Charged Lepton Flavor Violation

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International Neutrino Summer School

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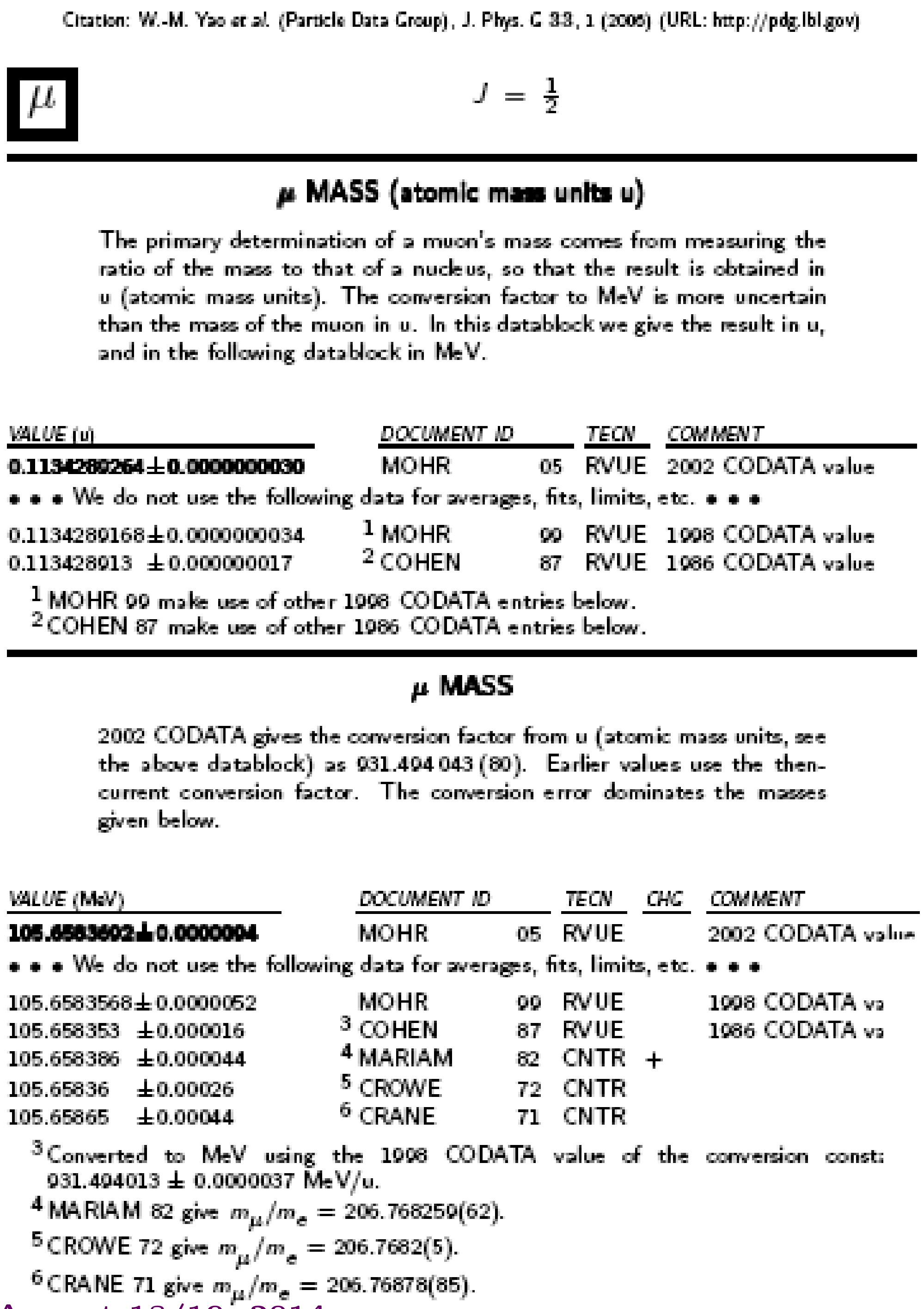
Tentative Outline

1. Introduction: Very Brief History;
2. Standard Model Expectations (?);
3. Main Experimental Observables and Challenges;
4. Connections to Neutrinos and to New Physics at the Weak Scale;
5. Connections to Other Observables: Comment on the Muon $g-2$.

Some References

- Y. Kuno, Y. Okada, hep-ph/9909265;
- J. Aysto *et al*, hep-ph/0109217 (NuFact CERN Document);
- AdG, P. Vogel, arXiv:1303.4097;
- R. Bernstein, P. Cooper, arXiv:1307.5787;
- J. Albrecht *et al*, arXiv:1311.5278 (Snowmass);
- B.L. Roberts and W.J. Marciano, “Lepton dipole moments,” (Advanced series on directions in high energy physics. 20).

“Who Ordered That?”



Citation: W.-M. Yao et al. (Particle Data Group), J. Phys. G 33, 1 (2006) (URL: http://pdg.lbl.gov)

μ $J = \frac{1}{2}$

μ MASS (atomic mass units u)

The primary determination of a muon's mass comes from measuring the ratio of the mass to that of a nucleus, so that the result is obtained in u (atomic mass units). The conversion factor to MeV is more uncertain than the mass of the muon in u. In this datablock we give the result in u, and in the following datablock in MeV.

VALUE (u)	*DOCUMENT ID*		*TECN*	*COMMENT*
0.1134289264±0.0000000030	MOHR	05	RVUE	2002 CODATA value
• • • We do not use the following data for averages, fits, limits, etc. • • •				
0.1134289168±0.0000000034	[1] MOHR	99	RVUE	1998 CODATA value
0.113428913 ±0.000000017	[2] COHEN	87	RVUE	1986 CODATA value

[1] MOHR 99 make use of other 1998 CODATA entries below.
[2] COHEN 87 make use of other 1986 CODATA entries below.

μ MASS

2002 CODATA gives the conversion factor from u (atomic mass units, see the above datablock) as 931.494 043 (80). Earlier values use the then-current conversion factor. The conversion error dominates the masses given below.

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
105.6583692±0.0000094	MOHR	05	RVUE		2002 CODATA value
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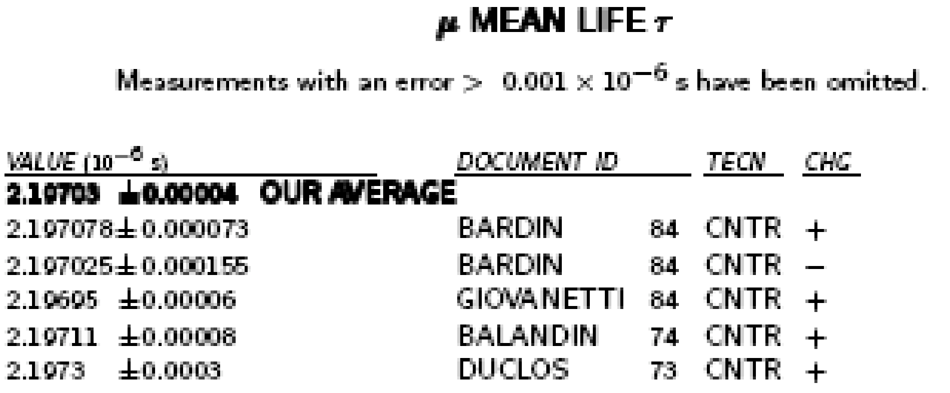
105.6583568±0.0000052	MOHR	99	RVUE		1998 CODATA va
105.658353 ±0.000016	[3] COHEN	87	RVUE		1986 CODATA va
105.658386 ±0.000044	[4] MARIAM	82	CNTR	+	
105.65836 ±0.00026	[5] CROWE	72	CNTR		
105.65865 ±0.00044	[6] CRANE	71	CNTR		

[3] Converted to MeV using the 1998 CODATA value of the conversion const: 931.494013 ± 0.0000037 MeV/u.
[4] MARIAM 82 give $m_\mu/m_e = 206.768259(62)$.
[5] CROWE 72 give $m_\mu/m_e = 206.7682(5)$.
[6] CRANE 71 give $m_\mu/m_e = 206.76878(85)$.

The muon is the best known unstable fundamental particle.

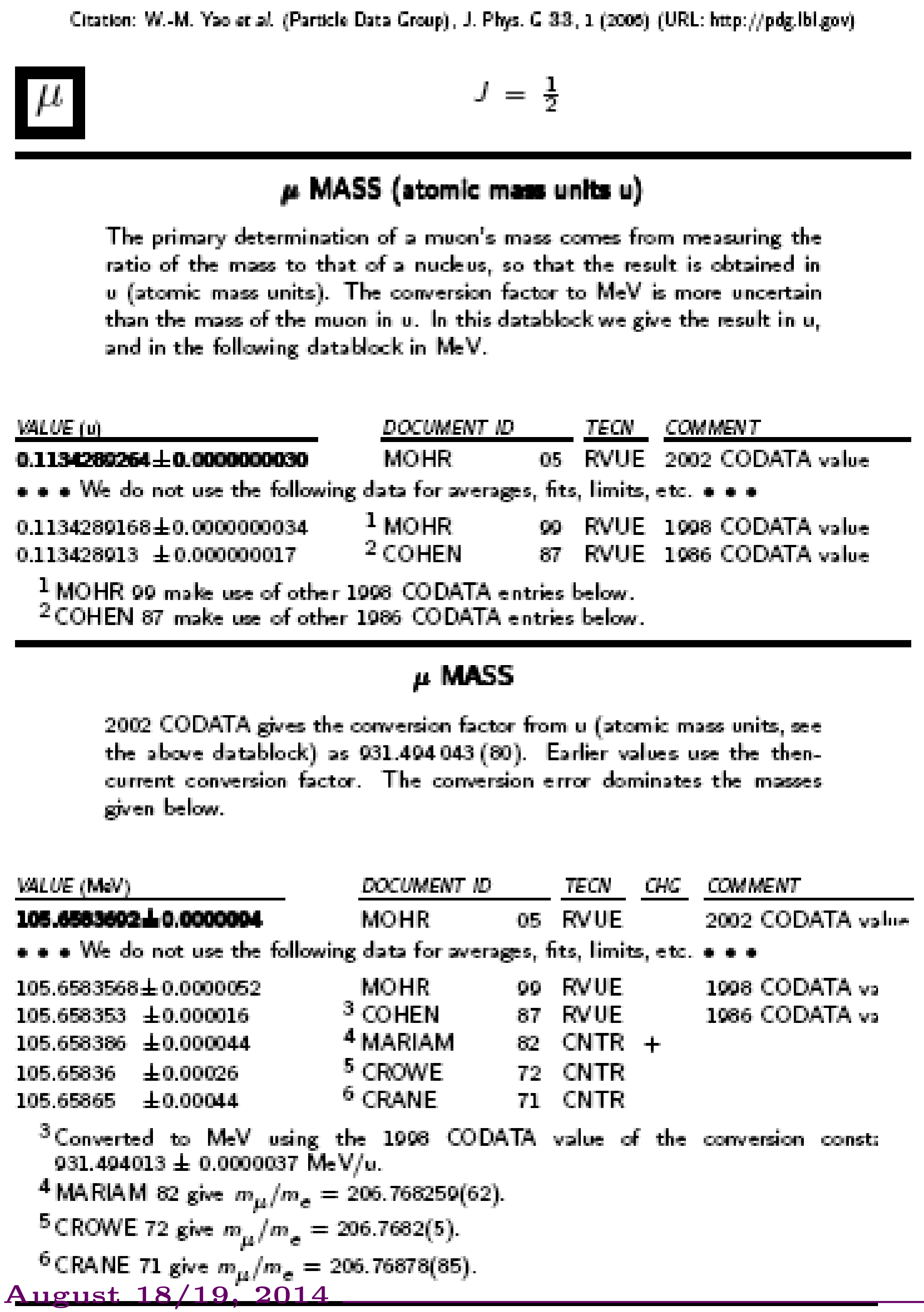
The muon is also the heaviest fundamental particle we can directly work with. It is a unique, priceless resource for physicists.



μ MEAN LIFE τ

Measurements with an error > 0.001×10^{-6} s have been omitted.

VALUE (10^{-6} s)	*DOCUMENT ID*		*TECN*	*CHG*
2.19703 ±0.00004 OUR AVERAGE				
2.197078±0.000073	BARDIN	84	CNTR	+
2.197025±0.000155	BARDIN	84	CNTR	−
2.19695 ±0.00006	GIOVANETTI	84	CNTR	+
2.19711 ±0.00008	BALANDIN	74	CNTR	+
2.1973 ±0.0003	DUCLOS	73	CNTR	+



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"Who Ordered That?"

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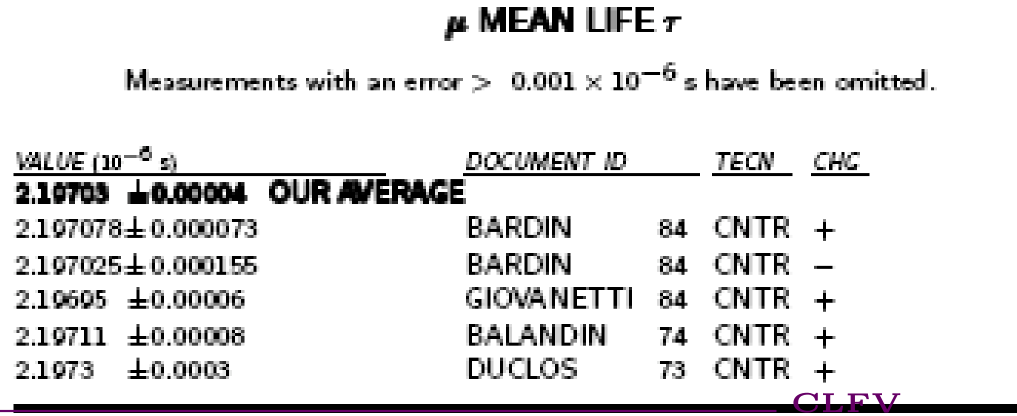
ANS: "We did!"



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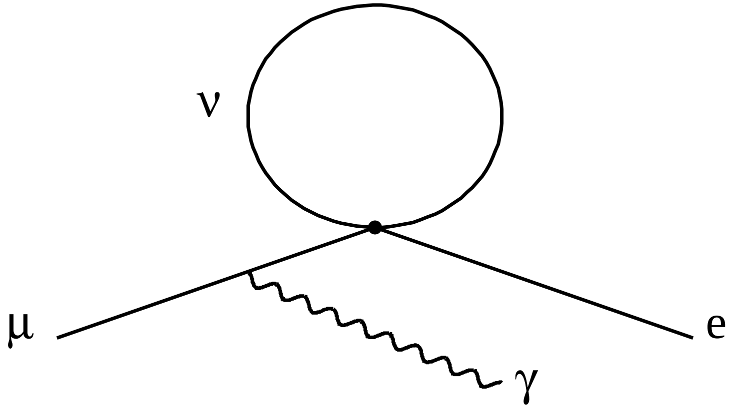
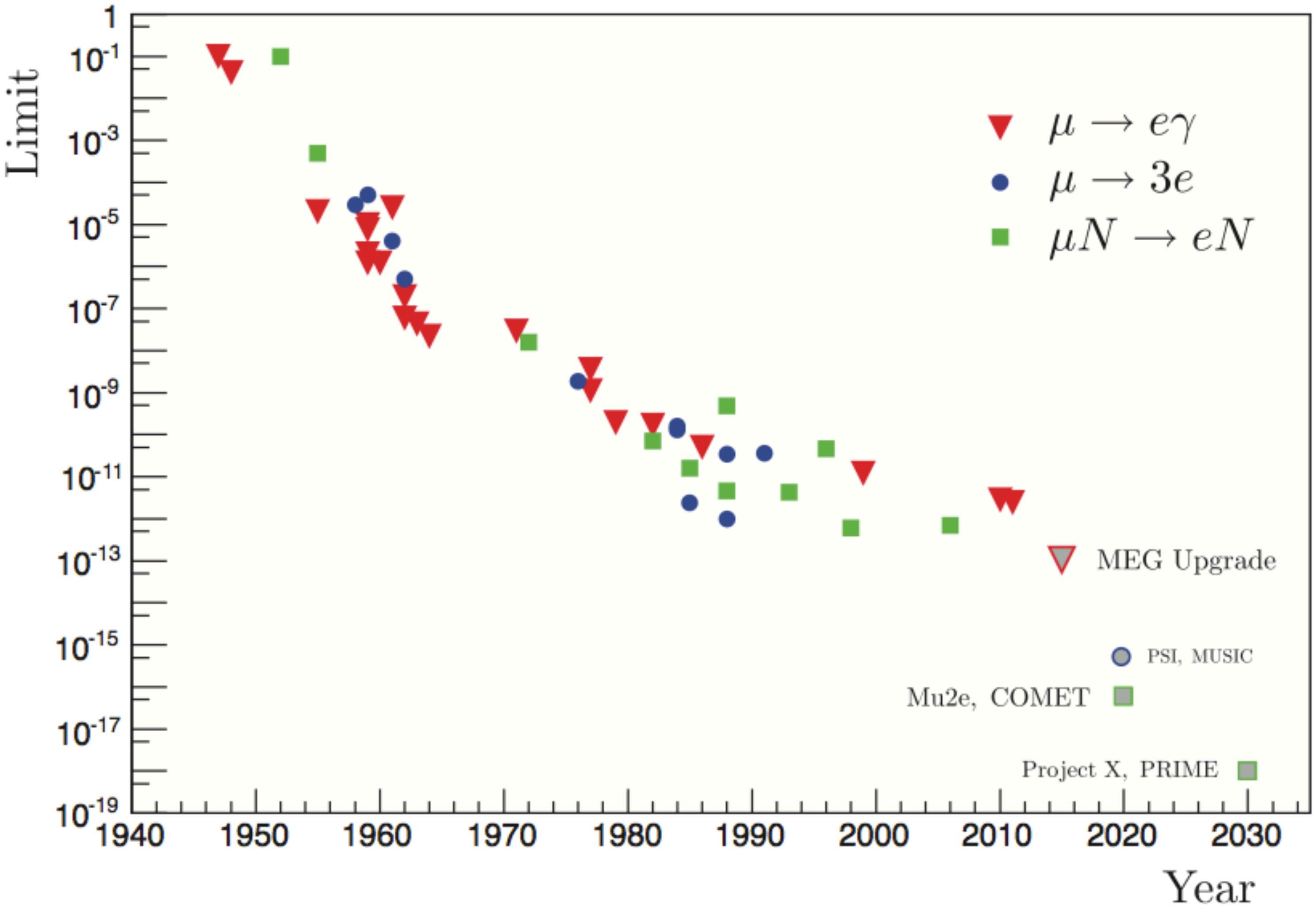
Ever since it was established that $\mu \rightarrow e\nu\bar{\nu}$, people have searched for $\mu \rightarrow e\gamma$, which was thought to arise at one-loop, like this:



The fact that $\mu \rightarrow e\gamma$ did not happen, led one to postulate that the two neutrino states produced in muon decay were distinct, and that $\mu \rightarrow e\gamma$, and other similar processes, were forbidden due to symmetries.

To this date, these so-called individual lepton-flavor numbers seem to be conserved in the case of charged lepton processes, in spite of many decades of (so far) fruitless searching…

History of $\mu \to e\gamma$, $\mu N \to eN$, and $\mu \to 3e$



[R. Bernstein, P. Cooper, arXiv 1307.5787]

Figure 3: The history of CLFV searches in muons (not including muonium.) One sees a steady improvement in all modes and then a flattening of the rate improvement throughout the 1990s. MEG has upgrade plans for the $\mu \to e\gamma$ search. The two next generations of $\mu N \to eN$, Mu2e/COMET at FNAL

SM Expectations?

In the old SM, the rate for charged lepton flavor violating processes is trivial to predict. It vanishes because individual lepton-flavor number is conserved:

- $N_\alpha(\text{in}) = N_\alpha(\text{out})$, for $\alpha = e, \mu, \tau$.

But individual lepton-flavor number are NOT conserved– ν oscillations!

Hence, in the νSM (the old Standard Model plus operators that lead to neutrino masses) $\mu \to e\gamma$ is allowed (along with all other charged lepton flavor violating processes).

These are Flavor Changing Neutral Current processes, observed in the quark sector ($b \to s\gamma$, $K^0 \leftrightarrow \bar{K}^0$, etc).

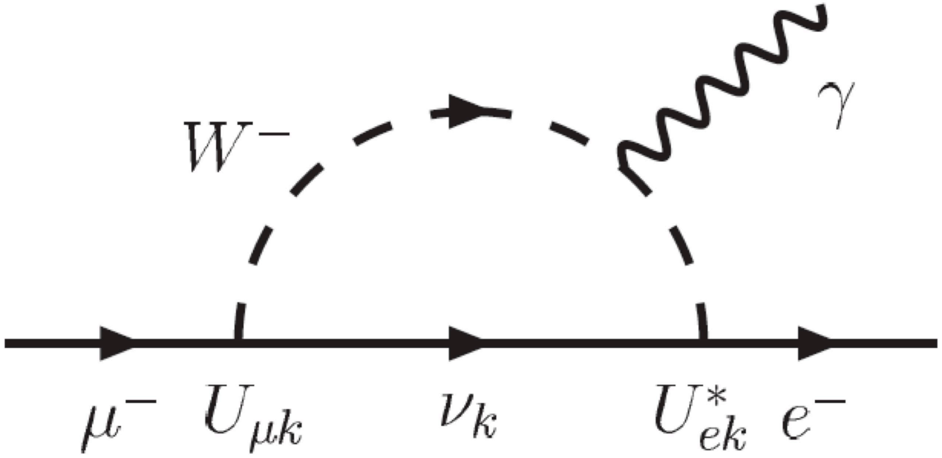
Unfortunately, we do not know the νSM expectation for charged lepton flavor violating processes → **we don't know the νSM Lagrangian !**

One contribution known to be there: active neutrino loops (same as quark sector).

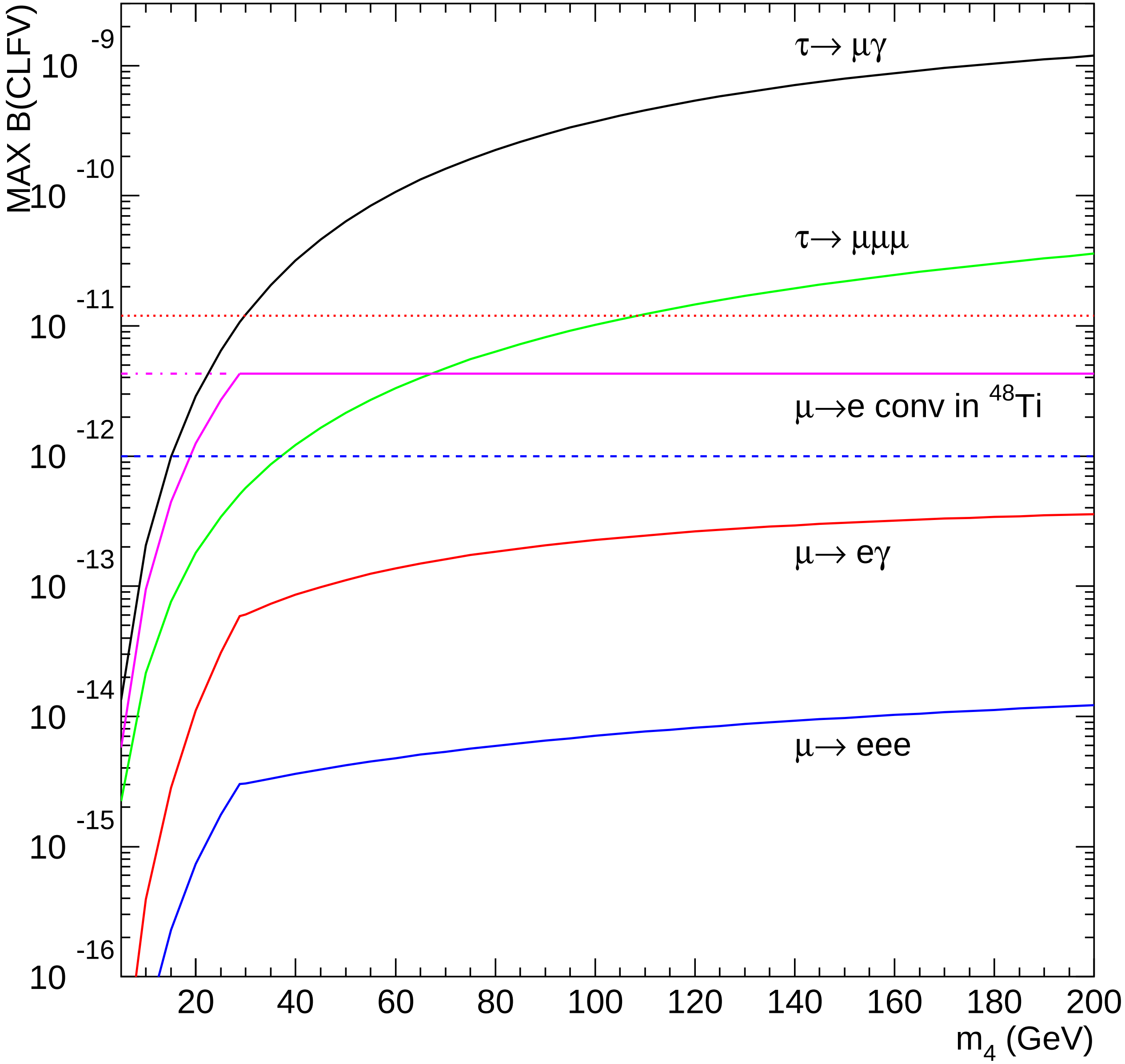
In the case of charged leptons, the **GIM suppression is very efficient**…

e.g.: $Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi} \left| \sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{1i}}{M^2_W} \right|^2 < 10^{-54}$

[$U_{\alpha i}$ are the elements of the leptonic mixing matrix,
$\Delta m^2_{1i} \equiv m^2_i - m^2_1$, $i = 2, 3$ are the neutrino mass-squared differences]



e.g.: SeeSaw Mechanism [minus "Theoretical Prejudice"]



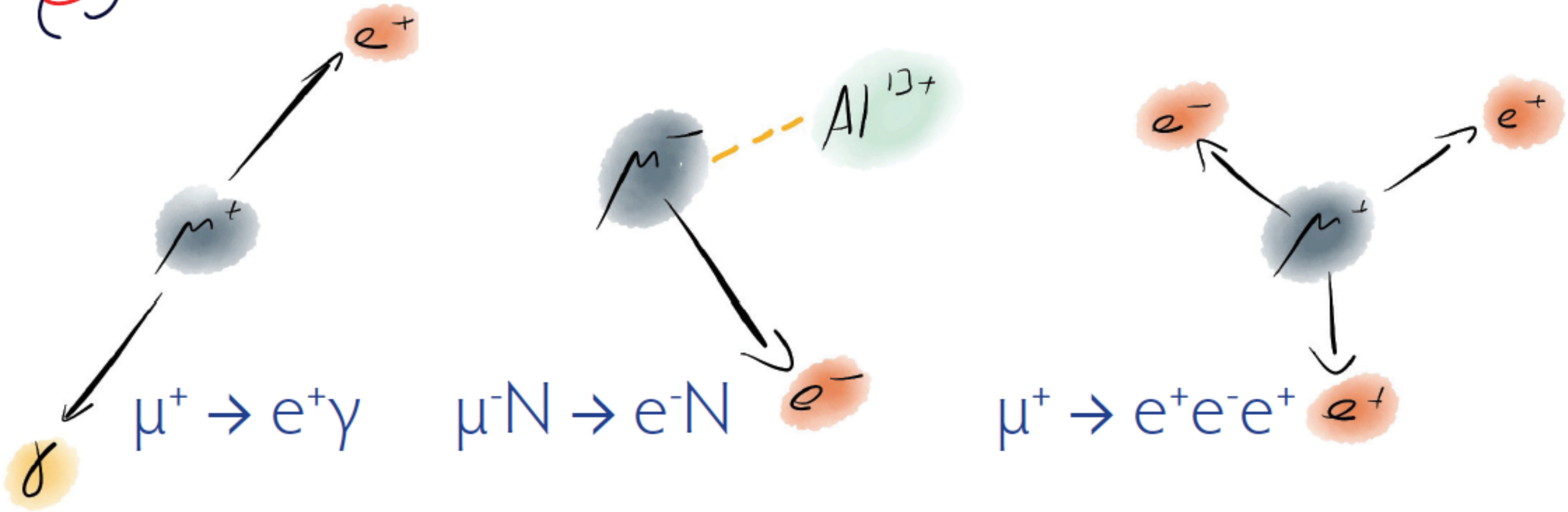
arXiv:0706.1732 [hep-ph]

Muon CLFV Processes of Interest:

- $\mu^+ \to e^+\gamma$ (‘mu to e gamma’);
- $\mu^+ \to e^+e^+e^-$ (‘mu to three e’);
- $\mu^- N \to e^- N$ (‘mu to e conversion’).

There are many other CLFV processes. These include rare meson decays ($K_L \to \mu e$, $K^+ \to \pi^+\mu e$, etc) and rare tau decay processes ($\tau \to \mu\gamma$, $\tau \to \eta e$, etc). I will briefly comment on the latter tomorrow, time permitting.

LFV Muon Decays: Experimental Situation



$\mu^+ \to e^+\gamma$ $\mu^-N \to e^-N$ $\mu^+ \to e^+e^-e^+$

MEG (PSI)

$B(\mu^+ \to e^+\gamma) < 5.7 \cdot 10^{-13}$
(2013)

upgrading

SINDRUM II (PSI)

$B(\mu^- Au \to e^- Au) < 7 \cdot 10^{-13}$
(2006)

Mu2e/Comet

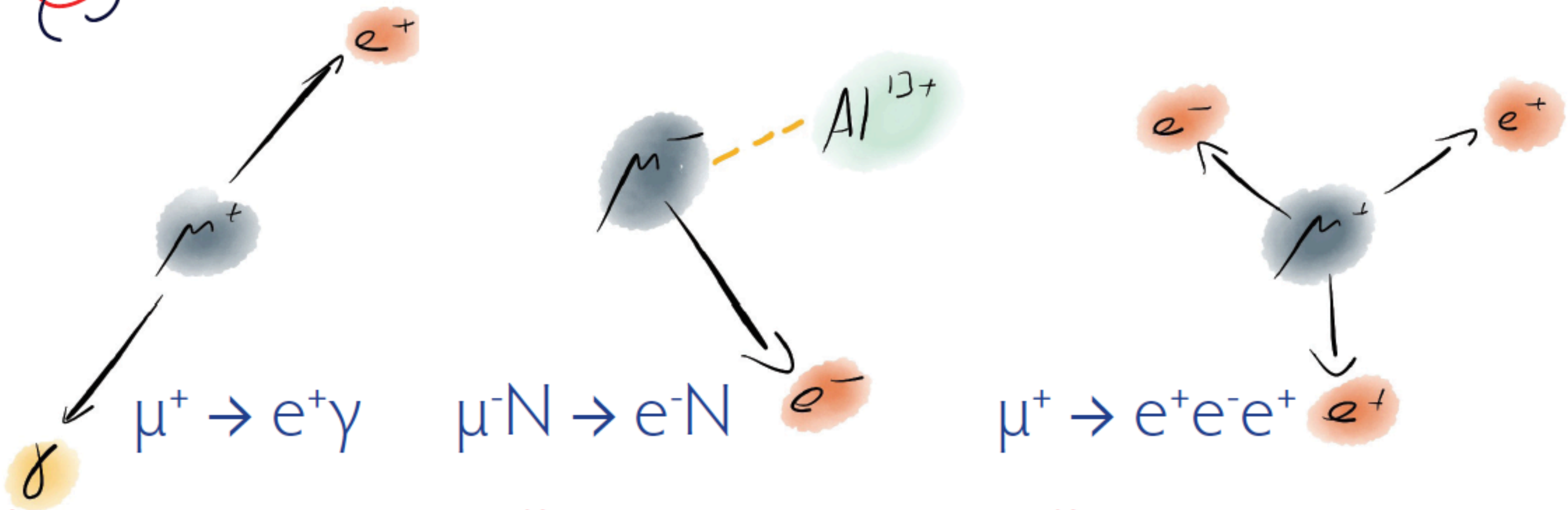
SINDRUM (PSI)

$B(\mu^+ \to e^+e^-e^+) < 1.0 \cdot 10^{-12}$
(1988)

Mu3e

[Berger, talk at Lepton Moments 2014]

LFV Muon Decays: Experimental signatures



$\mu^+ \to e^+\gamma$

$\mu^-N \to e^-N$

$\mu^+ \to e^+e^-e^+$

Kinematics

- 2-body decay
- Monoenergetic e^+, γ
- Back-to-back

Background

- Accidental background

Kinematics

- Quasi 2-body decay
- Monoenergetic e^-
- Single particle detected

Background

- Decay in orbit
- Antiprotons, pions, cosmics

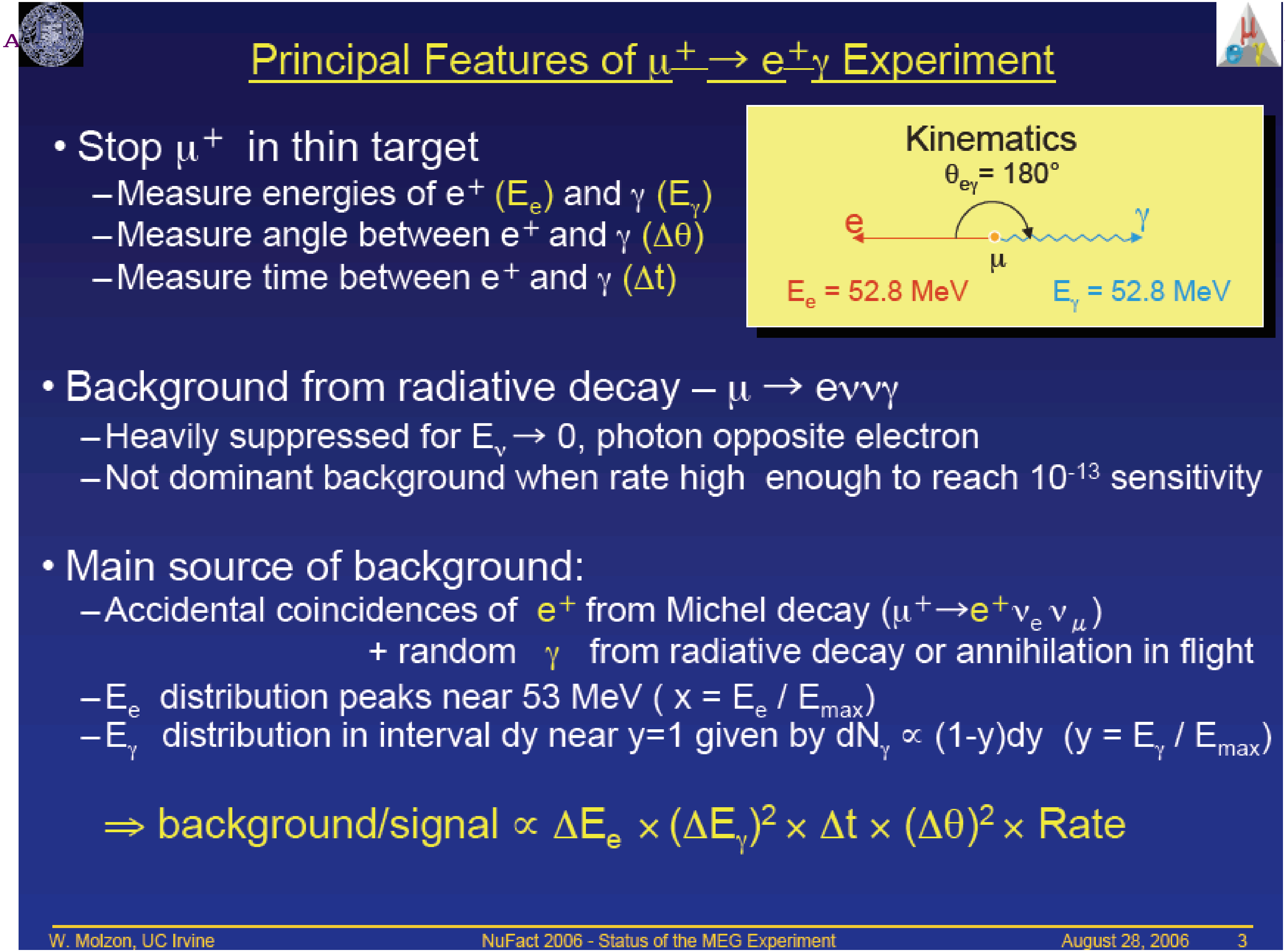
Kinematics

- 3-body decay
- Invariant mass constraint
- $\Sigma p_i = 0$

Background

- Radiative decay
- Accidental background

[Berger, talk at Lepton Moments 2014]



Principal Features of $\mu^+ \to e^+\gamma$ Experiment

- Stop μ^+ in thin target
  - Measure energies of e^+ (E_e) and γ (E_γ)
  - Measure angle between e^+ and γ ($\Delta\theta$)
  - Measure time between e^+ and γ (Δt)

Kinematics

$\theta_{e\gamma} = 180°$

$E_e = 52.8$ MeV

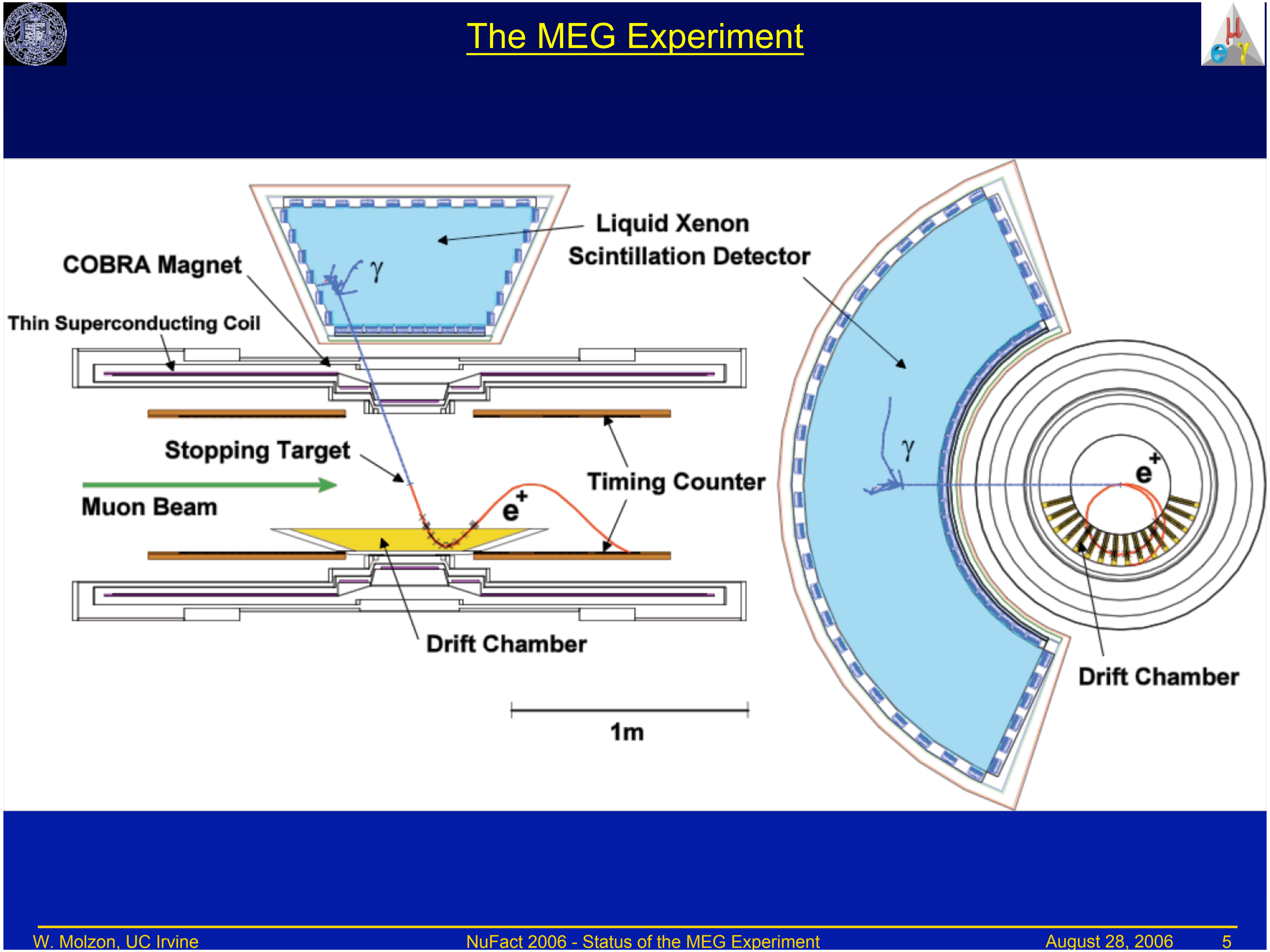
$E_\gamma = 52.8$ MeV

- Background from radiative decay – $\mu \to e\nu\nu\gamma$
  - Heavily suppressed for $E_\nu \to 0$, photon opposite electron
  - Not dominant background when rate high enough to reach 10^{-13} sensitivity

- Main source of background:
  - Accidental coincidences of e^+ from Michel decay ($\mu^+ \to e^+ \nu_e \nu_\mu$)
    + random γ from radiative decay or annihilation in flight
  - E_e distribution peaks near 53 MeV ($x = E_e / E_{max}$)
  - E_γ distribution in interval dy near y=1 given by $dN_\gamma \propto (1-y)dy$ ($y = E_\gamma / E_{max}$)

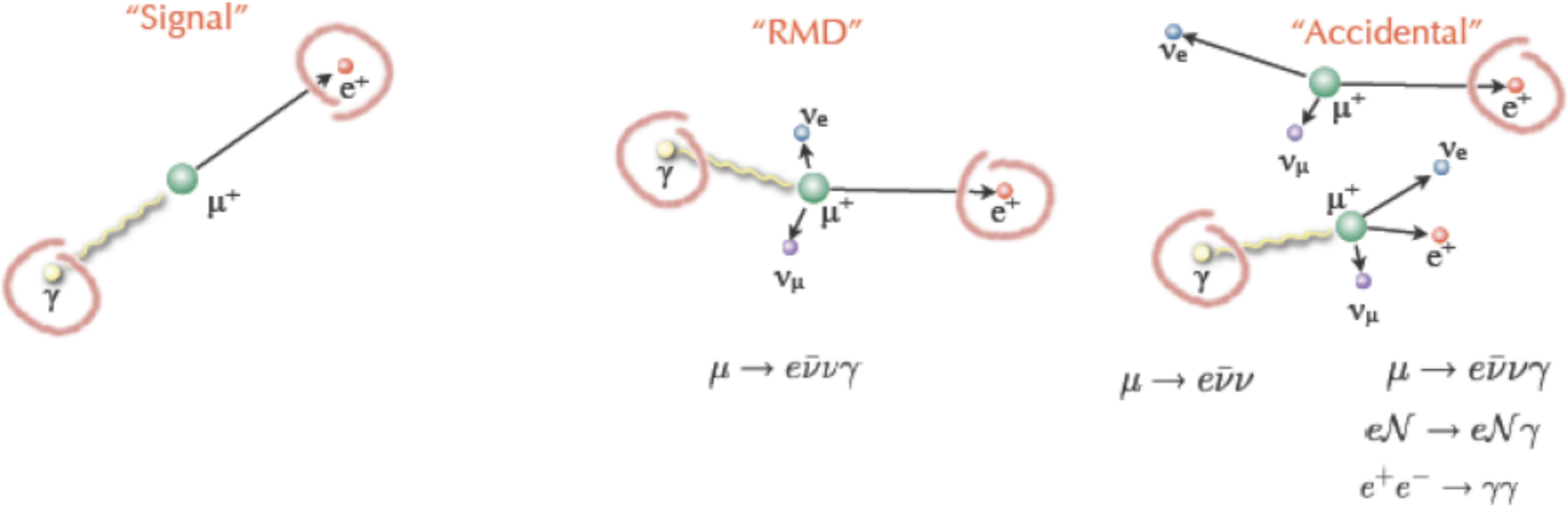
$$\Rightarrow \text{background/signal} \propto \Delta E_e \times (\Delta E_\gamma)^2 \times \Delta t \times (\Delta\theta)^2 \times \text{Rate}$$

W. Molzon, UC Irvine NuFact 2006 - Status of the MEG Experiment August 28, 2006 3



The MEG Experiment

W. Molzon, UC Irvine NuFact 2006 - Status of the MEG Experiment August 28, 2006 5



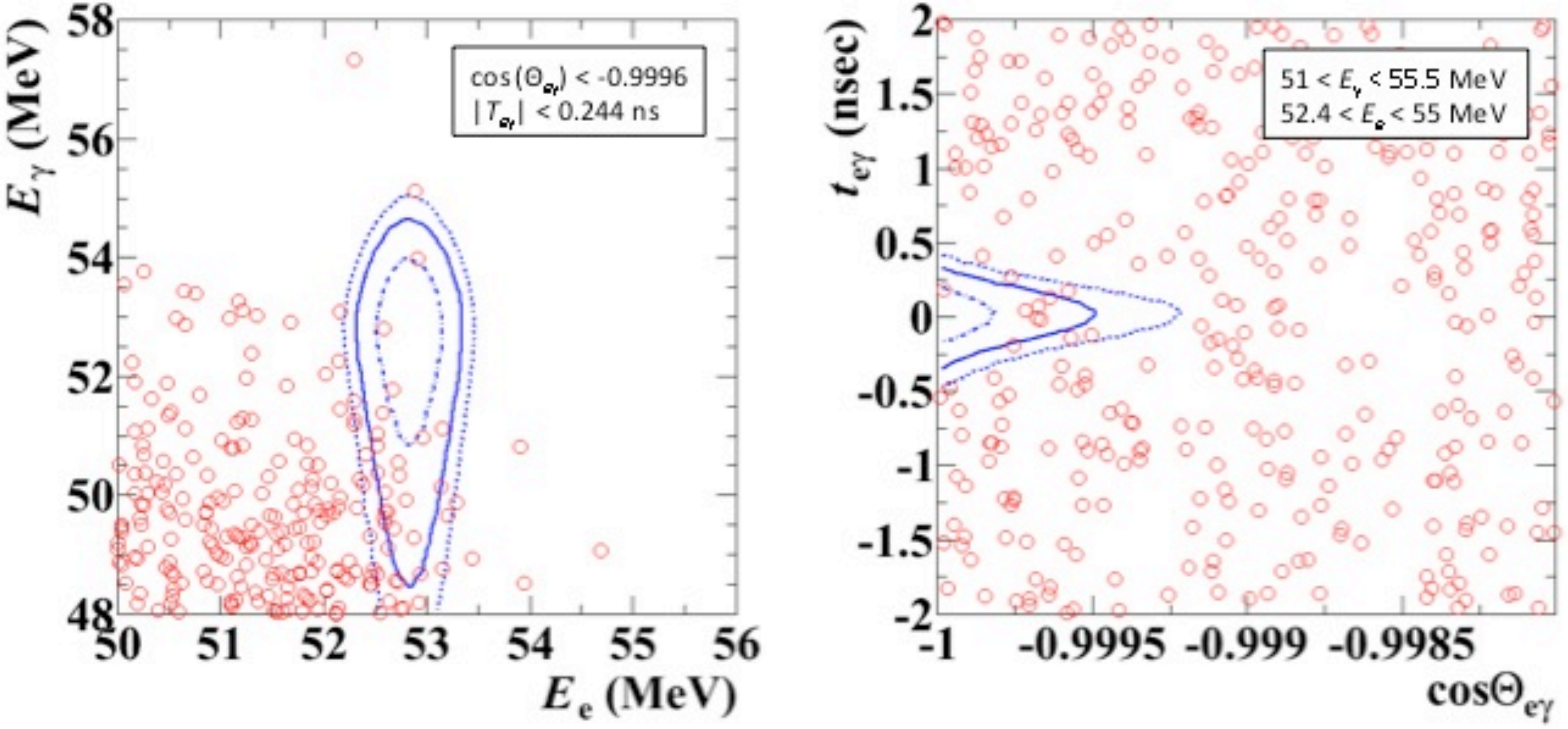
$\Uparrow$

Dominant Background

PRL 110 (2013) 201801

Event distributions

Signal PDF contours at 1, 1.64 and 2 sigma (68%, 90% and 95%)



No excess of events in the signal region

Conclusion

Most recent analysis:

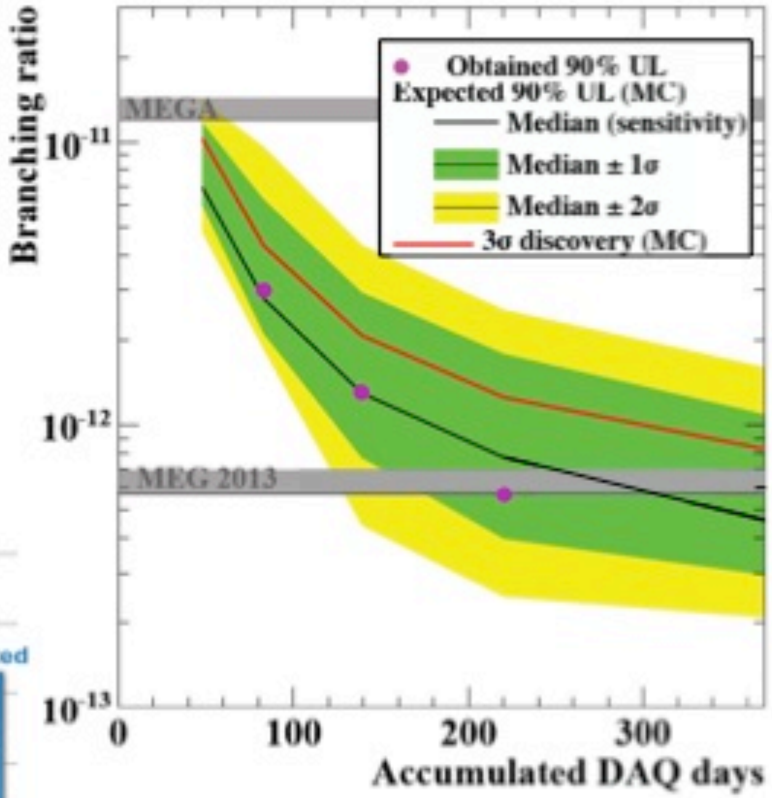
Combined 2009-2011 analysis did not show a significant excess of signal over background, resulting in a factor 4 improvement of the world's most stringent BR($\mu \to e\gamma$) upper limit:

$$\mathrm{Br}(\mu^+ \to e^+\gamma) \; < \; 5.7 \cdot 10^{-13} \; (90\,\% \; \mathrm{C.L.})$$

Outlook:

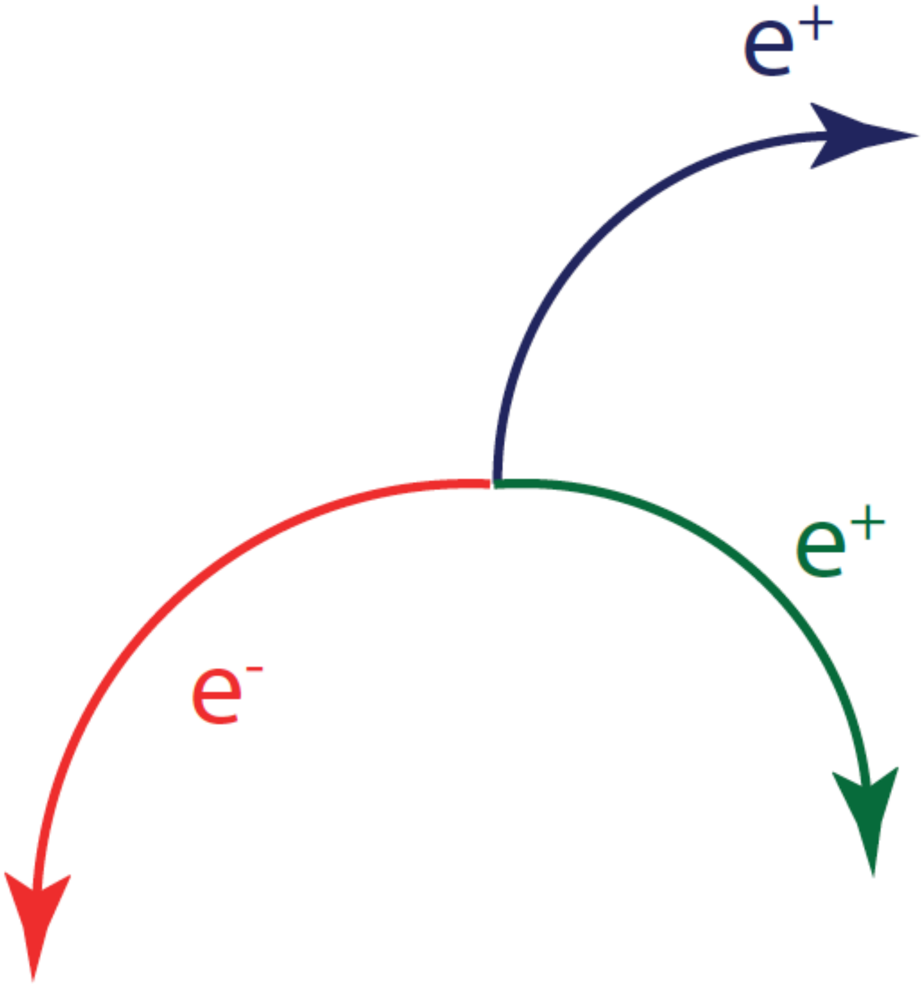
- Data taking finished September 2013
- Total statistics incl. 2012+2013 data is expected to double
- New results coming end of this year

- Observed BR limits & sensitivity:



But that's not all..

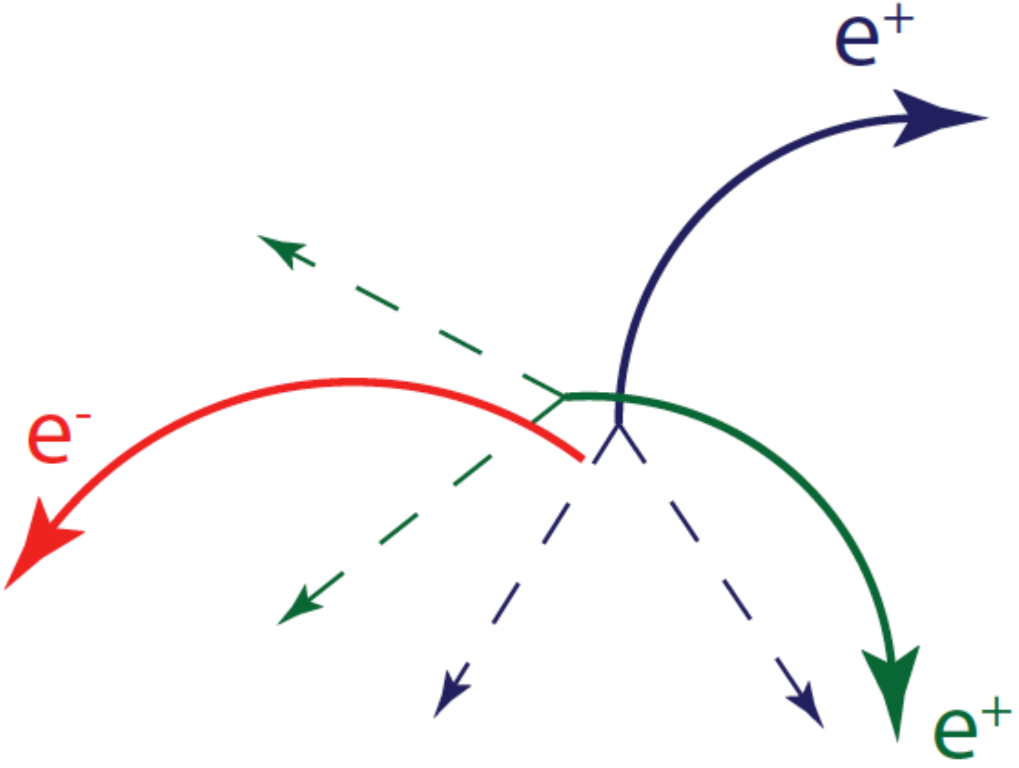
The signal



- $\mu^+ \rightarrow e^+e^-e^+$
- Two positrons, one electron
- From same vertex
- Same time
- Sum of 4-momenta corresponds to muon at rest
- Maximum momentum: $\frac{1}{2} m_\mu = 53$ MeV/c

[Berger, talk at Lepton Moments 2014]

Accidental Background

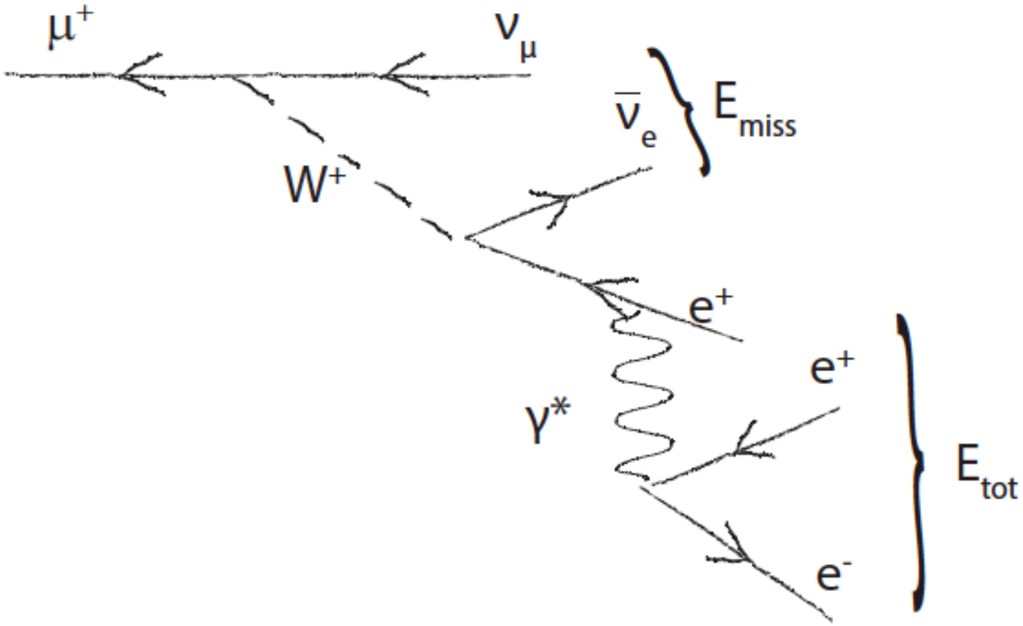


- Combination of positrons from ordinary muon decay with electrons from:
  - photon conversion,
  - Bhabha scattering,
  - Mis-reconstruction

- Need very good timing, vertex and momentum resolution

[Berger, talk at Lepton Moments 2014]

Internal conversion background

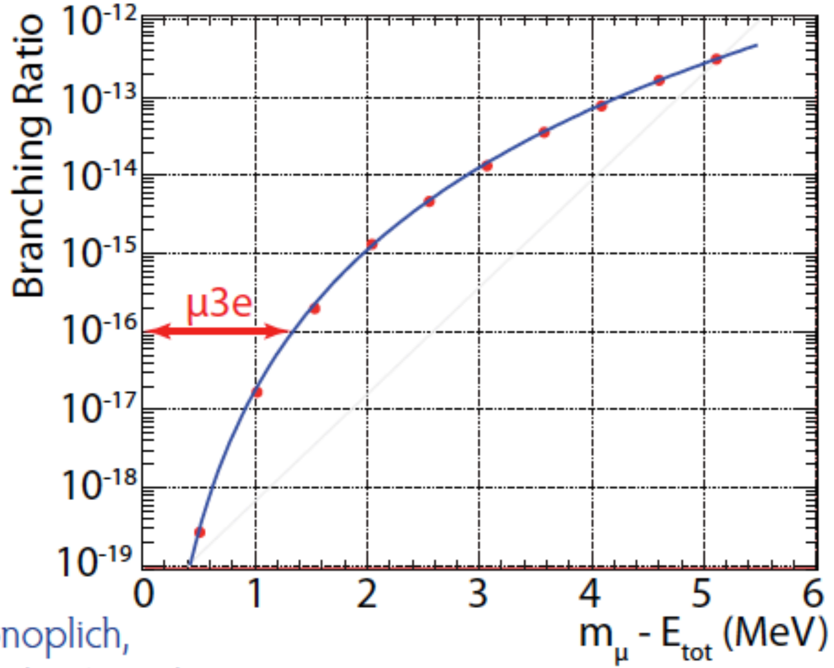


- Allowed radiative decay with internal conversion:

$$\mu^+ \rightarrow e^+e^-e^+\nu\bar{\nu}$$

- Only distinguishing feature: Missing momentum carried by neutrinos

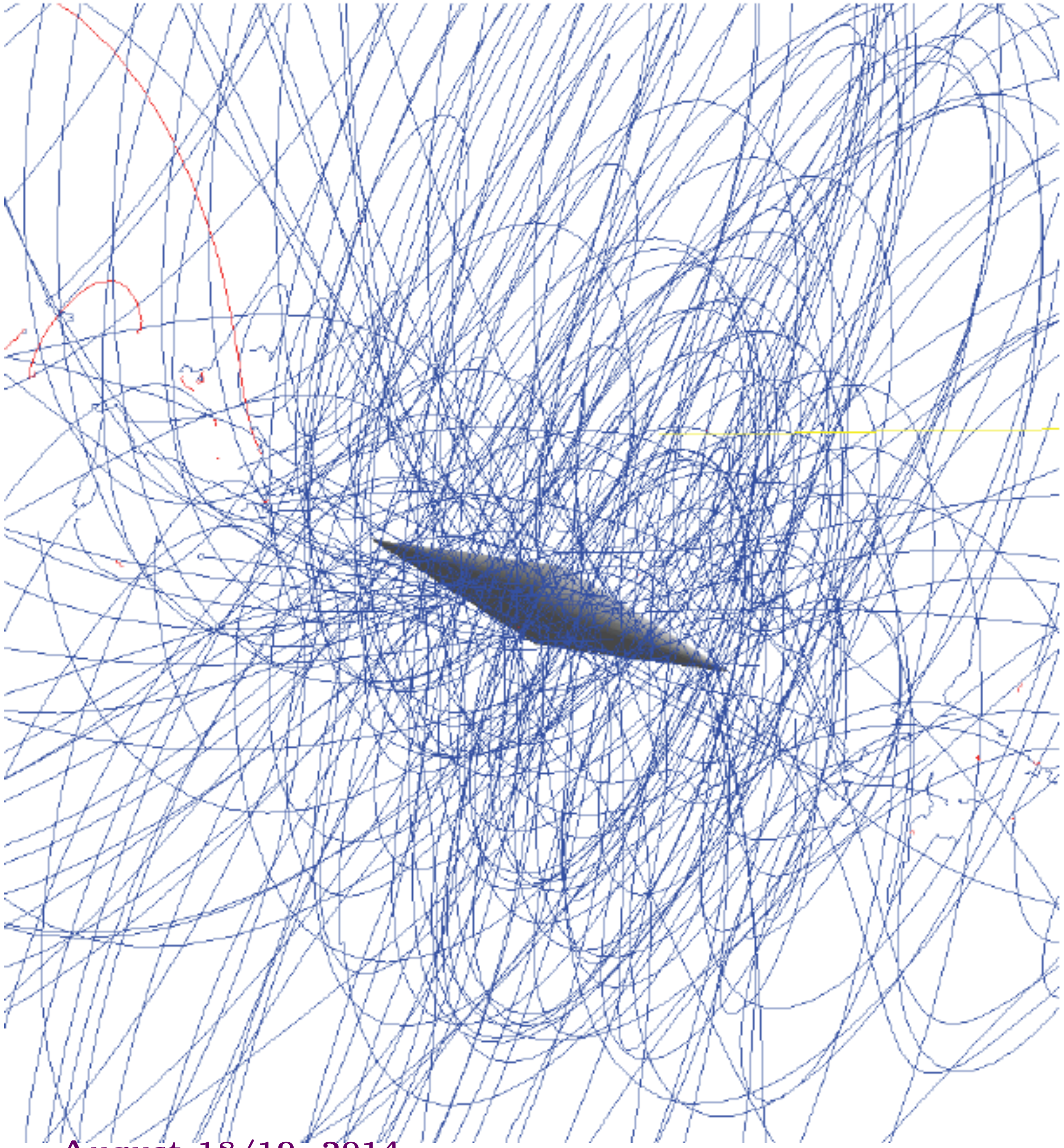
- Need excellent momentum resolution



(R. M. Djilkibaev, R. V. Konoplich, Phys.Rev. D79 (2009) 073004)

[Berger, talk at Lepton Moments 2014]

Detector Technology



- High granularity (occupancy)
- Close to target (vertex resolution)
- 3D space points (reconstruction)
- Minimum material (momenta below 53 MeV/c)

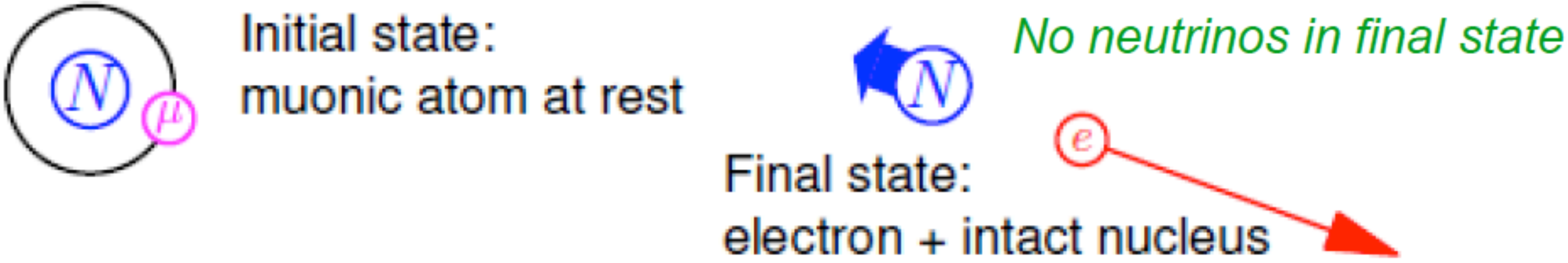
- Gas detectors do not work (space charge, aging, 3D)
- Silicon strips do not work (material budget, 3D)
- Hybrid pixels (as in LHC) do not work (material budget)

[Berger, talk at Lepton Moments 2014]

Muon to Electron Conversion

Charged lepton flavor violating process

Nucleus nearby for conservation of momentum and energy



Initial state:
muonic atom at rest

No neutrinos in final state

Final state:
electron + intact nucleus

- Signal is monoenergetic electron

$$E_e = m_\mu - E_b - E_{\text{recoil}} \approx 104.97 \text{ MeV for Al}$$

- Conventional signal normalization

$$R_{\mu e} = \frac{\Gamma[\mu^- + N \to e^- + N]}{\Gamma[\mu^- + N \to \text{all captures}]}$$

[J. Miller, talk at Lepton Moments 2014]

Muon to Electron Conversion: Present and Future Precision

Current limits: $$R_{\mu e} = \frac{\mu^- Au \to e^- Au}{\mu^- Au \to \text{capture}} < 7\text{x}10^{-13} \text{ (SINDRUM II)}$$

Also: $$R_{\mu e} = \frac{\mu^- Ti \to e^- Ti}{\mu^- Ti \to \text{capture}} < 4.3\text{x}10^{-12} \text{ (SINDRUM II)}$$

$$R_{\mu e} = \frac{\mu^- Ti \to e^- Ti}{\mu^- Ti \to \text{capture}} < 4.6\text{x}10^{-12} \text{ (TRIUMF)}$$

Mu2e goal: $$R_{\mu e} = \frac{\mu^- Al \to e^- Al}{\mu^- Al \to \text{capture}} < 6\text{x}10^{-17} \text{ (90\% c.l.)}$$

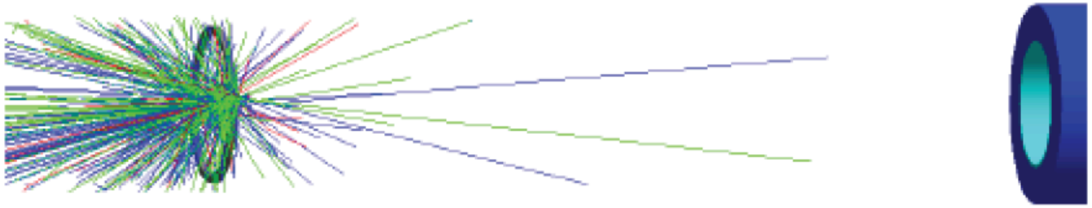
four orders of magnitude improvement over current limit!

[J. Miller, talk at Lepton Moments 2014]

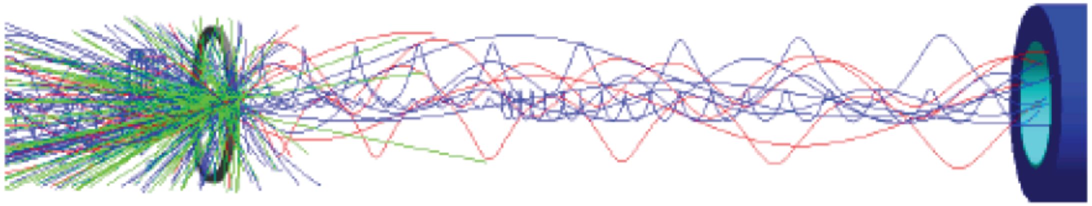
- Increase in sensitivity by 10^4
  - Mu2e single event sensitivity goal 2.5×10^{-17}
- Mu2e needs ~few x 10^{17} stopped muons
  - Best available beam line ~ 10^8 Hz (PSI 1.3 MW beam)
  - Mu2e goal ~few x 10^{10} Hz
  - We are not proposing a 1 GW beam…

Develop a more efficient muon beamline

Instead of this



Do this

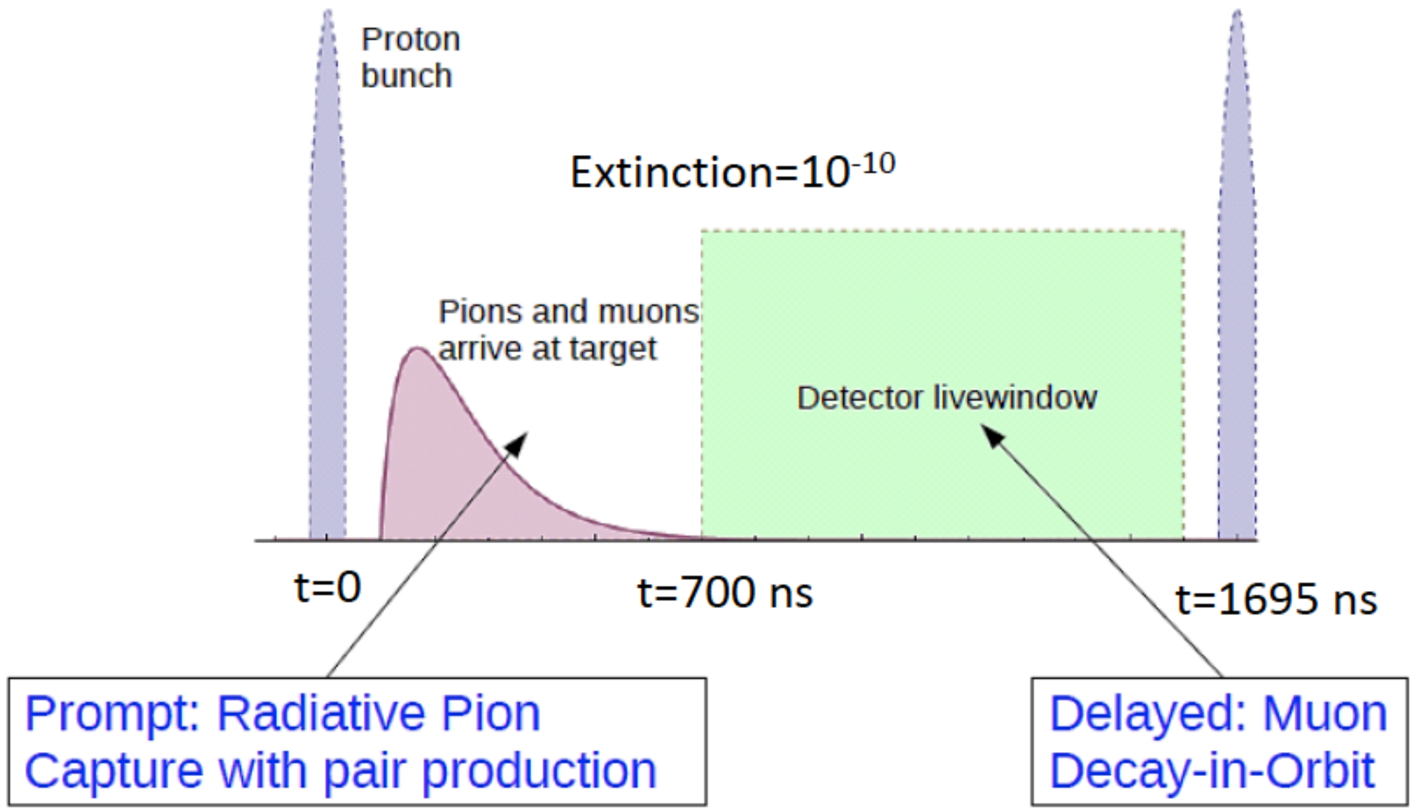


Solenoidal B field confines soft pions. Collect their decay muons.
Mu2e: > 10^{10} Hz stopped muons from only 8 kW of beam of protons

[J. Miller, talk at Lepton Moments 2014]

The two most dangerous backgrounds have very different timing properties

The FNAL accelerator complex produces proton beams with a pulsed structure



Prompt: Radiative Pion Capture with pair production

Delayed: Muon Decay-in-Orbit

Also low energy backgrounds from muon captures In stopping target. Per capture: ~1.2 neutrons, ~0.1 protons, ~2 gammas

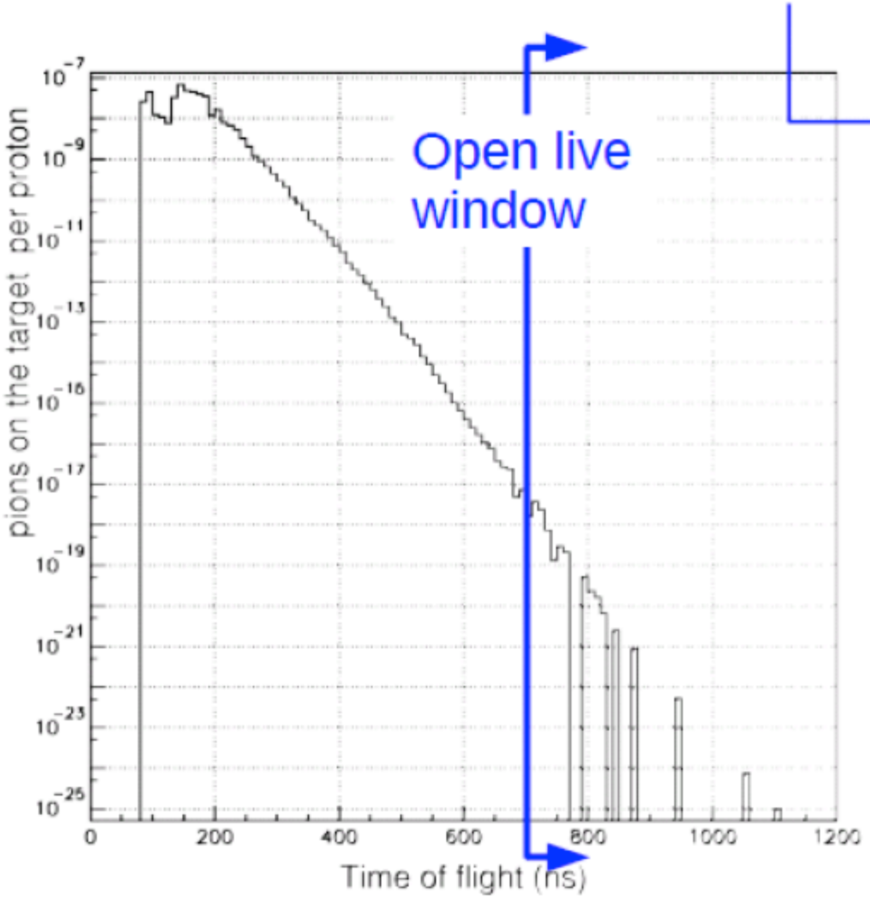
[J. Miller, talk at Lepton Moments 2014]

Radiative Pion Capture

can produce electrons near the conversion energy

$$\pi^- A(Z,N) \to \gamma A(Z-1,N+1)^*$$

$\gamma \to e^+e^-$



The electron spectrum extends out to nearly m_π=139.6 MeV

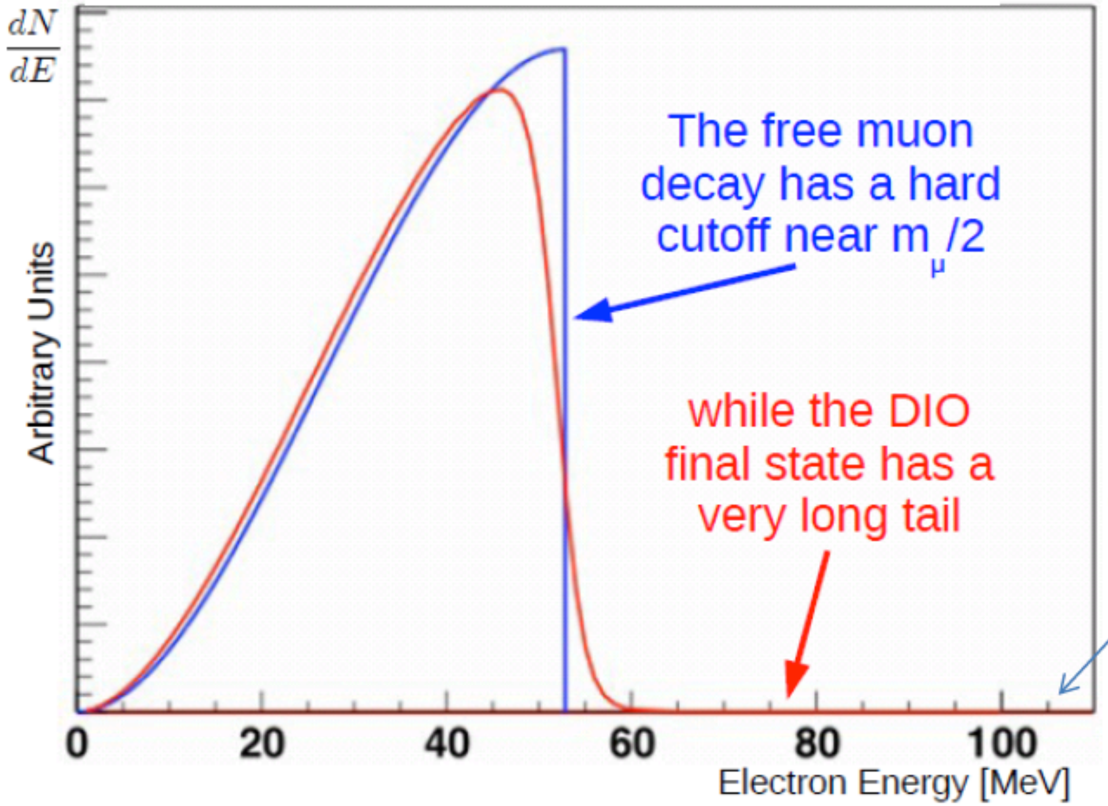
This (and other) prompt backgrounds are eliminated by keeping the DAQ dead until the background rate has fallen below some threshold.

[J. Miller, talk at Lepton Moments 2014]

Decay-in-Orbit is the major source of delayed background in the live window

$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$$

$$A(Z,N)\mu^- \rightarrow A(Z,N)e^- \nu_\mu \bar{\nu}_e$$



The free muon decay has a hard cutoff near $m_\mu/2$

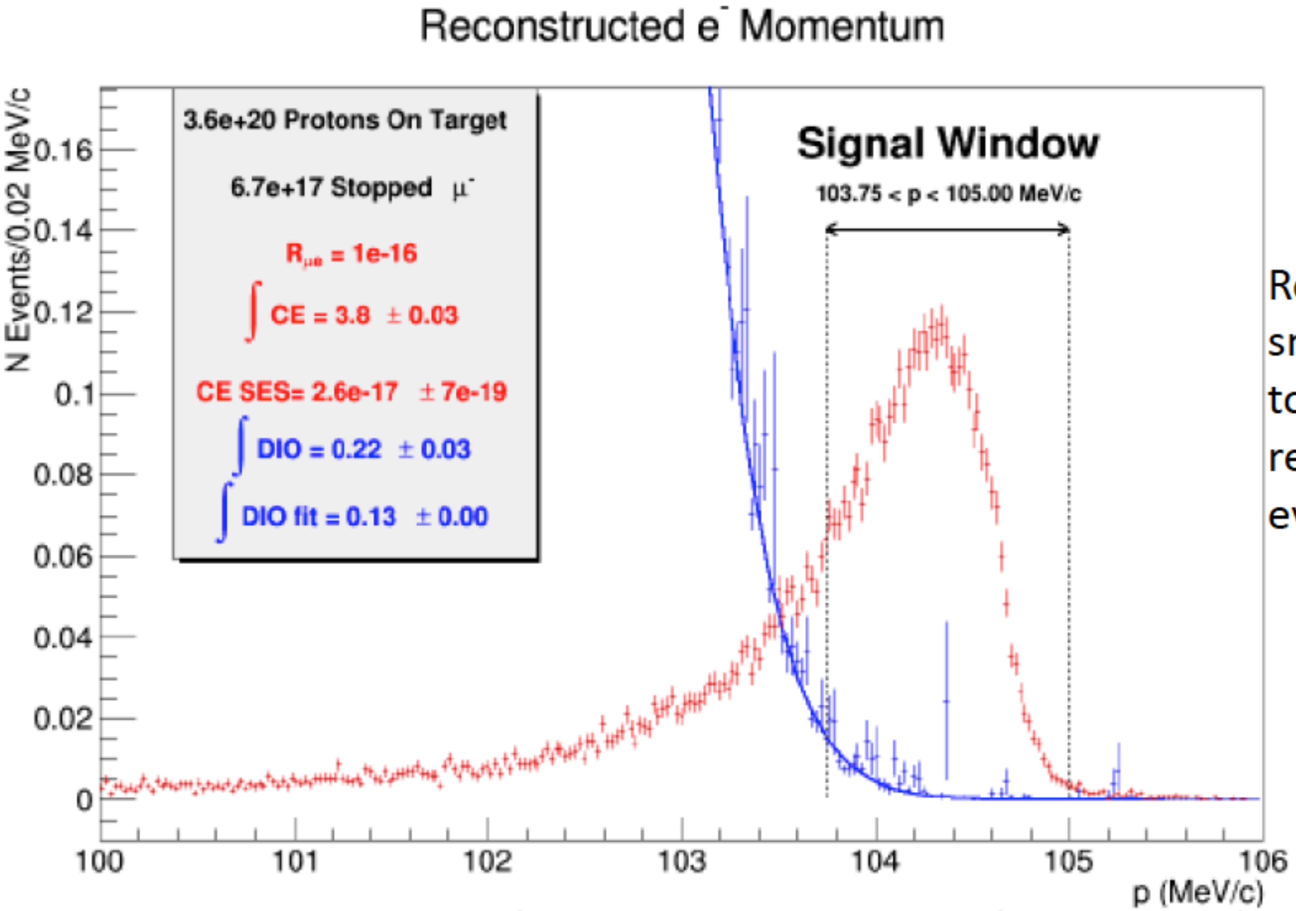
while the DIO final state has a very long tail

Conversion electron energy

[J. Miller, talk at Lepton Moments 2014]

Simulation of DIO + conversion electron energy distributions

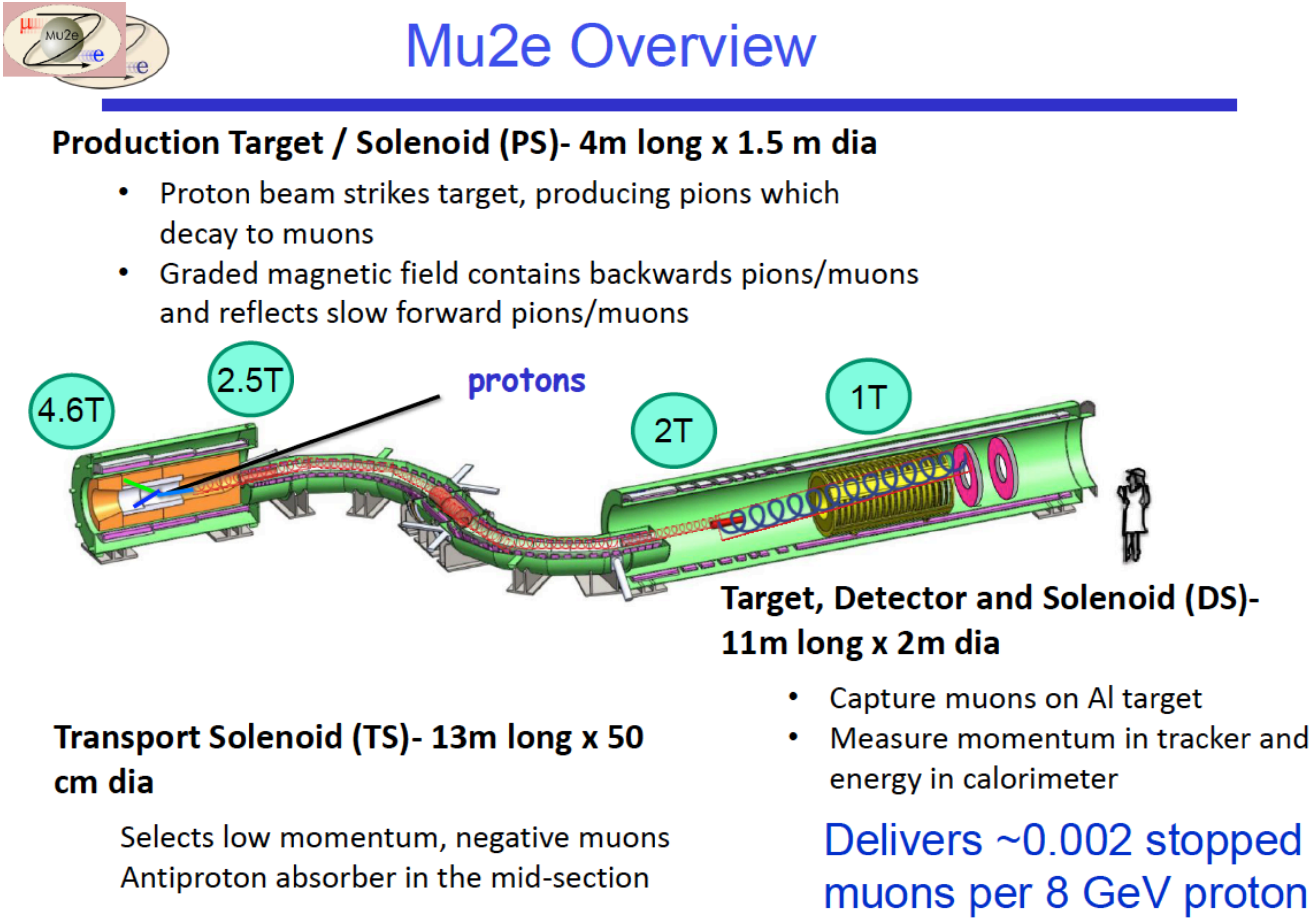
- *Assuming* $R_{\mu e}=10^{-16}$
- FWHM ~ 1 MeV, ~ 4 events in 103.7 MeV<E< 105 MeV)



Realistic scattering losses smear the distributions to lower energy, while resolution can push events to higher energy

DIO Spectrum: A. Czarnecki, X. Tormo, W. Marciano, Phys. Rev. D84 (2011) 013006

[J. Miller, talk at Lepton Moments 2014]



Mu2e Overview

Production Target / Solenoid (PS)- 4m long x 1.5 m dia

- Proton beam strikes target, producing pions which decay to muons
- Graded magnetic field contains backwards pions/muons and reflects slow forward pions/muons

Transport Solenoid (TS)- 13m long x 50 cm dia

Selects low momentum, negative muons
Antiproton absorber in the mid-section

Target, Detector and Solenoid (DS)- 11m long x 2m dia

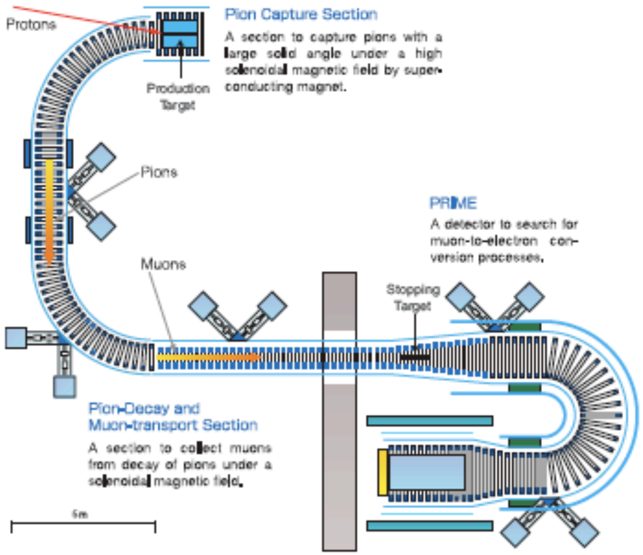
- Capture muons on Al target
- Measure momentum in tracker and energy in calorimeter

Delivers ~0.002 stopped muons per 8 GeV proton

[J. Miller, talk at Lepton Moments 2014]

Staging Approach
to Search for Muon to Electron Conversion

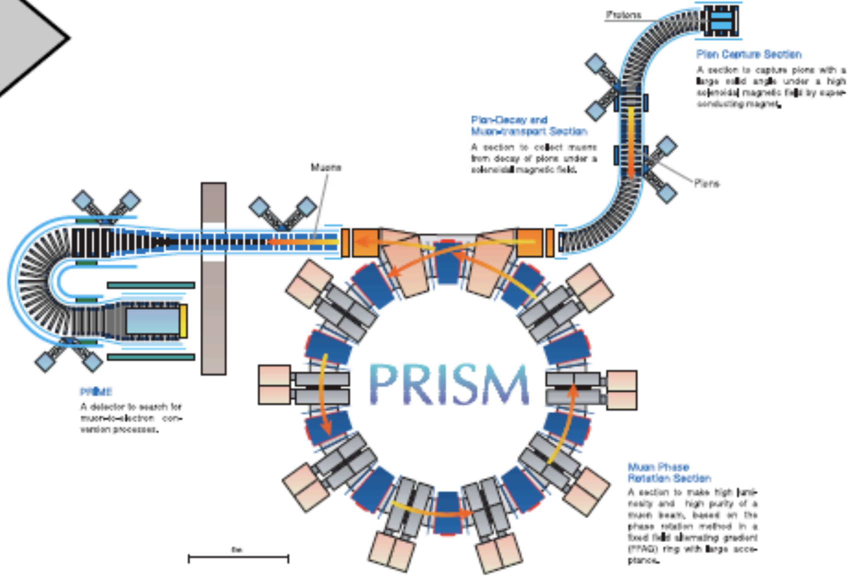
Phase 1 : COMET



$$B(\mu^- + Al \rightarrow e^- + Al) < 10^{-16}$$

- without a muon storage ring.
- use a slowly-extracted pulsed proton beam.
- medium proton beam power (60 kW)
- can be done at the J-PARC NP Hall.
- Early realization

Phase 2 : PRISM



$$B(\mu^- + Ti \rightarrow e^- + Ti) < 10^{-18}$$

- with a muon storage ring.
- use a fast-extracted pulsed proton beam.
- very high beam pwer (>1 MW)
- need a new beamline of fast extraction.
- Ultimate search

Independent from neutrino masses, there are strong theoretical reasons to believe that the expected rate for flavor changing violating processes is much, much larger than naive νSM predictions and that discovery is just around the corner.

Due to the lack of SM "backgrounds," searches for rare muon processes, including $\mu \to e\gamma$, $\mu \to e^+e^-e$ and $\mu + N \to e + N$ (μ-e–conversion in nuclei) are considered ideal laboratories to probe effects of new physics at or even above the electroweak scale.

Indeed, if there is new physics at the electroweak scale (as many theorists will have you believe) and if mixing in the lepton sector is large "everywhere" the question we need to address is quite different:

Why haven't we seen charged lepton flavor violation yet?

Model Independent Approach

As far as charged lepton flavor violating processes are concern, new physics effects can be parameterized via a handful of higher dimensional operators. For example, say that the following effective Lagrangian dominates CLFV phenomena:

$$\mathcal{L}_{\rm CLFV} = \frac{m_\mu}{(\kappa+1)\Lambda^2}\bar{\mu}_R\sigma_{\mu\nu}e_L F^{\mu\nu} + \frac{\kappa}{(1+\kappa)\Lambda^2}\bar{\mu}_L\gamma_\mu e_L\left(\bar{u}_L\gamma^\mu u_L + \bar{d}_L\gamma^\mu d_L\right)$$

First term: mediates $\mu\to e\gamma$ and, at order α, $\mu\to eee$ and $\mu+Z\to e+Z$

Second term: mediates $\mu+Z\to e+Z$ and, at one-loop, $\mu\to e\gamma$ and $\mu\to eee$

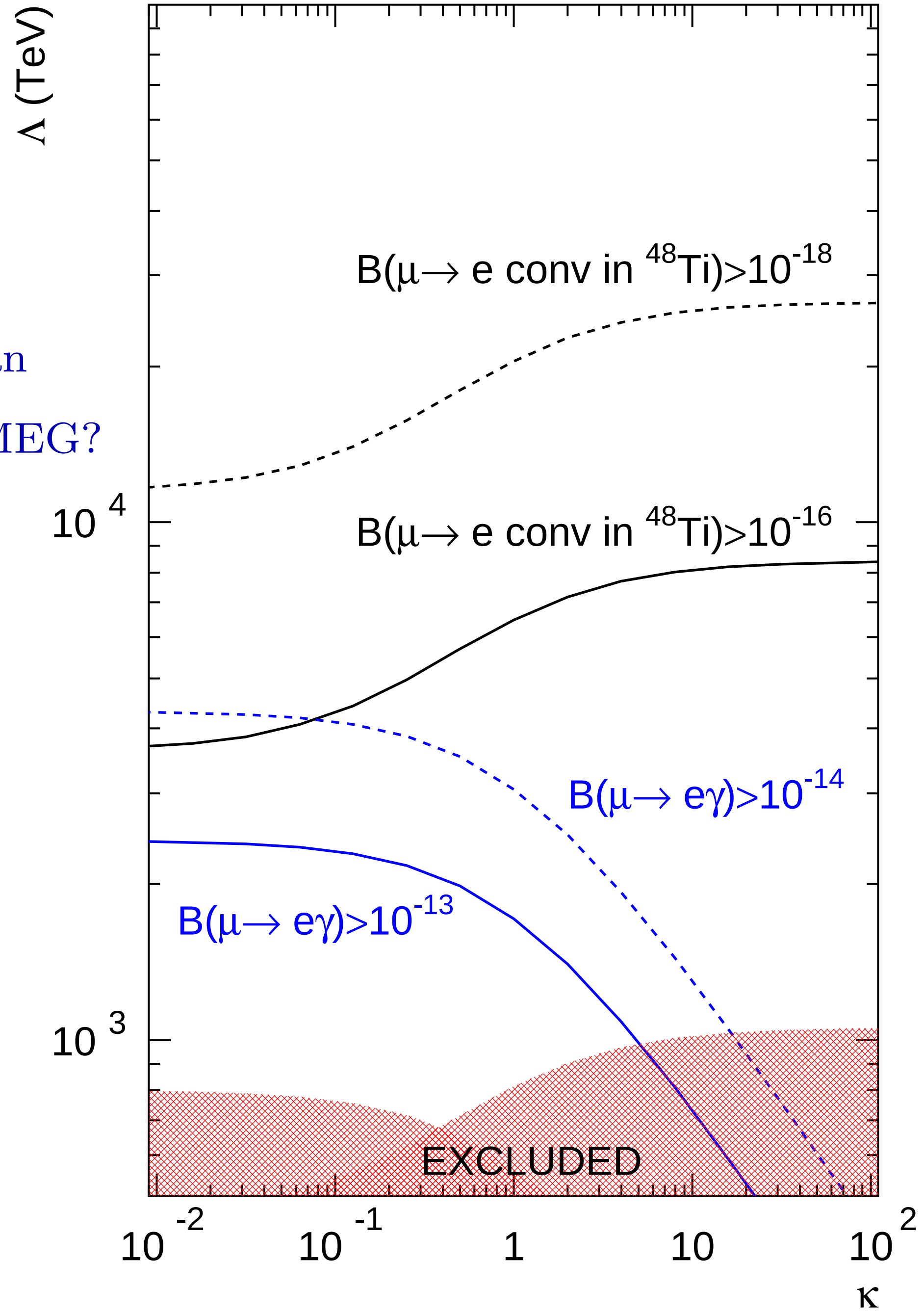
Λ is the "scale of new physics". κ interpolates between transition dipole moment and four-fermion operators.

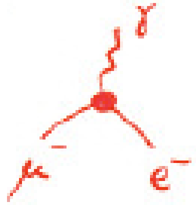
Which term wins? → Model Dependent

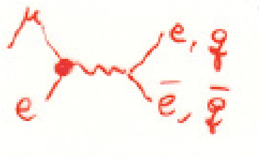
- $\mu \to e$-conv at 10^{-17} "guaranteed" deeper probe than $\mu \to e\gamma$ at 10^{-14}.

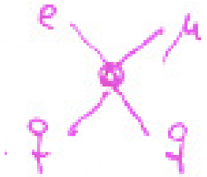
- It is really hard to do $\mu \to e\gamma$ much better than 10^{-14}. $\mu \to e$–conv "best" way forward after MEG?

- If the LHC does not discover new states $\mu \to e$-conv among very few process that can access 10,000+ TeV new physics scale: tree-level new physics: $\kappa \gg 1$, $\frac{1}{\Lambda^2} \sim \frac{g^2\theta_{e\mu}}{M^2_{\rm new}}$.







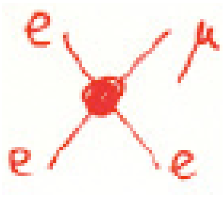


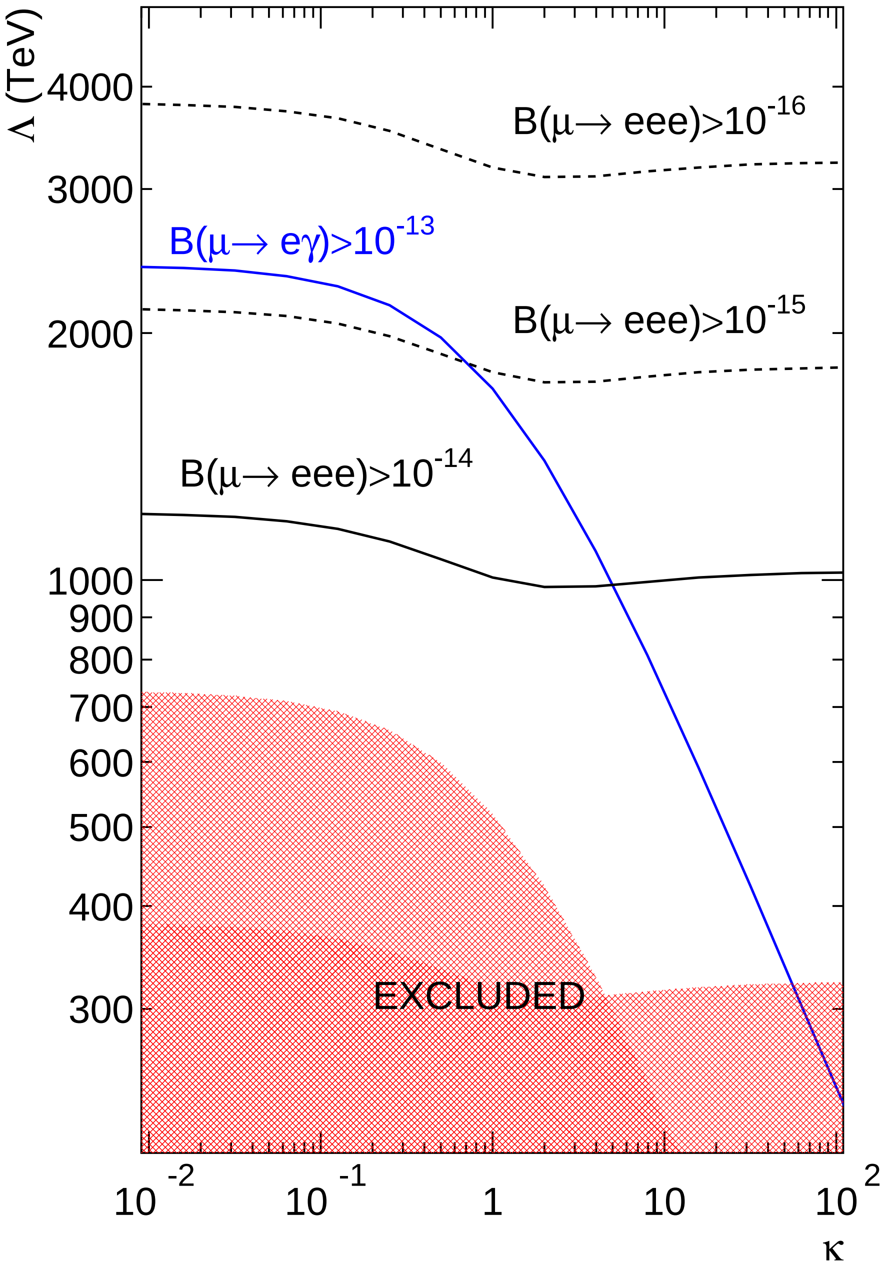
Other Example: $\mu \to e e^+ e^-$

$$\mathcal{L}_{\rm CLFV} = \frac{m_\mu}{(\kappa+1)\Lambda^2}\bar{\mu}_R\sigma_{\mu\nu}e_L F^{\mu\nu} +$$

$$+ \frac{\kappa}{(1+\kappa)\Lambda^2}\bar{\mu}_L\gamma_\mu e_L \bar{e}\gamma^\mu e$$

- $\mu \to eee$-conv at 10^{-16} "guaranteed" deeper probe than $\mu \to e\gamma$ at 10^{-14}.
- $\mu \to eee$ another way forward after MEG?
- If the LHC does not discover new states $\mu \to eee$ among very few process that can access 1,000+ TeV new physics scale: tree-level new physics: $\kappa \gg 1$, $\frac{1}{\Lambda^2} \sim \frac{g^2\theta_{e\mu}}{M^2_{\rm new}}$.





What does "Λ" mean?

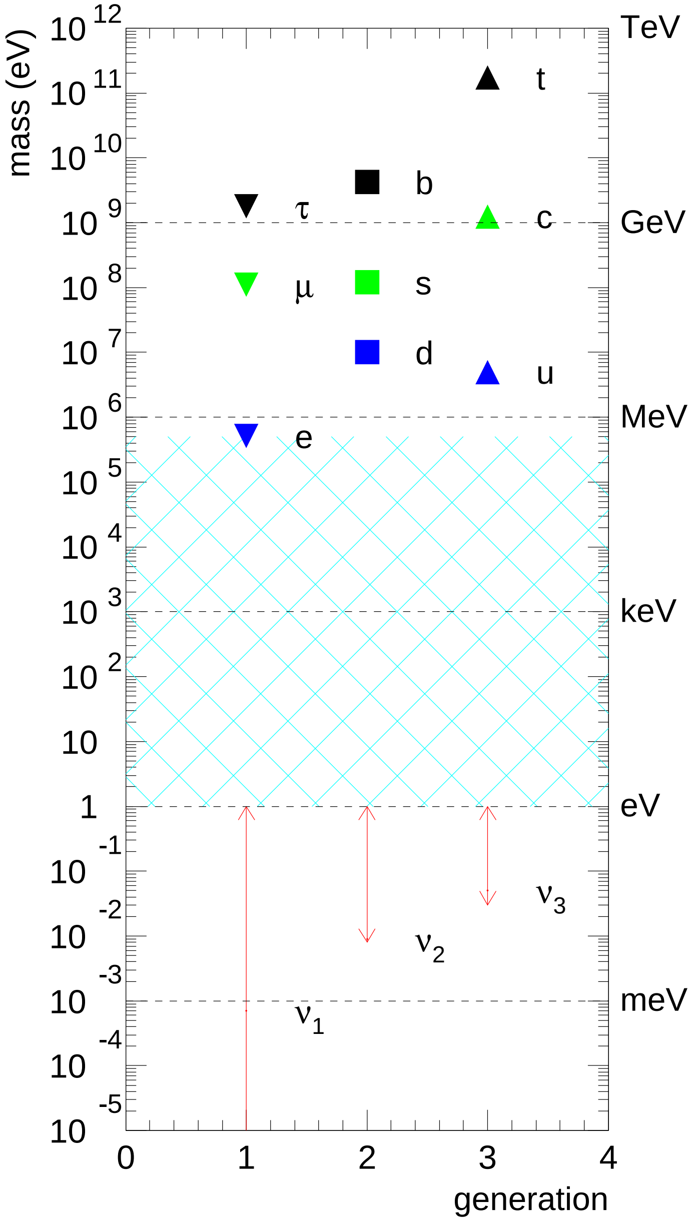
This is clearly model dependent! However, some general issues are easy to identify…

- $\mu \to e\gamma$ always occurs at the loop level, and is suppressed by the E&M coupling e. Also chiral suppression (potential for "$\tan\beta$" enhancement).

$$\frac{1}{\Lambda^2} \sim \frac{e}{16\pi^2}\frac{\tan\beta}{M_{\rm new}^2}$$

- $\mu \to eee$ and $\mu \to e$-conversion in nuclei can happen at the tree-level

$$\frac{1}{\Lambda^2} \sim \frac{y_{\rm new}^2}{M_{\rm new}^2}$$



What We Are Trying To Understand:

⇐ NEUTRINOS HAVE TINY MASSES

⇓ LEPTON MIXING IS “WEIRD” ⇓

$$V_{MNS} \sim \begin{pmatrix} 0.8 & 0.5 & \mathbf{0.2} \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix} \qquad V_{CKM} \sim \begin{pmatrix} 1 & 0.2 & {\scriptstyle 0.001} \\ 0.2 & 1 & 0.01 \\ {\scriptstyle 0.001} & 0.01 & 1 \end{pmatrix}$$

What Does It Mean?

Neutrino Masses, EWSB, and a New Mass Scale of Nature

The LHC has revealed that the minimum SM prescription for electroweak symmetry breaking — the one Higgs double model — is at least approximately correct. What does that have to do with neutrinos?

The tiny neutrino masses point to three different possibilities.

1. Neutrinos talk to the Higgs boson very, very **weakly** (Dirac neutrinos);
2. Neutrinos talk to a **different Higgs** boson – there is a new source of electroweak symmetry breaking! (Majorana neutrinos);
3. Neutrino masses are small because there is **another source of mass** out there — a new energy scale indirectly responsible for the tiny neutrino masses, a la the seesaw mechanism (Majorana neutrinos).

Searches for $0\nu\beta\beta$ help tell (1) from (2) and (3), the LHC, charged-lepton flavor violation, *et al* may provide more information.

Piecing the Neutrino Mass Puzzle

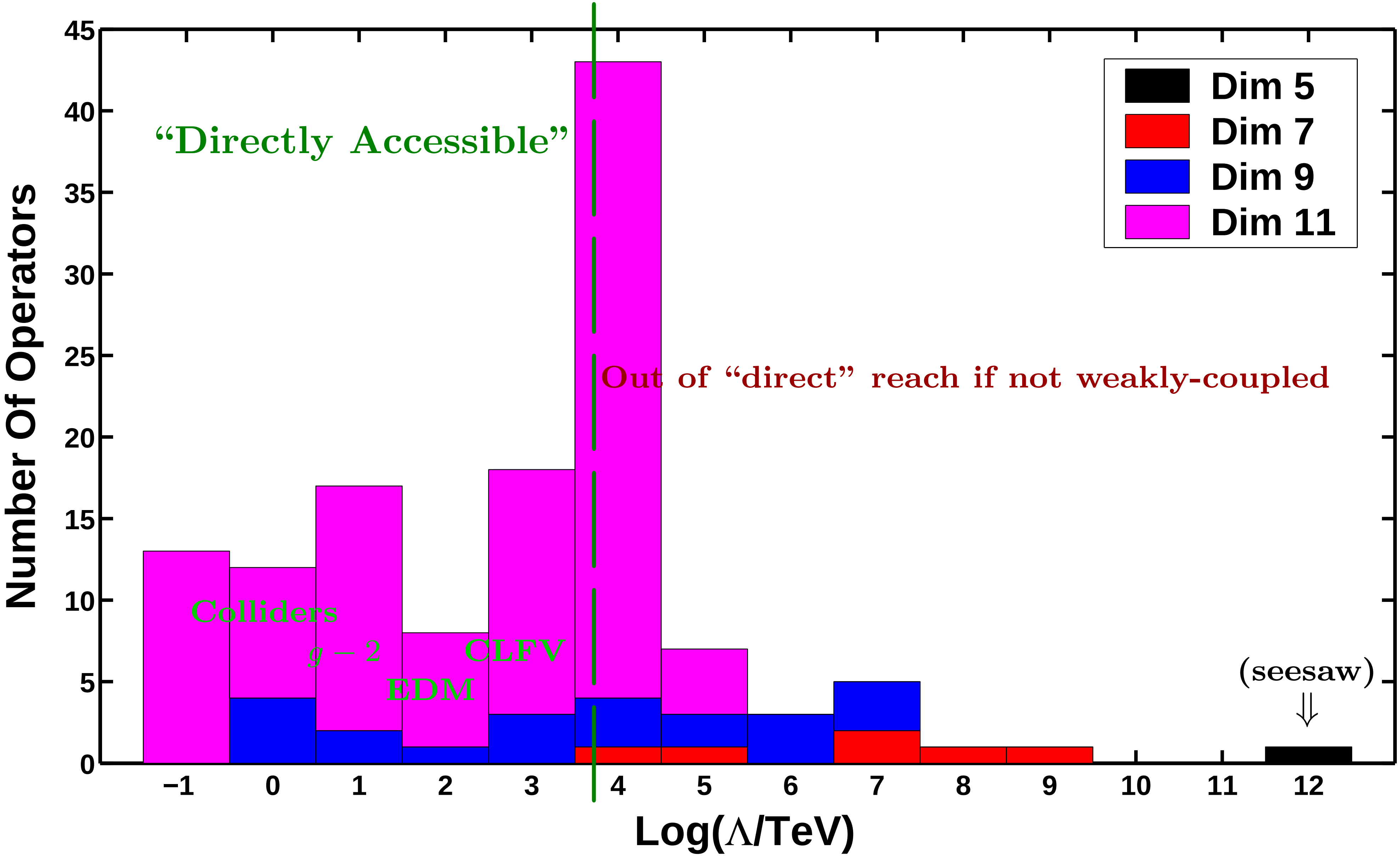
Understanding the origin of neutrino masses and exploring the new physics in the lepton sector will require unique **theoretical** and **experimental** efforts, including . . .

- understanding the fate of lepton-number. Neutrinoless double beta decay!
- a comprehensive long baseline neutrino program, towards precision oscillation physics.
- other probes of neutrino properties, including neutrino scattering.
- **precision studies of charged-lepton properties ($g-2$, edm), and searches for charged-lepton flavor violating processes.**
- collider experiments. The LHC and beyond may end up revealing the new physics behind small neutrino masses.
- cosmic surveys. Neutrino properties affect, in a significant way, the history of the universe. Will we learn about neutrinos from cosmology, or about cosmology from neutrinos?
- searches for baryon-number violating processes.

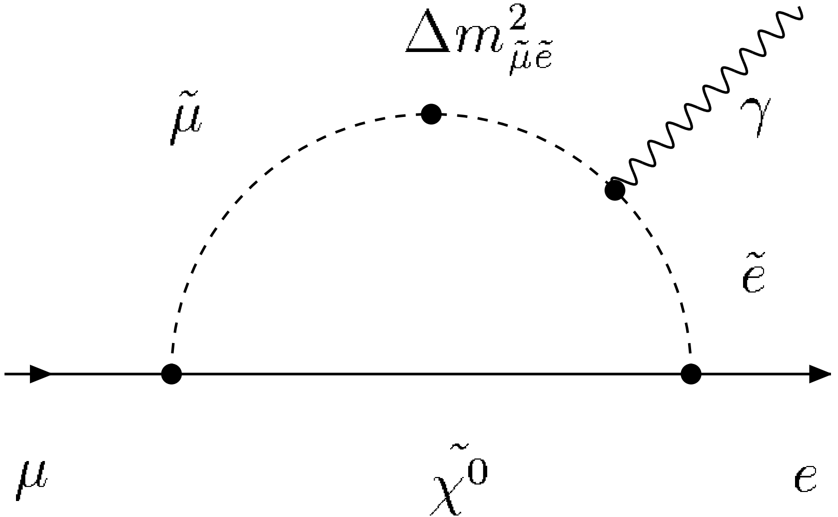
Connections Between CLFV and Neutrinos

Neutrinos are leptons too! The main question is how does the physics responsible for the neutrino mass manifest itself in CLFV. Generically, there are two possibilities:

- **Direct**. The same physics that leads to nonzero neutrino masses leads to observable rates for CLFV. In this case, CLFV searches help reveal the mechanism responsible for nonzero neutrino masses. Challenge: 10^{14} GeV versus 10^{6} GeV.
- **Indirect**. Observable CLFV is a consequence of new degrees of freedom at intermediate mass scales not related to neutrino masses. These new degrees of freedom interact with the origin of nonzero neutrino masses and inherit some of its properties, especially the flavor structure. Challenge: How does one reveal the connection?


45
40
35
30
25
20
15
10
5
0
Number Of Operators
−1
0
1
2
3
4
5
6
7
8
9
10
11
12
Log(Λ/TeV)
Dim 5
Dim 7
Dim 9
Dim 11
"Directly Accessible"
Out of "direct" reach if not weakly-coupled
Colliders
$g-2$
EDM
CLFV
(seesaw)
⇓

“Bread and Butter” SUSY plus High Energy Seesaw



$$\to \quad \theta_{\tilde{e}\tilde{\mu}} \sim \frac{\Delta m^2_{\tilde{e}\tilde{\mu}}}{\tilde{m}}$$

$Br(\mu\to e\gamma) \simeq \frac{\alpha^3\pi}{G_F^2\tilde{m}^4}\theta^2_{\tilde{e}\tilde{\mu}}$, $\tilde{m}^2$ is a typical supersymmetric mass.

$\theta_{\tilde{e}\tilde{\mu}}$ measures the “amount” of flavor violation.

For $\tilde{m}$ around 1 TeV, $\theta_{\tilde{e}\tilde{\mu}}$ is severely constrained. Very big problem.

“Natural” solution: $\boxed{\boxed{\theta_{\tilde{e}\tilde{\mu}} = 0}}$ → modified by quantum corrections.

The Seesaw Mechanism

$\mathcal{L} \supset -y_{i\alpha} L^i H N^\alpha - \frac{M_N^{\alpha\beta}}{2} N_\alpha N_\beta + H.c.,$ $\Rightarrow$ N^α gauge singlet fermions, $y_{i\alpha}$ dimensionless Yukawa couplings, $M_N^{\alpha\beta}$ (very large) mass parameters.

At low energies, integrate out the “right-handed neutrinos” N_α:

$$\mathcal{L} \supset \left(y M_N^{-1} y^t\right)_{ij} L^i H L^j H + \mathcal{O}\left(\frac{1}{M_N^2}\right) + H.c.$$

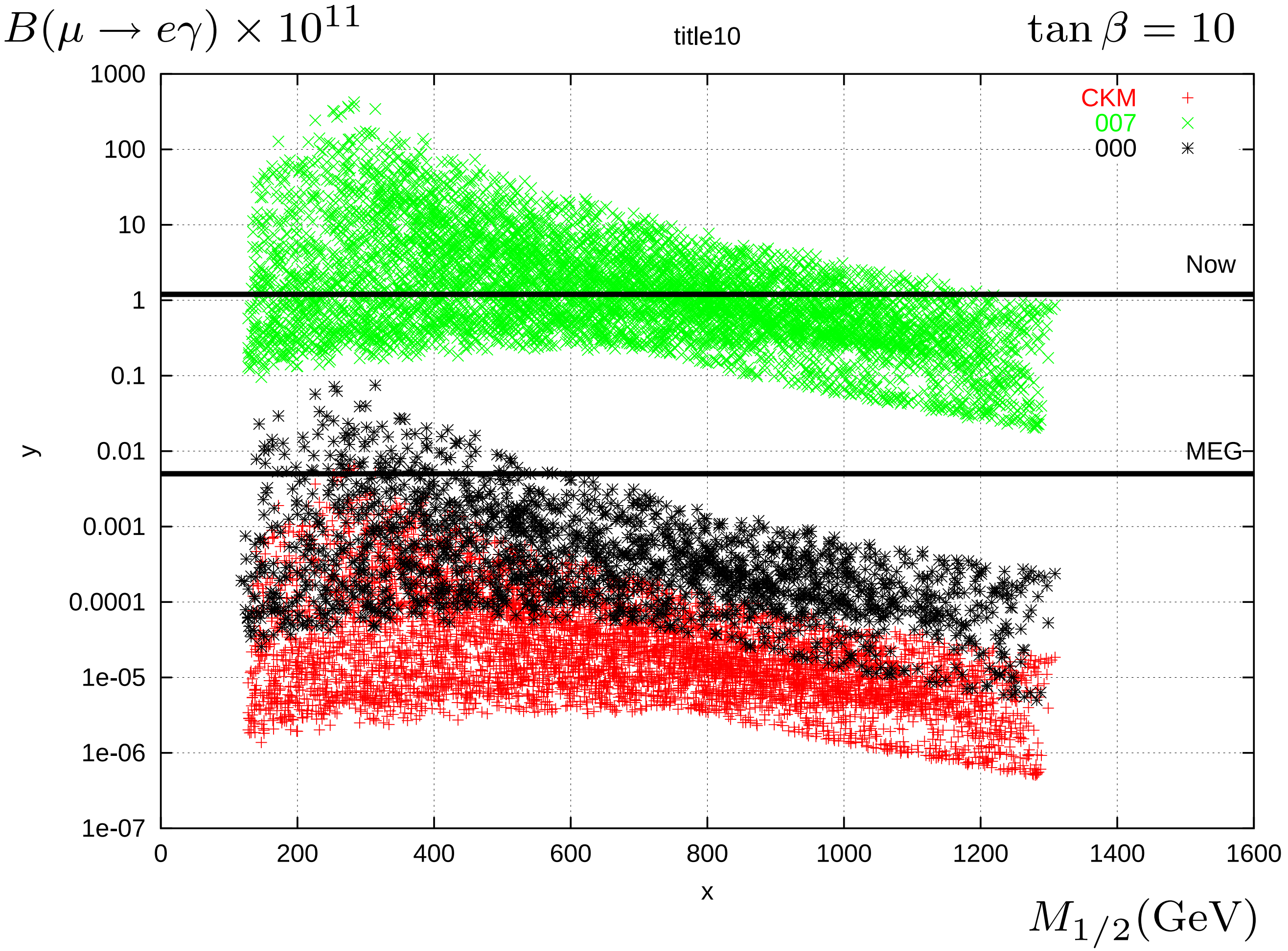
y are not diagonal → right-handed neutrino loops generate non-zero $\Delta m^2_{\tilde{e}\tilde{\mu}}$

$$\left(m^2_{\tilde{\ell}_L}\right)_{ij} \simeq -\frac{3m_0^2 + A_0^2}{8\pi^2} \sum_k (y)^*_{ki} (y)_{kj} \ln \frac{M_X}{M_{N_k}} , \qquad X = \text{Planck}, GUT, \text{etc}$$

If this is indeed the case, CLFV would serve as another channel to probe neutrino Yukawa couplings, which are not directly accessible experimentally.

Fundamentally important for “testing” the seesaw, leptogenesis, GUTs, etc

What are the neutrino Yukawa couplings → ansatz needed!



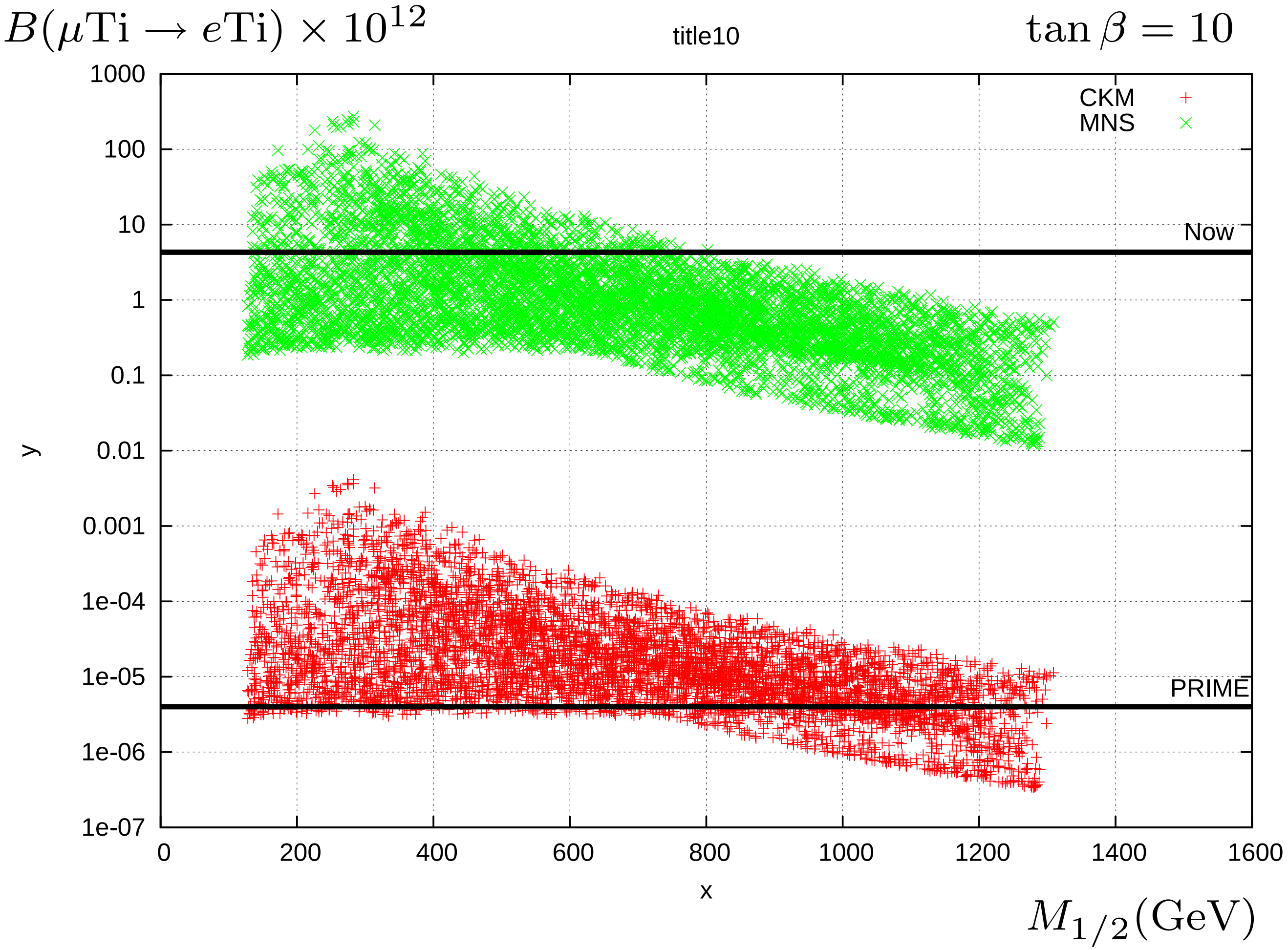
$SO(10)$ inspired model.

remember B scales with y^2.

$B(\mu \to e\gamma) \propto M_R^2[\ln(M_{Pl}/M_R)]^2$

[Calibbi, Faccia, Masiero, Vempati, hep-ph/0605139]

$\mu \to e$ conversion is at least as sensitive as $\mu \to e\gamma$



$SO(10)$ inspired model.

remember B scales with y^2.

$B(\mu \to e\gamma) \propto M_R^2[\ln(M_{Pl}/M_R)]^2$

[Calibbi, Faccia, Masiero, Vempati, hep-ph/0605139]

Input From/To Leptogenesis (⇒)

In the case of the seesaw mechanism, the matter-antimatter asymmetry generated via leptogenesis is (yet another) function of the neutrino Yukawa couplings:

If one is to hope to ever reconstruct the seesaw Lagrangian and test leptogenesis, LFV needs to be measured.

Note that this is VERY ambitious, and we need to get lucky a few times:

- Weak scale SUSY has to exist;
- "Precision" measurement of $\mu \to e$, $\tau \to \mu$, $\tau \to e$;
- "Precision" measurement of SUSY masses;
- Very good understanding of mechanism of SUSY breaking;
- There are no other relevant degrees of freedom between the weak scale and $> 10^9$ GeV;
- etc

Other ways to do this would be much appreciated!

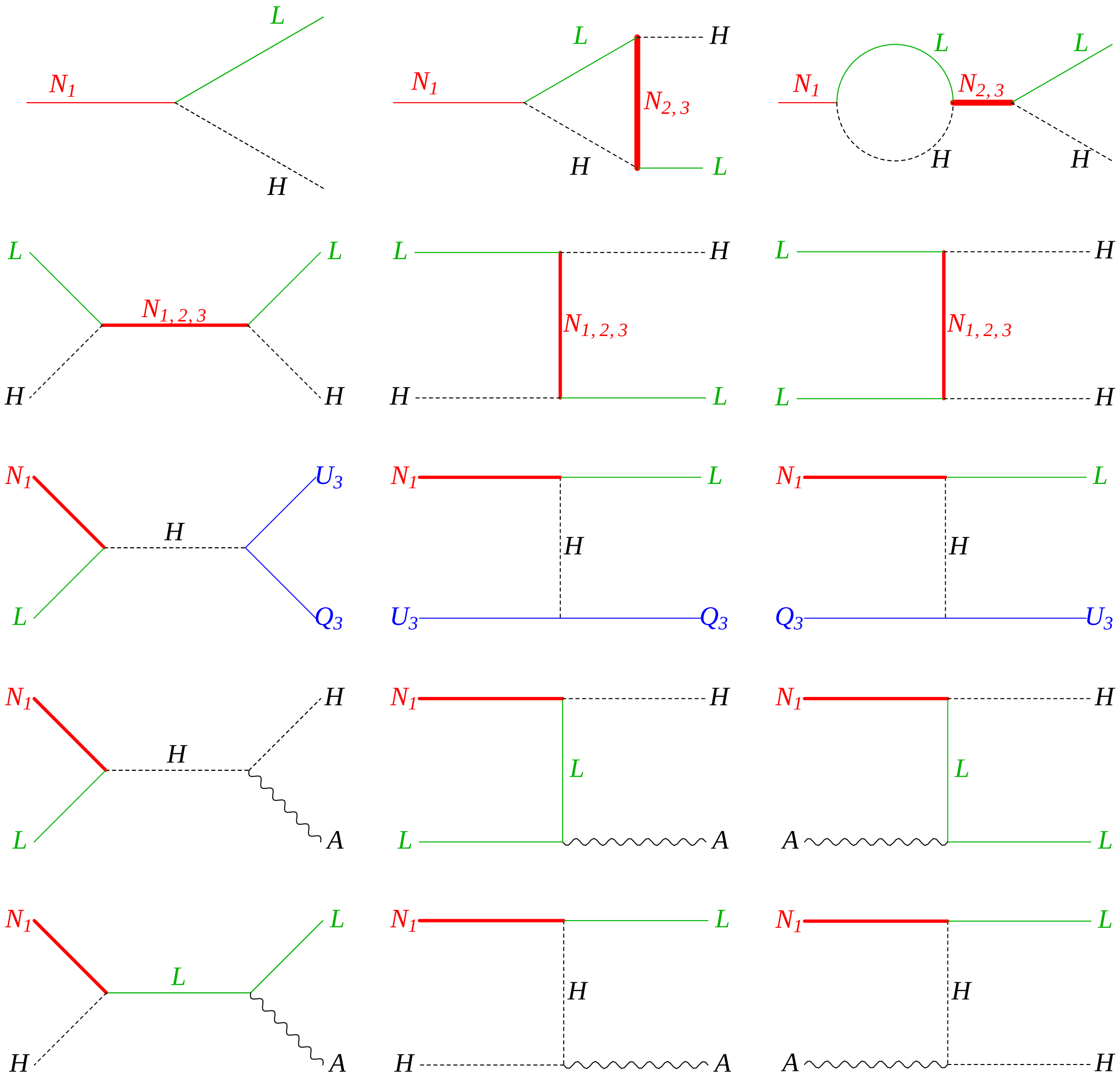
In the old SM, (electroweak) baryogenesis does not work – not enough CP-invariance violation, Higgs boson too light.

Neutrinos help by providing all the necessary ingredients for successful baryogenesis via leptogenesis.

- Violation of lepton number, which later on is transformed into baryon number by nonperturbative, finite temperature electroweak effects (in one version of the νSM, lepton number is broken at a high energy scale M).
- Violation of C-invariance and CP-invariance (weak interactions, plus new CP-odd phases).
- Deviation from thermal equilibrium (depending on the strength of the relevant interactions).

E.g. – thermal, seesaw leptogenesis, $\boxed{\mathcal{L} \supset -y_{i\alpha} L^i H N^\alpha - \frac{M_N^{\alpha\beta}}{2} N_\alpha N_\beta + H.c.}$

[Fukugita, Yanagida]



- L-violating processes
- $y \Rightarrow$ CP-violation
- deviation from thermal eq. constrains combinations of M_N and y.
- need to yield correct m_ν

not trivial!

[G. Giudice *et al*, hep-ph/0310123]

SUSY with R-parity Violation

The MSSM Lagrangian contains several marginal operators which are allowed by all gauge interactions but violate baryon and lepton number.

A subset of these (set λ'' to zero to prevent proton decay, and ignore bi-linear terms, which do not contribute as much to CLFV) is:

$$\begin{aligned}\mathcal{L} &= \lambda_{ijk}\left(\bar{\nu}^c_{Li}e_{Lj}\tilde{e}^*_{Rk}+\bar{e}_{Rk}\nu_{Li}\tilde{e}_{Lj}+\bar{e}_{Rk}e_{Lj}\tilde{\nu}_{Li}\right)\\ &+ \lambda'_{ijk}V^{j\alpha}_{KM}\left(\bar{\nu}^c_{Li}d_{L\alpha}\tilde{d}^*_{Rk}+\bar{d}_{Rk}\nu_{Li}\tilde{d}_{L\alpha}+\bar{d}_{Rk}d_{L\alpha}\tilde{\nu}_{Li}\right)\\ &- \lambda'_{ijk}\left(\bar{u}^c_{j}e_{Li}\tilde{d}^*_{Rk}+\bar{d}_{Rk}e_{Li}\tilde{u}_{Lj}+\bar{d}_{Rk}u_{Lj}\tilde{e}_{Li}\right)+\text{h.c.},\end{aligned}$$

The presence of different combinations of these terms leads to very distinct patterns for CLFV. Proves to be an excellent laboratory for probing all different possibilities.

[AdG, Lola, Tobe, hep-ph/0008085]

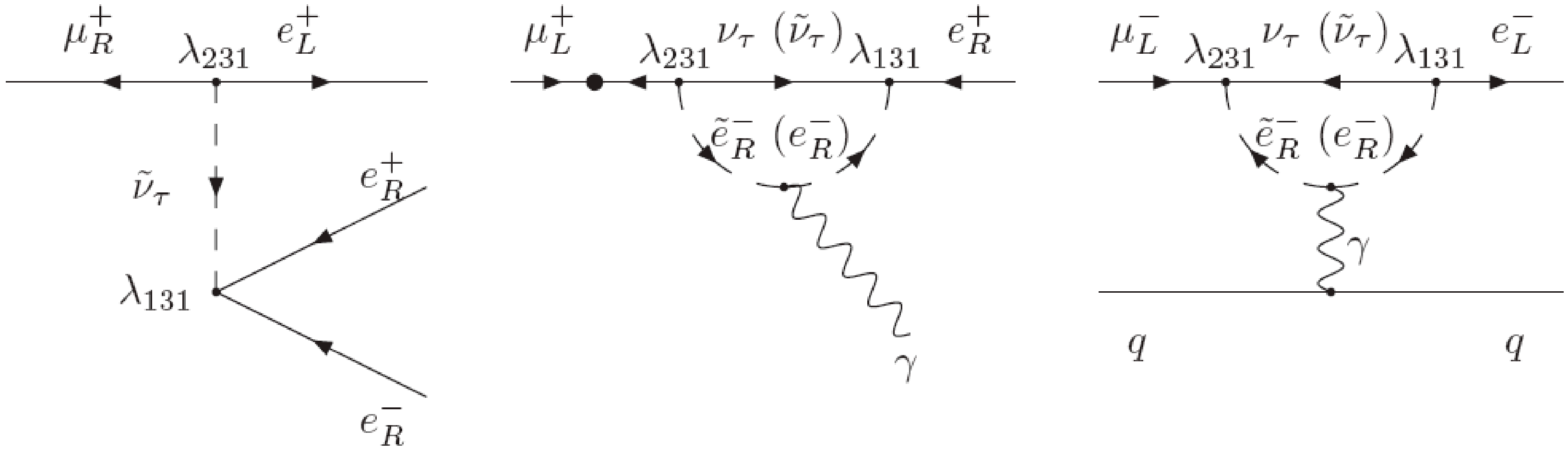


Figure 1: *Lowest order Feynman diagrams for lepton flavour violating processes induced by $\lambda_{131}\lambda_{231}$ couplings (see Eq. (2.1)).*

$$\frac{\mathrm{Br}(\mu^+\to e^+\gamma)}{\mathrm{Br}(\mu^+\to e^+e^-e^+)} = \frac{4\times10^{-4}\left(1-\frac{m^2_{\tilde\nu_\tau}}{2m^2_{\tilde e_R}}\right)^2}{\beta} \simeq 1\times10^{-4} \qquad (\beta\sim 1)$$

$$\frac{\mathrm{R}(\mu^-\to e^-\ \text{in Ti (Al)})}{\mathrm{Br}(\mu^+\to e^+e^-e^+)} = \frac{2\ (1)\times10^{-5}}{\beta}\left(\frac{5}{6}+\frac{m^2_{\tilde\nu_\tau}}{12m^2_{\tilde e_R}}+\log\frac{m_e^2}{m^2_{\tilde\nu_\tau}}+\delta\right)^2 \simeq 2\ (1)\times10^{-3},$$

$\mu^+ \to e^+e^-e^+$ most promising channel!

[AdG, Lola, Tobe, hep-ph/0008085]

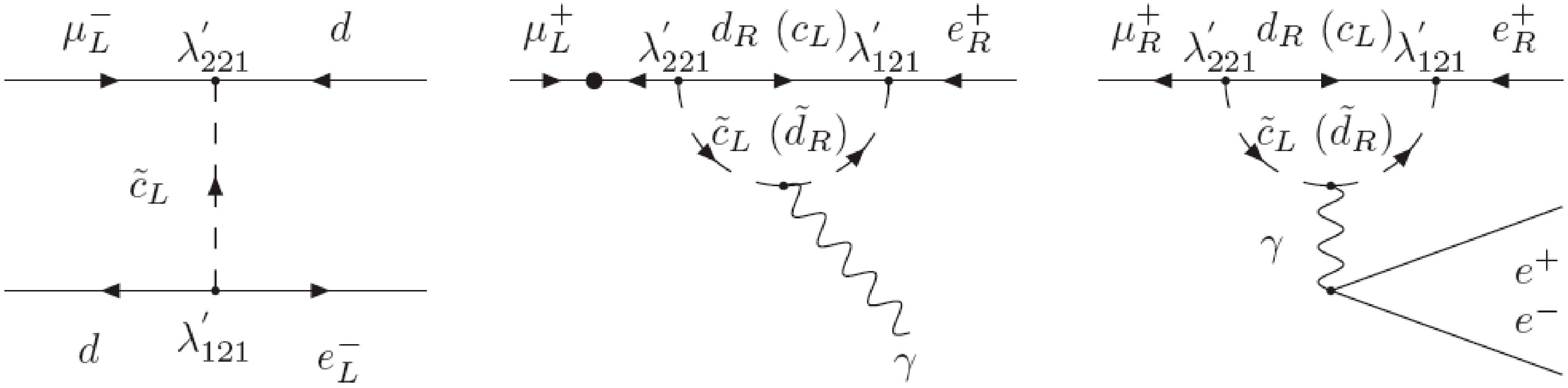


Figure 4: *Lowest order Feynman diagrams of lepton flavour violating processes induced by $f'_{121} f'_{221}$ couplings (see Eq. (2.1)).*

$$\frac{\mathrm{Br}(\mu^+ \to e^+ \gamma)}{\mathrm{Br}(\mu^+ \to e^+ e^- e^+)} = 1.1$$

$$(m_{\tilde{d}_R} = m_{\tilde{c}_L} = 300\ \mathrm{GeV})$$

$$\boxed{\frac{\mathrm{R}(\mu^- \to e^- \ \text{in Ti (Al)})}{\mathrm{Br}(\mu^+ \to e^+ e^- e^+)} = 2\ (1) \times 10^5}$$

$\mu - e$-conversion "only hope"!

[AdG, Lola, Tobe, hep-ph/0008085]

Type-II Seesaw: SM plus $SU(2)$ Triplet Higgs, $Y_T = 1$

$$\mathcal{L} \in \frac{\lambda_{\alpha\beta}}{2} L^{\alpha} L^{\beta} T.$$

Neutrino Majorana masses if T develops a vev …

$$m_{\alpha\beta} = \lambda_{\alpha\beta} v_T$$

$\mu \to e\gamma$, $\mu \to e$-conversion at the loop-level. However, $\mu \to eee$ at the tree level (note direct connection to neutrino mass-matrix flavor structure)…

$$\frac{1}{\Lambda^2} = \frac{m_{ee} m_{\mu e}}{v_T^2 M_T^2}$$

Key issue: are neutrino masses small because λ are small or because v_T is small (or both)? EWPD already push v_T below ~ 1 GeV…

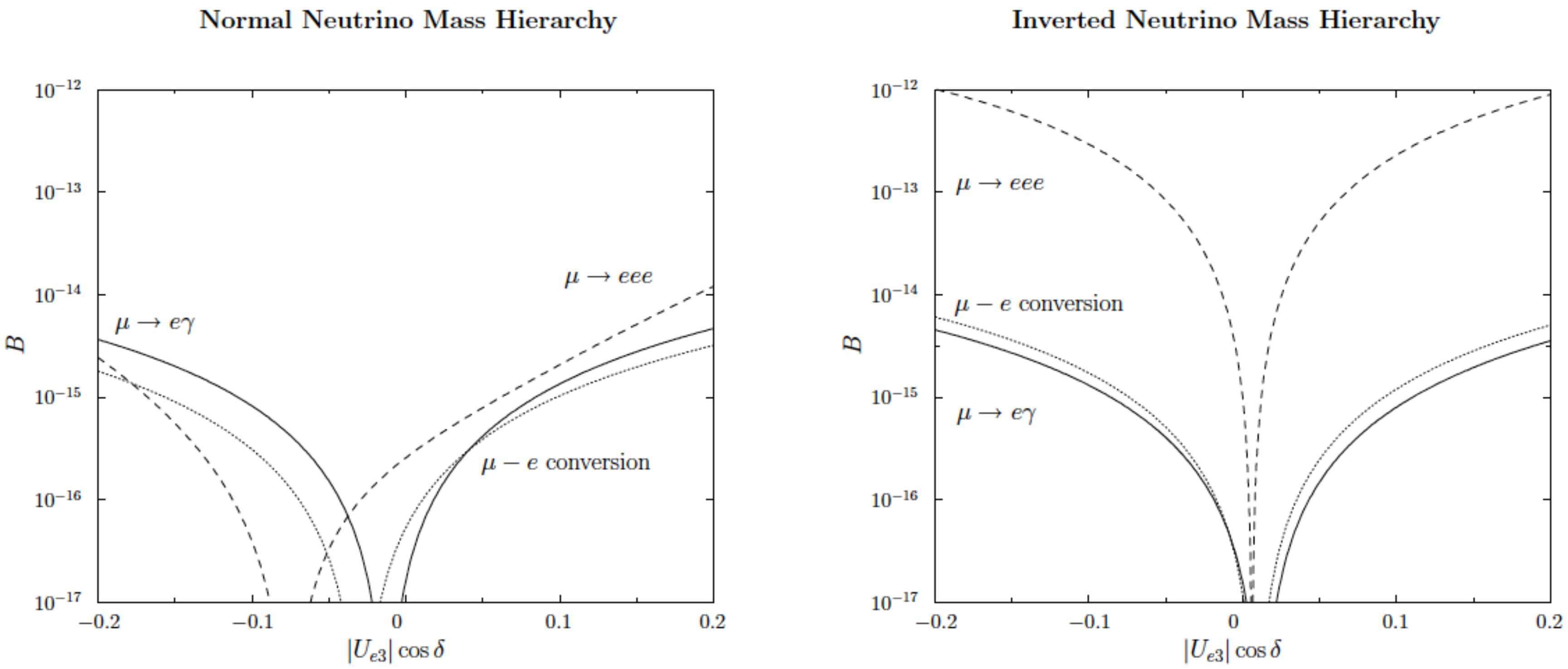


FIG. 1: The branching ratios B for $\mu \to e\gamma$ (solid line) and $\mu \to eee$ (dashed line), and the normalized capture rate B for $\mu \to e$-conversion in Ti (dotted line) as a function of $|U_{e3}|\cos\delta$ in a scenario where neutrino masses arise as a consequence of the presence of a triplet Higgs field with a small vacuum expectation value. The lightest neutrino mass is assumed to be negligible while the neutrino mass hierarchy is assumed to be normal (left-hand side) and inverted (right-hand side). See [1] for details.

Kakizaki, Ogura, Shima, PLB566, 210 (2003)

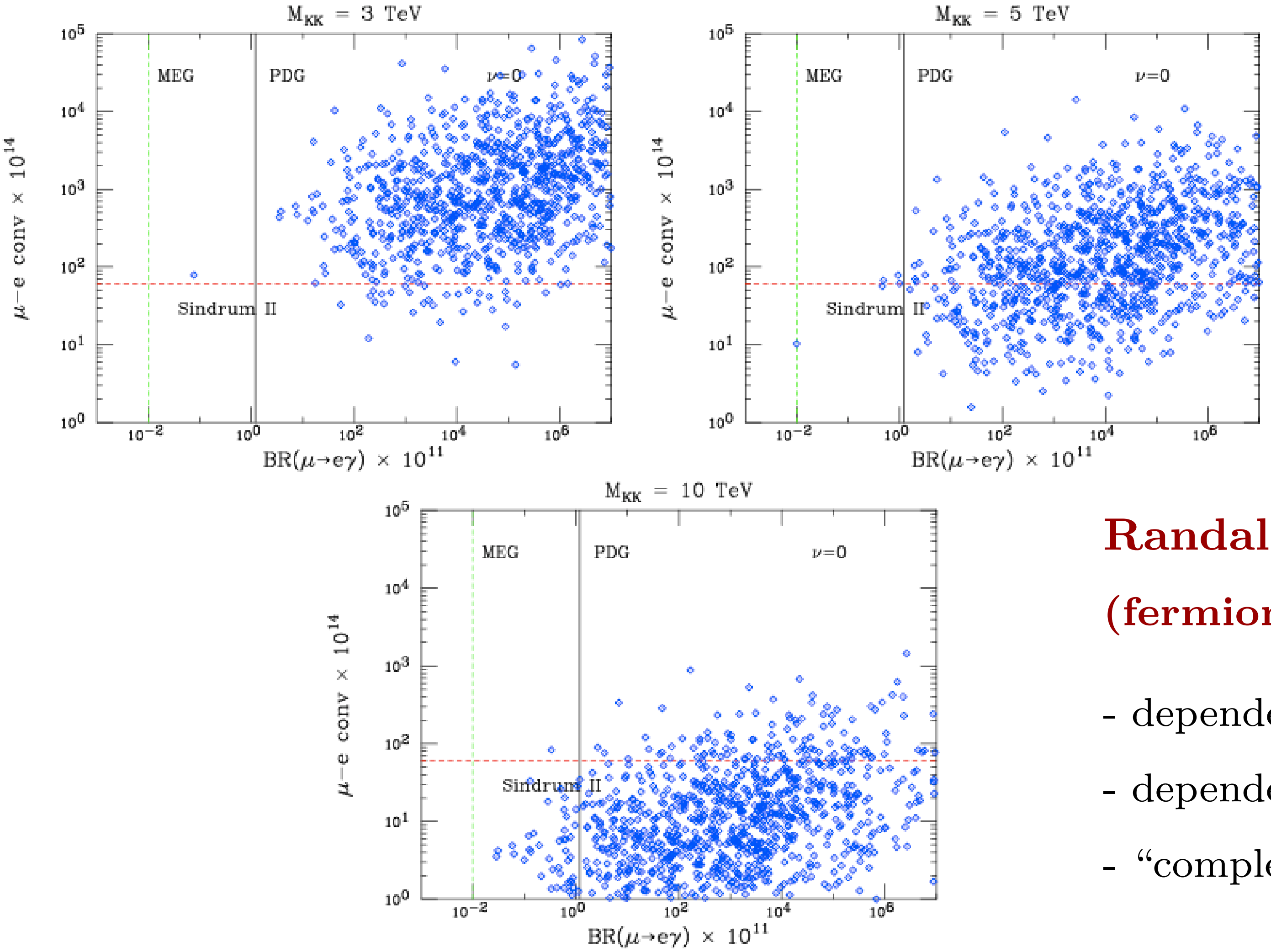


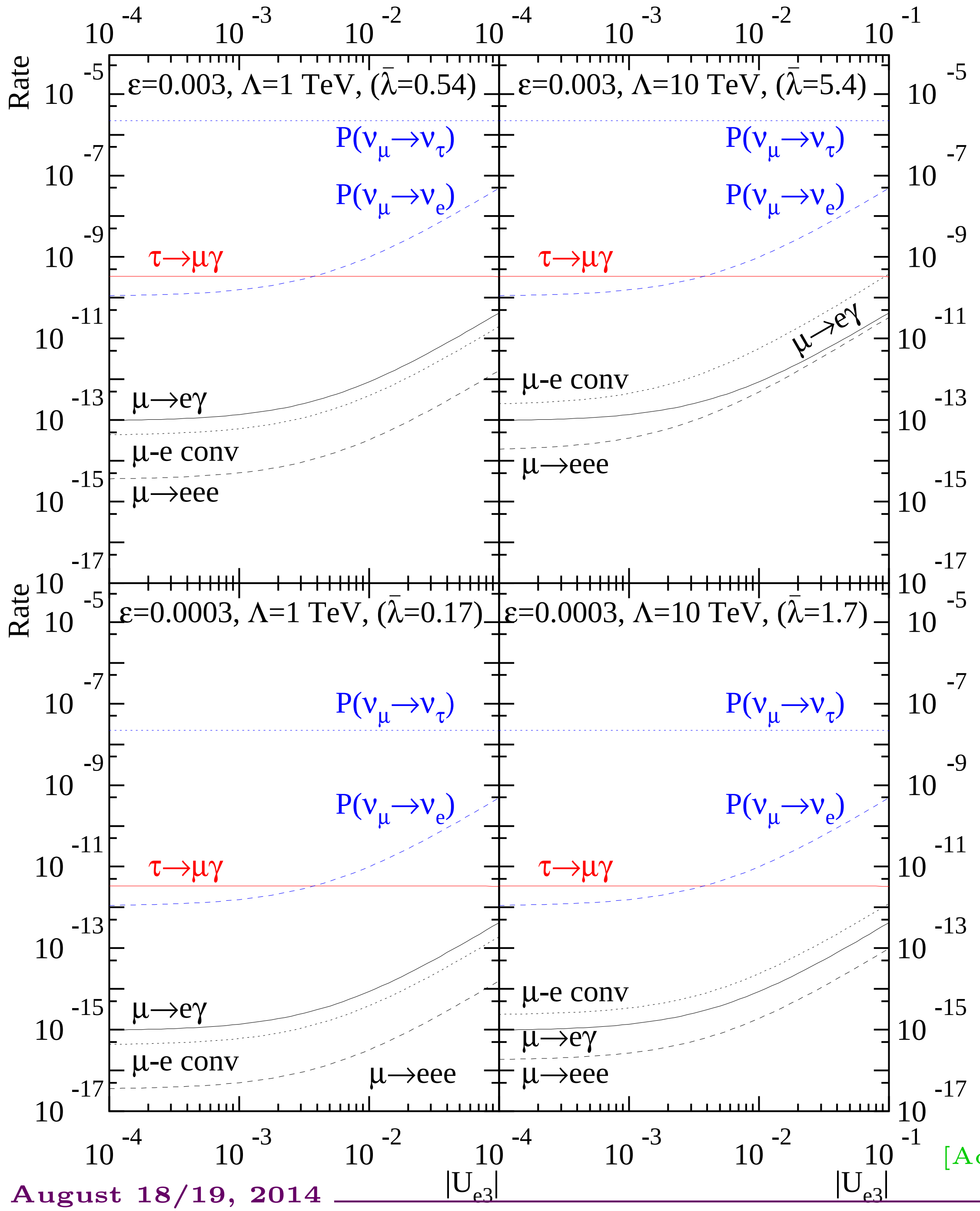
FIG. 6: Scan of the $\mu \to e\gamma$ and $\mu - e$ conversion predictions for $M_{KK} = 3, 5, 10$ TeV and $\nu = 0$. The solid line denotes the PDG bound on $BR(\mu \to e\gamma)$, while the dashed lines indicate the SINDRUM II limit on $\mu - e$ conversion and the projected MEG sensitivity to $BR(\mu \to e\gamma)$.

Randall-Sundrum Model

(fermions in the bulk)

- dependency on UV-completion(?)
- dependency on Yukawa couplings
- "complementarity" between $\mu \to e\gamma$, $\mu - e$ conv

[Agashe, Blechman, Petriello, hep-ph/0606021]



Large Extra-Dimensions

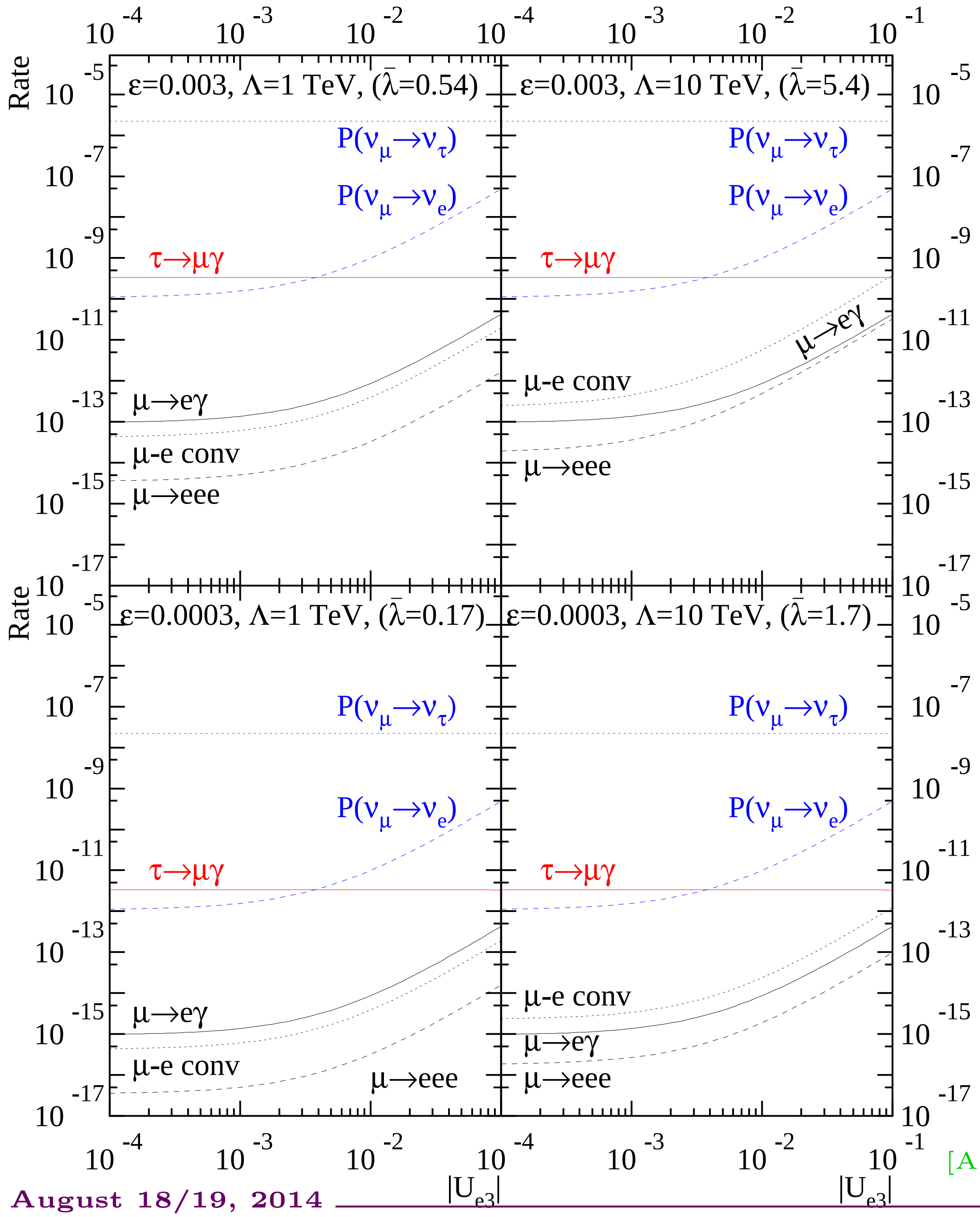
(right-handed neutrinos in the bulk)

-no ambiguity in y (neutrinos Dirac)

-dependency on UV-completion

$$\frac{B(\mu \to e\gamma)}{B(\mu - e\ \text{conv})} \in [0.1 - 10]$$

[AdG, Giudice, Strumia, Tobe, hep-ph/0107156]



Large Extra-Dimensions

-no ambiguity in y (neutrinos Dirac)

-dependency on UV-completion

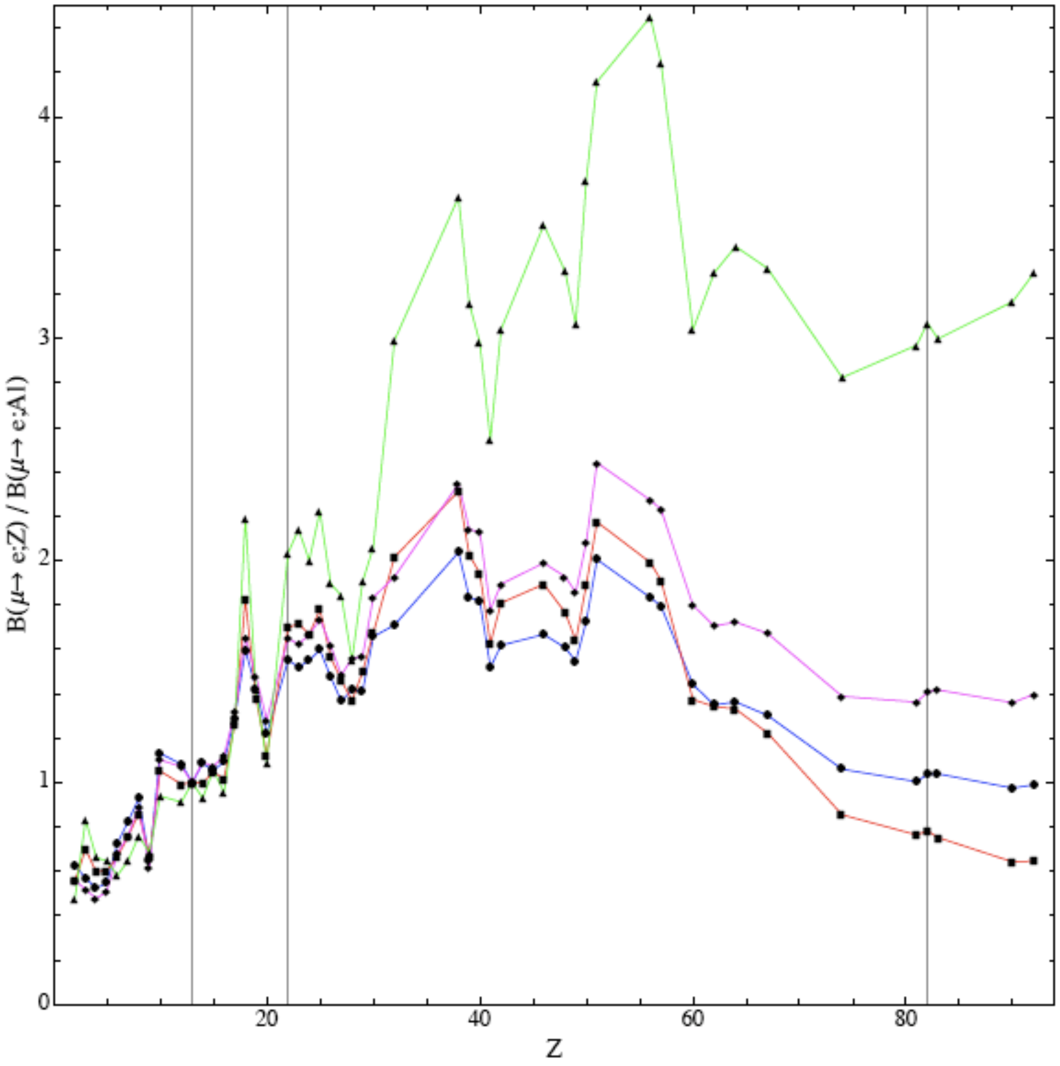
[AdG, Giudice, Strumia, Tobe, hep-ph/0107156]

What is This Really Good For?

While specific models (see last slides) provide estimates for the rates for CLFV processes, the observation of one specific CLFV process cannot determine the underlying physics mechanism (this is always true when all you measure is the coefficient of an effective operator).

Real strength lies in combinations of different measurements, including:

- kinematical observables (e.g. angular distributions in $\mu \to eee$);
- other CLFV channels;
- neutrino oscillations;
- measurements of $g-2$ and EDMs;
- collider searches for new, heavy states;
- etc.



Vector 4-Fermion Interaction (Z)

$\propto (\bar{\mu}\gamma_\alpha e)(\bar{q}\gamma^\alpha q)$

Vector 4-Fermion Interaction (γ)

Dipole ($\propto \bar{\mu}\sigma_{\alpha\beta} e F^{\alpha\beta}$)

Scalar 4-Fermion Interaction

$\propto (\bar{\mu}e)(\bar{q}q)$

Figure 3: Target dependence of the $\mu \to e$ conversion rate in different single-operator dominance models. We plot the conversion rates normalized to the rate in Aluminum ($Z = 13$) versus the atomic number Z for the four theoretical models described in the text: D (blue), S (red), $V^{(\gamma)}$ (magenta), $V^{(Z)}$ (green). The vertical lines correspond to $Z = 13$ (Al), $Z = 22$ (Ti), and $Z = 83$ (Pb).

[Cirigliano, Kitano, Okada, Tuzon, 0904.0957]

The Muon Magnetic Dipole Moment

The magnetic moment of the muon is defined by $\vec{M} = g_\mu \frac{e}{2m_\mu}\vec{S}$.

The Dirac equation predicts $g_\mu = 2$, so that the anomalous magnetic moment is defined as (note: dimensionless)

$$a_\mu \equiv \frac{g_\mu - 2}{2}$$

In the standard model, the (by far) largest contribution to a_μ comes from the one-loop QED vertex diagram, first computed by Schwinger:

$$a_\mu^{QED}(1-\text{loop}) = \frac{\alpha}{2\pi} = 116,140,973.5 \times 10^{-11}$$

The theoretical estimate has been improved significantly since then, mostly to keep up with the impressive experimental reach of measurements of the $g-2$ of the muon.

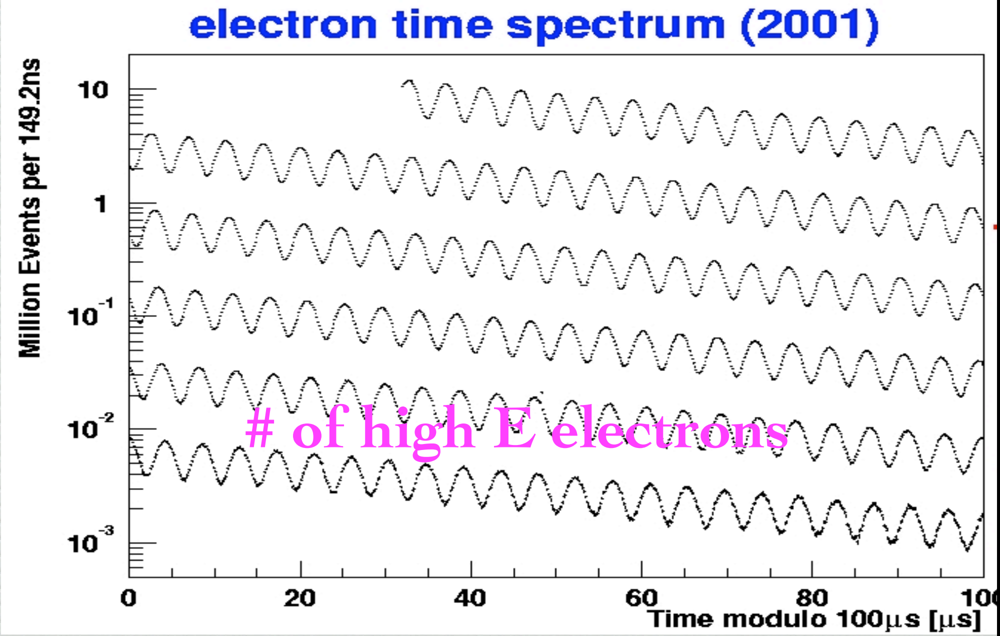
Spin Precession w.r.t. Momentum Vector

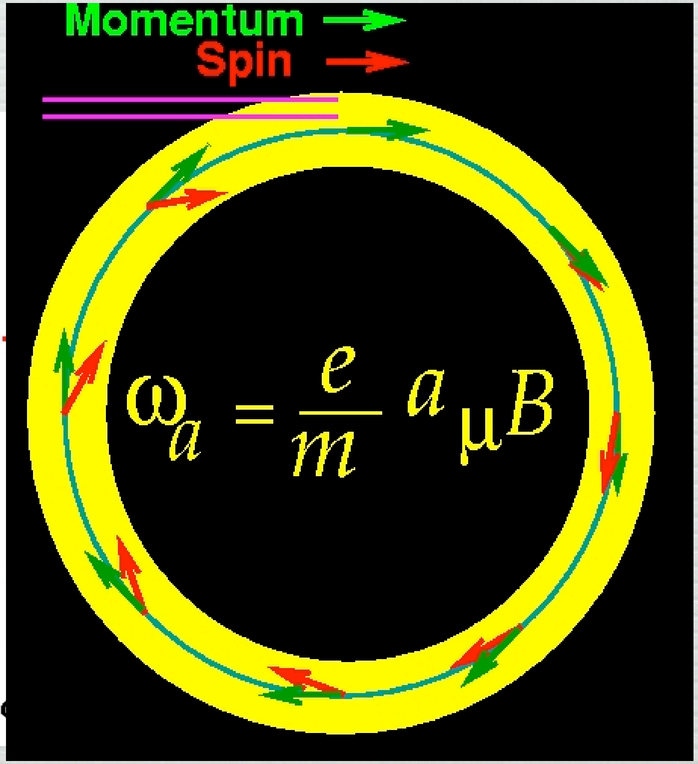
$$\vec{\omega}_a = -\frac{e}{m}\left[a_\mu \vec{B} - \cancel{\left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c}}\right]$$

(g-2)/2

$\gamma_{\text{magic}} = 29.3$

$p_{\text{magic}} = 3.09$ GeV/c





$$\omega_a = \frac{e}{m} a_\mu B$$

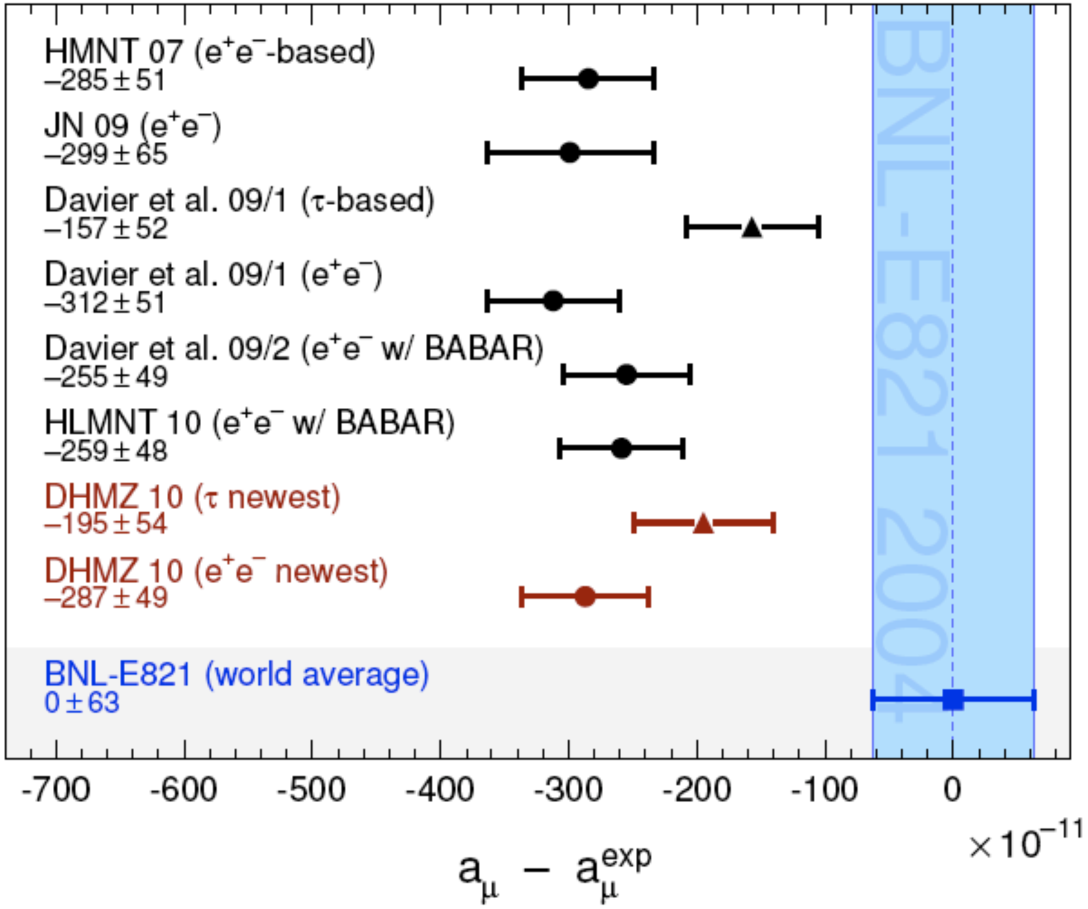


FIG. 9: Compilation of recent results for $a_\mu^{\rm SM}$ (in units of 10^{-11}), subtracted by the central value of the experimental average [12, 57]. The shaded vertical band indicates the experimental error. The SM predictions are taken from: this work (DHMZ 10), HLMNT (unpublished) [58] (e^+e^- based, including BABAR and KLOE 2010 $\pi^+\pi^-$ data), Davier *et al.* 09/1 [15] (τ-based), Davier *et al.* 09/1 [15] (e^+e^--based, not including BABAR $\pi^+\pi^-$ data), Davier *et al.* 09/2 [10] (e^+e^--based including BABAR $\pi^+\pi^-$ data), HMNT 07 [59] and JN 09 [60] (not including BABAR $\pi^+\pi^-$ data).

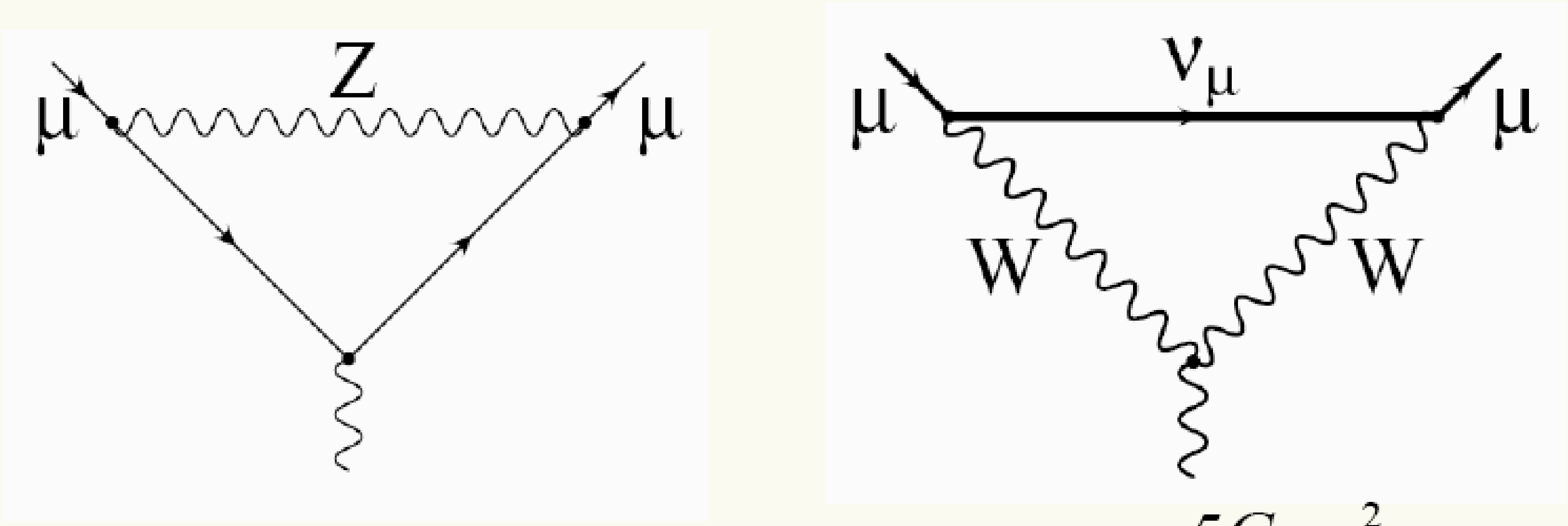
NOTE: $a_\mu^{LbL} = 105 \pm 26 \times 10^{-11}$

[Davier *et al*, 1010.4180]

[talk by A. Czarnecki at CIPANP 2006]

Electroweak effects

Small part of the total g-2: $154(3)\times 10^{-11}$



$$\left(-1+2\right)\cdot\frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\simeq 195\cdot 10^{-11}$$

very similar to New Physics!

(more on this later)

Dependence on muon mass;
that's why muons so much more sensitive
to New Physics than the electron

Sensitivity to New Physics

If there is new ultra-violate physics, it will manifest itself, as far as a_μ is concerned, via the following effective operator (dimension 6):

$$\frac{\lambda H}{\Lambda^2}\bar{\mu}\sigma_{\mu\nu}\mu F^{\mu\nu} \rightarrow \frac{m_\mu}{\Lambda^2}\bar{\mu}\sigma_{\mu\nu}\mu F^{\mu\nu},$$

where Λ is an estimate for the new physics scale. (dependency on muon mass is characteristic of several (almost all?) models. It is NOT guaranteed)

Contribution to a_μ from operator above is

$$\delta a_\mu = \frac{4m_\mu^2}{e\Lambda^2}$$

Current experimental sensitivity: $\Lambda \sim 10$ TeV.

Note that, usually, new physics scale can be much lower due to loop-factors, gauge couplings, etc. In the SM the heavy gauge boson contribution yields

$$\frac{1}{\Lambda^2} \sim \frac{eg^2}{16\pi^2 M_W^2} \quad \longrightarrow \quad \delta a_\mu \sim \frac{m_\mu^2 G_F}{4\pi^2}$$

Not A Bad Estimate!

Δa_μ: we need to dig a little more!

This could be the greatest discovery of the century. Depending, of course, on how far down it goes.

Model Independent Comparison Between $g-2$ and CLFV:

The dipole effective operators that mediate $\mu \to e\gamma$ and contribute to a_μ are virtually the same:

$$\frac{m_\mu}{\Lambda^2}\bar{\mu}\sigma^{\mu\nu}\mu F_{\mu\nu} \qquad \times \qquad \theta_{e\mu}\frac{m_\mu}{\Lambda^2}\bar{\mu}\sigma^{\mu\nu}eF_{\mu\nu}$$

$\theta_{e\mu}$ measures how much flavor is violated. $\theta_{e\mu} = 1$ in a flavor indifferent theory, $\theta_{e\mu} = 0$ in a theory where indiviadual lepton flavor number is exactly conserved.

If $\theta_{e\mu} \sim 1$, $\mu \to e\gamma$ is a much more stringent probe of Λ.

On the other hand, if the current discrepancy in a_μ is due to new physics, $\theta_{e\mu} \ll 1$ ($\theta_{e\mu} < 10^{-4}$). [Hisano, Tobe, hep-ph/0102315]

e.g., in SUSY models, $Br(\mu \to e\gamma) \simeq 3 \times 10^{-5} \left(\frac{10^{-9}}{\delta a_\mu}\right)\left(\frac{\Delta m^2_{\tilde{e}\tilde{\mu}}}{\tilde{m}^2}\right)^2$

Comparison restricted to dipole operator. If four-fermion operators are relevant, they will "only" enhance rate for CLFV with respect to expectations from $g-2$.

Electroweak Contribution with Dirac Neutrinos and no other new States as Another (Quite Unfortunate) Example:

$$\theta_{e\mu} \sim \sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{1i}}{M^2_W} < 10^{-25}$$

Why is that? Neutrino masses are the only source of flavor violation. If the neutrino masses vanish, so do all flavor violating effects. This is true despite the fact that the mixing angles ($U_{\alpha i}$'s) are large.

Any "other" source of lepton-flavor violation is guaranteed to dominate over this. This may, for example, already be imbedded in the physics responsible for generating neutrino masses.

What we can learn from CLFV and other searches for new physics at the TeV scale (a_μ and Colliders):

$g-2$	CLFV	What Does it Mean?
YES	YES	New Physics at the TeV Scale; Some Flavor Violation
YES	NO	New Physics at the TeV Scale; Tiny Flavor Violation
NO	YES	New Physics Above TeV Scale; Some Flavor Violation – How Large?
NO	NO	No New Physics at the TeV Scale; CLFV only way forward?

Colliders	CLFV	What Does it Mean?
YES	YES	New Physics at the TeV Scale; Info on Flavor Sector!
YES	NO	New Physics at the TeV Scale; New Physics Very Flavor Blind. Why?
NO	YES	New Physics “Leptonic” or Above TeV Scale; Which one?
NO	NO	No New Physics at the TeV Scale; CLFV only way forward?

On CLFV Processes Involving τ Leptons (Brief Comment)

Current Bound On Selected τ CLFV Processes (All from the B-Factories):

- $B(\tau \to e\gamma) < 1.1 \times 10^{-7}$; $B(\tau \to \mu\gamma) < 4.5 \times 10^{-8}$. $(\mu \to e\gamma)$
- $B(\tau \to e\pi) < 8.0 \times 10^{-8}$; $B(\tau \to \mu\pi) < 1.1 \times 10^{-7}$. $(\mu \to e$–conversion$)$
- $B(\tau \to eee) < 3.6 \times 10^{-8}$; $B(\tau \to ee\mu) < 2.0 \times 10^{-8}$, $(\mu \to eee)$
- $B(\tau \to e\mu\mu) < 2.3 \times 10^{-8}$; $B(\tau \to \mu\mu\mu) < 3.2 \times 10^{-8}$. $(\mu \to eee)$

Relation to $\mu \to e$ violating processes is model dependent. Typical enhancements, at the amplitude-level, include:

- Chirality flipping: $m_\tau \gg m_\mu$;
- Lepton mixing effects: $U_{\tau 3} \gg U_{e3}$;
- Mass-Squared Difference effects: $\Delta m^2_{13} \gg \Delta m^2_{12}$;
- etc

Future: LHC (perhaps), and SuperB-factories (for sure?), will get to 10^{-9} level.

Summary and Conclusions

- We know that charged lepton flavor violation must occur. Effects are, however, really tiny in "simplest" realizations of the νSM (neutrino masses too small).
- If there is new physics at the electroweak scale, there is every reason to believe that CLFV is well within the reach of next generation experiments. Indeed, it is fair to ask: 'Why haven't we seen it yet?'
- It is fundamental to probe all CLFV channels. While in many scenarios $\mu \to e\gamma$ is the "largest" channel, there is no theorem that guarantees this (and many exceptions).
- CLFV may be intimately related to new physics unveiled with the discovery of non-zero neutrino masses. It may play a fundamental role in our understanding of the seesaw mechanism, GUTs, the baryon-antibaryon asymmetry of the Universe. We won't know for sure until we see it!