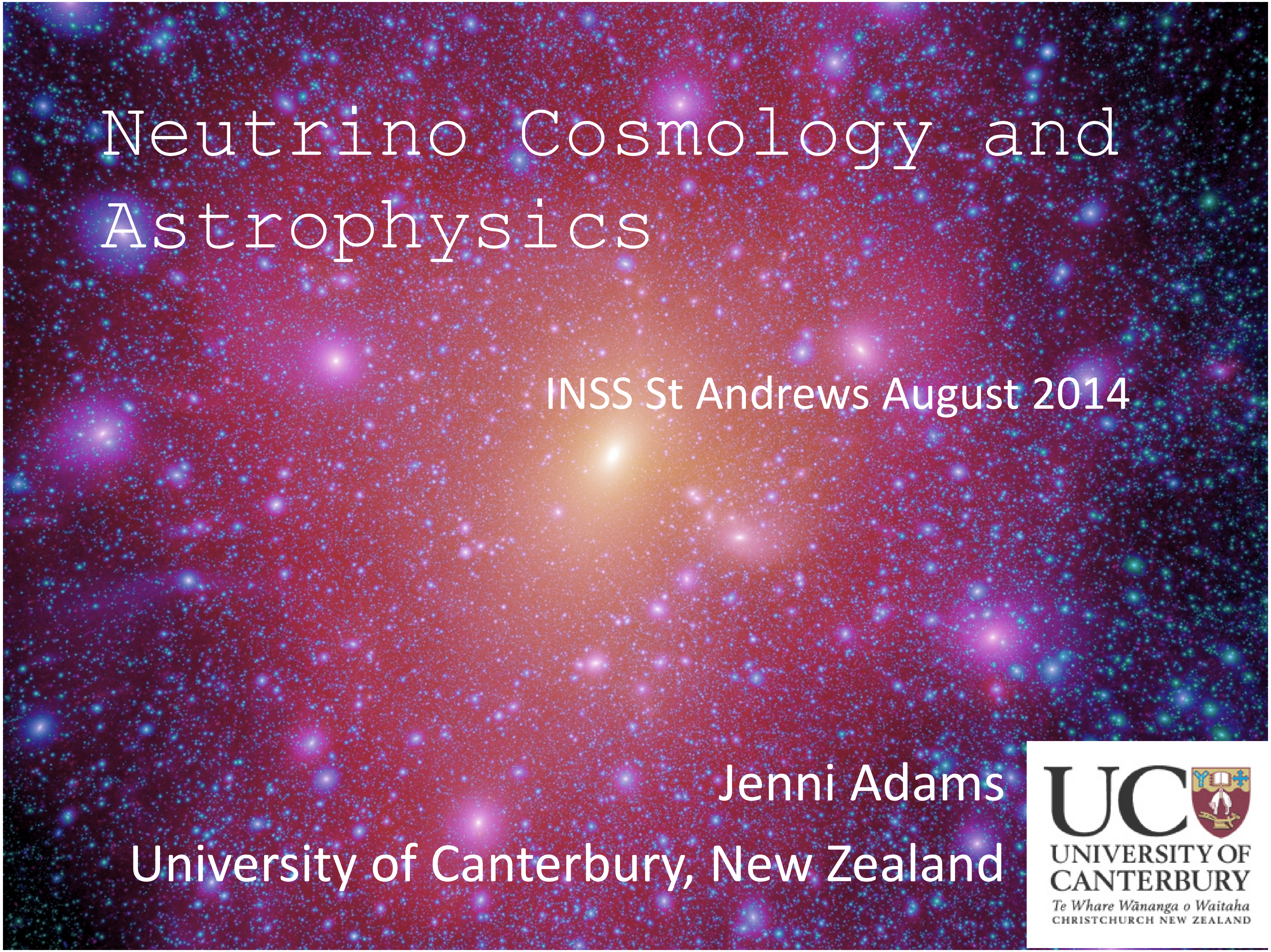
Neutrino Cosmology and Astrophysics

INSS St Andrews August 2014

Jenni Adams
University of Canterbury, New Zealand

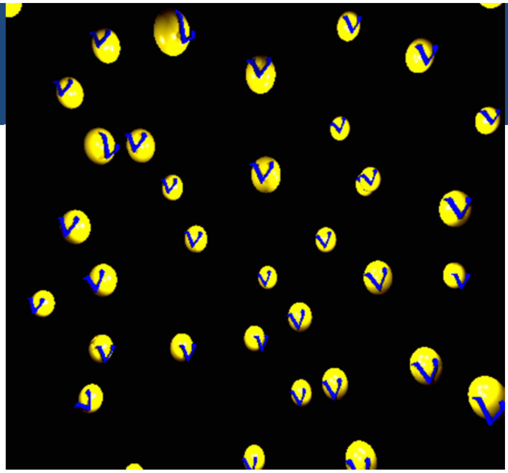


Lecture Plan:

1. Neutrino Cosmology
2. High Energy Astrophysical Neutrinos
3. Supernova Neutrinos

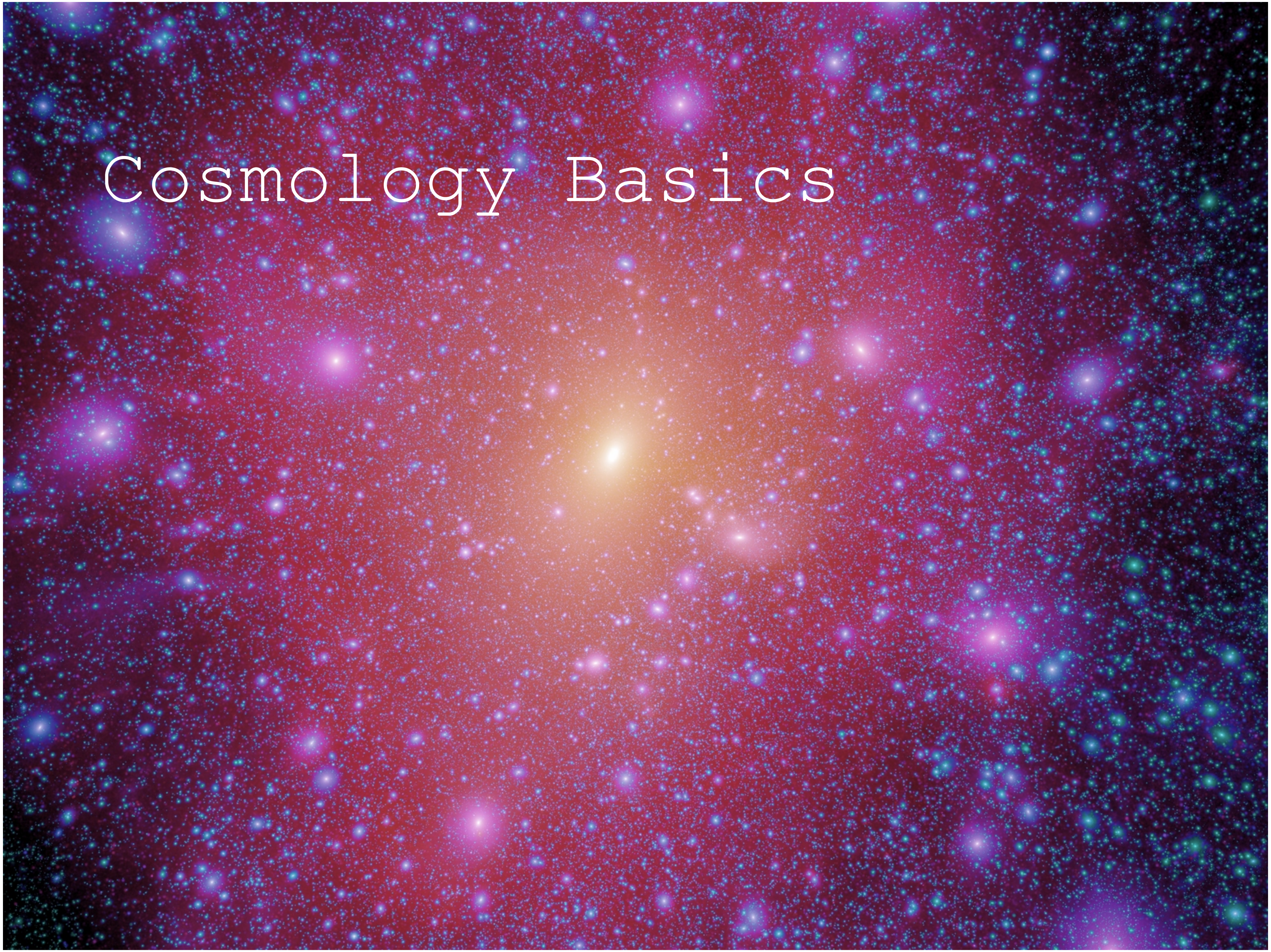
In preparing these lectures I have borrowed from the excellent lectures by Georg Raffelt at the NBI Neutrino School June 2014

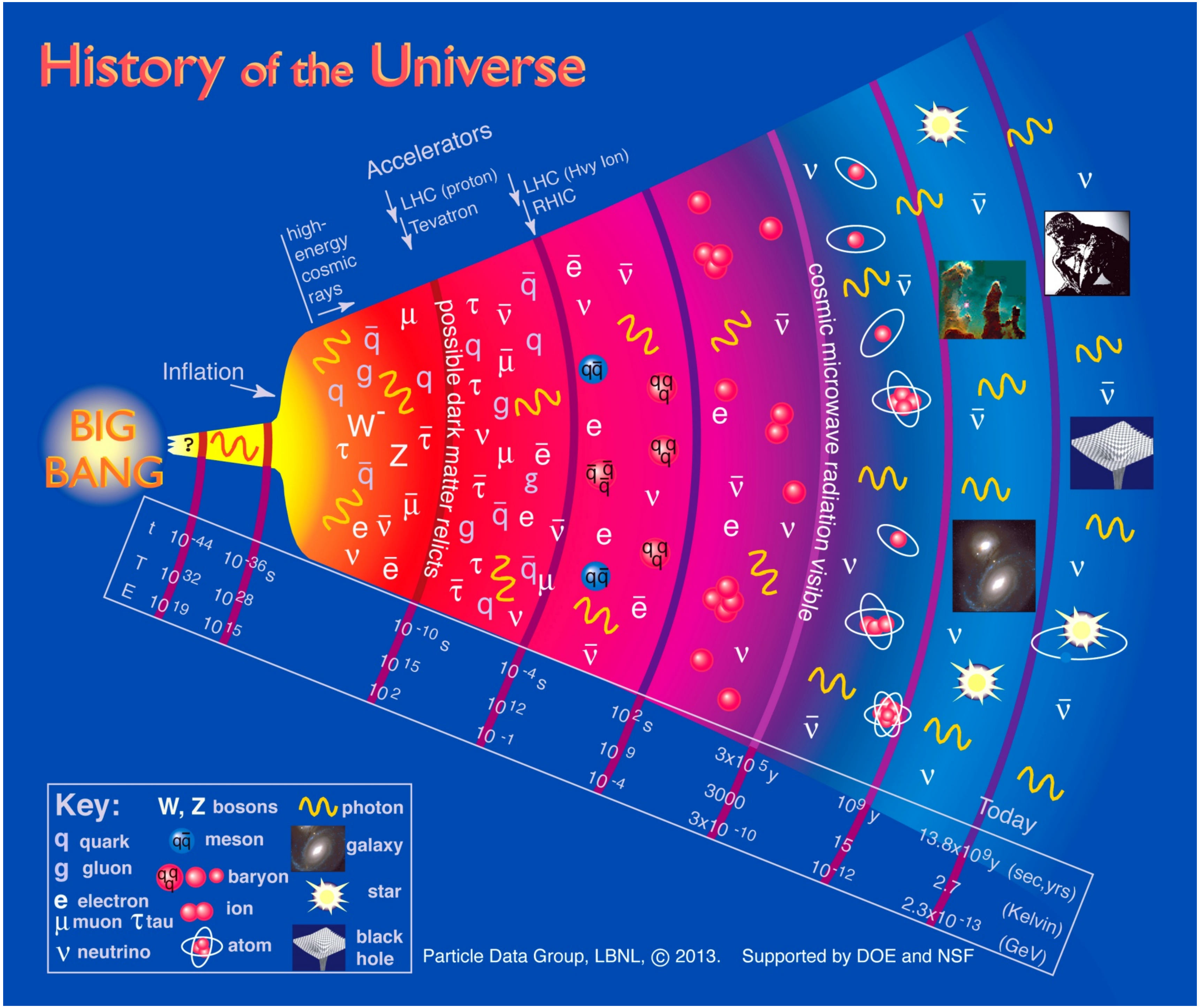
Neutrino Cosmology



- There is a cosmic background neutrino population which is a relic from the early universe
- The neutrino background affects cosmological processes
  - Primordial nucleosynthesis
  - Cosmic microwave background
  - Large structure formation
- Observations probing these processes give us information about neutrinos
- It is important to include the neutrino background effects to be able to interpret observations and learn about other constituents of the universe

Cosmology Basics




History of the Universe
Accelerators
LHC (proton)
Tevatron
LHC (Hvy Ion)
RHIC
high-energy cosmic rays
Inflation
BIG BANG
possible dark matter relicts
cosmic microwave radiation visible
Today
t 10^{-44} 10^{-36} s 10^{-10} s 10^{-4} s 10^{2} s 3×10^{5} y 10^{9} y 13.8×10^{9} y (sec,yrs)
T 10^{32} 10^{28} 10^{15} 10^{12} 10^{9} 3000 15 2.7 (Kelvin)
E 10^{19} 10^{15} 10^{2} 10^{-1} 10^{-4} 3×10^{-10} 10^{-12} 2.3×10^{-13} (GeV)
Key:
W, Z bosons
photon
q quark
meson
galaxy
g gluon
baryon
star
e electron
ion
μ muon τ tau
ν neutrino
atom
black hole
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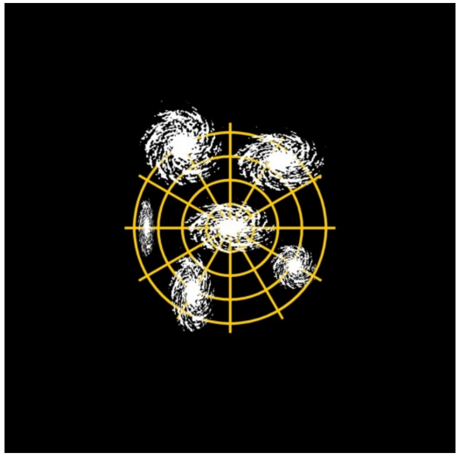
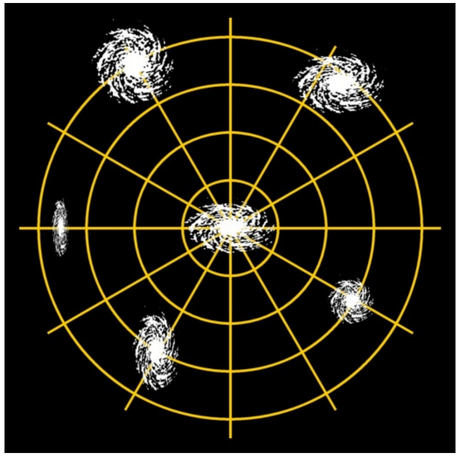
Cosmic Expansion

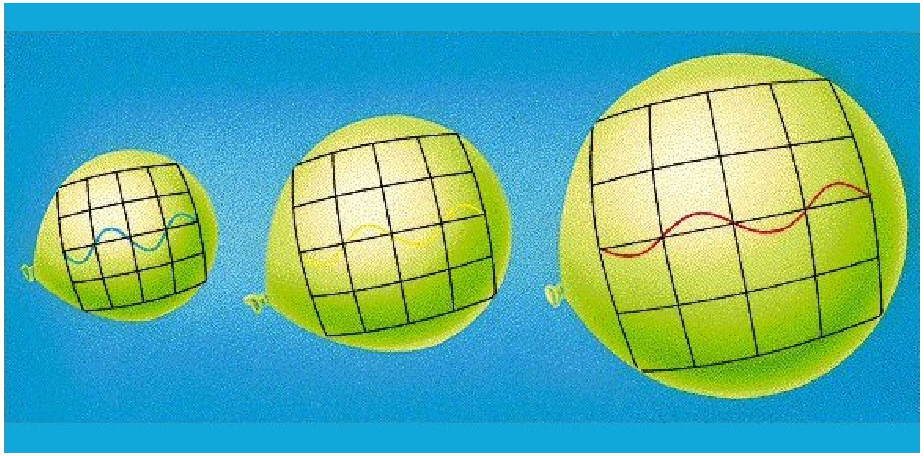
Cosmic Scale Factor





- Space between galaxies grows
- Galaxies (stars, people) stay the same (dominated by local gravity or by electromagnetic forces)
- Cosmic scale factor today: $a = 1$

Cosmic Redshift



- Wavelength of light is "stretched"
- Suffers redshift $z + 1 = \frac{\lambda_{\text{today}}}{\lambda_{\text{then}}}$
- Redshift today: $z = 0$

$$\boldsymbol{z + 1 = \frac{\lambda_{\text{today}}}{\lambda_{\text{then}}} = \frac{a_{\text{today}}}{a_{\text{then}}}}$$

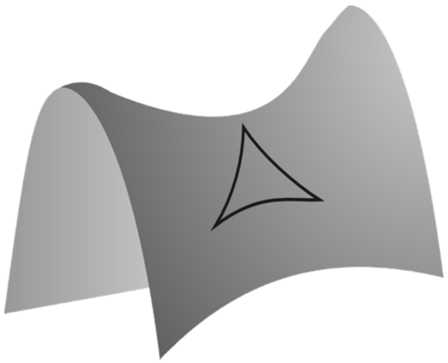
Friedman-Robertson-Walker-Lemaître Cosmology

- On scales $\gtrsim$ 100 Mpc, space is maximally symmetric (homogeneous & isotropic)

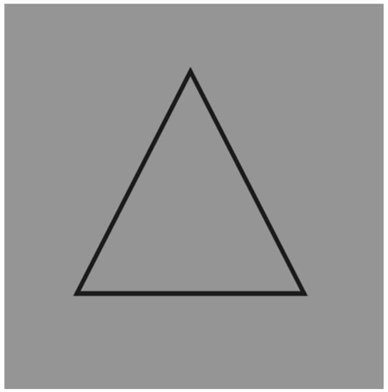
- The corresponding Robertson-Walker metric is

$$ds^2 = dt^2 - a^2(\mathrm{t})\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

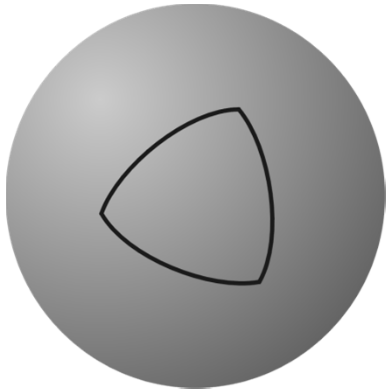
- dt ↑ Clock time of co-moving observer
- $a(\mathrm{t})$ ↑ Cosmic scale factor
- k ↑ Curvature k = 0, ±1
- (r, θ, ϕ) ↑ Co-moving spherical coordinates r is dimensionless



k = –1



k = 0



k = +1

Friedman-Robertson-Walker-Lemaître Cosmology

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- $(d\theta^2 + \sin^2\theta\, d\phi^2)$ ↑ Co-moving spherical coordinates; r is dimensionless

- Energy-momentum tensor: perfect fluid with density ρ, pressure p

$$T^{\mu\nu} = \begin{pmatrix} \rho & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix} \qquad T^{\mu\nu}_{\text{vac}} = \rho\, g^{\mu\nu} \begin{pmatrix} \rho & & & \\ & -\rho & & \\ & & -\rho & \\ & & & -\rho \end{pmatrix}$$

Critical Density and Density Parameter

- Evolution of the cosmic scale factor $a(t)$ is governed by the Friedman Equation

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G_\mathrm{N}\rho - \frac{k}{a^2 R_C^2}$$

Note rate depends on density

- In a flat universe ($k = 0$), the relationship between H and ρ is unique

$$\rho_\mathrm{crit} = \frac{3H^2}{8\pi G_\mathrm{N}} = \frac{3}{8\pi}(H m_\mathrm{Pl})^2$$

critical density

- Cosmic density always expressed in terms of density parameters

$$\Omega = \frac{\rho}{\rho_\mathrm{crit}} = \frac{8\pi G_\mathrm{N}\rho}{3H^2}$$

- With the present-day Hubble parameter $H_0 = 67.3\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$ we have

$$\rho_\mathrm{crit} = 8.51 \times 10^{-30}\mathrm{g\ cm^{-3}} = 5\ \mathrm{GeV\ m^{-3}} = (2.5\ \mathrm{meV})^4$$

Most of this in the form of “dark energy”

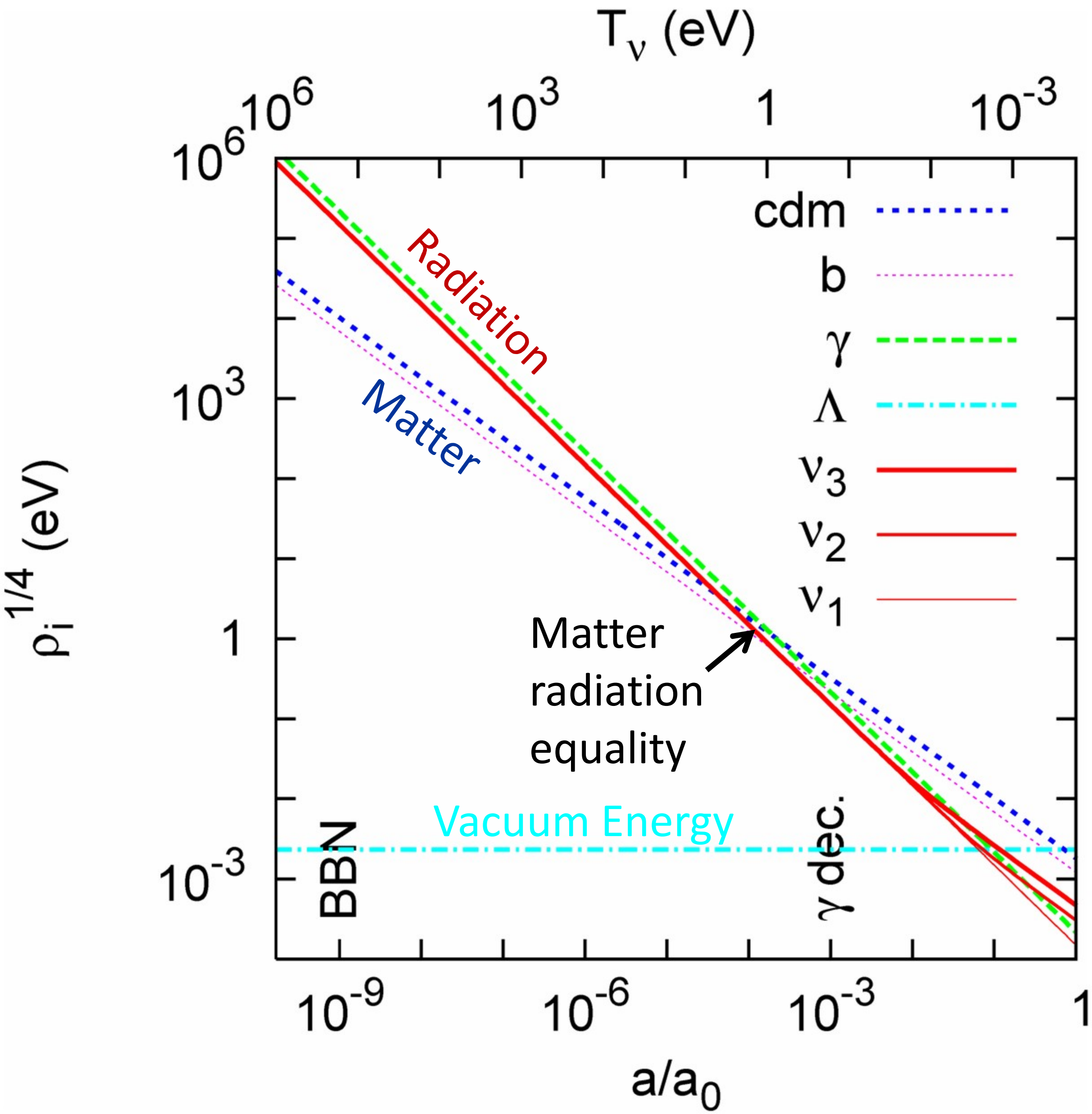
Generic Solutions of Friedman Equation

	Equation of state	Behavior of energy-density under cosmic expansion		Evolution of cosmic scale factor
Radiation	$p = \frac{\rho}{3}$	$\rho \propto a^{-4}$	Dilution of radiation and redshift of energy	$a(t) \propto t^{1/2}$
Matter	$p = 0$	$\rho \propto a^{-3}$	Dilution of matter	$a(t) \propto t^{2/3}$
Vacuum energy	$p = -\rho$	$\rho = \text{const}$	Vacuum energy not diluted by expansion	$a(t) \propto e^{\sqrt{\Lambda/3}\,t}$ $\Lambda = 8\pi G_\text{N}\rho_\text{vac}$

Energy-momentum tensor of a perfect fluid with density ρ and pressure p

$$T^{\mu\nu} = \begin{pmatrix} \rho & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix} \qquad T^{\mu\nu}_\text{vac} = \rho\, g^{\mu\nu} \begin{pmatrix} \rho & & & \\ & -\rho & & \\ & & -\rho & \\ & & & -\rho \end{pmatrix}$$

Evolution of Cosmic Density Components



Assumed neutrino masses

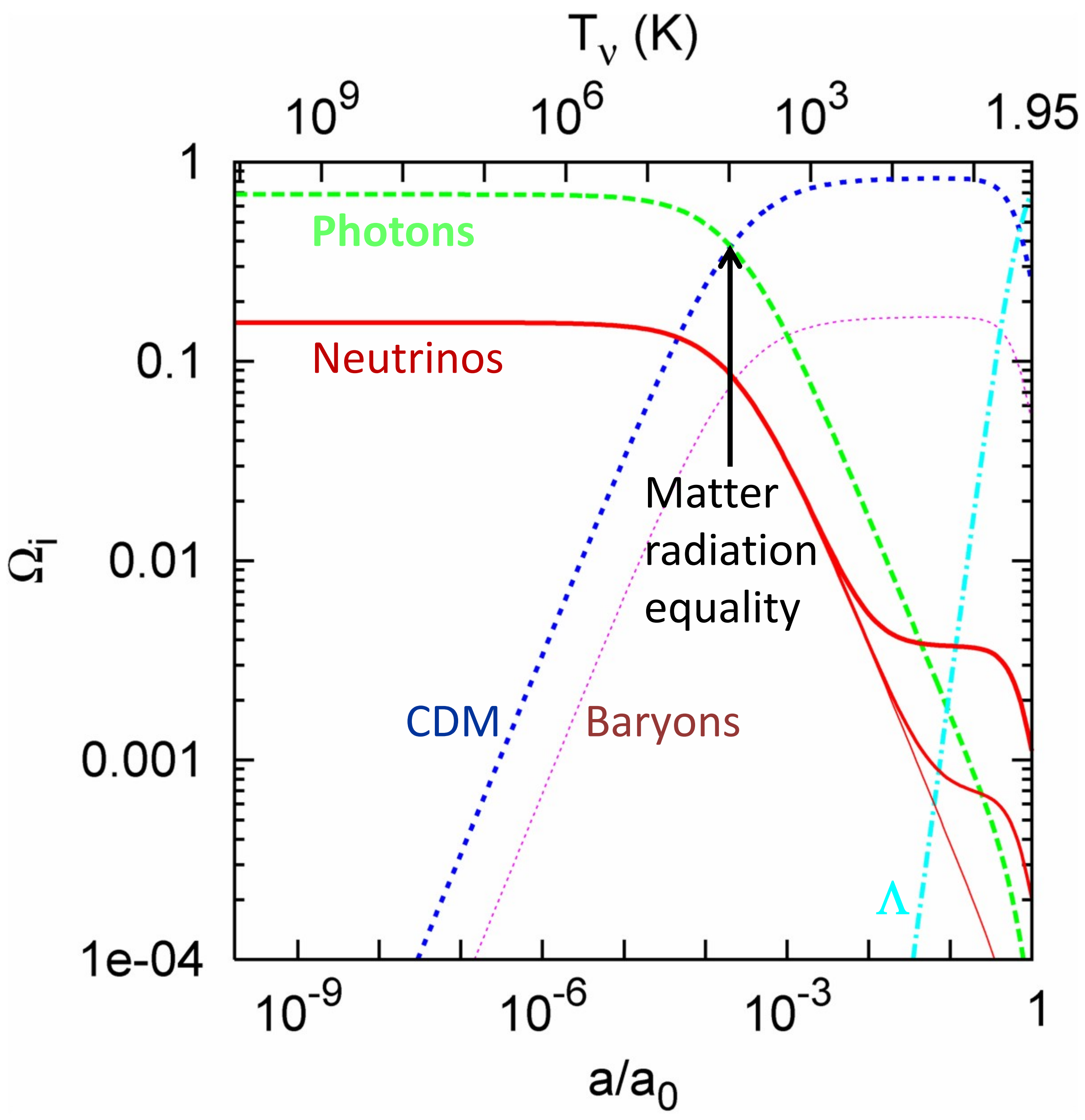
m_3 = 50 meV

m_2 = 9 meV

m_1 = 0

Lesgourgues & Pastor
astro-ph/0603494

Evolution of Cosmic Density Components



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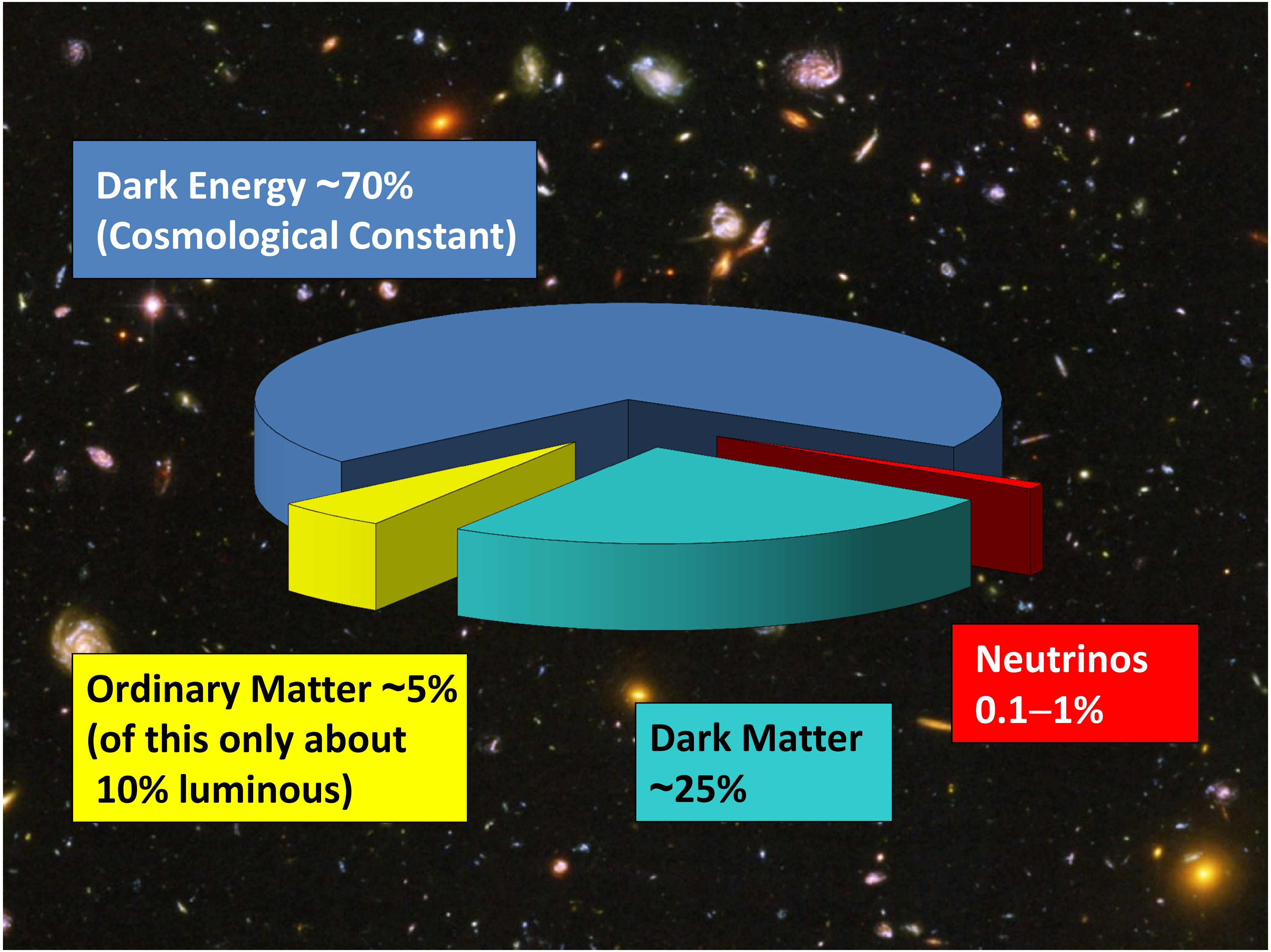
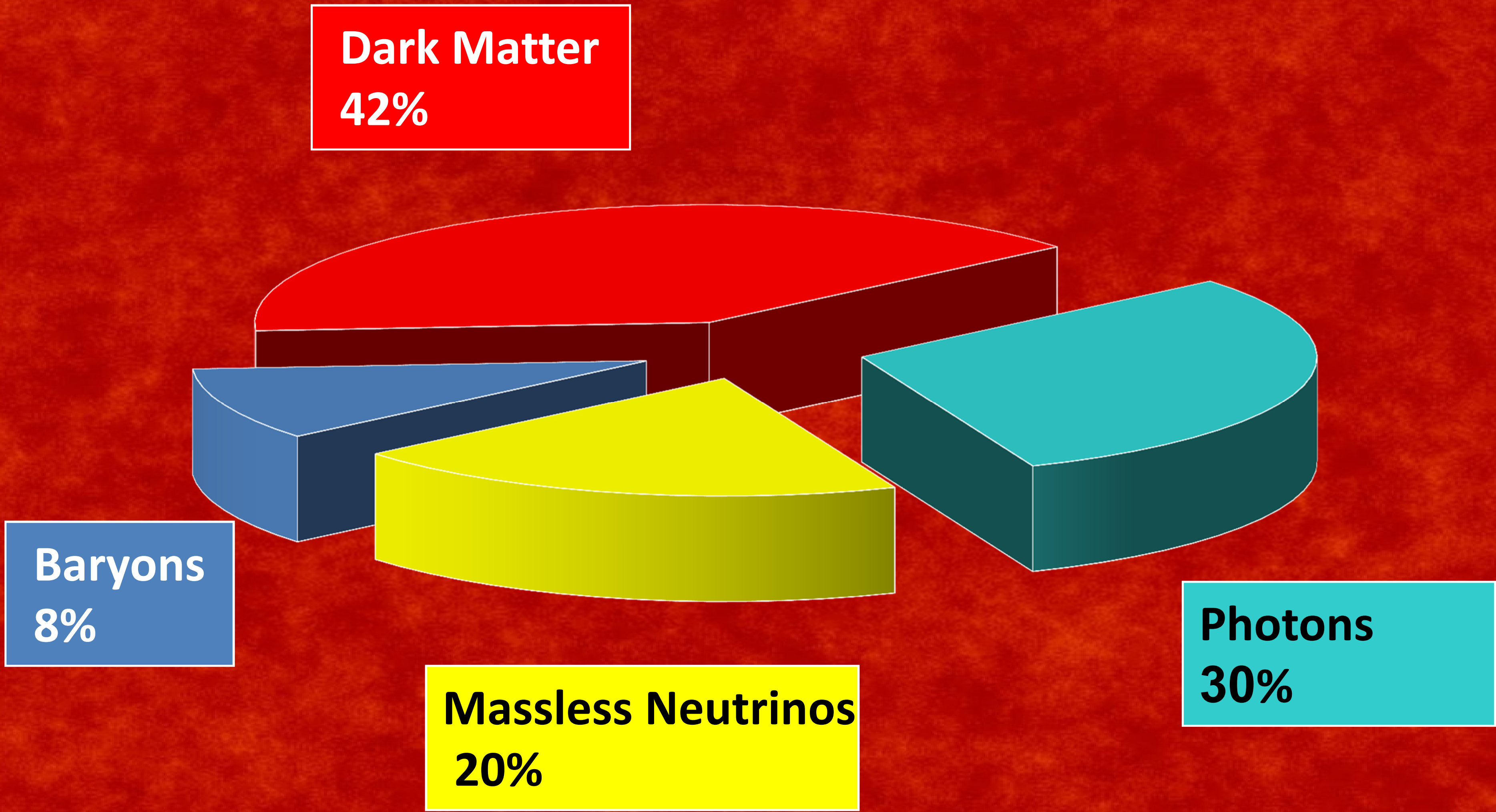
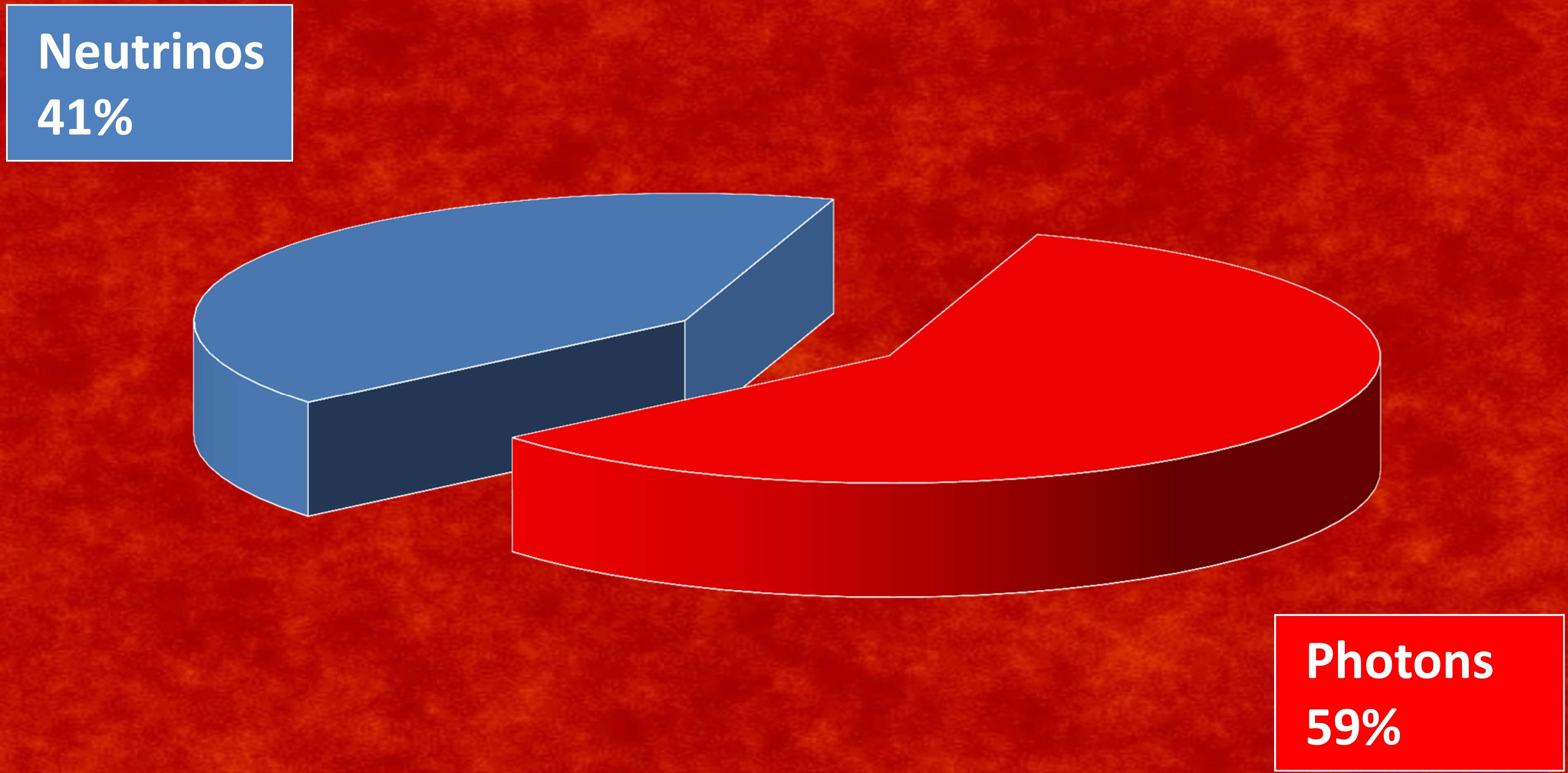

Dark Energy ~70%
(Cosmological Constant)
Ordinary Matter ~5%
(of this only about
10% luminous)
Dark Matter
~25%
Neutrinos
0.1–1%

Matter-Radiation Equality (Redshift 3400)

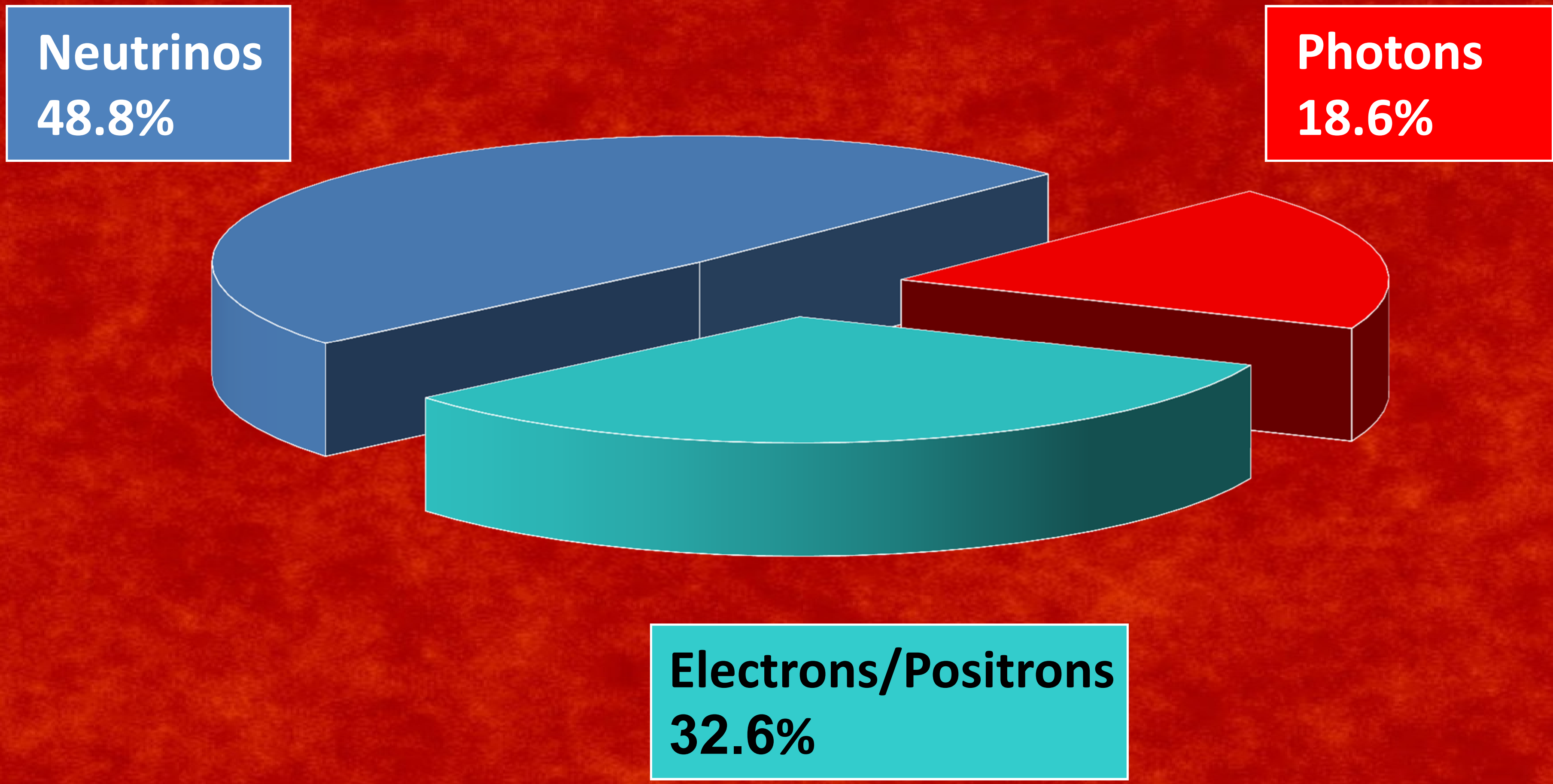


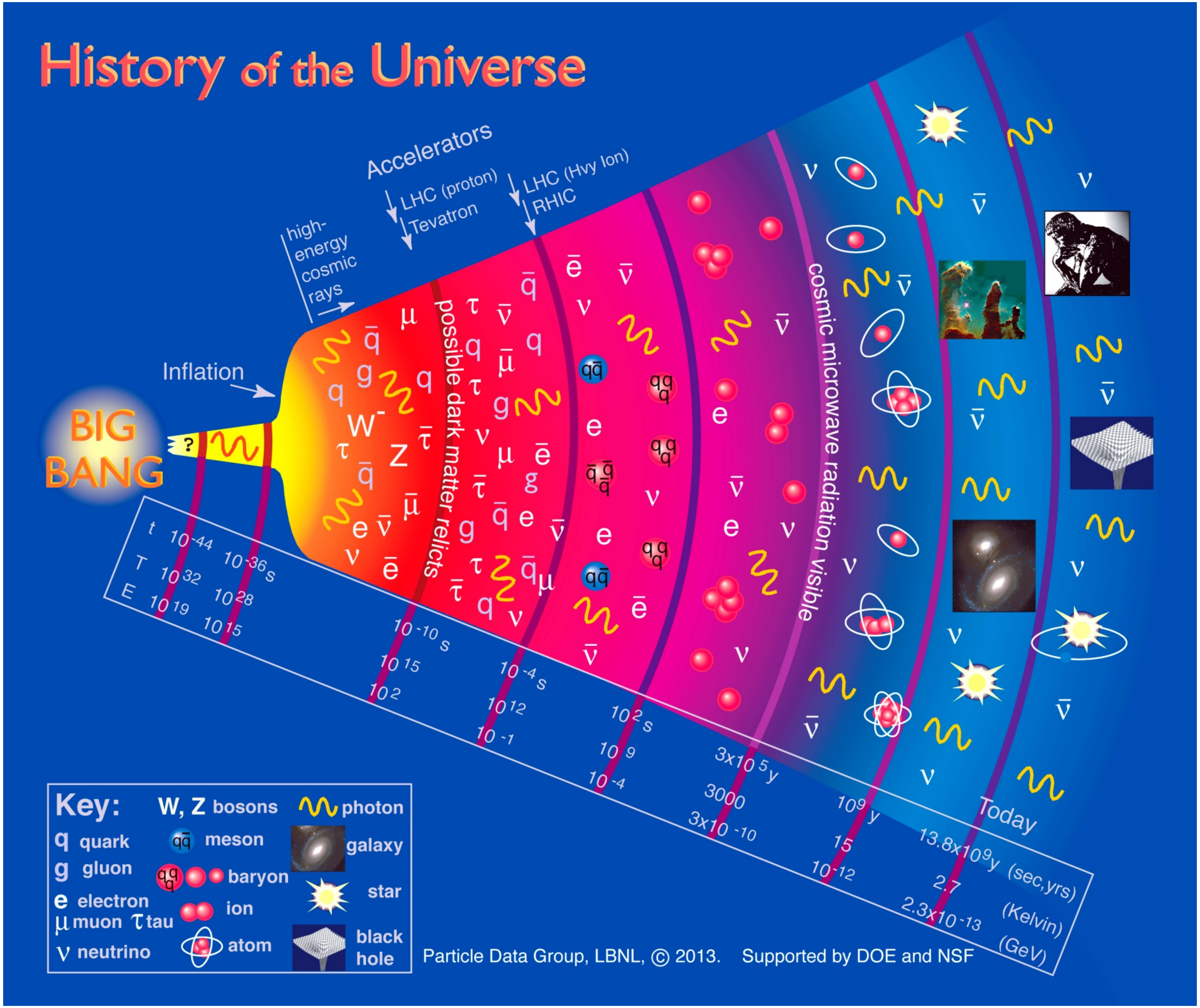
After Electron-Positron Annihilation (T = 100 keV)



Relevant for Big Bang Nucleosynthesis (BBN)

Before Electron-Positron Annihilation (T = 1 MeV)

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cosmic microwave radiation visible
t 10-44 10-36 s 10-10 s 10-4 s 10 2 s 3x10 5 y 10 9 y
T 10 32 10 28 10 15 10 12 10 9 3000 15 2.7 (Kelvin)
E 10 19 10 15 10 2 10 -1 10 -4 3x10 -10 10-12 2.3x10 -13 (GeV)
Today
13.8x10 9 y (sec,yrs)
Key:
q quark
g gluon
e electron
μ muon τ tau
ν neutrino
W, Z bosons
meson
baryon
ion
atom
photon
galaxy
star
black hole
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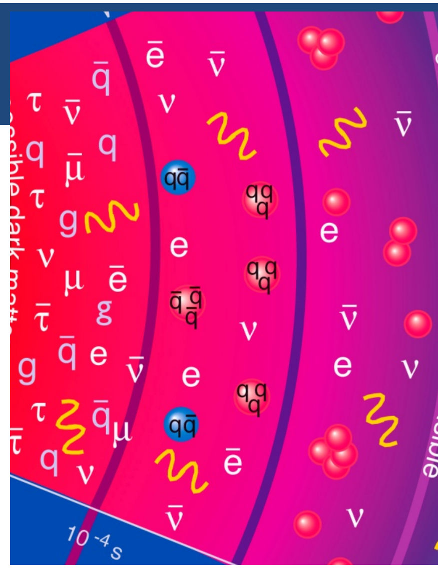
Equilibrium Particle Interactions



- Boltzmann equation governs distributions

$$\frac{\mathrm{d}f_X}{\mathrm{d}t} + 3\frac{\dot{a}}{a} f_X + \langle\sigma_A v\rangle\left({f_X}^2 - {f_{Xeq}}^2\right) = 0$$

- Two regimes:

$\Gamma = \langle\sigma_A v\rangle n_X > H$ Thermal equilibrium

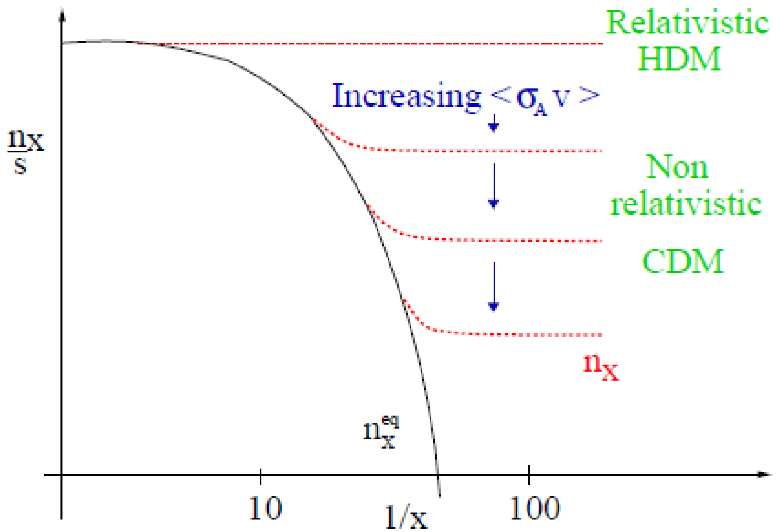
Interaction rate > Expansion rate

$$f_{eq}(\boldsymbol{p}) = \frac{1}{e^{E_{\boldsymbol{p}}/T} \pm 1}$$

+ Fermions, - Bosons

$\Gamma \ll H$ Freezeout

Distribution constant at freezeout level, only redshifted



Thermal Radiation

	General	Bosons	Fermions
Number density n	$g\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{e^{E_{\boldsymbol{p}}/T}\pm 1}$	$g_B\frac{\zeta_3}{\pi^2}T^3$	$\frac{3}{4}\,g_F\frac{\zeta_3}{\pi^2}T^3$
Energy density ρ	$g\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{E_{\boldsymbol{p}}}{e^{E_{\boldsymbol{p}}/T}\pm 1}$	$g_B\frac{\pi^2}{30}T^4$	$\frac{7}{8}\,g_F\frac{\pi^2}{30}T^4$
Pressure P	$g\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{\lvert\boldsymbol{p}^2\rvert}{E_{\boldsymbol{p}}}\frac{1}{e^{E_{\boldsymbol{p}}/T}\pm 1}$	$\frac{\rho}{3}$	
Entropy density s	$\frac{\rho+P}{T}=\frac{4}{3}\,\frac{\rho}{T}$	$g_B\frac{2\pi^2}{45}T^3$	$\frac{7}{8}\,g_F\frac{2\pi^2}{45}T^3$

↕

$$\mathrm{d}E = T\mathrm{d}S - P\mathrm{d}V$$
$$T\mathrm{d}S = (\rho + P)dV$$

using integrals

$$\int_0^\infty \frac{x^2dx}{\exp(x)-1} = 2\zeta(3),$$
$$\int_0^\infty \frac{x^2dx}{\exp(x)+1} = \frac{6}{8}\zeta(3),$$
$$\int_0^\infty \frac{x^3dx}{\exp(x)-1} = 6\zeta(4) = \frac{\pi^4}{15},$$
$$\int_0^\infty \frac{x^3dx}{\exp(x)+1} = \frac{7}{48}\zeta(4) = \frac{7}{8}\frac{\pi^4}{15}$$

Riemann Zeta Function
$\zeta = 1.2020569\ldots$

Thermal Degrees of Freedom

$$g_* = g_B + \frac{7}{8} g_F$$

Mass threshold		Particles	g_B	g_F	g_*
	low	γ, 3ν	2	6	(7.25)
m_e	0.5 MeV	$e^{\pm}$	2	10	10.75
m_μ	105 MeV	$\mu^{\pm}$	2	14	14.25
m_π	135 MeV	π^0, $\pi^{\pm}$	5	14	17.25
Λ_{QCD}	~170 MeV	u, d, s, gluons	18	50	61.75
$m_{c,\tau}$	2 GeV	c, τ	18	66	75.75
m_b	6 GeV	$b^{\pm}$	18	78	86.25
$m_{W,Z}$	90 GeV	Z^0, $W^{\pm}$	27	78	92.25
m_H	126 GeV	Higgs	28	78	93.25
m_t	170 GeV	t	28	90	106.75
Λ_{SUSY}	~ 1 TeV ?	SUSY particles	118	118	213.50

Neutrino Background

Neutrino Thermal Equilibrium

Neutrino reaction rate

Examples of neutrino processes

$$e^+ + e^- \leftrightarrow \overline{\nu} + \nu$$
$$\overline{\nu} + \nu \leftrightarrow \overline{\nu} + \nu$$
$$\nu + e^\pm \leftrightarrow \nu + e^\pm$$

Reaction rate in a thermal medium

for $T \ll m_{W,Z}$

$$\Gamma \sim G_F^2 T^5$$

Cosmic expansion rate

Friedmann equation (flat universe)

$$H^2 = \frac{8\pi}{3}\frac{\rho}{m_{Pl}^2} \qquad \left(G_N = \frac{1}{m_{Pl}^2}\right)$$

Radiation dominates

$$\rho \sim T^4$$

Expansion rate

$$H \sim \frac{T^2}{m_{Pl}}$$

Condition for thermal equilibrium: $\Gamma > H$

$$T > \left(m_{Pl} G_F^2\right)^{-1/3} \sim [10^{19}\text{GeV}\,(10^{-5}\text{GeV}^{-2})^2]^{-1/3} = 1\ \text{MeV}$$

Neutrinos are in thermal equilibrium for $T \gtrsim 1$ MeV
corresponding to $t \lesssim 1$ sec

Present-Day Neutrino Density

Neutrino decoupling (freeze out)	$H \sim \Gamma$ $T \approx 2.4$ MeV (electron flavour) $T \approx 3.7$ MeV (other flavours)
Redshift of Fermi-Dirac distribution ("nothing changes at freeze-out")	$\frac{dn_{\nu\bar{\nu}}}{dE} = \frac{1}{\pi^2} \frac{E^2}{e^{E/T}+1}$ Temperature scales with redshift $T_\nu = T_\gamma \propto (z+1)$
Electron-positron annihilation beginning at T ≈ m_e = 0.511 MeV	• Entropy of e^+e^- transfered to photons $g_* T_\gamma^3 \big\vert_{\text{before}} = g_* T_\gamma^3 \big\vert_{\text{after}}$ $\overbrace{2 + \frac{7}{8} 4 = \frac{11}{2}}$ $\quad \overbrace{2}$ $T_\gamma^3 \big\vert_{\text{before}} = \mathbf{\frac{4}{11}} T_\gamma^3 \big\vert_{\text{after}}$
Redshift of neutrino and photon thermal distributions so that today we have	$n_{\nu\bar{\nu}}(1\ \text{flavor}) = \frac{4}{11} \times \frac{3}{4} \times n_\gamma = \frac{3}{11} n_\gamma \approx 112\ \text{cm}^{-3}$ $T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \approx 1.95\ \text{K}$ for massless neutrinos

Present-Day Neutrino Distribution

Minimal neutrino masses from oscillation experiments	Normal $m_3 \gtrsim 50$ meV $m_2 \gtrsim 8$ meV $m_1 \geq 0$	Inverted $m_1 \approx m_2 \gtrsim 50$ meV $m_3 \geq 0$
Temperature of massless cosmic background neutrinos	T = 1.95 K = 0.17 meV	
Cosmic redshift of **momenta** (not energies)	$\frac{dn_{\nu\bar{\nu}}}{dp} = \frac{1}{\pi^2} \frac{p^2}{e^{p/T}+1}$	Not a thermal distribution unless $T \gg m$
Average velocity for $m \gg T$	$\langle v \rangle \approx \frac{3T}{m}$	
Normal hierarchy neutrinos	$\langle v_3 \rangle < 1 \times 10^{-2} c$	$\langle v_2 \rangle < 6 \times 10^{-2} c$

Cosmic radiation density after e^+e^- annihilation

Radiation density for $N_\nu = 3$ standard neutrino flavors

$$\rho_{\rm rad} = \rho_\gamma + \rho_\nu = \frac{\pi^2}{15}\left(T_\gamma^4 + N_\nu \frac{7}{8}\, T_\nu^4\right) = \left[1 + \mathrm{N}_\nu \frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\right]\rho_\gamma$$

Cosmic radiation density is expressed in terms of
"effective number of thermally excited neutrino species" $N_{\rm eff}$

$$\rho_{\rm rad} = \left[1 + \mathrm{N}_{\rm eff} \frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\right]\rho_\gamma = [1 + \mathrm{N}_{\rm eff}\, 0.2271]\rho_\gamma$$

$N_{\rm eff}$ is a measure for the radiation density, not necessarily related to neutrinos

Residual neutrino heating by e^+e^- annihilation and corrections for finite temperature

QED effects and neutrino flavor oscillations means T_ν *not* $\left(\frac{4}{11}\right)^{1/3} T_\gamma$

$N_{\rm eff} = 3.046$ Standard value

$$\rho_{\rm rad} = (1 + 0.6918 + 0.2271\, \Delta\mathrm{N}_{\rm eff})\, \rho_\gamma$$

Of course, the number of known neutrino species ν_e, ν_μ, ν_τ is exactly 3

Cosmological Limit on Neutrino Masses

Cosmic neutrino "sea" $\sim 112\ \mathrm{cm}^{-3}$ neutrinos + anti-neutrinos per flavor

$$\Omega_\nu h^2 = \sum \frac{m_\nu}{93\ \mathrm{eV}}$$

closure bound $\Omega_\nu h^2 < 1$ $\sum m_\nu < 90\ \mathrm{eV}$

JETP Lett. 4 (1966) 120

REST MASS OF MUONIC NEUTRINO AND COSMOLOGY

S. S. Gershtein and Ya. B. Zel'dovich

Submitted 4 June 1966
ZhETF Pis'ma 4, No. 5, 174-177, 1 September 1966

Low-accuracy experimental estimates of the rest mass of the neutrino [1] yield $m(\nu_e) < 200\ \mathrm{eV}/c^2$ for the electronic neutrino and $m(\nu_\mu) < 2.5 \times 10^6\ \mathrm{eV}/c^2$ for the muonic neutrino.

Cosmological considerations connected with the hot model of the Universe [2] make it possible to strengthen greatly the second inequality. Just as in the paper by Ya. B. Zel'dovich and Ya. A. Smorodinskii [3], let us consider the gravitational effect of the neutrinos on the dynamics of the expanding Universe. The age of the known astronomical objects is not smaller than 5×10^9 years, and Hubble's constant H is not smaller than 75 km/sec-Mparsec $= (13 \times 10^9\ \text{years})^{-1}$. It follows therefore that the density of all types of matter in the Universe is at the present time 1)

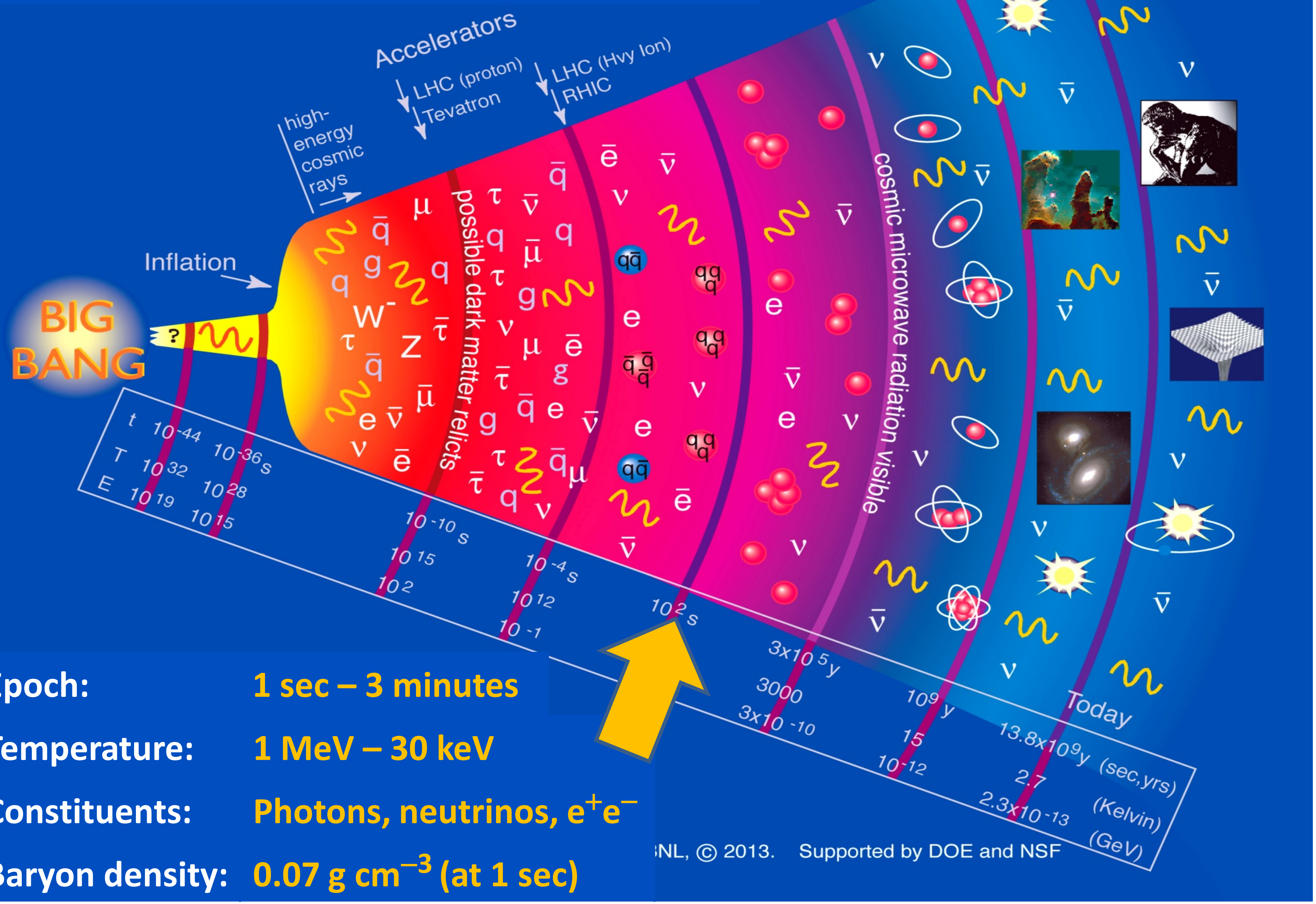
$$\rho < 2 \times 10^{-28}\ \mathrm{g/cm}^3.$$

Evidence for cosmic neutrinos....indirect

- Big Bang Nucleosynthesis
- Imprint on CMB and large scale structure

- Can use observations to constrain N_{eff} and the Σ mass

BBN Nucleosynthesis



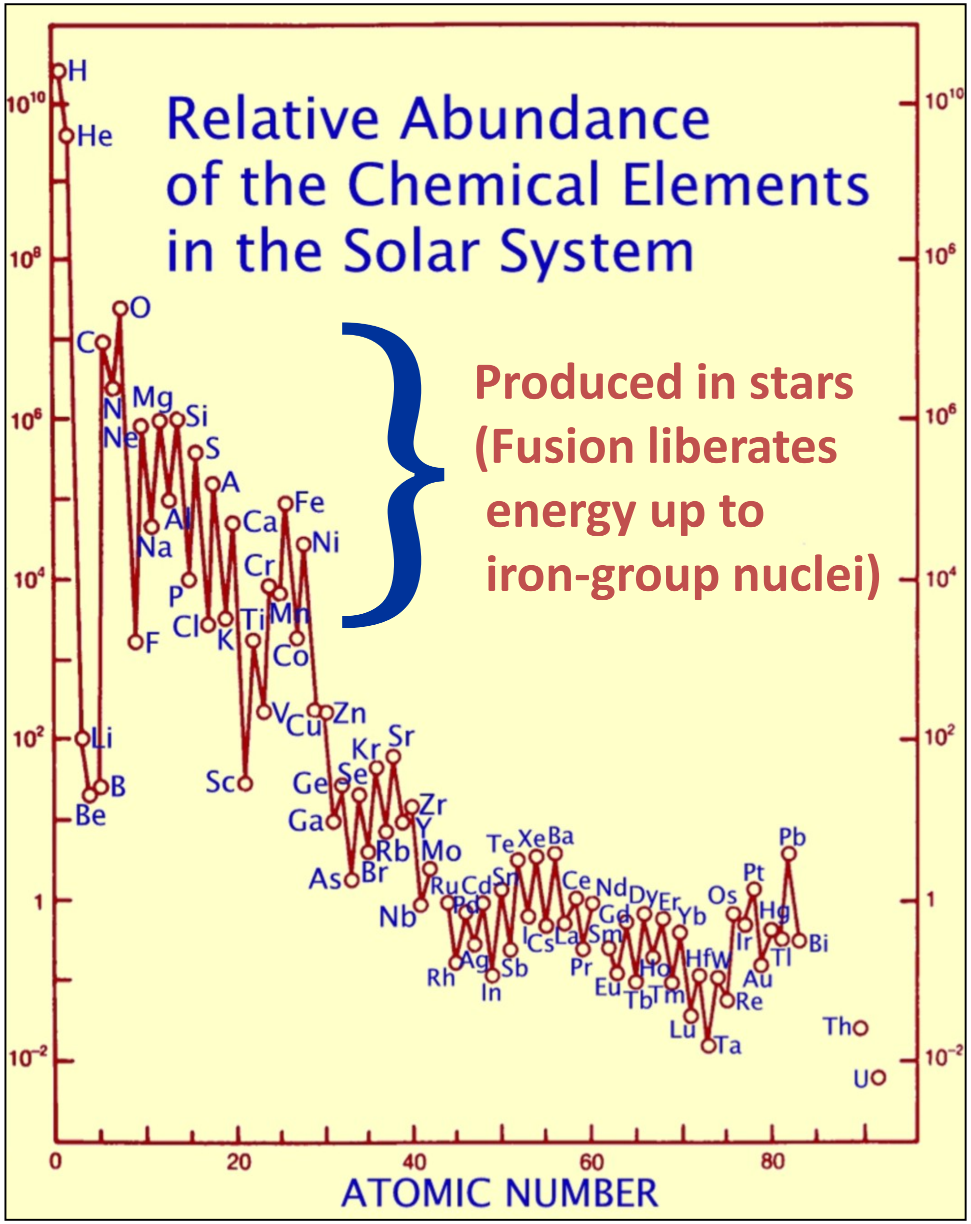
Epoch: 1 sec – 3 minutes

Temperature: 1 MeV – 30 keV

Constituents: Photons, neutrinos, $e^{+}e^{-}$

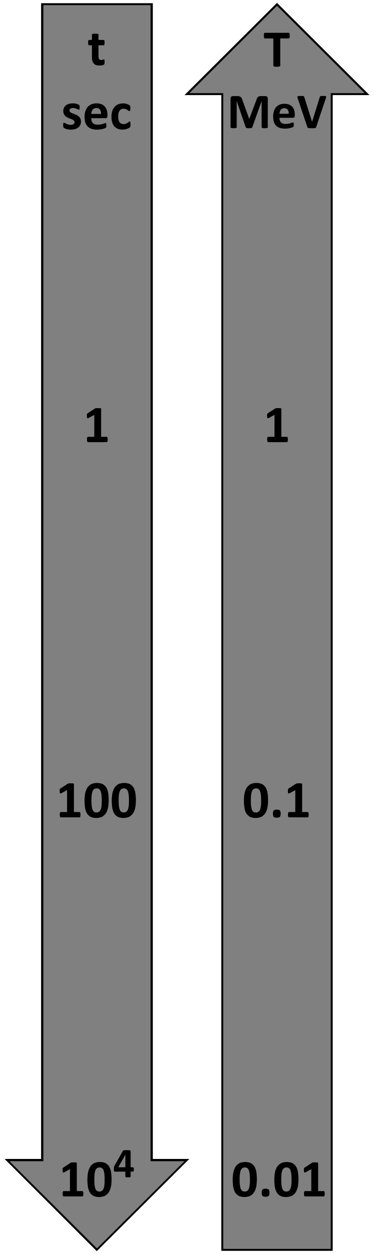
Baryon density: 0.07 g cm^{-3} (at 1 sec)

Origin of Elements

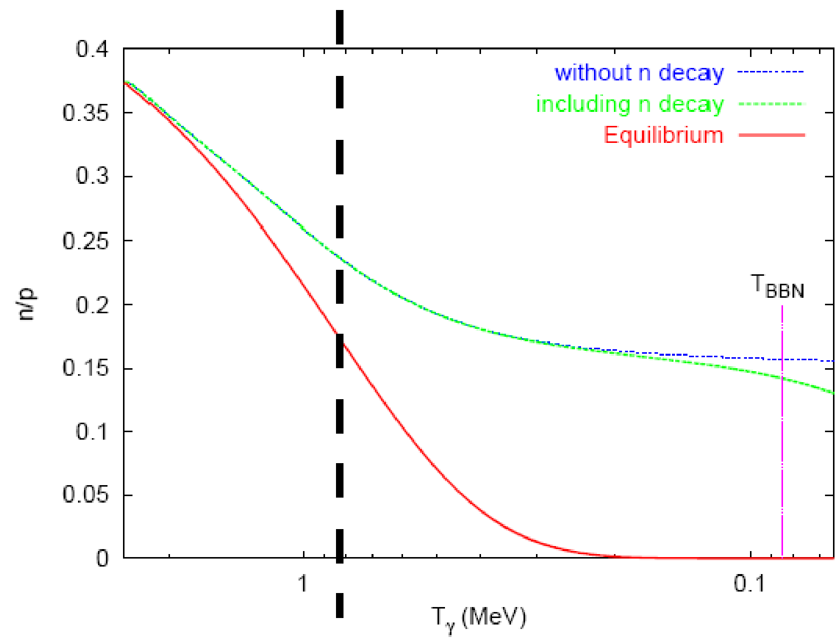


- Mass fraction of helium ~ 25% everywhere in the universe
- Most of it not produced in stars (far too little star light from liberated energy)
- Big-bang nucleosynthesis (BBN) is a pillar of modern cosmology
- Neutrinos play a crucial role

Helium Synthesis



β equilibrium	$p + e^- \leftrightarrow n + \nu_e$ $p + \bar{\nu}_e \leftrightarrow n + e^+$	$\frac{n}{p} = \exp\left(-\frac{m_n - m_p}{T}\right) \approx 1$ $m_n - m_p = 1.293$ MeV
β freeze-out	$\frac{n}{p} \approx \frac{1}{6}$	**β rates fall below expansion rate H**
• Neutron decay • Nuclear statist. equilibrium	$n_n \propto \exp\left(-\frac{t}{\tau_n}\right)$ $\tau_n = 880$ sec (?)	$2p + 2n \leftrightarrow {}^4He + \gamma$ **Helium suppressed by large entropy**
Neutrons freeze in helium	$\frac{n}{p} \approx$	
Thermonuclear reaction chains	**Prod** **D, ^{3}H**	
All done		

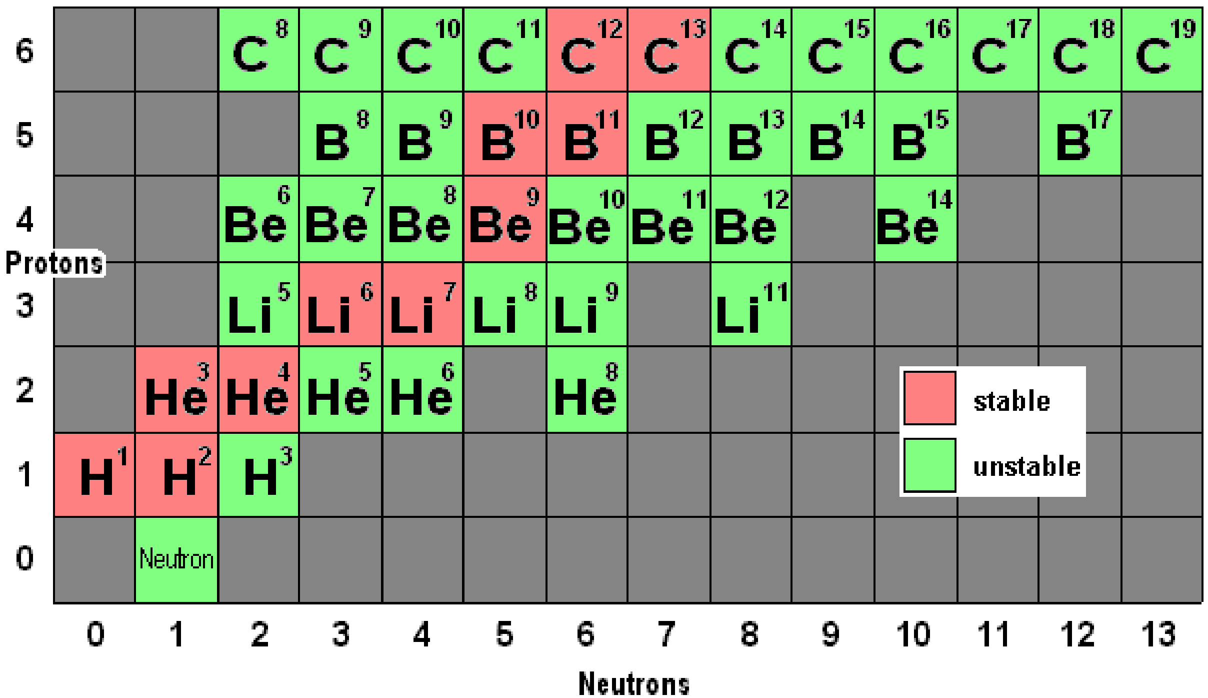


Why do nuclei form so late?

- Thermal equilibrium → all nuclei present
- Binding energies much larger than MeV, so why are they still dissociated at weak-interaction freeze-out? Why not everything in iron?
- Basic answer: High-entropy environment with $\sim 10^9$ photons per baryon

$$Nn + Zp \leftrightarrow (A,Z) + \text{photons}$$

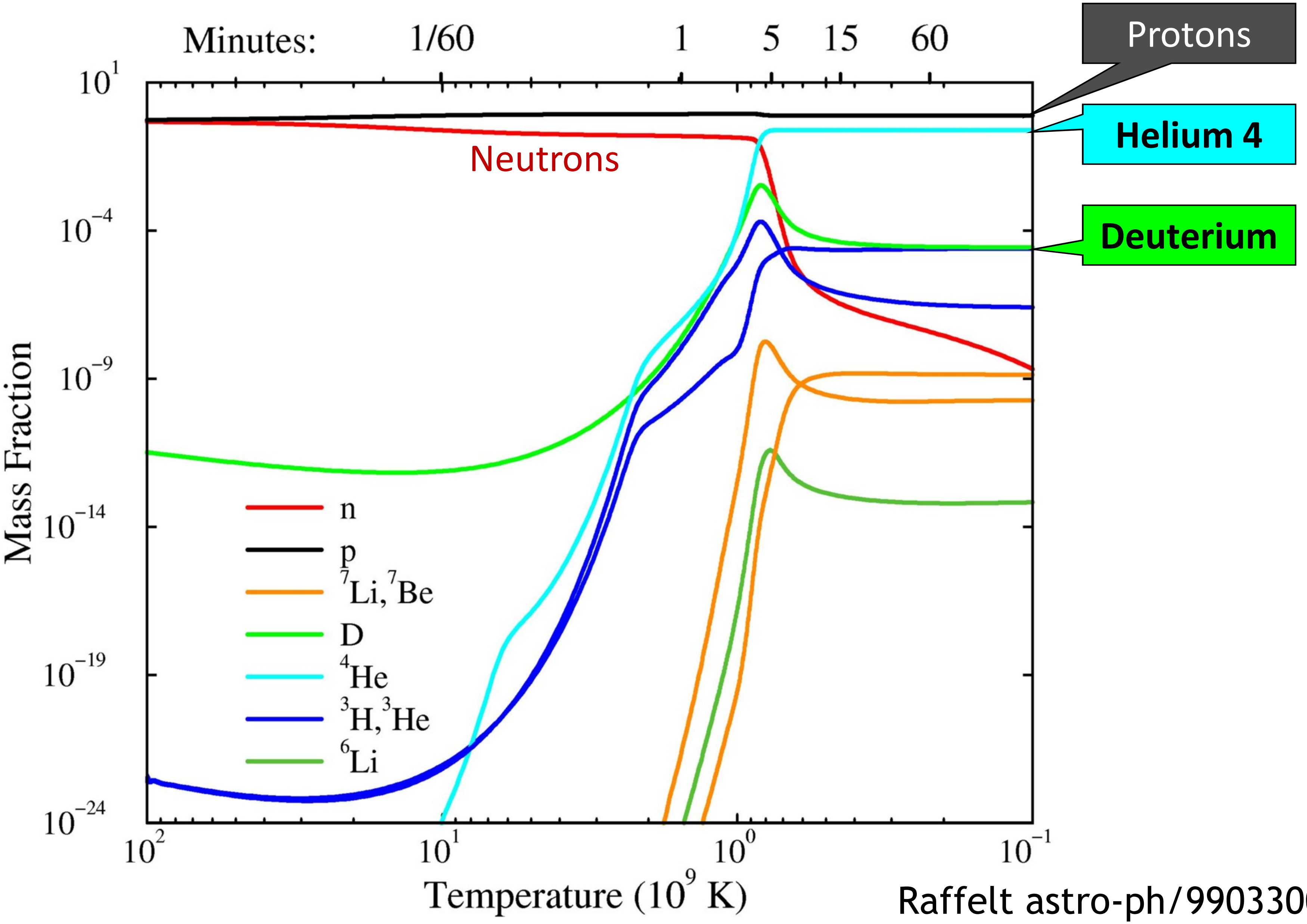
High-E tail of photon distribution enough to keep nuclei dissociated



	B (MeV)	B/A (MeV)
D	2.23	1.1
^{3}H	6.92	2.3
^{3}He	7.72	2.6
^{4}He	28.30	7.1
^{6}Li	31.99	5.3
^{7}Li	39.25	5.6
^{7}Be	37.60	5.4
^{12}C	92.2	7.7

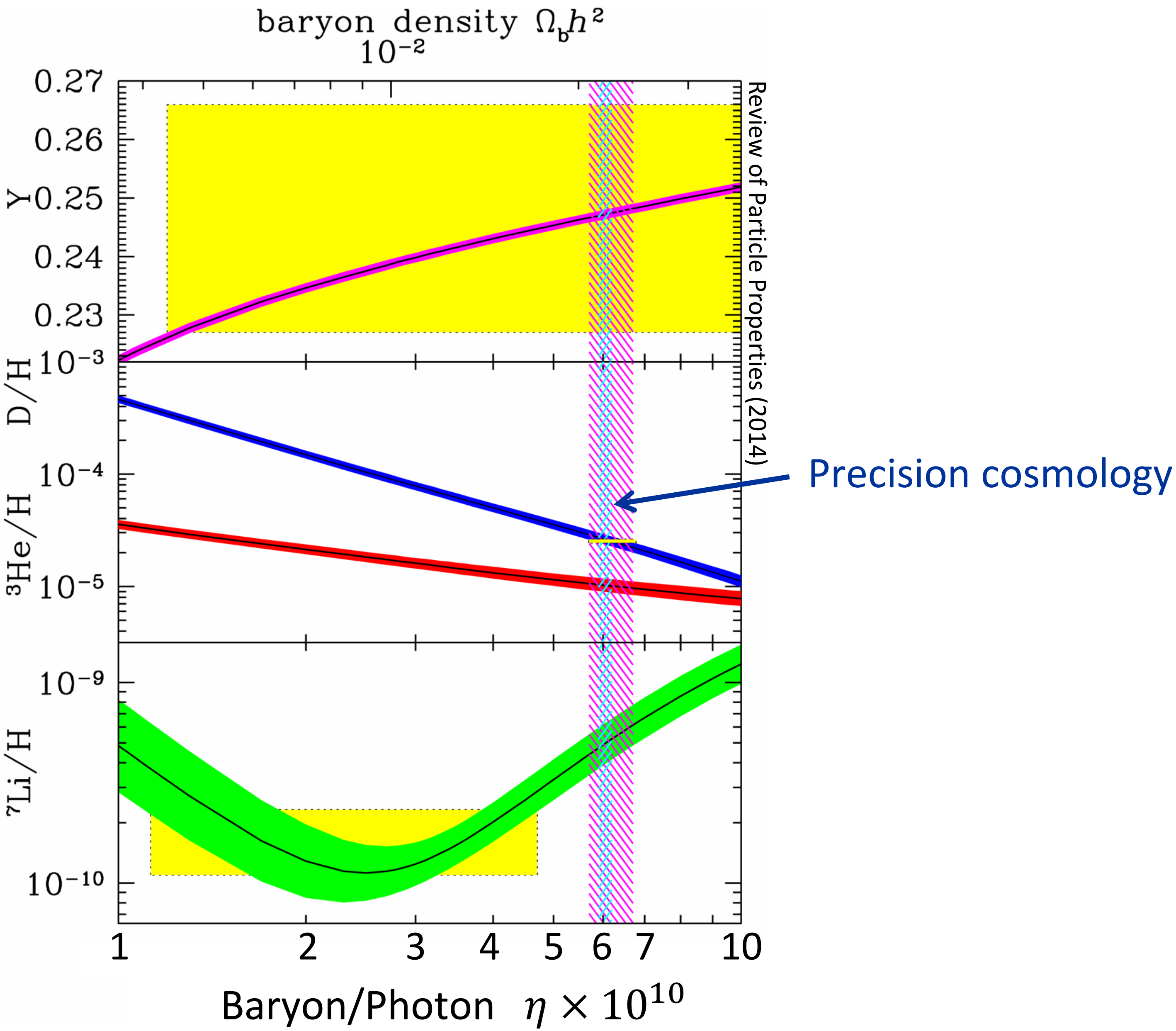
Predictions depend on baryon-photon ratio

Formation of Light Elements



Raffelt astro-ph/9903300

BBN Theory vs Observations



Neutrinos and Big Bang Nucleosynthesis

- Universe is radiation dominated
- The presence of neutrinos in radiation affects the expansion rate

$$H^2 = \frac{8\pi}{3}\frac{\rho}{m_{\mathrm{Pl}}^2}$$

- Higher expansion rate means higher freezeout temperature and greater n/p ratio

- Neutrino asymmetry would also affect the weak interactions (come back to this)

$$\bar{\nu}_e + p \rightarrow e^+ + n$$
$$\nu_e + n \rightarrow e^- + p$$

Cosmic radiation density after e^+e^- annihilation

Radiation density for $N_\nu = 3$ standard neutrino flavors

$$\rho_{\rm rad} = \rho_\gamma + \rho_\nu = \frac{\pi^2}{15}\left(T_\gamma^4 + N_\nu \frac{7}{8}\, T_\nu^4\right) = \left[1 + \mathrm{N}_\nu \frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\right]\rho_\gamma$$

Cosmic radiation density is expressed in terms of
"effective number of thermally excited neutrino species" $N_{\rm eff}$

$$\rho_{\rm rad} = \left[1 + \mathrm{N}_{\rm eff}\, \frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\right]\rho_\gamma = [1 + \mathrm{N}_{\rm eff}\, 0.2271]\rho_\gamma$$

$N_{\rm eff}$ is a measure for the radiation density, not necessarily related to neutrinos

Residual neutrino heating by e^+e^- annihilation and corrections for finite temperature

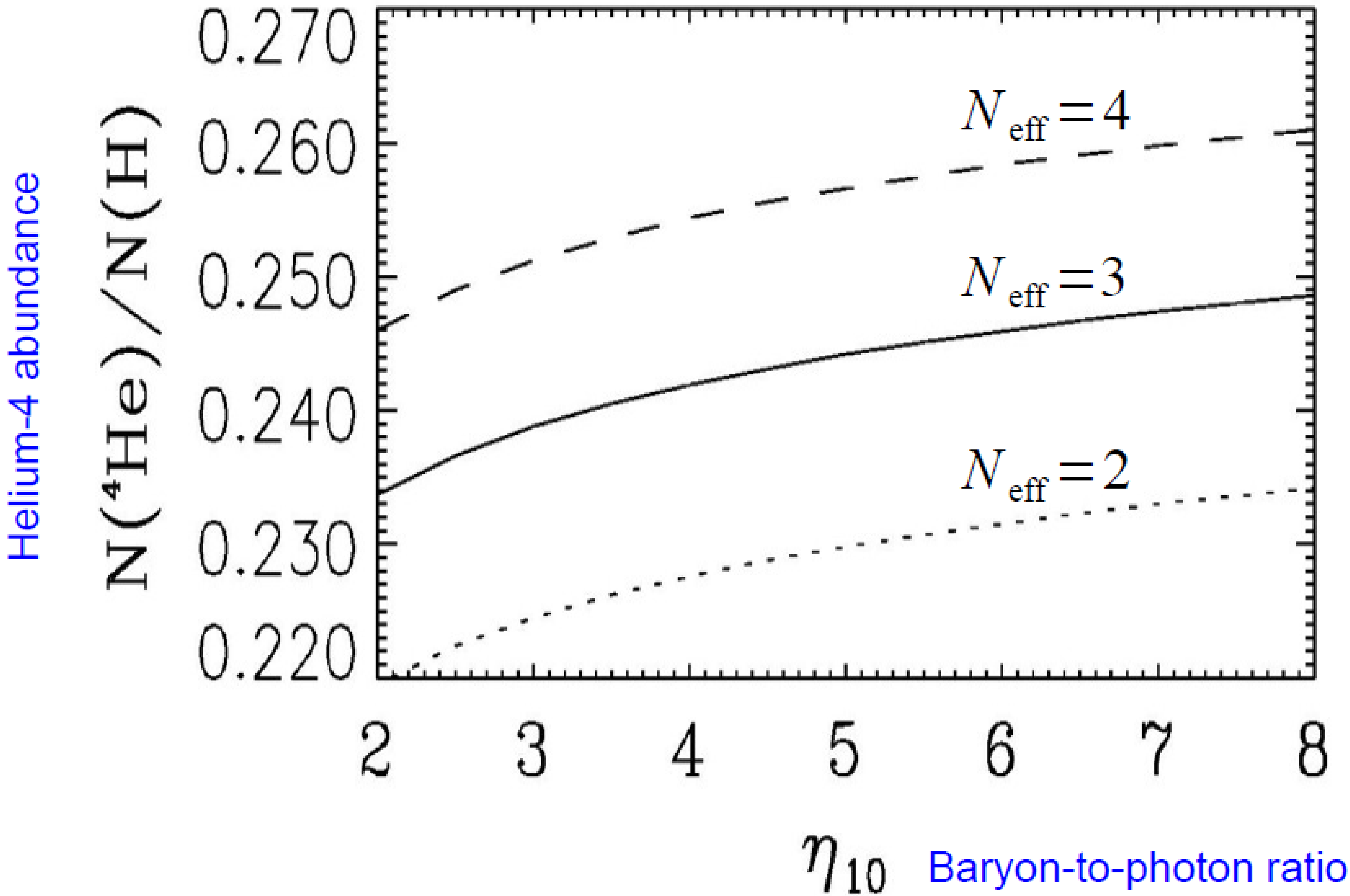
QED effects and neutrino flavor oscillations means T_ν *not* $\left(\frac{4}{11}\right)^{1/3} T_\gamma$

$N_{\rm eff} = 3.046$ Standard value

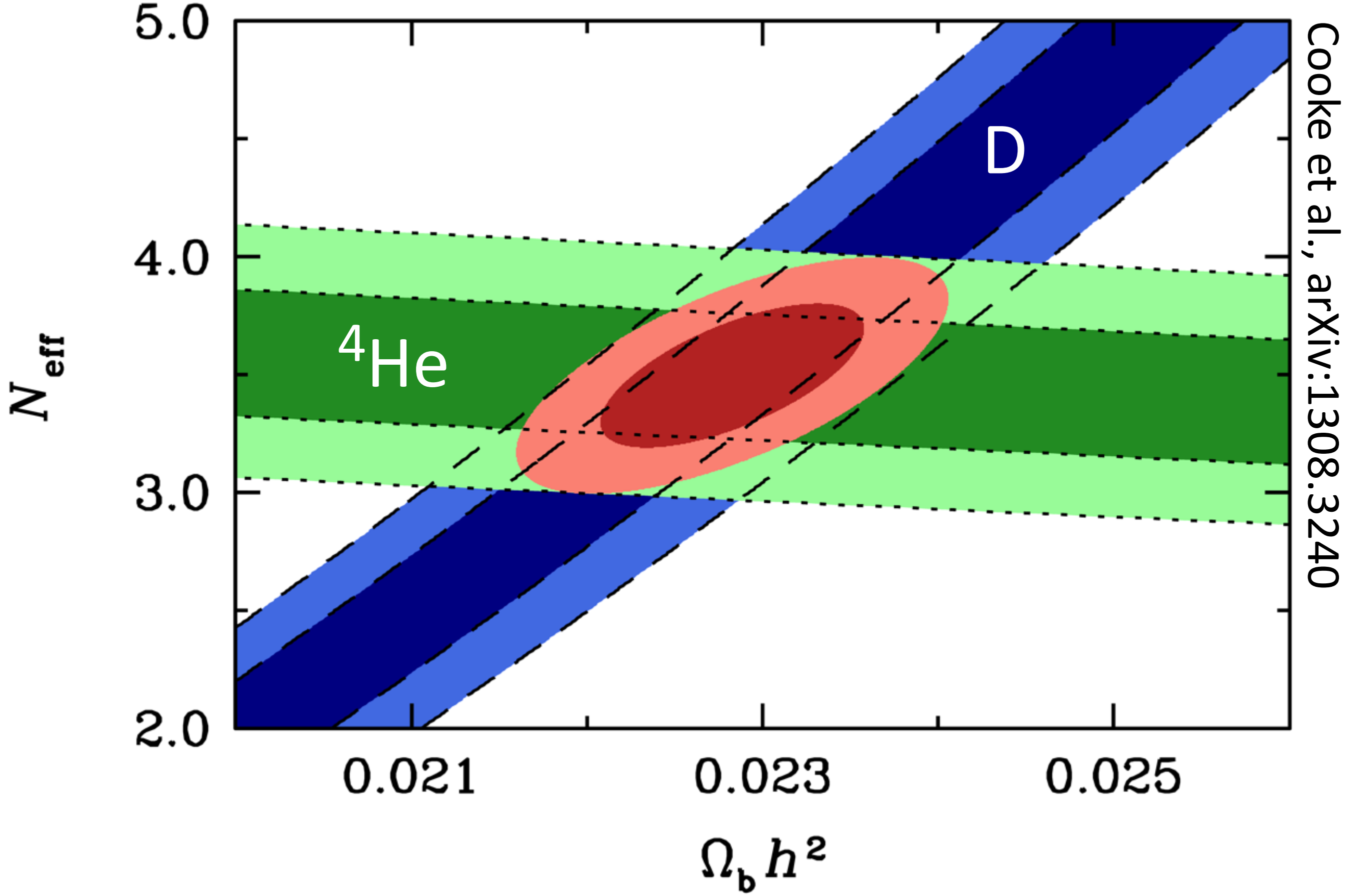
$$\rho_{\rm rad} = (1 + 0.6918 + 0.2271\, \Delta\mathrm{N}_{\rm eff})\, \rho_\gamma$$

Of course, the number of known neutrino species ν_e, ν_μ, ν_τ is exactly 3

Helium-4 is most sensitive to N_{eff}



Baryon and Radiation Density from BBN



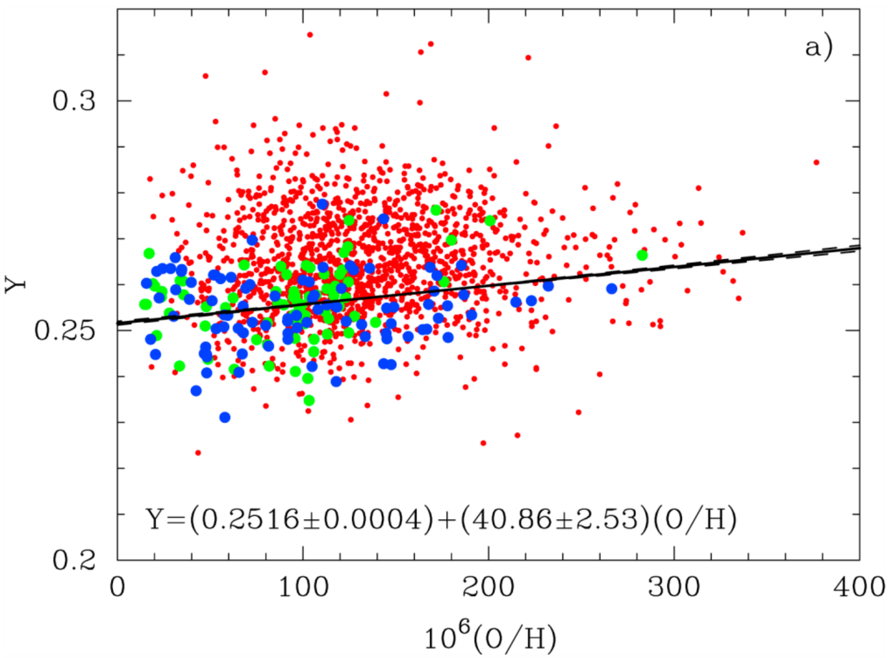
D abundance from Cook et al. (2013) and He-4 from Izotov et al. (2013)

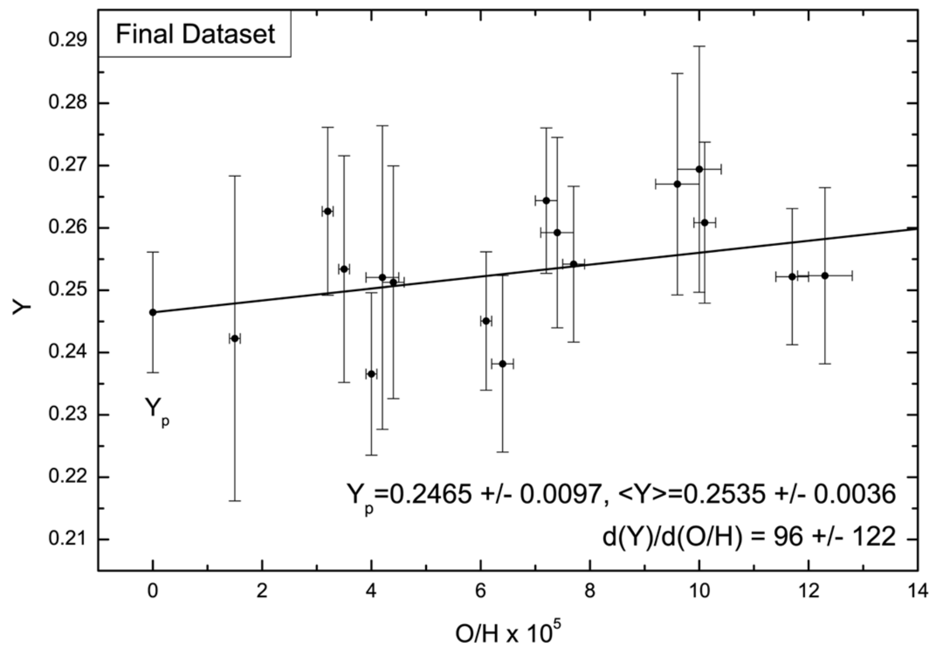
BBN hint for extra radiation (evidence driven by He abundance)

Helium Mass Fraction from HII Regions

Extrapolation to zero metalicity in many HII regions

Izotov, Stasinska & Guseva arXiv:1308.2100	Aver, Olive, Porter & Skillman arXiv:1309.0047



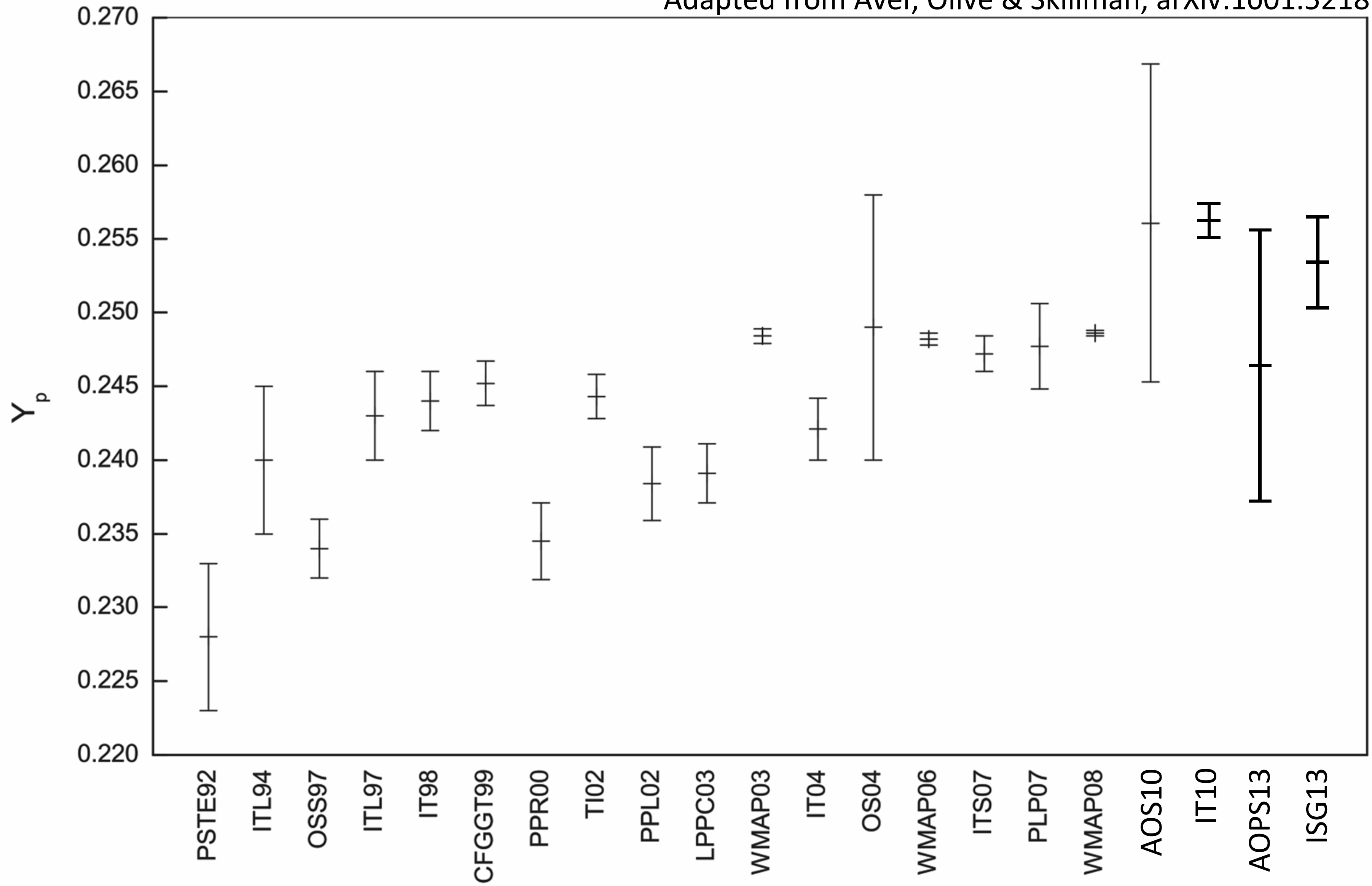


$Y_P = 0.254 \pm 0.003$

$Y_P = 0.2465 \pm 0.0097$

Progression of Best-Fit Helium Abundance

Adapted from Aver, Olive & Skillman, arXiv:1001.5218



Thermal Neutrino Distribution

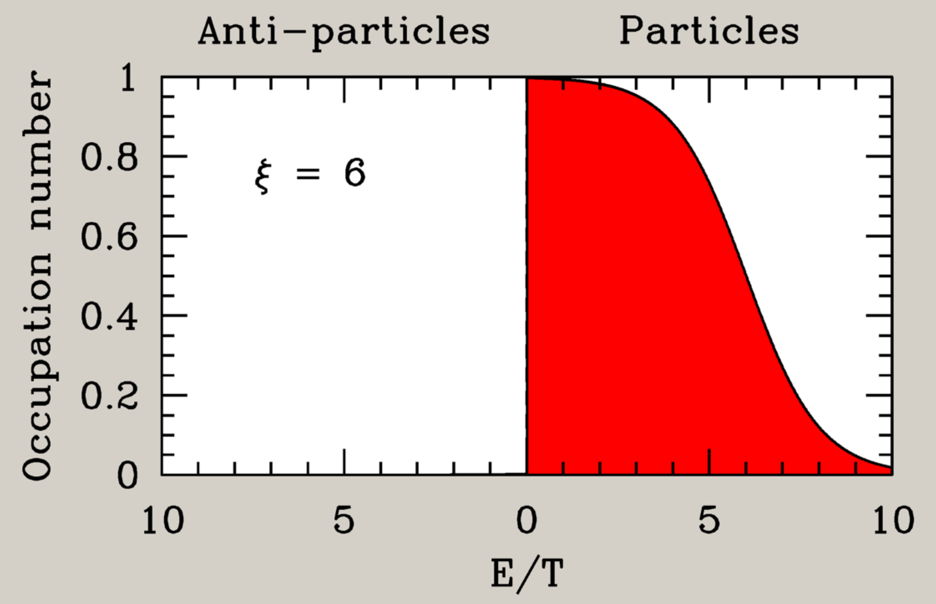
Fermi-Dirac distribution

- Temperature T
- Chemical potential μ

  $\mu > 0$ Particles

  $\mu < 0$ Anti-particles

$$f_{\boldsymbol{p}} = \frac{1}{e^{(E_{\boldsymbol{p}}-\mu)/T} + 1}$$



Degeneracy parameter $\xi = \frac{\mu}{T}$ Invariant under cosmic expansion

Difference in number density

$$n_\nu - n_{\bar{\nu}} = \int dE \, \frac{4\pi}{(2\pi)^3} \left(\frac{E^2}{1 + e^{E/T - \xi}} - \frac{E^2}{1 + e^{E/T + \xi}} \right)$$

$$= \frac{1}{6\pi^2} T^3 [\pi^2 \xi + \xi^3 + \cdots]$$

Thermal Neutrino Distribution

Fermi-Dirac distribution

- Temperature T
- Chemical potential μ

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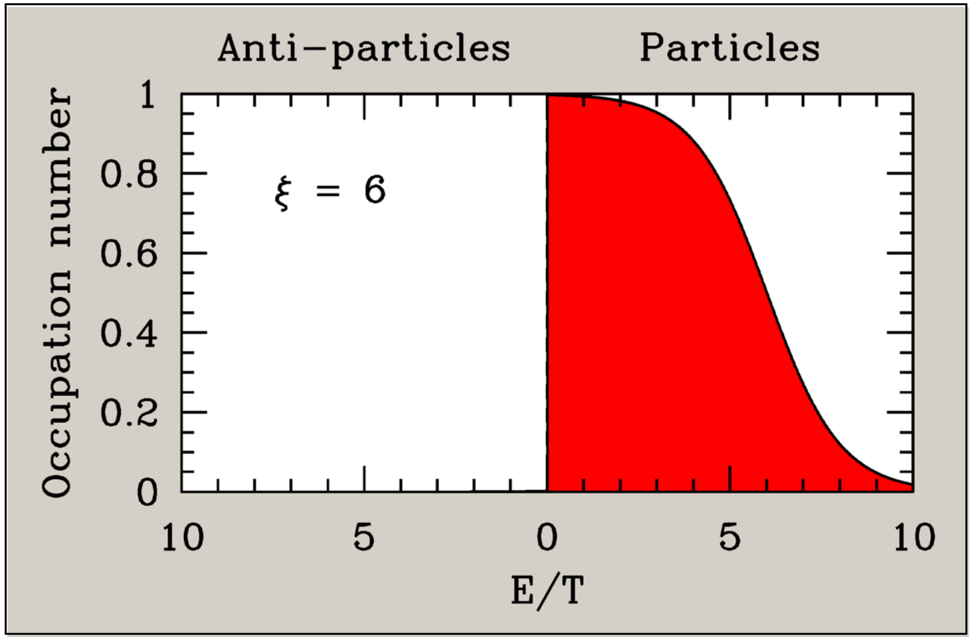
$$f_{\mathbf{p}} = \frac{1}{e^{(E_{\mathbf{p}}-\mu)/T} + 1}$$



Degeneracy parameter $\xi = \frac{\mu}{T}$ Invariant under cosmic expansion

Number density

$$n_{\nu\bar{\nu}} = \int dE \, \frac{4\pi}{(2\pi)^3} \left(\frac{E^2}{1 + e^{E/T-\xi}} + \frac{E^2}{1 + e^{E/T+\xi}} \right)$$

$$= \frac{3}{2\pi^2} T^3 \left[\zeta_3 + \frac{2\ln(2)}{3} \xi^2 + \frac{\xi^4}{72} + \cdots \right]$$

BBN and Neutrino Chemical Potentials

Expansion rate effect (all flavors)	Energy density in one neutrino flavor with degeneracy parameter $\xi = \eta/T$ $$\rho_{\nu\bar{\nu}} = \frac{7\pi^2}{120} T_\nu^4 \left[1 + \underbrace{\frac{30}{7}\left(\frac{\xi}{\pi}\right)^2 + \frac{15}{7}\left(\frac{\xi}{\pi}\right)^4}_{\Delta N_{\rm eff}}\right]$$
Beta equilibrium effect for electron flavor $n + \nu_e \leftrightarrow p + e^-$ $p + \bar{\nu}_e \leftrightarrow n + e^+$	Helium abundance essentially fixed by n/p ratio at beta freeze-out $$\frac{n}{p} = e^{-(m_n - m_p)/T_{\rm F} - \xi_{\nu_e}}$$ Effect on helium equivalent to $$\Delta N_{\rm eff} \sim -18\,\xi_{\nu_e}$$

- ν_e beta effect can compensate expansion-rate effect of $\nu_{\mu,\tau}$
- Naively, BBN limit only applies to ξ_{ν_e}
- However, flavor oscillations equalize chemical potentials before BBN

Chemical Potentials and Flavour Oscillations

Neutrino oscillations

↓

Flavor lepton numbers not conserved

↓

Only one common neutrino chemical potential

↓

Stringent ξ_{ν_e} limit applies to all flavors

$|\xi_{\nu_{e,\mu,\tau}}| < 0.07$

↓

Extra neutrino density $\Delta N_{eff} < 0.0064$

↓

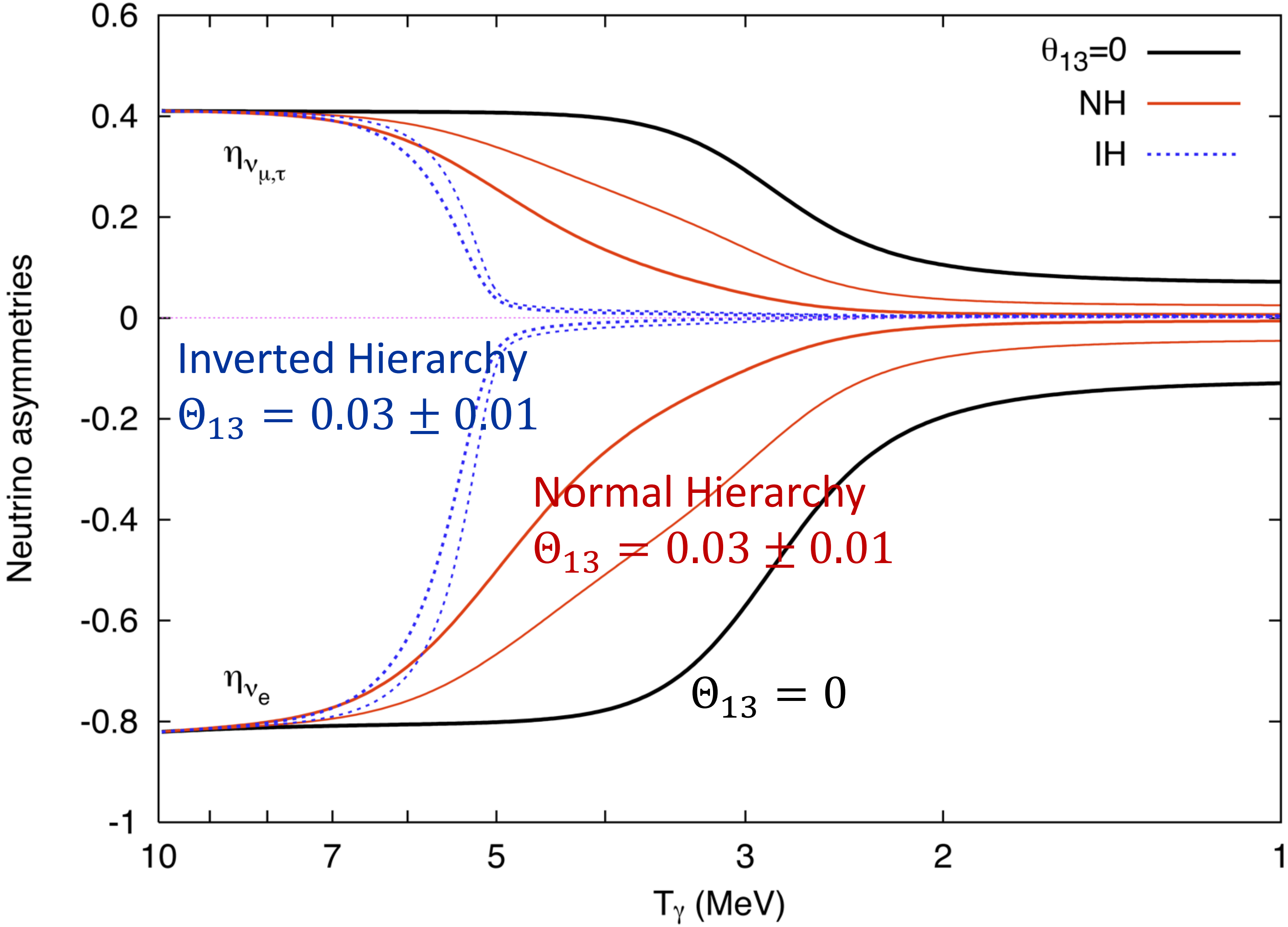
Cosmic neutrino density close to standard value

Flavour equilibrium before n/p freeze out assured because no mixing angle small

Our knowledge of the cosmic neutrino density depends on measured oscillation parameters!

arXiv:hep-ph/0012056 , hep-ph/0201287, astro-ph/0203442, hep-ph/0203180, arXiv:0808.3137, 1011.0916, 1110.4335

Flavor Conversion before BBN (Θ_{13} not small)



Mangano, Miele, Pastor, Pisanti & Sarikas, arXiv:1110.4335