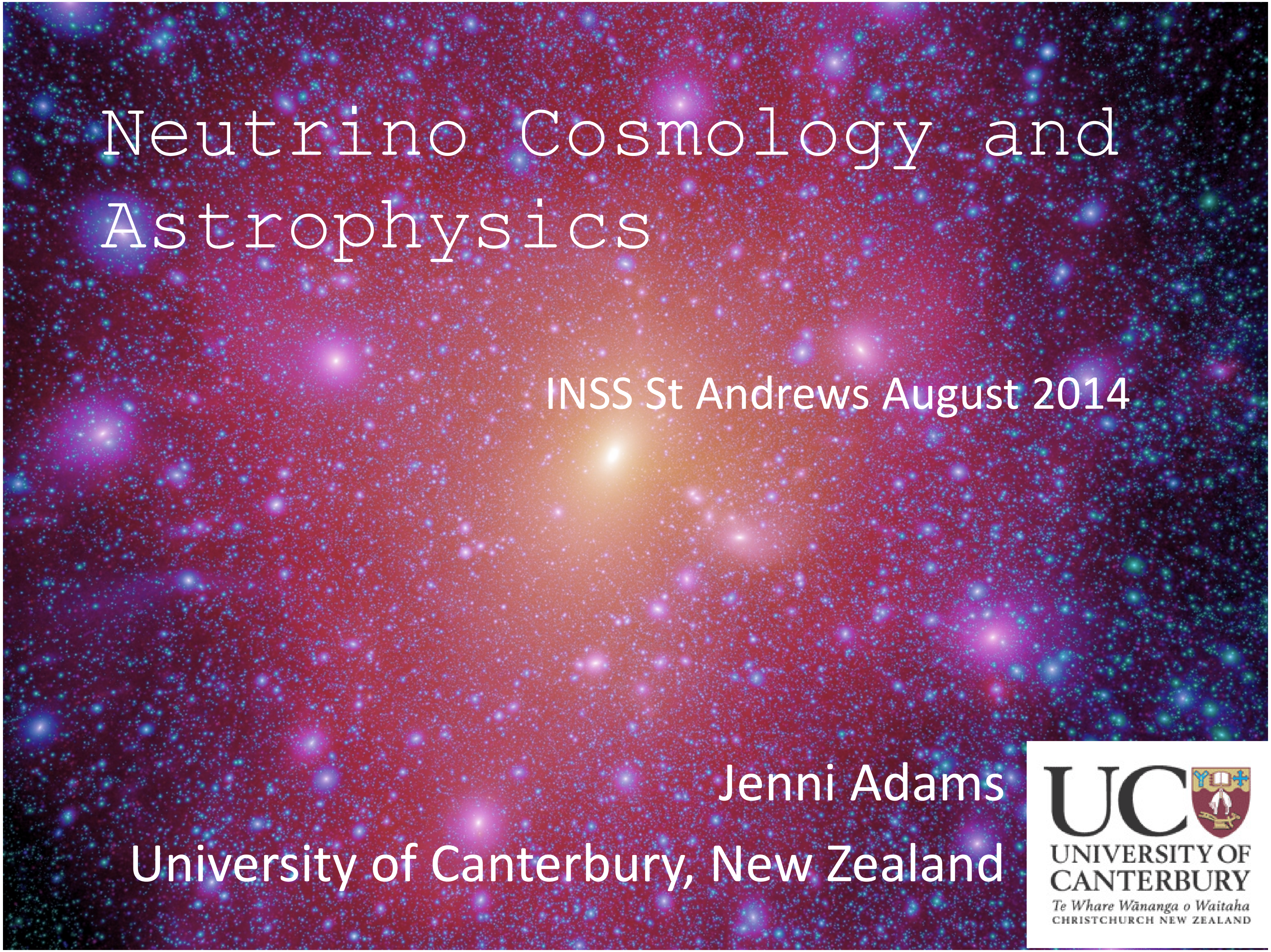

Neutrino Cosmology and Astrophysics

INSS St Andrews August 2014

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Neutrino Background

Thermal Radiation

	General	Bosons	Fermions
Number density n	$g\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{e^{E_{\boldsymbol{p}}/T}\pm 1}$	$g_B\frac{\zeta_3}{\pi^2}T^3$	$\frac{3}{4}\,g_F\frac{\zeta_3}{\pi^2}T^3$
Energy density ρ	$g\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{E_{\boldsymbol{p}}}{e^{E_{\boldsymbol{p}}/T}\pm 1}$	$g_B\frac{\pi^2}{30}T^4$	$\frac{7}{8}\,g_F\frac{\pi^2}{30}T^4$
Pressure P	$g\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{\lvert\boldsymbol{p}^2\rvert}{E_p}\frac{1}{e^{E_{\boldsymbol{p}}/T}\pm 1}$	$\frac{\rho}{3}$	
Entropy density s	$\frac{\rho+P}{T}=\frac{4}{3}\frac{\rho}{T}$	$g_B\frac{2\pi^2}{45}T^3$	$\frac{7}{8}\,g_F\frac{2\pi^2}{45}T^3$

↕

$$\mathrm{d}E = T\mathrm{d}S - P\mathrm{d}V$$
$$T\mathrm{d}S = (\rho + P)dV$$

using integrals

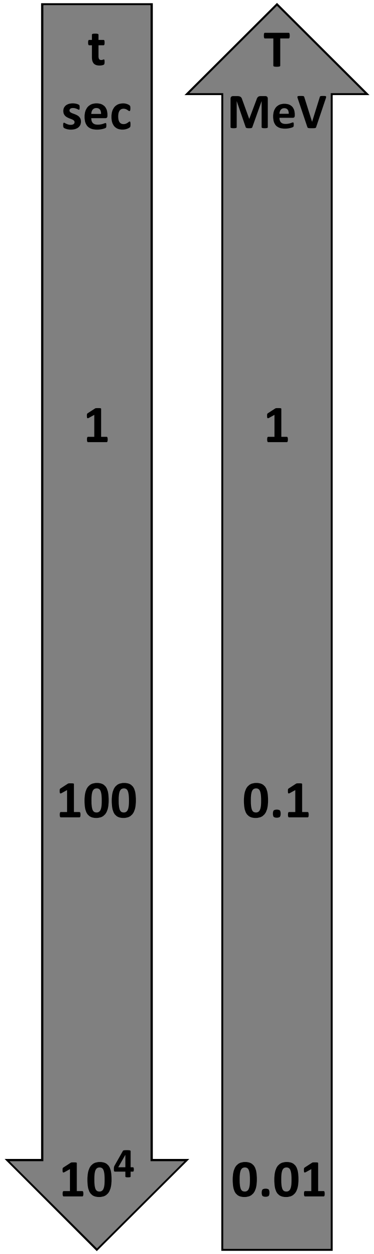
$$\int_0^\infty \frac{x^2dx}{\exp(x)-1} = 2\zeta(3),$$
$$\int_0^\infty \frac{x^2dx}{\exp(x)+1} = \frac{6}{8}\zeta(3),$$
$$\int_0^\infty \frac{x^3dx}{\exp(x)-1} = 6\zeta(4) = \frac{\pi^4}{15},$$
$$\int_0^\infty \frac{x^3dx}{\exp(x)+1} = \frac{7}{48}\zeta(4) = \frac{7}{8}\frac{\pi^4}{15}$$

Riemann Zeta Function
$\zeta = 1.2020569\ldots$

Present-Day Neutrino Density

Neutrino decoupling (freeze out)	$H \sim \Gamma$ $T \approx 2.4\ \text{MeV}$ (electron flavour) $T \approx 3.7\ \text{MeV}$ (other flavours)
Redshift of Fermi-Dirac distribution ("nothing changes at freeze-out")	$\frac{dn_{\nu\bar{\nu}}}{dE} = \frac{1}{\pi^2}\frac{E^2}{e^{E/T}+1}$ Temperature scales with redshift $T_\nu = T_\gamma \propto (z+1)$
Electron-positron annihilation beginning at T ≈ m_e = 0.511 MeV	• Entropy of e^+e^- transfered to photons $g_* T_\gamma^3 \big\vert_{\text{before}} = g_* T_\gamma^3 \big\vert_{\text{after}}$ $\left.\overbrace{2+\frac{7}{8}4=\frac{11}{2}} \qquad \overbrace{2}\ \right\} T_\gamma^3 \big\vert_{\text{before}} = \frac{\mathbf{4}}{\mathbf{11}} T_\gamma^3 \big\vert_{\text{after}}$
Redshift of neutrino and photon thermal distributions so that today we have	$n_{\nu\bar{\nu}}(1\ \text{flavor}) = \frac{4}{11} \times \frac{3}{4} \times n_\gamma = \frac{3}{11} n_\gamma \approx 112\ \text{cm}^{-3}$ $T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \approx 1.95\ \text{K}$ for massless neutrinos

Helium Synthesis



β equilibrium	$p + e^- \leftrightarrow n + \nu_e$ $p + \bar{\nu}_e \leftrightarrow n + e^+$	$\frac{n}{p} = \exp\left(-\frac{m_n - m_p}{T}\right) \approx 1$ $m_n - m_p = 1.293\ \mathrm{MeV}$
β freeze-out	$\frac{n}{p} \approx \frac{1}{6}$	**β rates fall below expansion rate H**
- Neutron decay - Nuclear statist. equilibrium	$n_n \propto \exp\left(-\frac{t}{\tau_n}\right)$ $\tau_n = 880\ \mathrm{sec}\ (?)$	$2\mathrm{p} + 2\mathrm{n} \leftrightarrow {}^4\mathrm{He} + \gamma$ **Helium suppressed by large entropy**
Neutrons freeze in helium	$\frac{n}{p} \approx \frac{1}{7}$	**Helium mass fraction $Y_P \approx 25\%$**
Thermonuclear reaction chains	**Production and destruction of traces of D, ^{3}He, ^{6}Li, ^{7}Li, ^{7}Be**	
All done		

Neutrinos and Big Bang Nucleosynthesis

- Universe is radiation dominated
- The presence of neutrinos in radiation affects the expansion rate

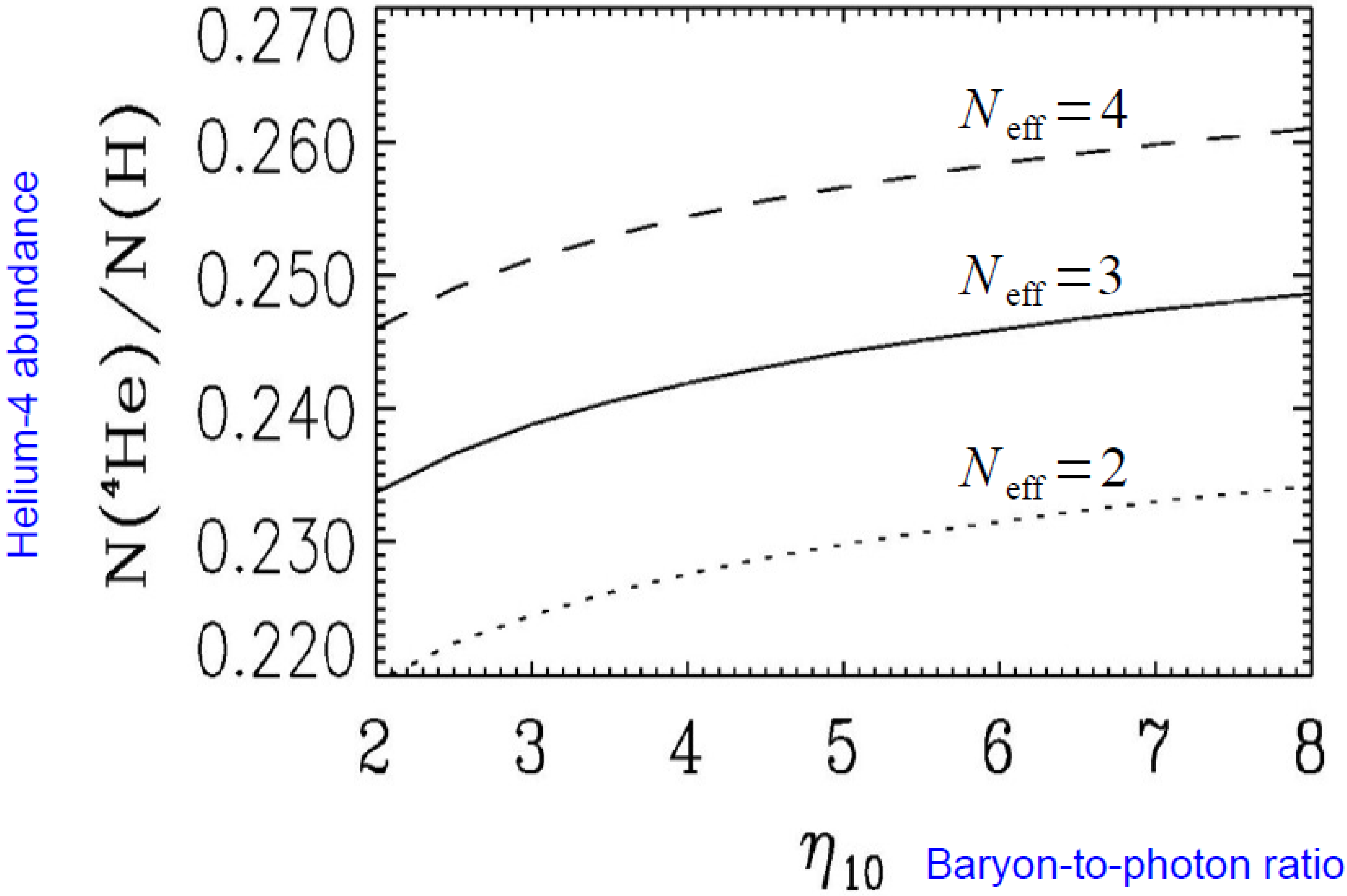
$$\mathrm{H}^2 = \frac{8\pi}{3}\frac{\rho}{m_{\mathrm{Pl}}^2} \quad \Longleftarrow$$

- Higher expansion rate means higher freezeout temperature and greater n/p ratio

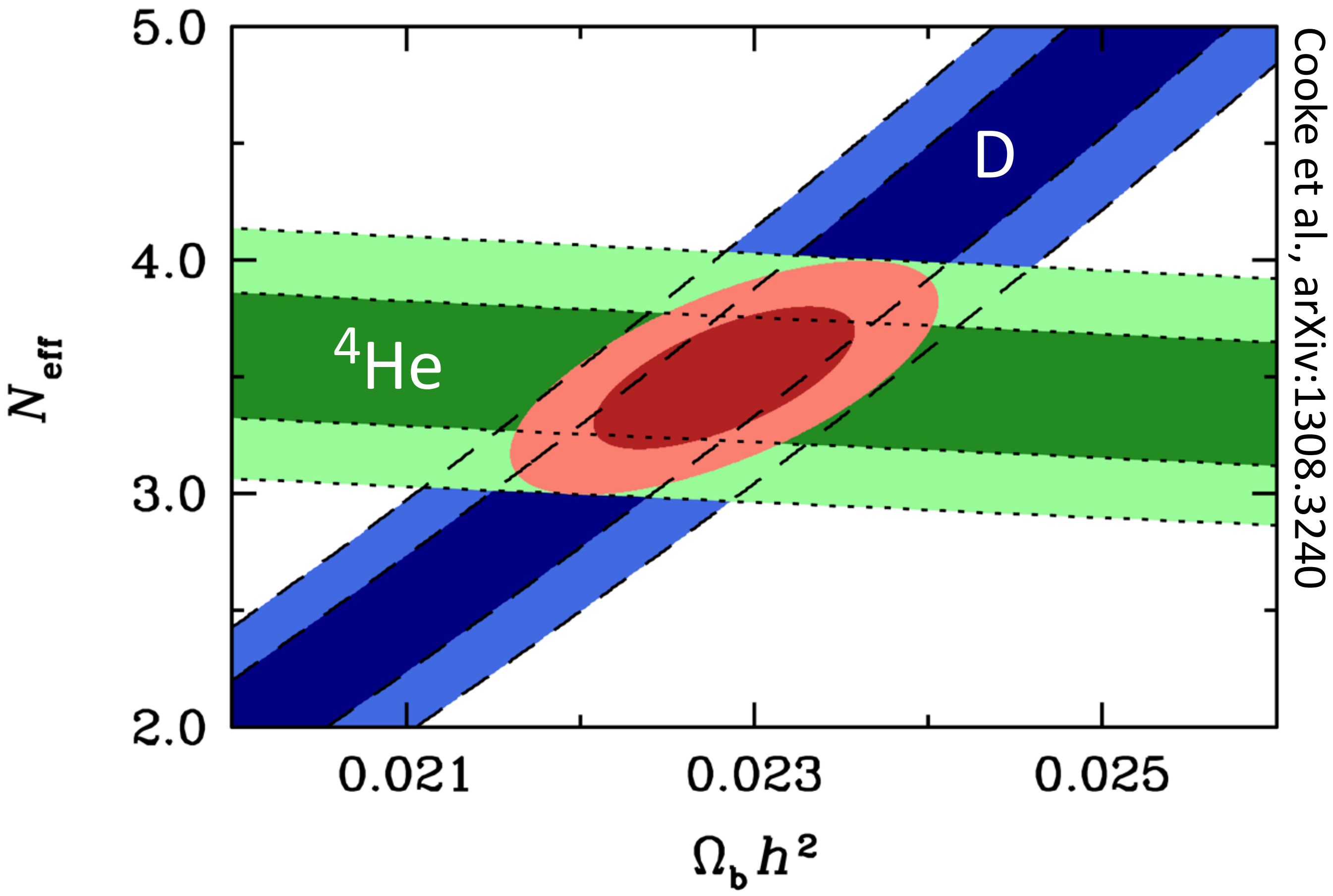
- Neutrino asymmetry would also affect the weak interactions (come back to this)

$$\begin{aligned} \bar{\nu}_e + p &\rightarrow e^+ + n \\ \nu_e + n &\rightarrow e^- + p \end{aligned}$$

Helium-4 is most sensitive to N_{eff}



Baryon and Radiation Density from BBN



Cooke et al., arXiv:1308.3240

D abundance from Cook et al. (2013) and He-4 from Izotov et al. (2013)

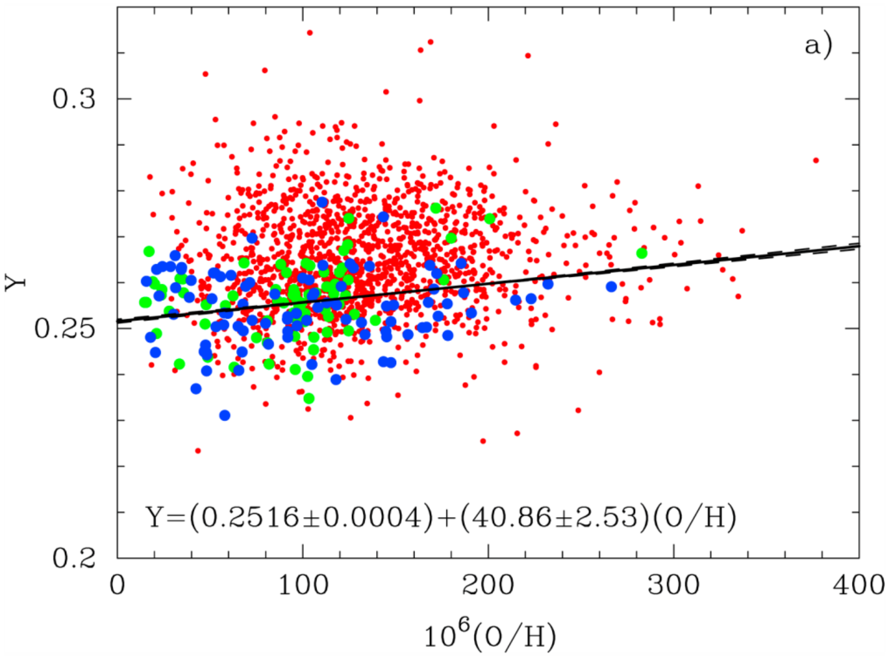
BBN hint for extra radiation (evidence driven by He abundance)

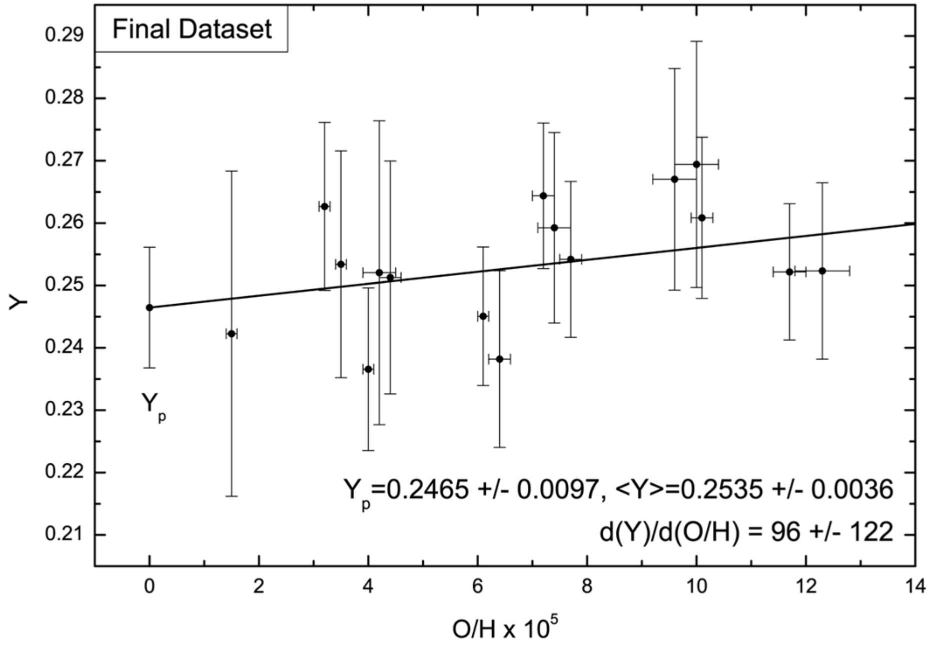
Helium Mass Fraction from HII Regions

Extrapolation to zero metalicity in many HII regions

Izotov, Stasinska & Guseva
arXiv:1308.2100

Aver, Olive, Porter & Skillman
arXiv:1309.0047



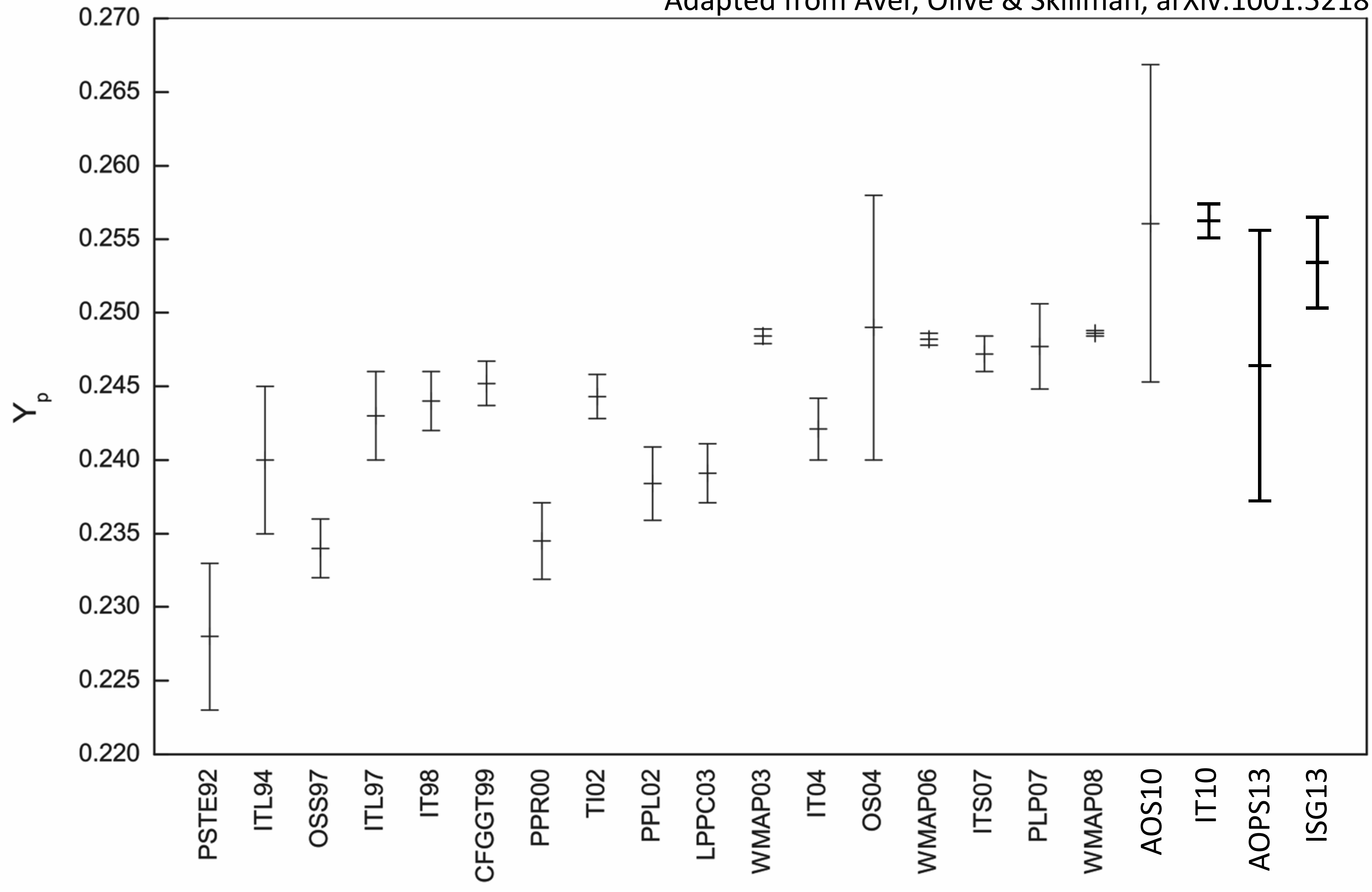


$$Y_P = 0.254 \pm 0.003$$

$$Y_P = 0.2465 \pm 0.0097$$

Progression of Best-Fit Helium Abundance

Adapted from Aver, Olive & Skillman, arXiv:1001.5218



Thermal Neutrino Distribution

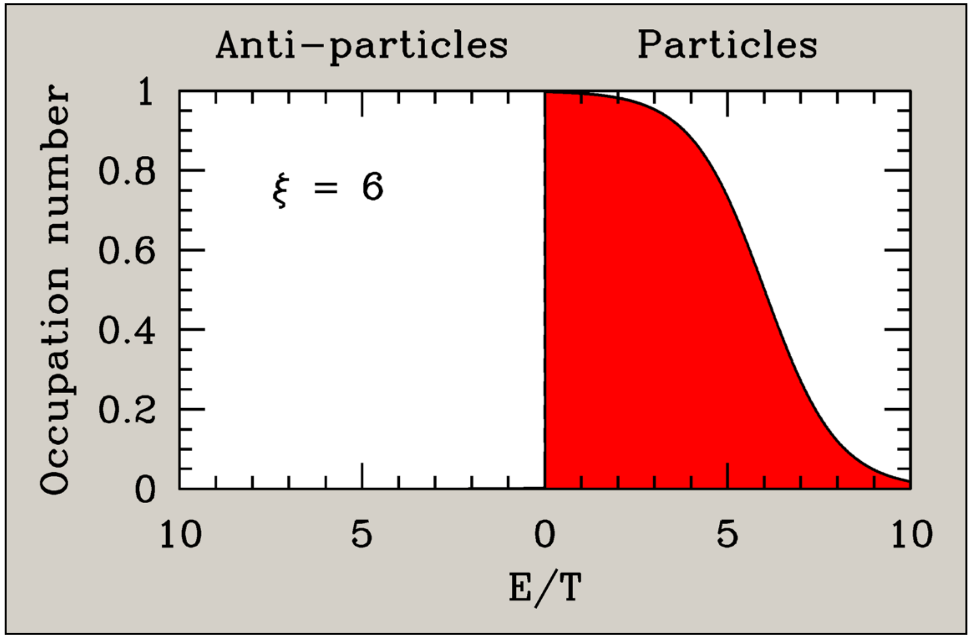
Fermi-Dirac distribution

- Temperature T
- Chemical potential μ

$\mu > 0$ Particles

$\mu < 0$ Anti-particles

$$f_p = \frac{1}{e^{(E_p - \mu)/T} + 1}$$



Degeneracy parameter $\xi = \frac{\mu}{T}$ Invariant under cosmic expansion

Difference in number density

$$n_\nu - n_{\bar{\nu}} = \int dE\, \frac{4\pi}{(2\pi)^3} \left(\frac{E^2}{1 + e^{E/T - \xi}} - \frac{E^2}{1 + e^{E/T + \xi}} \right)$$

$$= \frac{1}{6\pi^2} T^3 [\pi^2 \xi + \xi^3 + \cdots]$$

Thermal Neutrino Distribution

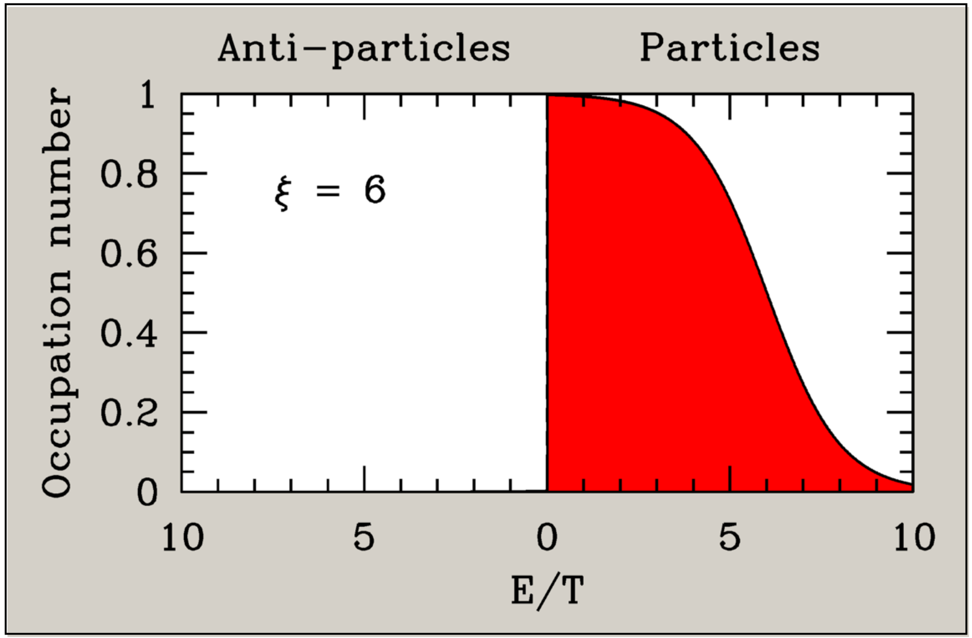
Fermi-Dirac distribution

- Temperature T
- Chemical potential μ

  $\mu > 0$ Particles

  $\mu < 0$ Anti-particles

$$f_{\boldsymbol{p}} = \frac{1}{e^{(E_{\boldsymbol{p}}-\mu)/T}+1}$$



Degeneracy parameter $\xi = \frac{\mu}{T}$ Invariant under cosmic expansion

Number density

$$n_{\nu\bar{\nu}} = \int dE\,\frac{4\pi}{(2\pi)^3}\left(\frac{E^2}{1+e^{E/T-\xi}}+\frac{E^2}{1+e^{E/T+\xi}}\right)$$

$$= \frac{3}{2\pi^2}T^3\left[\zeta_3+\frac{2\ln(2)}{3}\xi^2+\frac{\xi^4}{72}+\cdots\right]$$

BBN and Neutrino Chemical Potentials

Expansion rate effect (all flavors)

Energy density in one neutrino flavor with degeneracy parameter $\xi = \eta/T$

$$\rho_{\nu\bar{\nu}} = \frac{7\pi^2}{120}\, T_\nu^4 \left[1 + \underbrace{\frac{30}{7}\left(\frac{\xi}{\pi}\right)^2 + \frac{15}{7}\left(\frac{\xi}{\pi}\right)^4}_{\Delta N_{\rm eff}}\right]$$

Beta equilibrium effect for electron flavor

$n + \nu_e \leftrightarrow p + e^-$

$p + \bar{\nu}_e \leftrightarrow n + e^+$

Helium abundance essentially fixed by n/p ratio at beta freeze-out

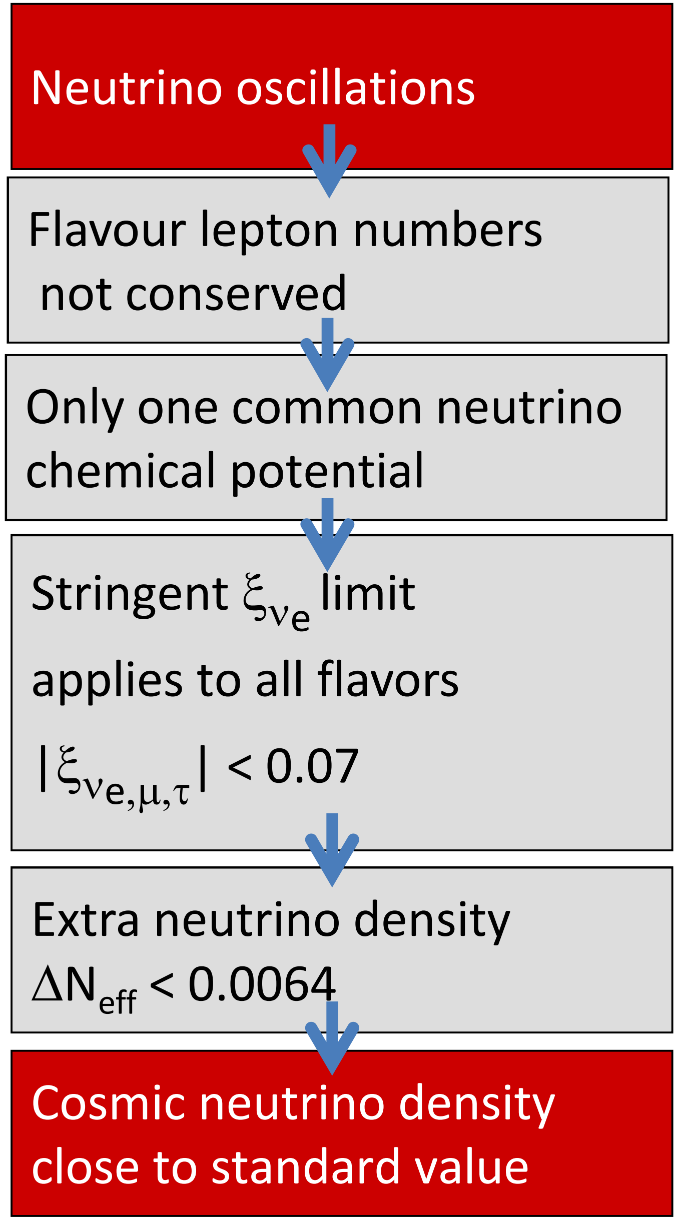
$$\frac{n}{p} = e^{-(m_n - m_p)/T_{\rm F} - \xi_{\nu_e}}$$

Effect on helium equivalent to

$$\Delta N_{\rm eff} \sim -18\, \xi_{\nu_e}$$

- ν_e beta effect can compensate expansion-rate effect of $\nu_{\mu,\tau}$
- Naively, BBN limit only applies to ξ_{ν_e}
- However, flavor oscillations equalize chemical potentials before BBN

Chemical Potentials and Flavour Oscillations



Neutrino oscillations

Flavour lepton numbers not conserved

Only one common neutrino chemical potential

Stringent ξ_{ν_e} limit applies to all flavors $|\xi_{\nu_{e,\mu,\tau}}| < 0.07$

Extra neutrino density $\Delta N_{\mathrm{eff}} < 0.0064$

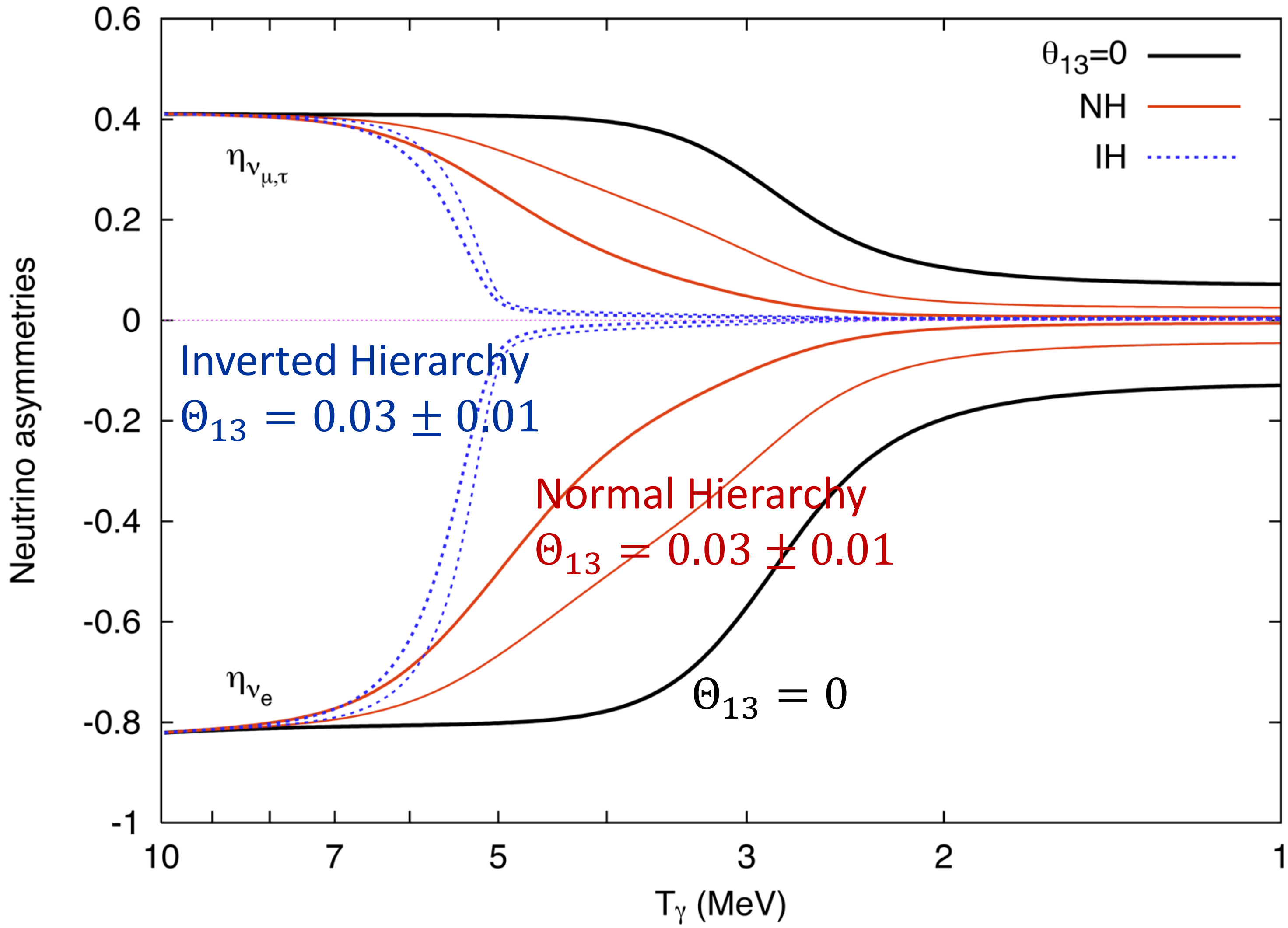
Cosmic neutrino density close to standard value

Flavour equilibrium before n/p freeze out assured because no mixing angle small

Our knowledge of the cosmic neutrino density depends on measured oscillation parameters!

arXiv:hep-ph/0012056 , hep-ph/0201287, astro-ph/0203442, hep-ph/0203180, arXiv:0808.3137, 1011.0916, 1110.4335

Flavor Conversion before BBN (Θ_{13} not small)

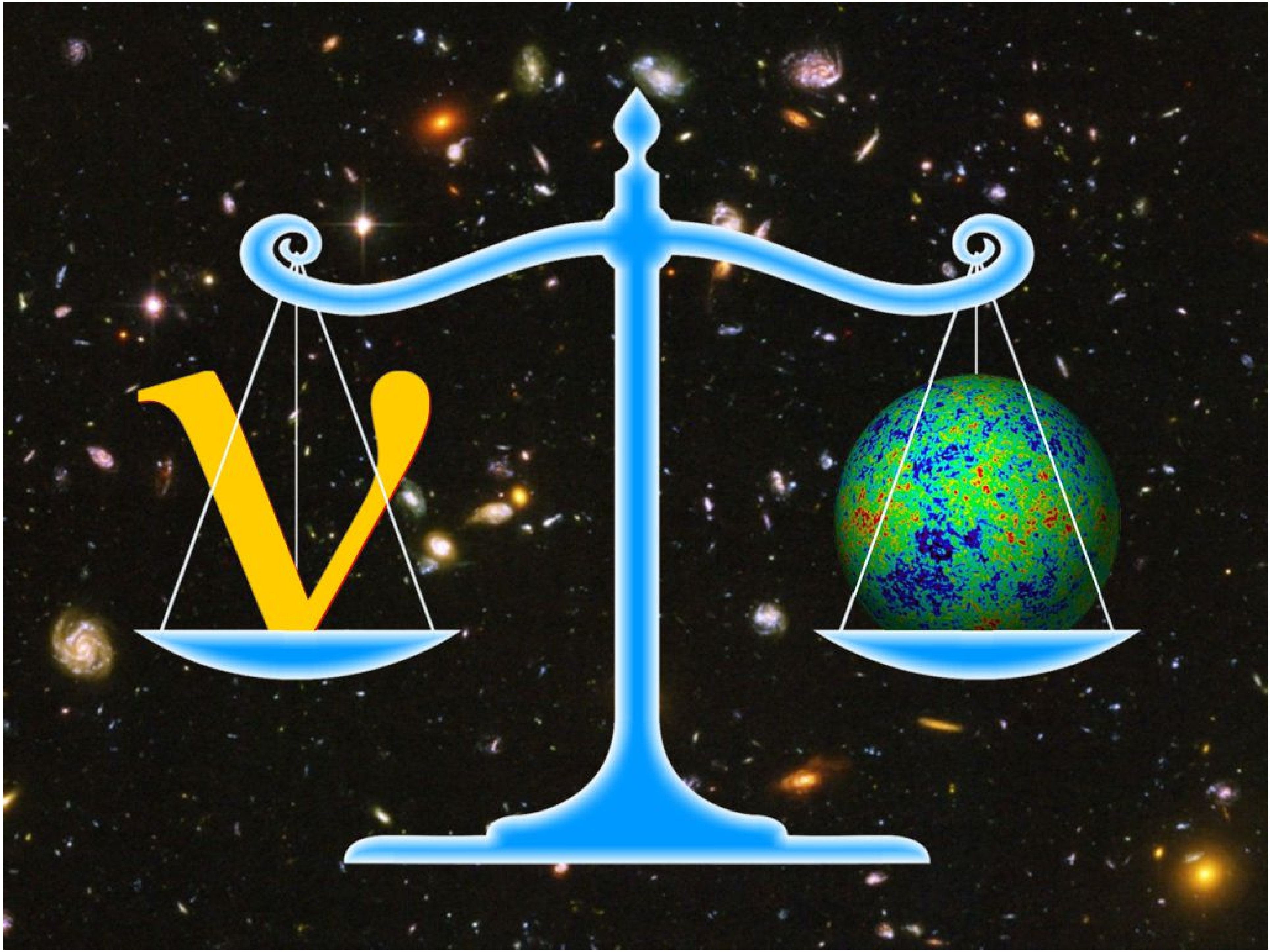


Inverted Hierarchy
$\Theta_{13} = 0.03 \pm 0.01$

Normal Hierarchy
$\Theta_{13} = 0.03 \pm 0.01$

$\Theta_{13} = 0$

Mangano, Miele, Pastor, Pisanti & Sarikas, arXiv:1110.4335

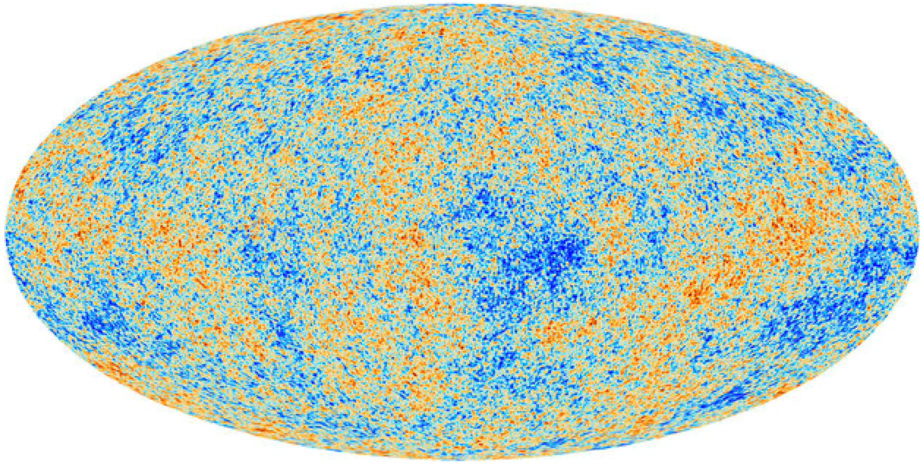


Basic Idea

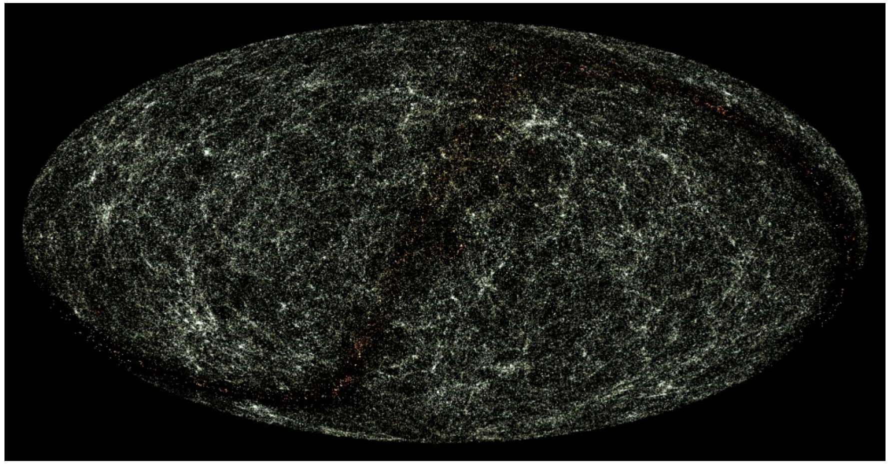
Comparison of theoretical predictions with observations of the anisotropy

(temperature (and polarisation) differences from isotropy)

of the cosmic microwave background and correlations in the large scale structure

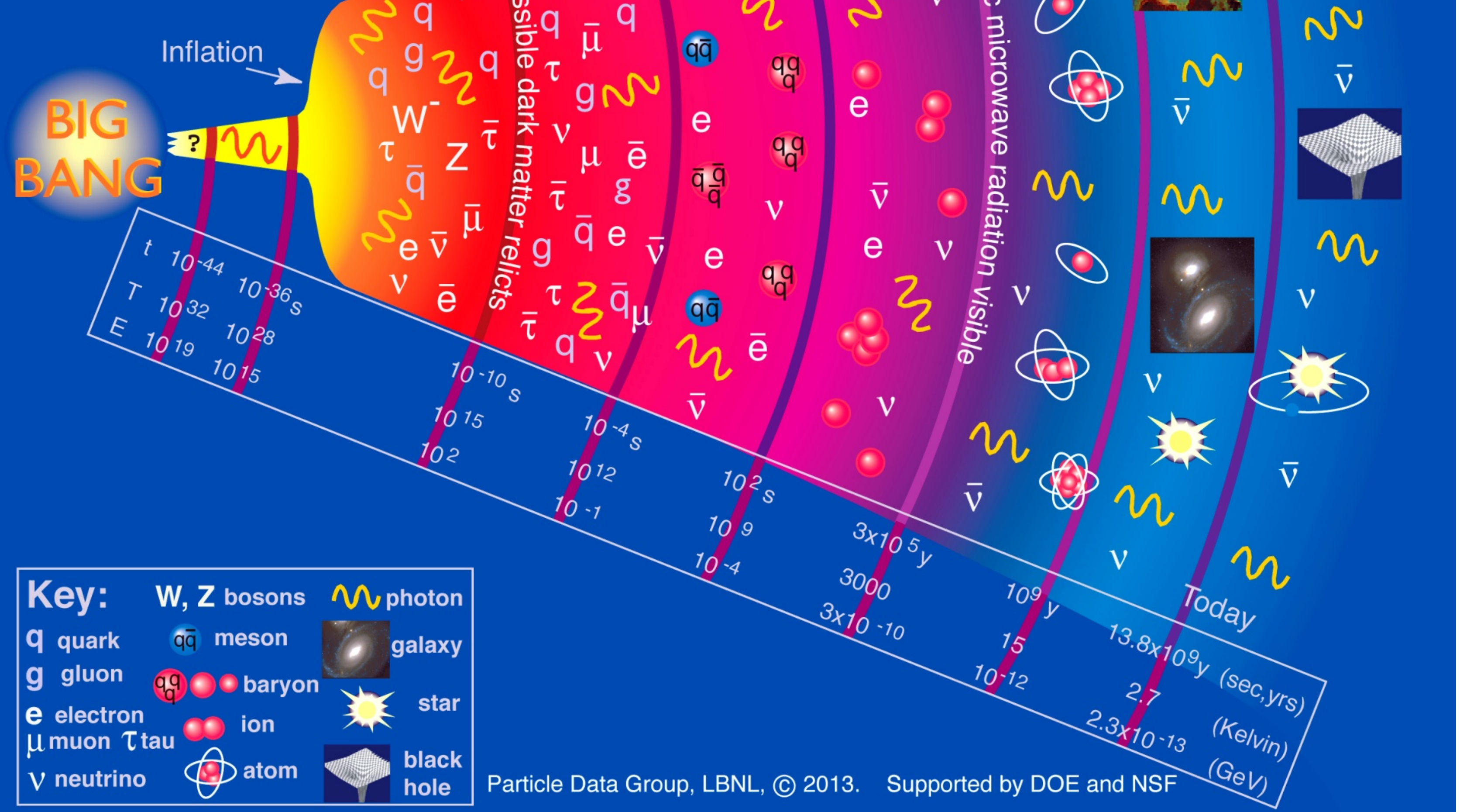


Wmap9 CMB
http://wmap.gsfc.nasa.gov/resources/cmbimages.html



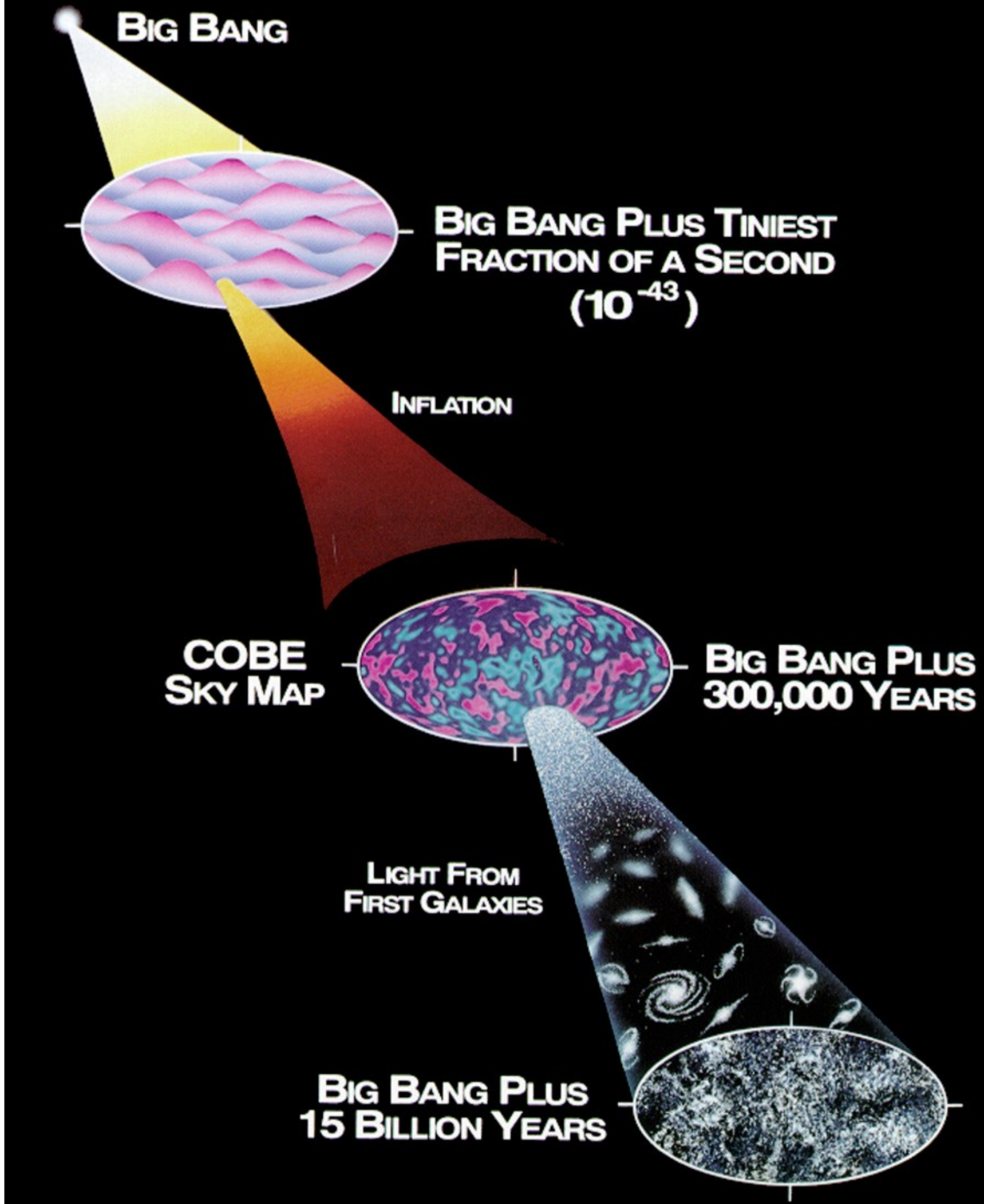
Sky Map of Galaxies (2MASS XSC)
http://spider.ipac.caltech.edu/staff/jarrett/2mass/XSC/
arrett_allsky.html

History of the Universe



Particle Data Group, LBNL, © 2013. Supported by DOE and NSF

Structure Formation in the Universe



Early phase of exponential expansion (Inflationary epoch)

Zero-point fluctuations of quantum fields are stretched and frozen

Structure grows by gravitational instability

Cosmic density fluctuations are frozen quantum fluctuations

Inhomogeneity Growth

- Obtain equations governing evolution of matter and metric inhomogeneities by perturbing Einstein equation (yields rate of growth (or not) in different regimes)

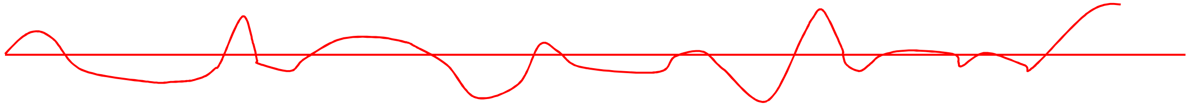
- Plus Boltzmann equations for various forms of matter/energy

  - See
    J. Lesgourges and S. Pastor, *Massive neutrinos and cosmology*, Phys. Rep. 429 (2006) [astro-ph/0603494]
    for a thorough account specifically highlighting the effect of neutrinos

Power Spectrum of Density Fluctuations

Field of density fluctuations of matter (e.g. dark matter)

$$\delta(\boldsymbol{x}) = \frac{\delta\rho(\boldsymbol{x})}{\bar{\rho}}$$



Fourier transform of density field

$$\delta_{\boldsymbol{k}} = \int d^3\boldsymbol{x}\, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\delta(\boldsymbol{x})$$

Power spectrum is essentially the square of the Fourier transform ($\hat{\delta}$ is δ-function)

$$\langle\delta_{\boldsymbol{k}}\delta_{\boldsymbol{k}'}\rangle = (2\pi)^3\hat{\delta}(\boldsymbol{k}-\boldsymbol{k}')P(\boldsymbol{k})$$

Power spectrum is Fourier transform of two-point correlation function ($x = x_2 - x_1$)

$$\xi(\boldsymbol{x}) = \langle\delta(\boldsymbol{x}_2)\delta(\boldsymbol{x}_1)\rangle = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}} P(\boldsymbol{k}) = \int \frac{d\Omega}{4\pi}\frac{dk}{k}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \underbrace{\frac{k^3P(\boldsymbol{k})}{2\pi^2}}_{\Delta^2(\boldsymbol{k})}$$

Gaussian random field (phases of δ_k uncorrelated) is fully characterized by

$P(k) = |\delta_k|^2$ Power Spectrum

or equivalently by

$$\Delta(k) = \left(\frac{k^3P(k)}{2\pi^2}\right)^{1/2} = \frac{k^{3/2}|\delta_k|}{\sqrt{2}\,\pi}$$

No “non-Gaussianities” in cosmological precision data (Planck CMBR results)

Gravitational Growth of Density Perturbations

The dynamical evolution of small perturbations

$$\delta(x) = \frac{\delta\rho(x)}{\bar{\rho}} \ll 1$$

is independent for each Fourier mode δ_k (linear regime)

- For pressureless, nonrelativistic matter (cold dark matter) naively expect exponential growth by gravitational instability
- But only power-law growth in expanding universe (competition between expansion and gravitational instability)

	Sub-horizon $\lambda \ll H^{-1}$	Super-horizon $\lambda \gg H^{-1}$
Radiation dominates $a \propto t^{1/2}$	$\delta_k \propto \text{const}$	$\delta_k \propto a^2 \propto t$
Matter dominates $a \propto t^{2/3}$	$\delta_k \propto a \propto t^{2/3}$	

Density contrast grows linearly with scale factor

Processed Power Spectrum in CDM Scenario

Primordial spectrum usually assumed to be a power law

$$P(k) = |\delta_k|^2 \propto k^{n_s}$$

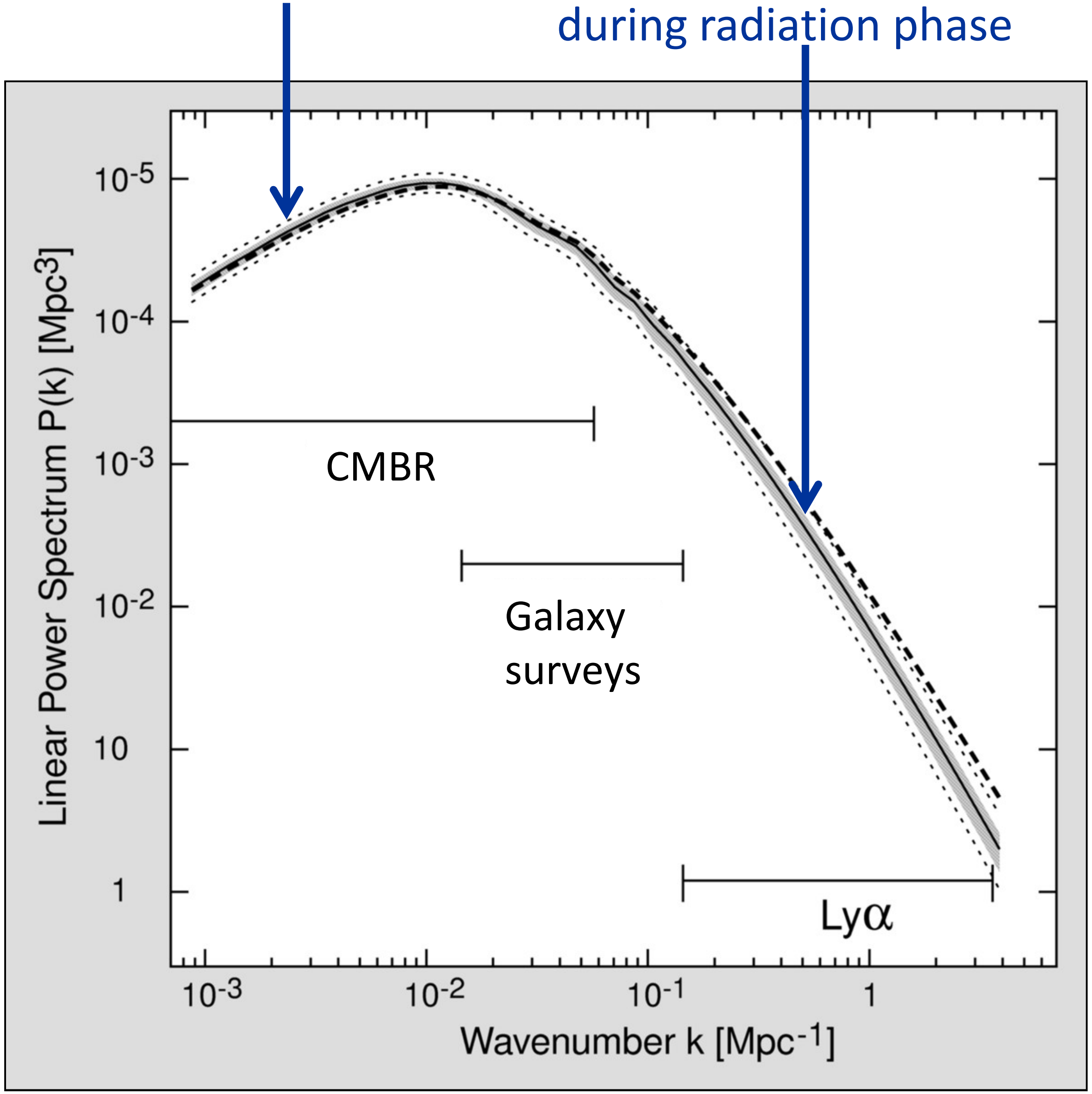
Harrison-Zeldovich spectrum ("flat") has $n_s = 1$

Precision cosmology provides

$$n_s = 0.960 \pm 0.007$$

in good agreement with simplest theories of inflation

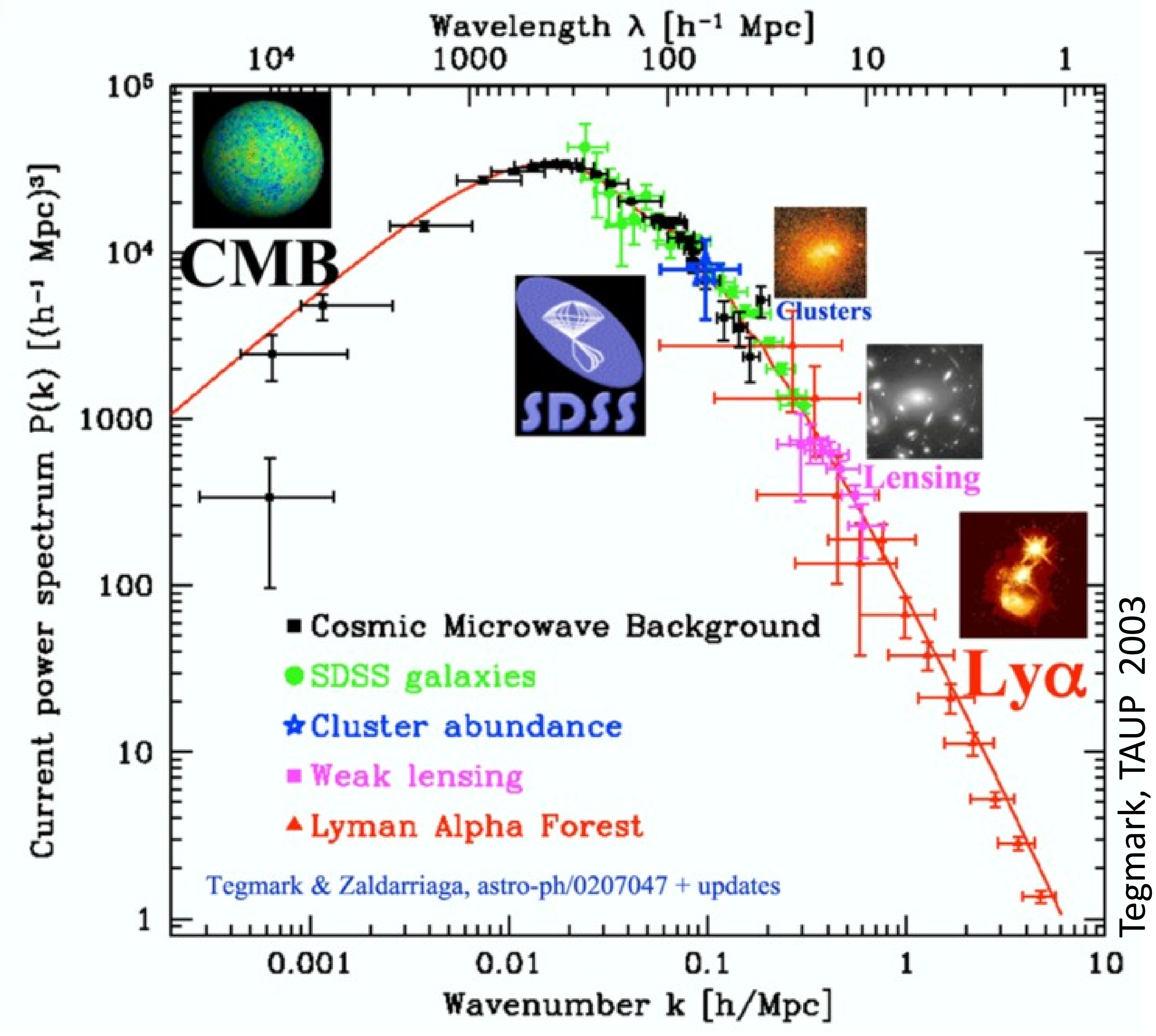
n < 1 less power on small scales



Primordial spectrum

Suppressed by stagnation during radiation phase

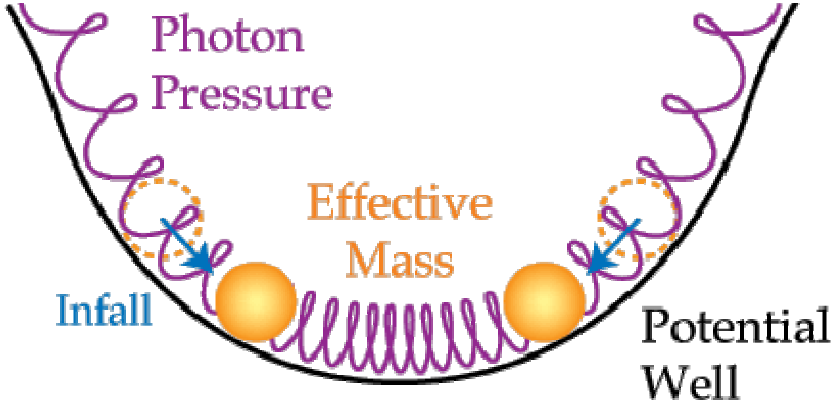
Power Spectrum of Cosmic Density Fluctuations



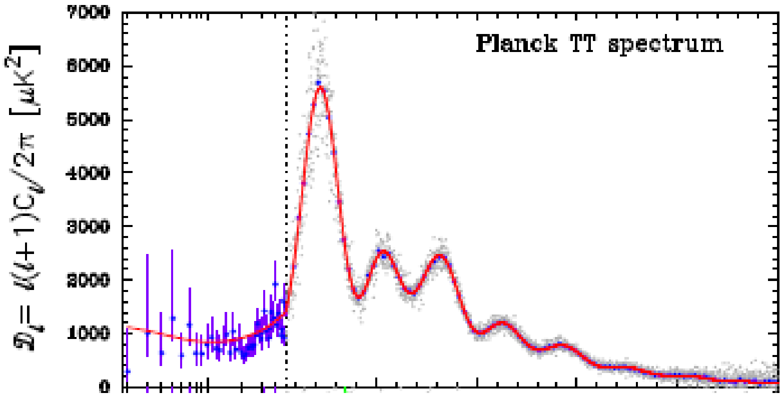
Tegmark, TAUP 2003

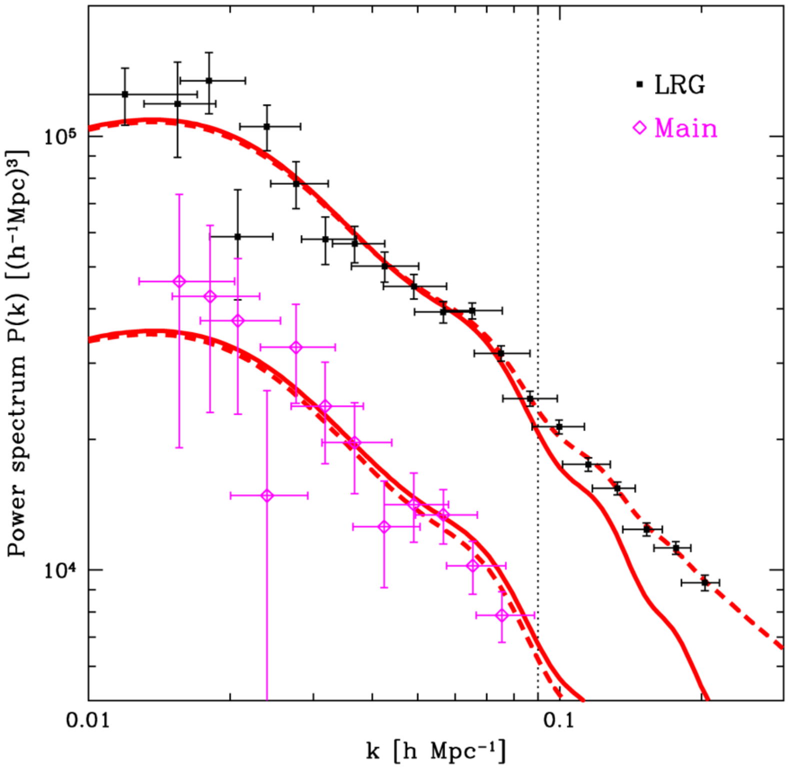
Acoustic-oscillations in photon-baryon fluid

- When photons and baryons are coupled, competition between gravitational attraction and radiation pressure leads to acoustic oscillations



- After decoupling baryons fall into dark matter potential wells
- Density contrast grows by gravitational instability





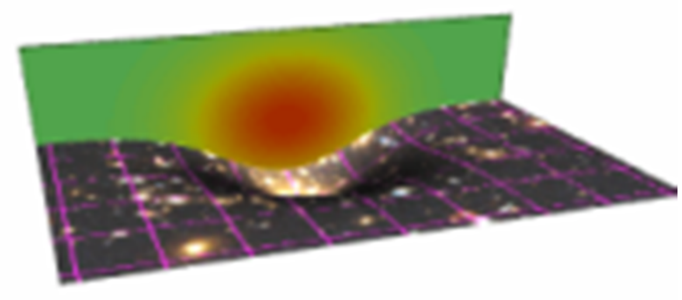
Temperature variations from density variations

Primary effects

- Denser regions hotter
- Photons climbing out of potential wells are redshifted (Sachs Wolfe effect)
- Doppler shift for photons scattered from moving electrons

Secondary effects

- Integrated Sachs-Wolfe
  - occurs between the surface of last scattering and observation and only occurs when the Universe is not dominated by matter. When the Universe is matter dominated gravitational potential wells do not evolve much in the time that photons traverse them (so blueshift as falling in and redshift climbing out cancel)
    - Early Integrated Sachs-Wolfe occurs soon after last scattering when the radiation density of the Universe is non-negligible
    - Late-time Integrated Sachs Wolfe occurs when the cosmological constant is dominant



- Gravitational Lensing

Power Spectrum of CMB Temperature Fluctuations

Sky map of CMBR temperature fluctuations

$$\Delta(\theta,\varphi) = \frac{T(\theta,\varphi) - \langle T\rangle}{\langle T\rangle}$$

Multipole expansion

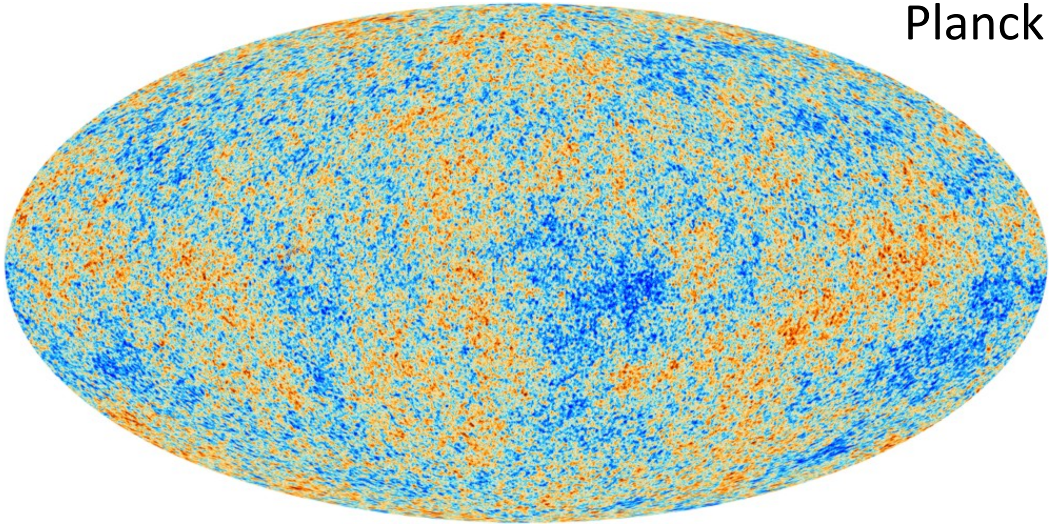
$$\Delta(\theta,\varphi) = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta,\varphi)$$

Angular power spectrum

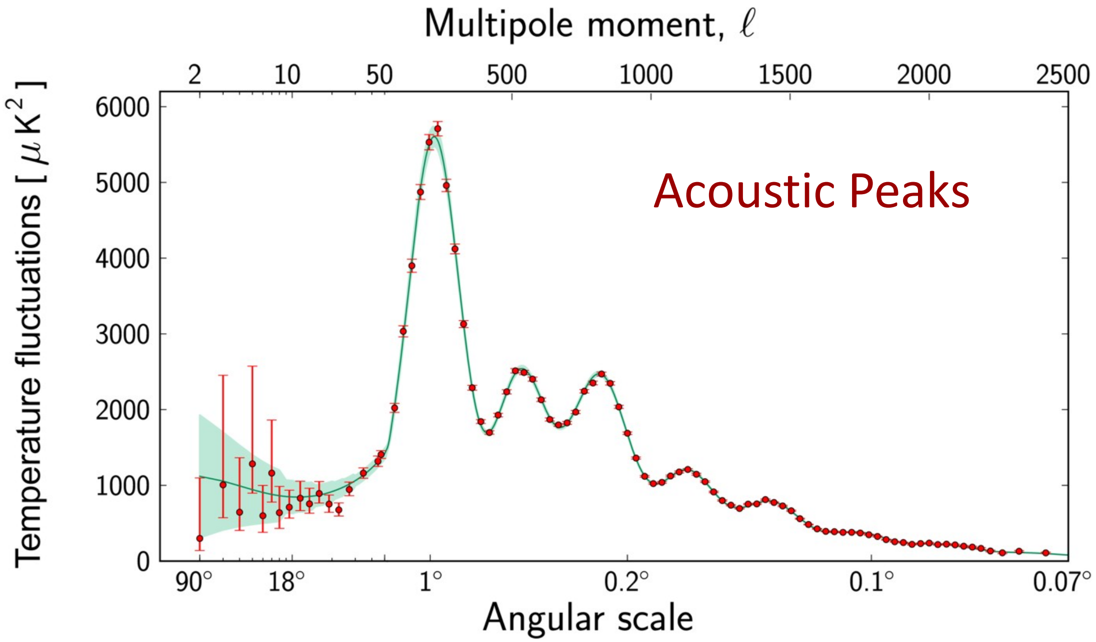
$$C_\ell = \langle a_{\ell m}^* a_{\ell m}\rangle$$

$$= \frac{1}{2\ell+1}\sum_{m=-\ell}^{\ell} a_{\ell m}^* a_{\ell m}$$

Provides “acoustic peaks” and a wealth of cosmological information



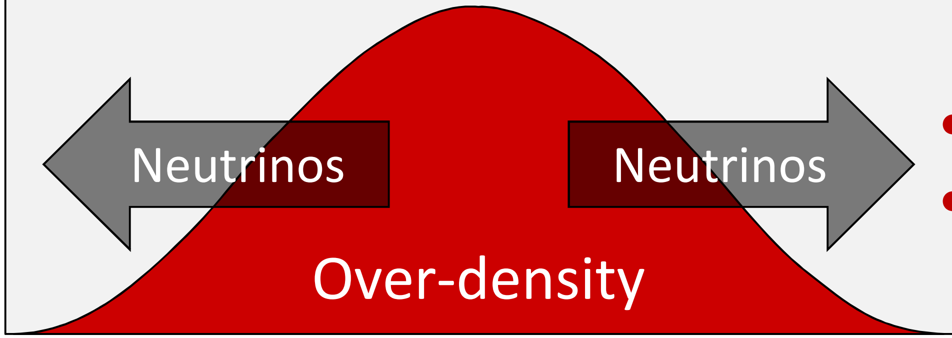
Planck



What is wrong with neutrino dark matter?

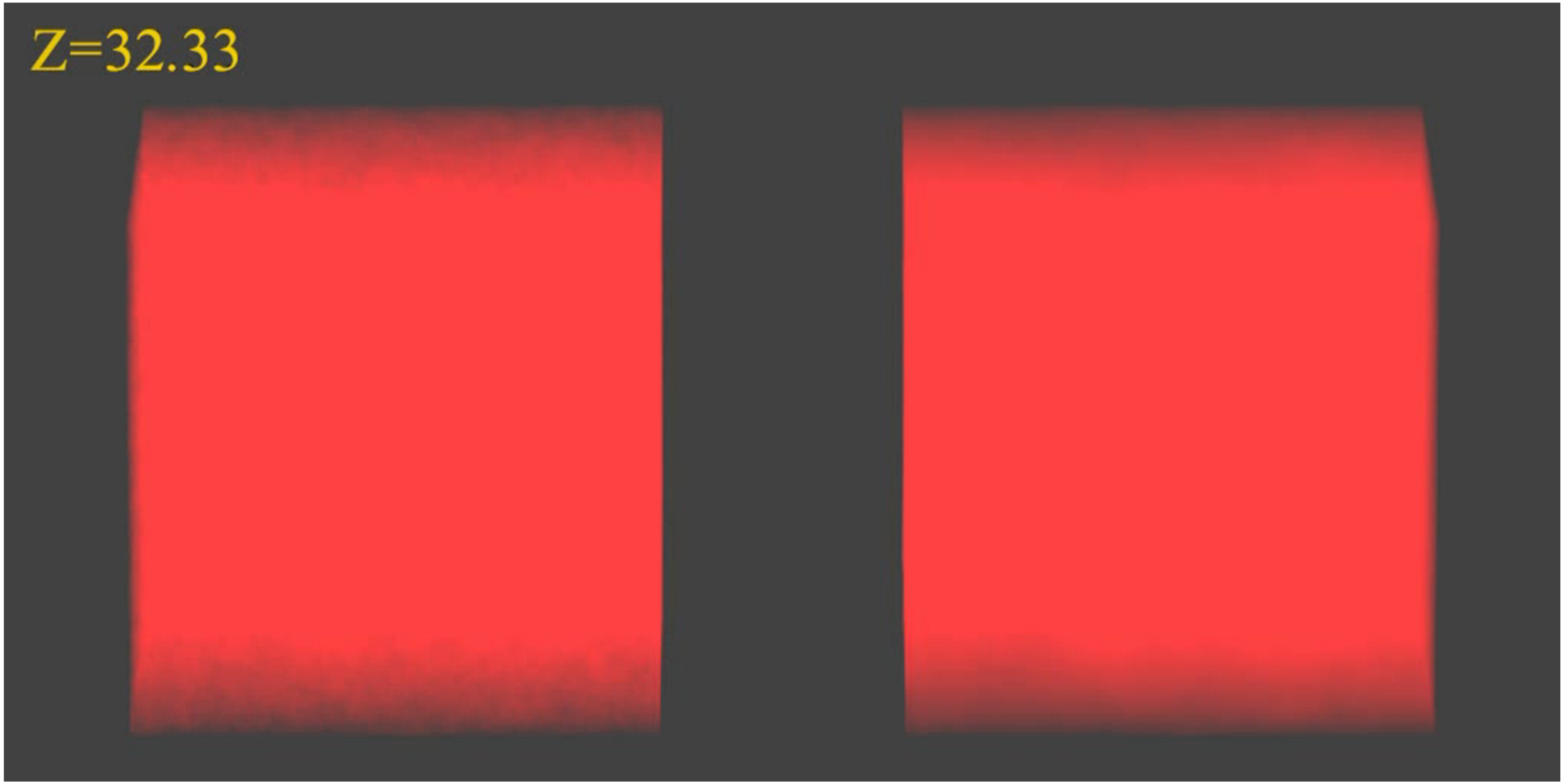
Neutrino Free Streaming

- At $T < 1\,\text{MeV}$ neutrino scattering in early universe is ineffective
- Stream freely until non-relativistic
- Wash out density contrasts on small scales



- Neutrinos are “Hot Dark Matter”
- Ruled out by structure formation

Neutrino effect on large scale structure growth



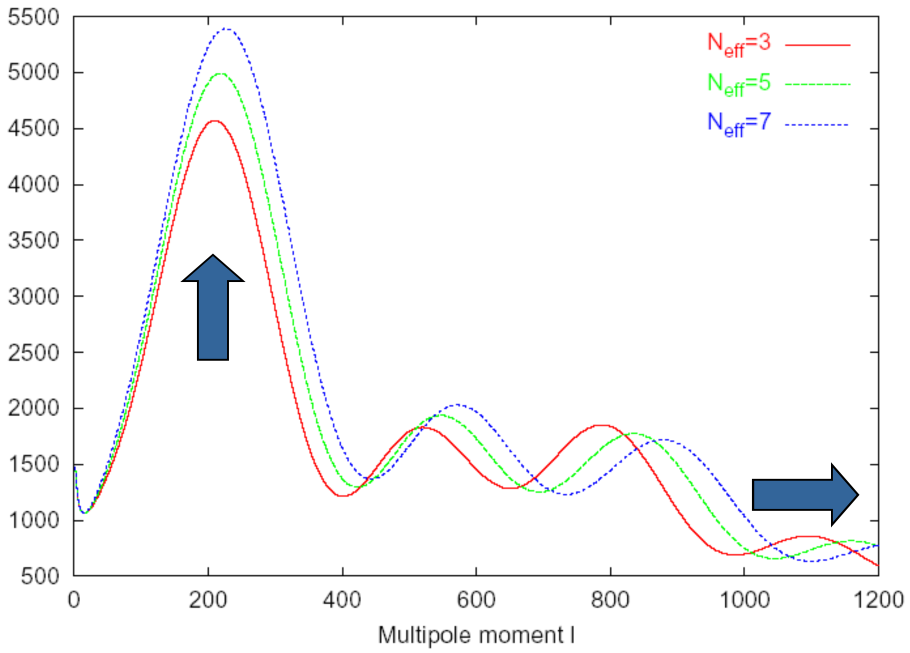
Standard ΛCDM Model

Neutrinos with Σm_{ν} = 6.9 eV

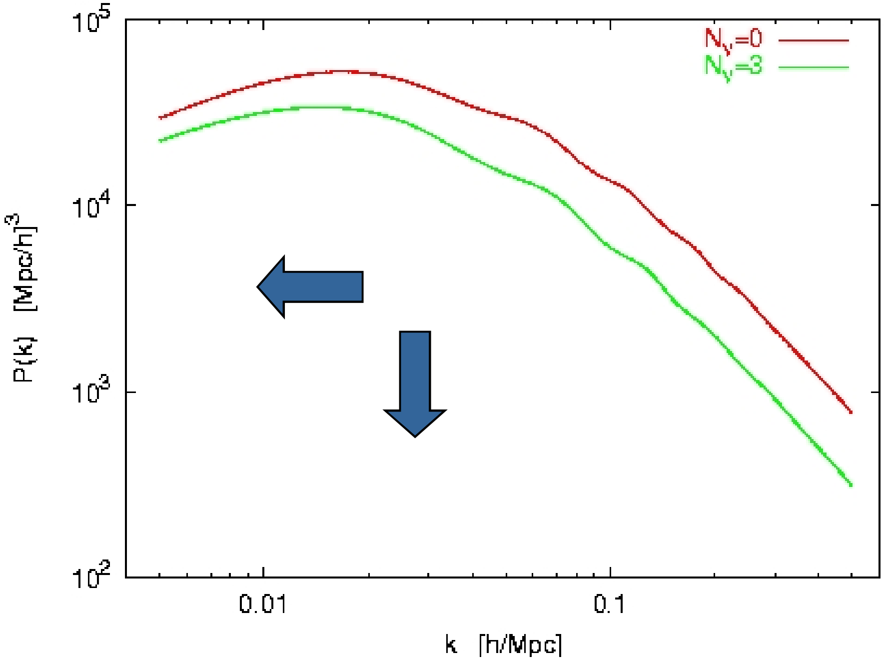
Troels Haugbølle, http://users-phys.au.dk/haugboel

Impact of extra radiation

Redshift of matter-radiation equality modified by N_{eff}



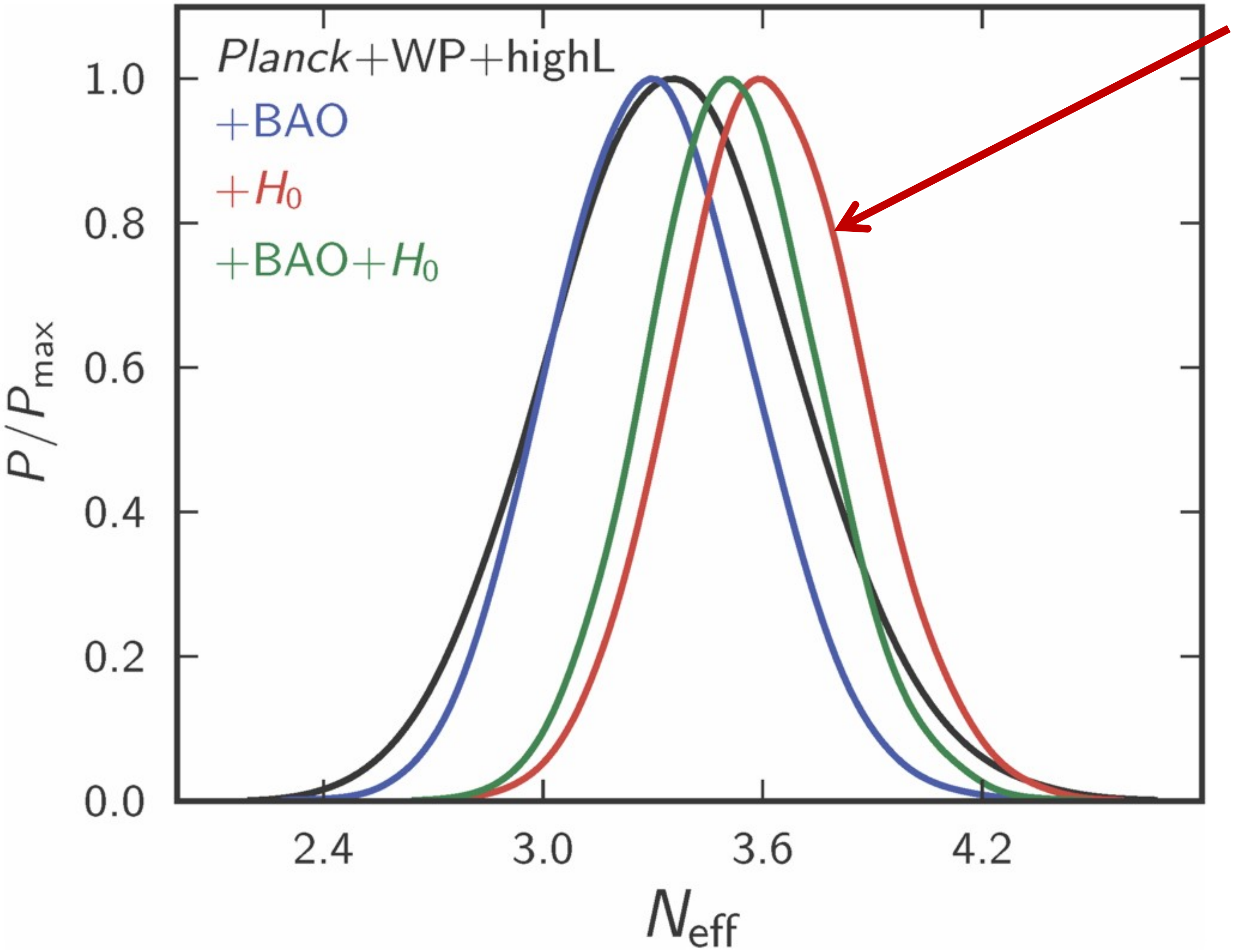
CMB angular power spectrum



Matter power spectrum

Hint for “Dark Radiation” in the Universe?

Depending on used data sets, indication for extra radiation at CMB decoupling
Same as pre-Planck situation based on WMAP-9 etc.

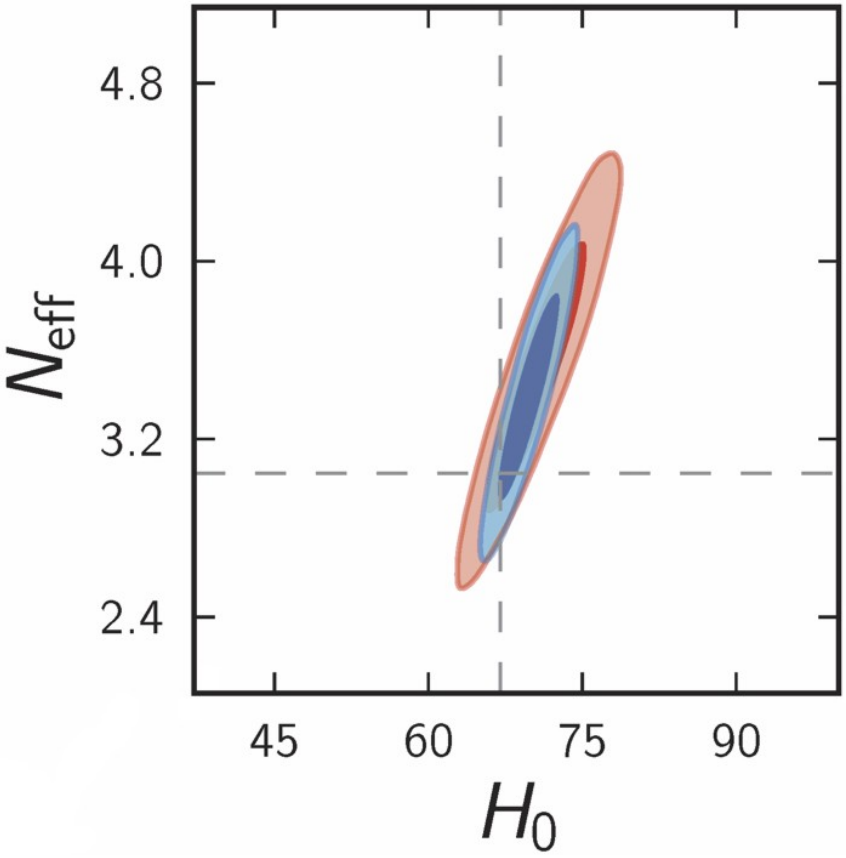


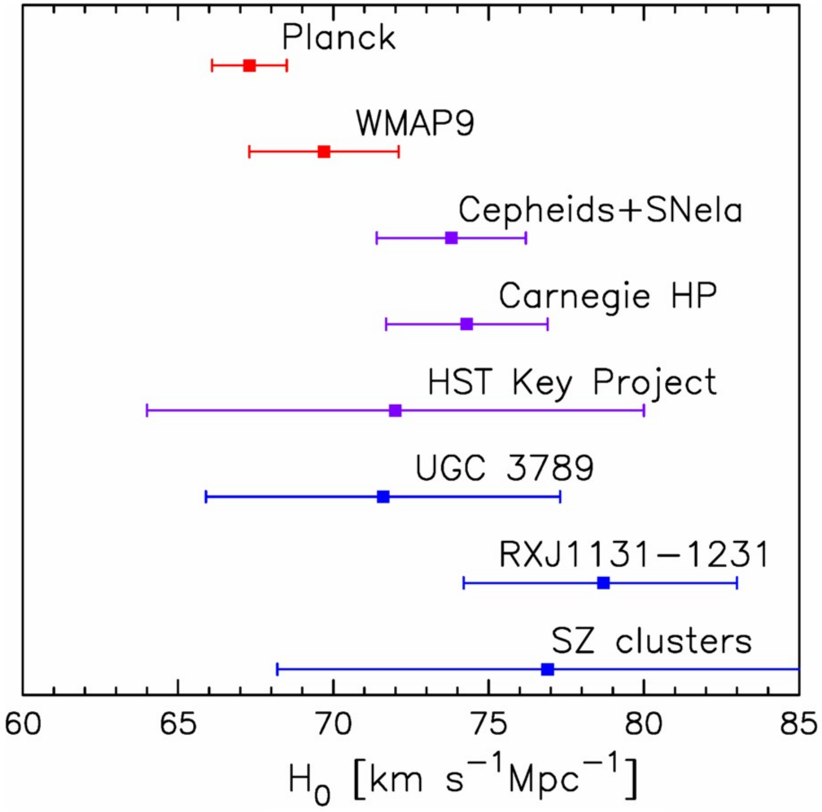
Including prior on H_0
provides
> 2σ “evidence”
for extra radiation
at CMB decoupling

Ade et al. (Planck Collaboration), arXiv:1303.5076

Degeneracy between N_{eff} and H_0

Extra radiation relaxes tension between H_0 determinations

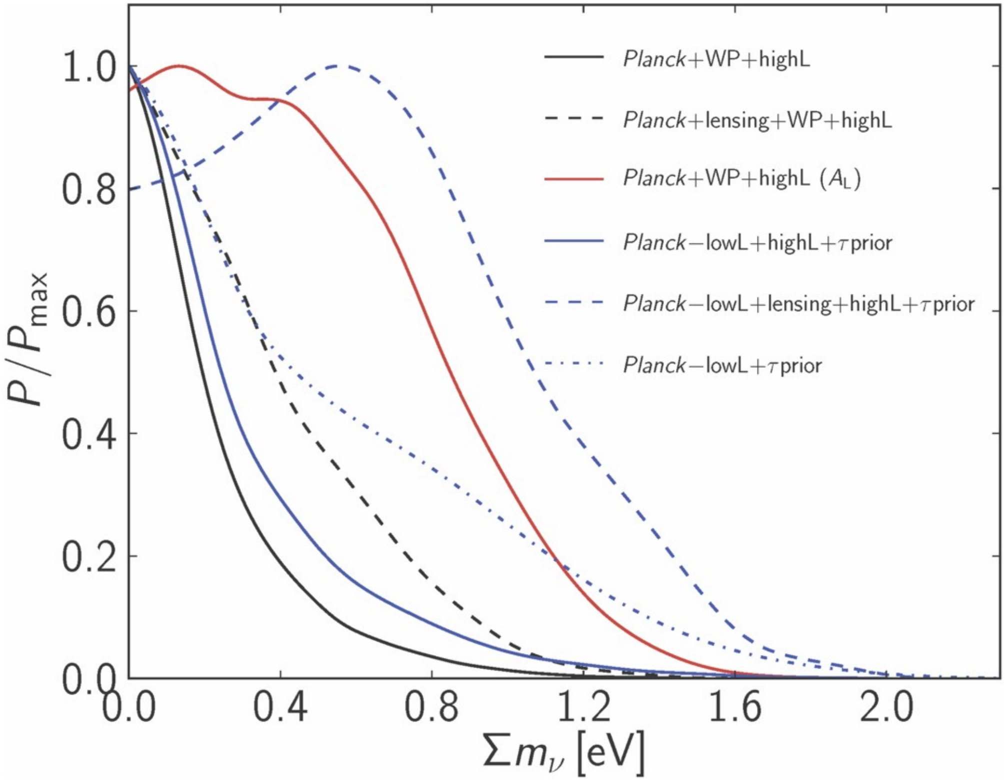




Ade et al. (Planck Collaboration), arXiv:1303.5076

Neutrino Mass Limits Post Planck (2013)

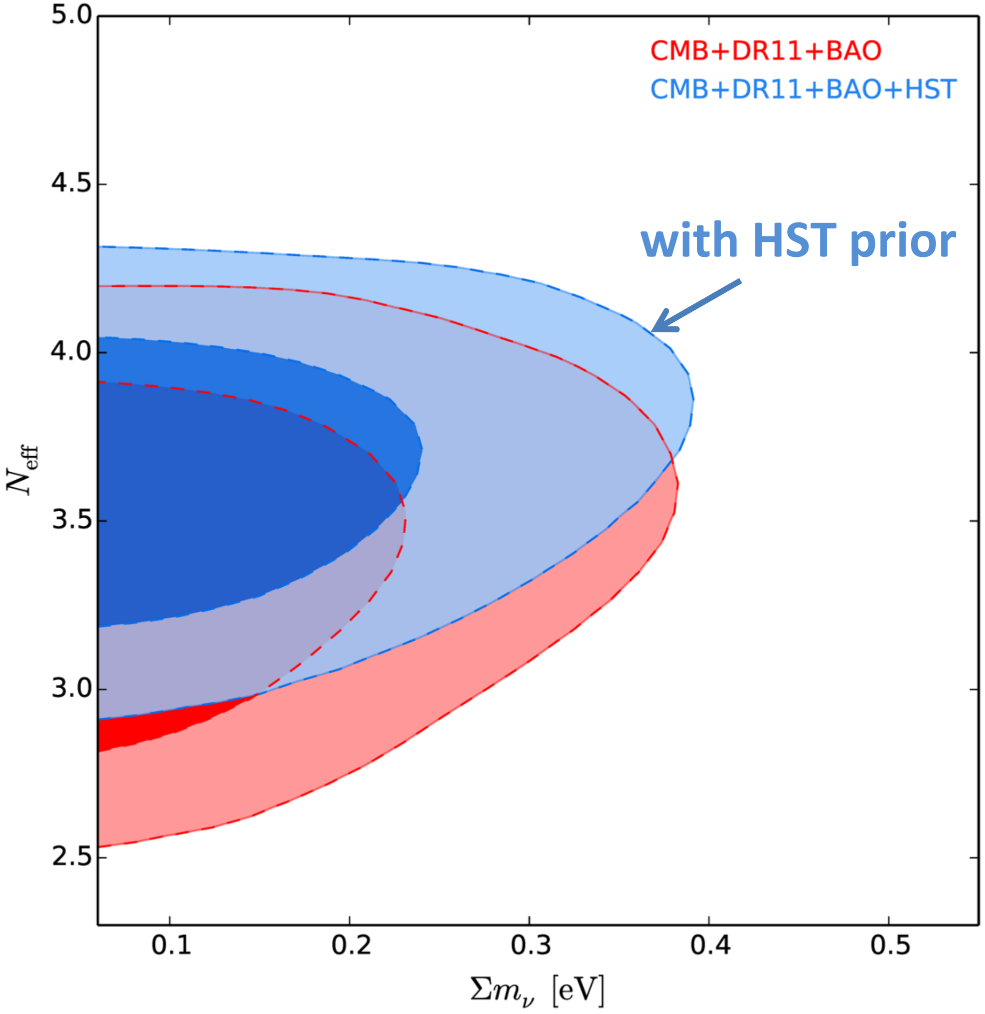
Depends on used data sets
Many different analyses in the literature



Planck alone: $\Sigma m_\nu < 1.08$ eV (95% CL)
CMB + BAO limit: $\Sigma m_\nu < 0.23$ eV (95% CL)

Ade et al. (Planck Collaboration), arXiv:1303.5076

Neutrino Mass and N_{eff} Limits



Giusarma, Di Valentino, Lattanzi, Melchiorri & Mena, arXiv:1403.4852

Neutrino Mass from Cosmology (Hannestad)

More data

	Minimal ΛCDM	$+N_\nu$	$+w+$……
+Ly-alpha		eV	0.2–0.3 eV
+ SNI-a +WL		eV	0.5–0.6 eV
+ SDSS	0.4 eV	~ 1 eV	
CMB only	1.1 eV	~ 2 eV	

Larger model space

References

- A.D. Dolgov, *Neutrinos in Cosmology*, Phys Rept. 370 (2002) 333 [hep=ph/0202122]
- J. Lesgourges and S. Pastor, *Massive neutrinos and cosmology*, Phys. Rep. 429 (2006) [astro-ph/0603494]
- S. Hannestad, *Primordial neutrinos*, Ann. Rev. Nucl. Part. Sci. 56 (2006) 17 [hep-ph/0602058]
- Y.Y.Y. Wong, *Neutrino mass in cosmology: status and prospects*, Ann. Rev. Nul. Part. Sci. 56 (2006) 137 [hep-ph/0602058]

Sterile Neutrinos in Cosmology Summary

- Fully thermalised sterile neutrino (eV-mass) excluded

- Partially thermalised allowed or even favoured, needs new ingredients

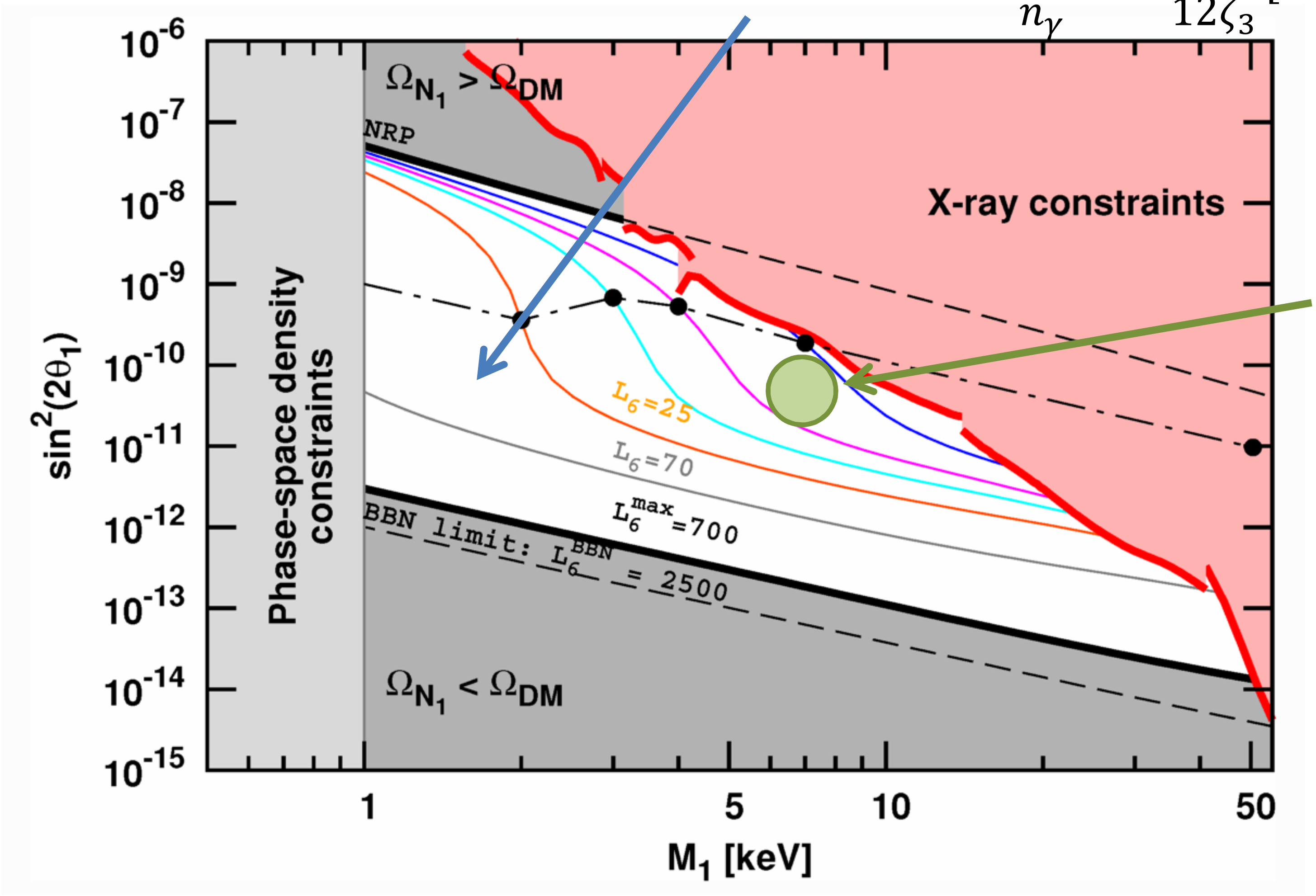
- keV-range sterile neutrinos possible as dark matter

- 3.55 keV x-ray line hint for this scenario?

Sterile Neutrino Dark Matter

Resonant production, requires large lepton asymmetry caused by other sterile neutrinos flavors

$$L = \frac{n_\nu - n_{\bar{\nu}}}{n_\gamma} = \frac{1}{12\zeta_3}\left[\pi^2\xi + \xi^3 + \cdots\right]$$

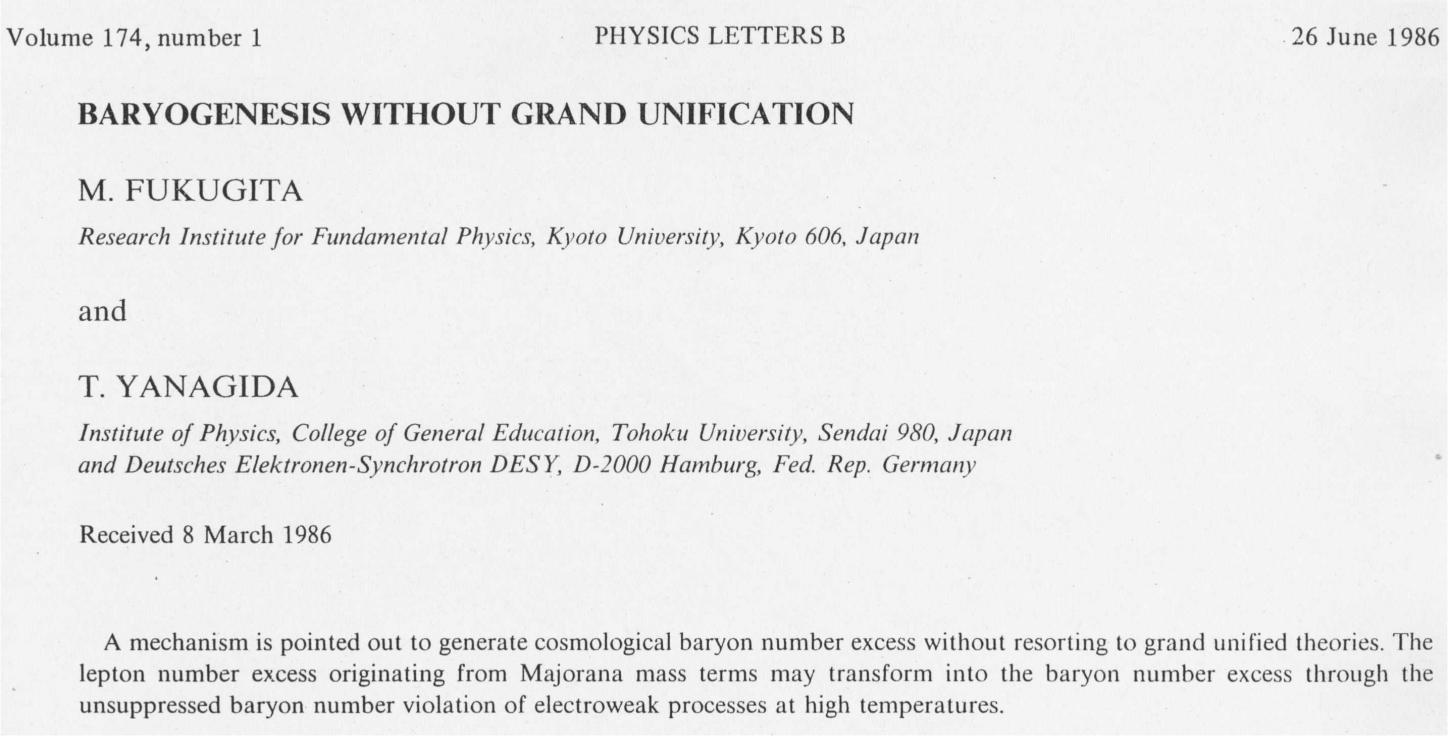


Unidentified tentative x-ray line (3.55 keV) arXiv:1402.2301 & 1402.4119

Sterile Neutrino White Paper, arXiv:1204.5379

Leptogenesis

- More matter than anti-matter in the universe (BAU – Baryon Asymmetry of the Universe)
- Not from initial conditions (inflationary universe)
- Should be generated by physical processes: "Baryogenesis"
- Requires an absolute difference between matter and anti-matter in physical laws



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BARYOGENESIS WITHOUT GRAND UNIFICATION

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A mechanism is pointed out to generate cosmological baryon number excess without resorting to grand unified theories. The lepton number excess originating from Majorana mass terms may transform into the baryon number excess through the unsuppressed baryon number violation of electroweak processes at high temperatures.