The Standard Model

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Textbook material:

- An Introduction to Quantum Field Theory, Peskin&Schroeder
- Gauge Theories of the Strong, Weak, Electromagnetic Interactions, C. Quigg

Overview of the SM

Lecture I: the SM tapestry

- Particles as Quantum Fields
- Particle zoo vs symmetry
- Gauge invariance and particle interactions
- The origin of mass: Spontaneous Symmetry breaking
- The flavour of the SM

Lecture II: the SM swiss watch

- Observables and field correlation functions
- How we calculate ? Perturbation Theory and beyond
- Precision tests of the SM (LEP-TEVATRON-B factories)

Lecture III: the open-ended SM

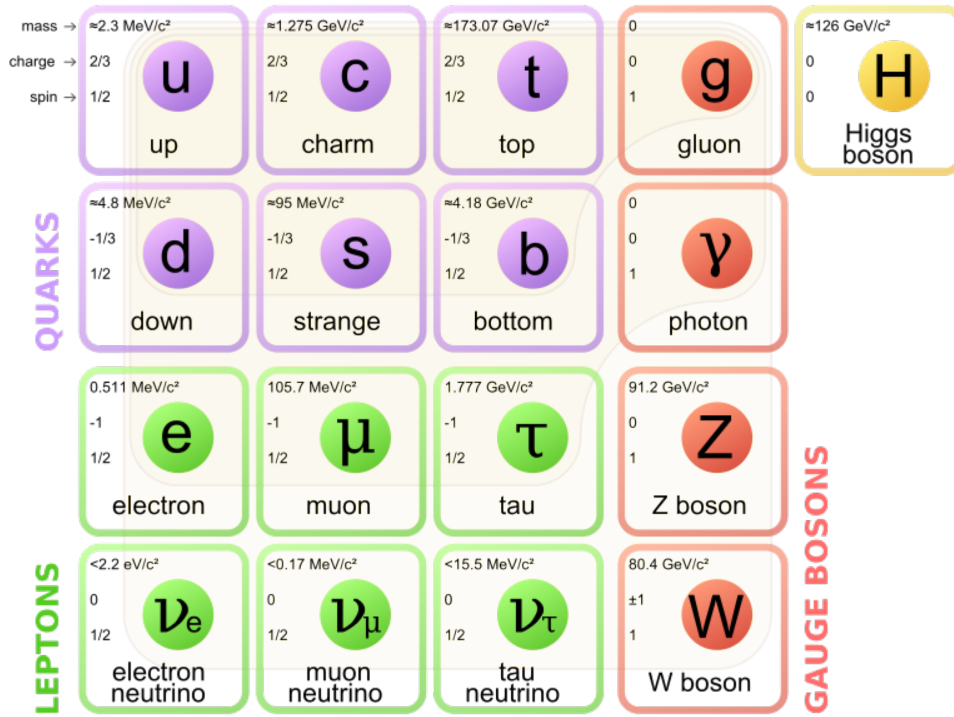
- The SM at the LHC: Higgs physics
- Open questions

Lecture I: the SM tapestry

"The hunt of the Unicorn"

The Standard Model (SM)

Describes the dynamics of elementary particles in the quantum/ relativistic domain: key in understanding atomic physics, nuclear physics, astrophysics…

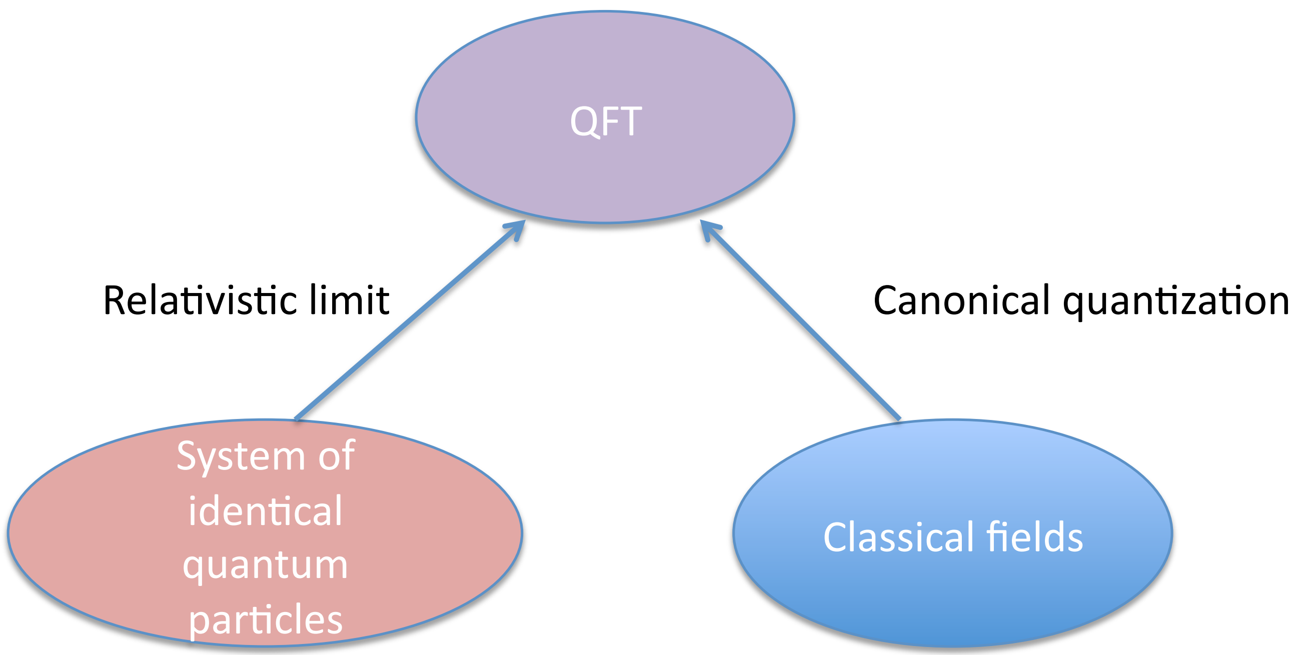


Elementary as far as we know….

SM is a Quantum Field Theory

Unifying picture of the concepts of

Interactions (fields): electromagnetic,… strong, weak
Matter (particles): electron, proton, neutron,…, neutrinos, muons, hadrons, quarks, …



QFT in a nutshell

Elementary particles or interactions represented by (complex) causal quantum fields (operators in Fock space):

$$\phi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\mathbf{p}}}} \left(u_s^a(\mathbf{p}) \hat{a}^s_{\mathbf{p},a} e^{-ipx} + v_s^a(\mathbf{p}) \hat{b}^{s\dagger}_{\mathbf{p},a} e^{ipx} \right) \Big|_{p^0 = E_{\mathbf{p}} \equiv \sqrt{\mathbf{p}^2 + m^2}}$$

a : flavour index

s : spin index

One (anti)particle states:

$$\begin{cases} |\mathbf{p}, s, a\rangle_+ = \hat{a}^{s\dagger}_{\mathbf{p},a} |0\rangle \\ |\mathbf{p}, s, a\rangle_- = \hat{b}^{s\dagger}_{\mathbf{p},a} |0\rangle \end{cases}$$

$u_s^a(\mathbf{p}), v_s^a(\mathbf{p})$ wave functions in spin and internal space

QFT in a nutshell

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Quantum relativistic particles (second quantization) → Fock Space: field operator creates/destroys a particle at x

=

Quantum fields (canonical quantization) → Harmonic osc.: a,a⁺ ladder operators create/destroy a quantum of energy

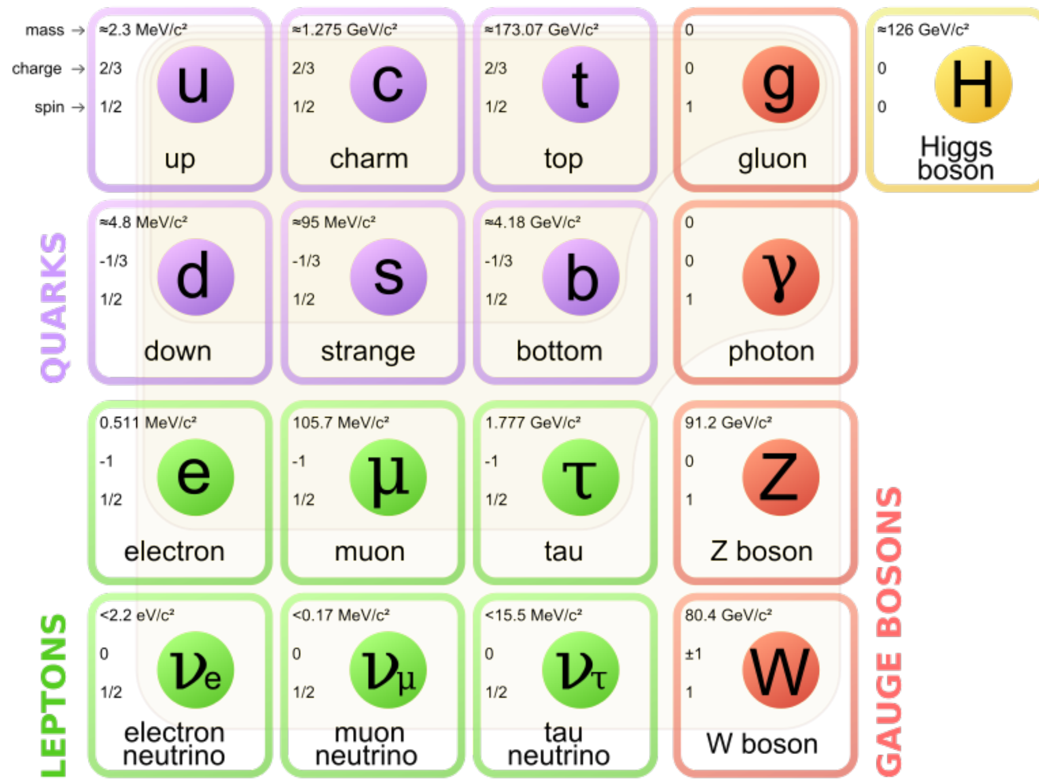
For each **p**, s, a, one quantum harmonic oscillator

$$E_N(\mathbf{p}, s, a) = (N + 1/2) E_{\mathbf{p}} \leftrightarrow \text{N particle state}$$

Particle zoo vs. symmetries

Symmetries are the underlying principle behind this structure:

- How many particles are there and number of degrees of freedom
- How particles interact



Particle zoo vs. symmetries

- Symmetries seem to be a fundamental principle of Nature
- Groups are the mathematical representation of symmetries
- Representation of symmetry groups tell us about the structure of Nature (how many dofs are there ?)

Elementary particles: vectors on which symmetry transformations can act (irreducibly, without leaving invariant subspaces)

Space-time symmetries: Lorentz group SO(1,3)
Discrete symmetries: eg. Parity
Internal symmetries: SU(2) isospin, SU(3) color

Lorentz group: index s

Elementary particles are irreducible representations of the group of rotations (spin)

$$j = \mathbb{Z}/2 \qquad dim(j) = 2j + 1$$

+ Boosts: (j_1, j_2) $dim(j_1, j_2) = (2j_1 + 1)(2j_2 + 1)$

+ Parity: $(j_1, j_2) \to^{P} (j_2, j_1)$

Rep.	Field	dim.	Spin	Parity
(0,0)	H	1	0	Yes
$(\frac{1}{2},0)$	q_R, l_R	2	$\frac{1}{2}$	No
$(0,\frac{1}{2})$	q_L, l_L	2	$\frac{1}{2}$	No
$(\frac{1}{2},\frac{1}{2})$	$A_\mu, G_\mu, W^{\pm}_\mu, Z_\mu$	4	1	Yes

$$\Psi_{L/R} \equiv P_{L/R}\Psi$$

$$P_{L/R} \equiv \frac{1 \mp \gamma_5}{2}$$

Causal relativistic free fields

Weyl fermion: $$\mathcal{L} = \bar{\Psi}_L (i\gamma_\mu \partial_\mu) \Psi_L$$

Two-component spinor: particle with negative helicity, antiparticle with negative one

Dirac fermion: $$\mathcal{L} = (\bar{\Psi}_L + \bar{\Psi}_R)(i\gamma_\mu \partial_\mu - m)(\Psi_L + \Psi_R)$$

Four-component spinor: particle and antiparticle with both helicities

Vector boson: $$\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \frac{m^2}{2} A_\mu^2$$

Massless: two polarizations, Massive: three polarizations

Internal Symmetries vs. Interactions

Two types of internal symmetries:

global: transforms in the same way fields at all space-time points

local: transform independently fields at each space-time point

Local symmetries imply the existence of some fields and dictate how elementary particles interact !

Gauge Symmetry: U(1)

Maxwell eqs. in terms of gauge potentials are invariant under

$$A_\mu(x) \to A_\mu(x) + \partial_\mu \alpha(x)$$

Eqs. of motion for any charged field will also remain invariant if

$$\Psi(x) \to e^{iq\alpha(x)}\Psi(x)$$

e.m. gauge invariance <-> U(1) local gauge transformation

$$D_\mu \Psi \equiv (\partial_\mu - iqA_\mu)\Psi \to e^{iq\alpha} D_\mu \Psi$$

U(1) invariants:

$$\bar{\Psi}\gamma_\mu D_\mu \Psi \qquad \bar{\Psi}\Psi \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Gauge Symmetry: U(1)

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}^2 + \bar{\Psi}(i\not{D} - m)\Psi$$

$$D_\mu = \partial_\mu - iqA_\mu$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

A mass term $-\frac{m_A^2}{2}A_\mu^2$ breaks symmetry: photon massless

Gauge Symmetry: SU(N)

Gauge transformation belongs to the special unitary group

$\Psi^{i=1,\ldots,N}$ fundamental rep.

$$\Psi(x) \to \Omega(x)\Psi(x)$$

$\Omega(x) \to \mathrm{N} \times \mathrm{N}$ unitary matrix

$$\Omega(x) = \exp\left(i\alpha_a T^a\right)$$

$$T^{a\dagger} = T^a \leftrightarrow \text{generators}, a = 1, \ldots, N^2 - 1$$

(For N=2 $\quad T^a = \frac{\sigma_a}{2}$)

$A_\mu^{a=1,..,N^2-1}$ Adjoint rep.

$$gA_\mu^a T^a \to \Omega(x) g A_\mu^a T^a \Omega^\dagger(x) + i\Omega(x)\partial_\mu \Omega^\dagger(x)$$

Gauge Symmetry SU(N)

$$D_\mu = \partial_\mu - igA_\mu^a T^a \qquad F_{\mu\nu}^a T^a = \frac{i}{g}[D_\mu, D_\nu]$$

$$D_\mu \Psi \to \Omega \; D_\mu \Psi \qquad F_{\mu\nu}^a T^a \to \Omega F_{\mu\nu}^a T^a \Omega^\dagger$$

$$\mathcal{L}_{SU(N)} = -\frac{1}{4}(F_{\mu\nu}^a)^2 + \bar{\Psi}\,(i\,\not{D} - m)\,\Psi$$

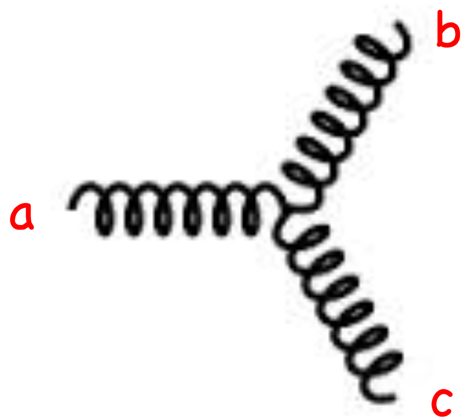
A mass term $-\frac{1}{2}m_A^2 A_\mu^a$ would also break the gauge symmetry

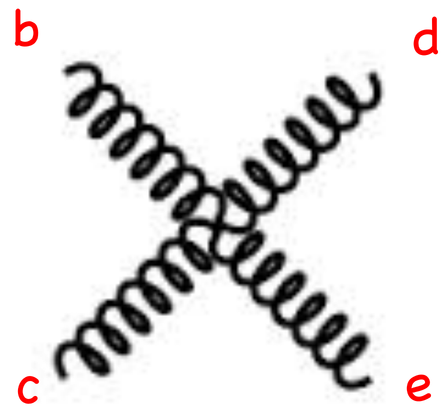
This is a gauge invariant Lagrangian,
but is it the only one ?

Gauge Symmetry SU(N)

$$\mathcal{L}_{SU(N)} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \bar{\Psi}\,(i\,\not{D} - m)\,\Psi$$

Mediators self-interactions:





$\propto \mathrm{Tr}[T^a[T^b, T^c]]$

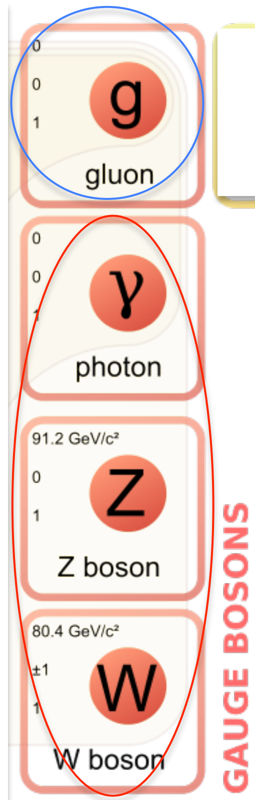
$\propto \mathrm{Tr}[T^a[T^b, T^c]]\mathrm{Tr}[T^a[T^d, T^e]]$

SM is a gauge theory

$$SU(3) \times SU(2) \times U(1)_Y$$

$$SU(3)$$

$$SU(2) \times U(1)_Y$$

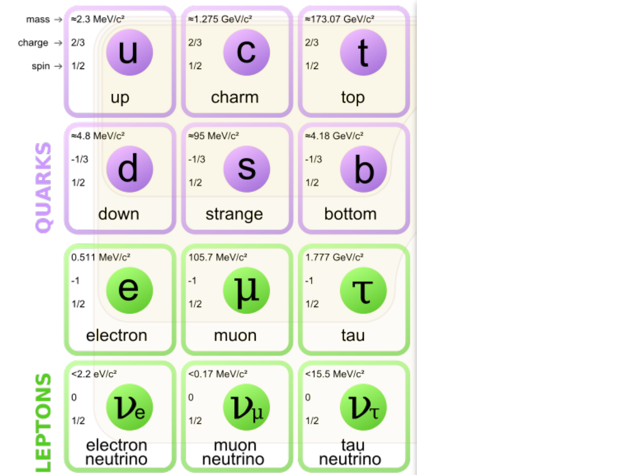


$$G_\mu^{a=1,..,8}$$

$$W_{\mu\nu}^{a=1,2,3}, B_\mu$$

SM gauge group

$$SU(3) \times SU(2) \times U(1)_Y$$



$$\Psi_{L/R} \equiv P_{L/R}\Psi$$

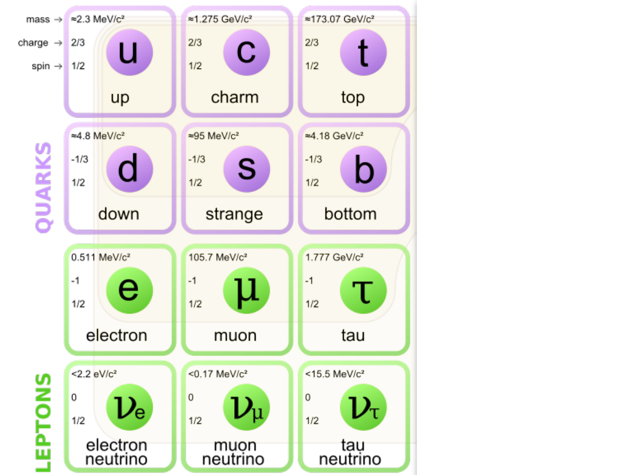
$(1,2)_{-\frac{1}{2}}$	$(3,2)_{\frac{1}{6}}$	$(1,1)_{-1}$	$(3,1)_{\frac{2}{3}}$	$(3,1)_{-\frac{1}{3}}$
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} u^i \\ d^i \end{pmatrix}_L$	e_R	u^i_R	d^i_R
$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} c^i \\ s^i \end{pmatrix}_L$	μ_R	c^i_R	s^i_R
$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$\begin{pmatrix} t^i \\ b^i \end{pmatrix}_L$	τ_R	t^i_R	b^i_R

Flavour/ family

Parity (~helicity) conjugate

SM gauge group

$$SU(3) \times SU(2) \times U(1)_Y$$



$$\Psi_{L/R} \equiv P_{L/R}\Psi$$

$(1,2)_{-\frac{1}{2}}$	$(3,2)_{\frac{1}{6}}$	$(1,1)_{-1}$	$(3,1)_{\frac{2}{3}}$	$(3,1)_{-\frac{1}{3}}$
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} u^i \\ d^i \end{pmatrix}_L$ (u^i: u1, u2, u3)	e_R	u^i_R (u1, u2, u3)	d^i_R
$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} c^i \\ s^i \end{pmatrix}_L$	μ_R	c^i_R	s^i_R
$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$\begin{pmatrix} t^i \\ b^i \end{pmatrix}_L$	τ_R	t^i_R	b^i_R

Flavour/ family

Parity (~helicity) conjugate

The puzzle in the 60's

- Particles with different names in the same gauge SU(2) multiplet
- Parity violation: L, R different charges, but fermions massive
- Three of the gauge fields not massless
- Weak interactions mix quark generations

The SU(2)xU(1) symmetry is hidden

Realization of continuous symmetries

Consider a field theory invariant under some symmetry group $[H, U] = 0$

Under infinitesimal transformation a field in a representation of the group with generators T^a:

$$\phi \to (I + i\epsilon^a T^a)\phi$$

Weyl-Wigner: $U|0\rangle = |0\rangle \quad \Rightarrow T^a \langle 0|\phi|0\rangle = 0$

And all the states in the multiplet have the same energy

for some a

Nambu-Goldstone $T^a \langle 0|\phi|0\rangle \neq 0 \quad \Rightarrow U|0\rangle \neq |0\rangle$

Goldstone theorem: as many massless modes as broken generators

Spontaneous Symmetry Breaking

If this is how the SU(2) is broken
where did the massless fields go ?

An important intuition came from the Meissner effect in superconductors

The photon becomes massive inside a superconductor due to the existence of a Cooper pair condensate which has e.m. charge

Anderson

SSB of Gauge symmetry

The effective field theory of superconductivity: a complex scalar field with charge q coupled to the U(1) gauge field

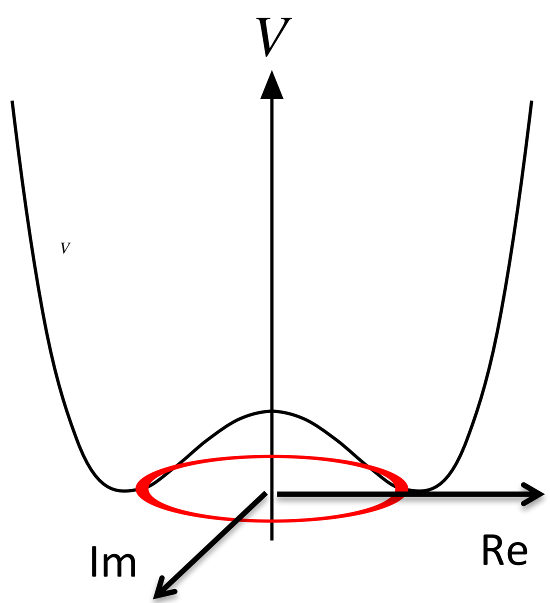
$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^\dagger D_\mu\phi - V(\phi) \qquad D_\mu = \partial_\mu - iqA_\mu$$

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

$$\lambda > 0$$

Lowest energy configuration (vacuum):

$$\phi^\dagger\phi = \frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2} \qquad \phi = \frac{v}{\sqrt{2}}e^{i\theta}$$



SSB of Gauge symmetry

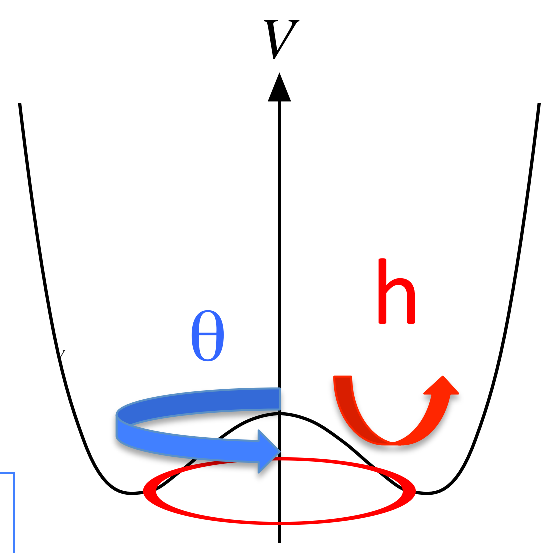
Perturbing around the true vacuum $$\phi = \frac{v + h(x)}{\sqrt{2}} e^{i\theta(x)}$$

$$\mathcal{L}(\phi, A_\mu) = -\tfrac{1}{4} F_{\mu\nu} F_{\mu\nu}$$

$$+ \quad \tfrac{1}{2}\partial_\mu h \partial_\mu h + \tfrac{v^2}{2}\partial_\mu \theta \partial_\mu \theta - q v^2 \partial_\mu \theta A_\mu + \tfrac{q^2 v^2}{2} A_\mu A_\mu$$

$$- \quad \mu^2 h^2 - \lambda v h^3 - \tfrac{\lambda}{4} h^4$$

θ is a Goldstone Boson



GB:field with only derivative couplings

The ABEGH^{2}KN Mechanism

Anderson-Brout-Englert-Guralnik-Hagen-Higgs-Kibble-Nambu

$$\mathcal{L}^{(2)} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{q^2v^2}{2}A_\mu A_\mu + \frac{1}{2}(\partial_\mu h\, \partial_\mu h - \mu^2 h^2) + \frac{v^2}{2}\partial_\mu\theta\, \partial_\mu\theta - qv^2\partial_\mu\theta A_\mu$$

Gauge transformation: $$A'_\mu = A_\mu - \frac{1}{q}\partial_\mu\theta$$

$$= -\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \frac{q^2v^2}{2}A'_\mu A'_\mu + \frac{1}{2}(\partial_\mu h\, \partial_\mu h - \mu^2 h^2)$$

Goldstone mode -> massive gauge field (ie. longitudinal polarization)

Goldstone mode "is eaten" by the gauge field to get massive: unitary gauge

Radial mode -> massive neutral scalar field

SM BEH mechanism

A complex doublet with quantum numbers Y=+1/2 and no color

$$\mathcal{L}_\phi = D_\mu \phi^\dagger D_\mu \phi - V(\phi)$$

$$D_\mu \phi = (\partial_\mu - igW_\mu^a \frac{\sigma^a}{2} - i\frac{g'}{2} B_\mu)\phi$$

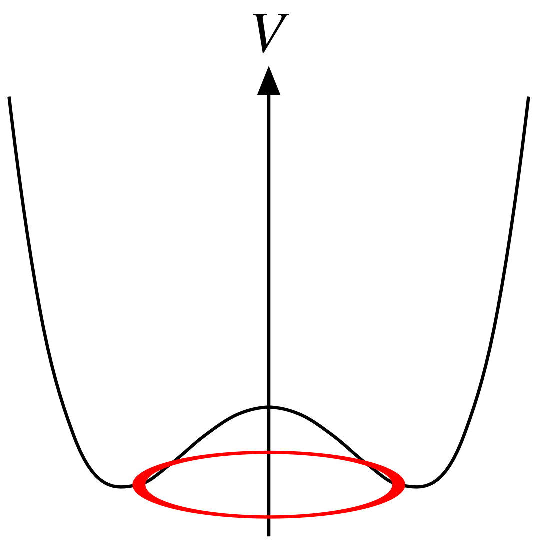
$$\phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_0 + i\phi_3 \end{pmatrix}$$

$$\phi \to e^{iT^a \alpha^a} \phi \quad T^a = \left(\frac{\sigma^0}{2}, \frac{\vec{\sigma}}{2}\right)$$

A potential with a minimum at

$$\langle \phi^\dagger \phi \rangle = \frac{v^2}{2} = \frac{\mu^2}{2\lambda}$$

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$



SM BEH mechanism

Of the full symmetry group: $\phi \to e^{iT^a\alpha^a}\phi \quad T^a = \left(\frac{\sigma^0}{2}, \frac{\vec{\sigma}}{2}\right)$

A U(1) subgroup remains unbroken

$$(T^0 + T^3)\langle\phi\rangle = 0$$

$$SU(2) \times U(1) \to U(1)_{em}$$

v

three broken generators: three massive gauge fields (W^{+-}, Z^0) and one massless photon

Gauge boson masses

$$\phi = e^{i\alpha^a(x)T^a} \frac{1}{\sqrt{2}} \left(\begin{array}{c} 0 \\ v + h(x) \end{array} \right)$$

Exercise: show that this is a general parametrization of the complex scalar field

Gauge boson masses

$$\phi = e^{i\alpha^a T^a} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

In unitary gauge:

$$D_\mu \phi^\dagger D_\mu \phi = \frac{1}{2}\partial_\mu h \, \partial_\mu h + \frac{1}{2}(0 \quad v+h) \left(g W_\mu^a \frac{\sigma^a}{2} + \frac{1}{2} g' B_\mu \right)^2 \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$= \frac{1}{2}\frac{v^2}{4}\left\{ g^2\left((W_\mu^1)^2 + (W_\mu^2)^2 \right) + (g' B_\mu - g W_\mu^3)^2 \right\} = m_W^2 W_\mu^+ W_\mu^- + \frac{1}{2} m_Z^2 Z_\mu Z_\mu + O(h)$$

Charged weak: $W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \pm i W_\mu^2 \right)$ $\qquad m_W \equiv g\frac{v}{2}$

Neutral weak $Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}}\left(g W_\mu^3 - g' B_\mu \right)$ $\qquad m_Z \equiv \sqrt{g^2 + g'^2}\frac{v}{2}$

Electromagnetic $A_\mu = \frac{1}{\sqrt{g^2 + g'^2}}\left(g' W_\mu^3 + g B_\mu \right)$

Question: why did we choose such normalization ?

Question: why did we choose such normalization ?

Exercise: check that the kinetic terms are properly normalized

Gauge Boson masses

Weak mixing angle

$$Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left(g W_\mu^3 - g' B_\mu \right)$$

$$A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left(g' W_\mu^3 + g B_\mu \right)$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

$$\cos\theta_W \equiv \frac{g}{\sqrt{g^2 + g'^2}} \quad \sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

v

$$m_W = m_Z \cos\theta_W$$

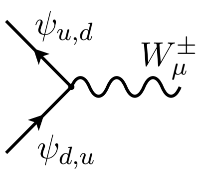
Neutral currents

All fermions are doublets $D_\mu \Psi = (\partial_\mu - igW_\mu^a \frac{\sigma^a}{2} - iY_\Psi g' B_\mu)\Psi$

$$D_\mu \Psi = \left(\partial_\mu - i\frac{g}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) - i\frac{g}{\cos\theta_W}(T^3 - \sin^2\theta_W Q_\Psi)Z_\mu - ieQ_\Psi A_\mu\right)\Psi$$

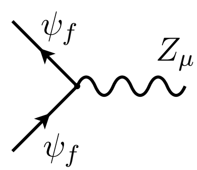
$$Q_\Psi \equiv T^3 + Y_\Psi \quad e \equiv \frac{gg'}{\sqrt{g^2 + g'^2}} = g\sin\theta_W$$

Off-diagonal in isospin!

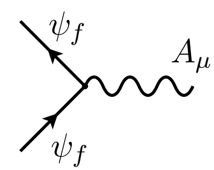


$$i\frac{g}{\sqrt{2}}\gamma_\mu \frac{1-\gamma_5}{2}$$

Diagonal in isospin!



$$i\frac{g}{\cos\theta_W}\gamma_\mu\left(g_V^f - g_A^f\gamma_5\right)$$



$$-ieQ_f\gamma_\mu$$

$$g_V^f = \frac{1}{2}T_f^3 - Q_f\sin^2\theta_W, \quad g_A^f = \frac{1}{2}T_f^3 .$$

Neutral currents

Predicts all fermion couplings to neutral currents in terms of their em charges:

$$Y_{l_L} = Q_e + \frac{1}{2} = Q_\nu - \frac{1}{2} = -\frac{1}{2}$$

$$Y_{q_L} = Q_d + \frac{1}{2} = Q_u - \frac{1}{2} = \frac{1}{6}$$

$$Y_{l_R} = Q_e = -1$$

$$Y_{u_R} = Q_u = \frac{2}{3}$$

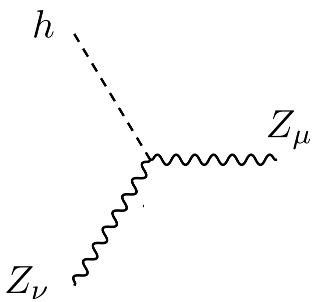
$$Y_{d_R} = Q_d = -\frac{1}{3}$$

If there were right-handed neutrinos they would have

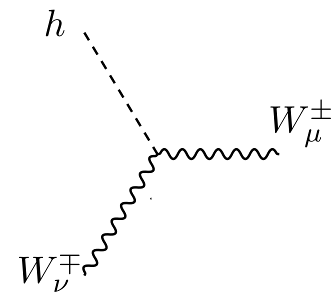
$$Y_{\nu_R} = Q_\nu = 0$$

Higgs-Gauge couplings

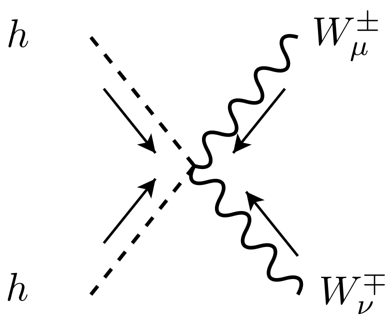
$$(1+h/v)^2(m_W^2 W_\mu^+ W_\mu^- + \frac{1}{2} m_Z^2 Z_\mu Z_\mu)$$



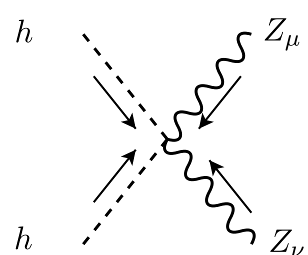
$$i\,\frac{g}{\cos\theta_W}\,m_Z\,g_{\mu\nu}$$



$$ig\,m_W\,g_{\mu\nu}$$



$$\frac{i}{2}\,g^2\,g_{\mu\nu}$$



$$\frac{i}{2}\frac{g^2}{\cos^2\theta_W}g_{\mu\nu}$$

Fermion masses

Dirac fermion of mass m:

$$-\mathcal{L}_m^{\rm Dirac} = m\bar\psi\psi = m(\overline{\psi_L+\psi_R})(\psi_L+\psi_R) = m(\overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L)$$

Breaks SU(2)xU(1) gauge invariance!

But we can have other invariants with the conjugated scalar doublet:

$$\tilde\phi \equiv \sigma^2\phi^*, \quad \tilde\phi : (1,2,-\frac{1}{2}), \quad \langle\tilde\phi\rangle = \begin{pmatrix} \frac{v}{\sqrt{2}} \\ 0 \end{pmatrix}$$

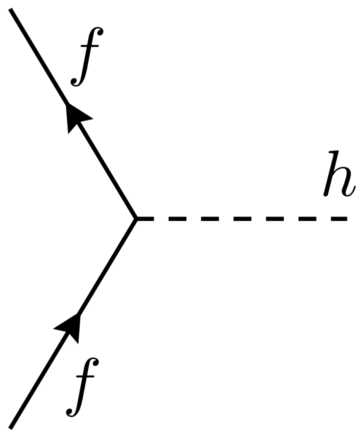
$$\mathcal{L}_{SM} \supset -Y_d\bar q_L\phi d_R - Y_u\bar q_L\tilde\phi u_R - Y_l\bar l_L\phi l_R$$
$$\to -m_d\bar d_L d_R - m_u\bar u_L u_R - m_l\bar l_l l_R + O(h)$$

Exercise: check that the charge assignment of the tilde field is correct

$$\tilde{\phi} \equiv \sigma^2 \phi^*, \quad \tilde{\phi} : (1, 2, -\frac{1}{2}), \quad \langle \tilde{\phi} \rangle = \begin{pmatrix} \frac{v}{\sqrt{2}} \\ 0 \end{pmatrix}$$

Higgs-fermion couplings

$$(1 + h/v)\left(-m_d \bar{d}_L d_R - m_u \bar{u}_L u_R - m_l \bar{l}_L l_R\right) + h.c.$$



$$-i \, \frac{g}{2} \, \frac{m_f}{m_W}$$

Flavour mixing

No mixing between different families as it stands...but it turns out there are three families, why can't these Yukawa interactions mix families ?

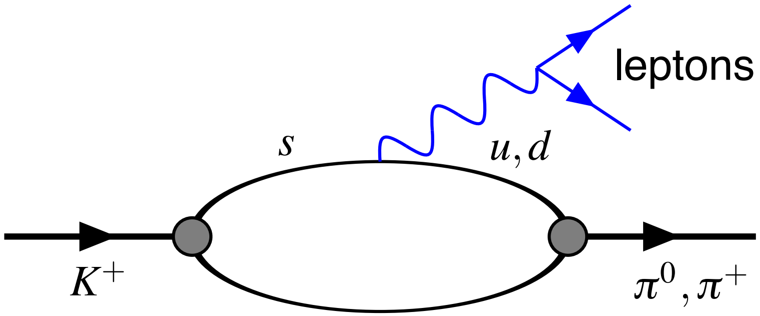
Exercise: show that in the absence of Yukawa couplings, the Lagrangian has a flavour/family symmetry:

$$U(3)_{q_L} \times U(3)_{l_L} \times U(3)_{u_R} \times U(3)_{d_R} \times U(3)_{l_R}$$

Quark mixing

There is flavour changing in charged currents: s -> u, but very suppressed in neutral currents

$$Br(K^+ \to \pi^0 e^+ \nu_e) \simeq 5\% \qquad Br(K^+ \to \pi^+ e^+ e^-) \simeq 3 \times 10^{-7}$$



How to explain mixing in CC without that in NC ?

Glashow-Illiopoulos-Maiani mechanism

Quark mixing: Cabbibo-Kobayashi-Maskawa

The Yukawa couplings are generic matrices in flavour space:

basis where CC and NC diagonal ≠ mass eigenbasis

$$\mathcal{L}_{SM} \supset -(\bar{d}_L, \bar{s}_L, \bar{b}_L) \underbrace{m_d}_{3\times 3} \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} - (\bar{u}_L, \bar{c}_L, \bar{t}_L) \underbrace{m_u}_{3\times 3} \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} - (\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L) \underbrace{m_u}_{3\times 3} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$$

$$m_i = U^\dagger_{Li} \mathrm{Diag}(m_i) V_{Ri}$$

$$u'_L = U_{Lu} u_L, \quad d'_L = U_{Ld} d_L, \quad l'_L = U_{Ll} l_L, \quad u'_R = V_{Ru} u_R, \quad d'_R = V_{Rd} d_R, \quad l'_R = V_{Rl} l_R$$

$$\mathcal{L}^{CC}_{SM} \supset -\frac{g}{\sqrt{2}} \bar{u}'_{Li} \underbrace{(U_{Lu} U^\dagger_{Ld})_{ij}}_{CKM} \gamma_\mu W^+_\mu d'_{Lj} + h.c.$$

Quark mixing: Cabbibo-Kobayashi-Maskawa

The Yukawa couplings are generic matrices in flavour space:

basis where CC and NC diagonal ≠ mass eigenbasis

$$\mathcal{L}_{SM} \supset -(\bar{d}_L, \bar{s}_L, \bar{b}_L) \underbrace{m_d}_{3\times 3} \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} - (\bar{u}_L, \bar{c}_L, \bar{t}_L) \underbrace{m_u}_{3\times 3} \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} - (\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L) \underbrace{m_u}_{3\times 3} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$$

$$m_i = U^\dagger_{Li} \mathrm{Diag}(m_i) V_{Ri}$$

$$u'_L = U_{Lu} u_L, \quad d'_L = U_{Ld} d_L, \quad l'_L = U_{Ll} l_L, \quad u'_R = V_{Ru} u_R, \quad d'_R = V_{Rd} d_R, \quad l'_R = V_{Rl} l_R$$

$$\mathcal{L}^{NC}_{SM} \supset -\frac{g}{\cos\theta_W} \bar{d}'_{Li} \left(-\frac{1}{2} + \frac{1}{3} \sin^2\theta_W \right) \underbrace{(U_{Ld} U^\dagger_{Ld})_{ij}}_{\delta_{ij}} \gamma_\mu Z_\mu d'_{Lj}$$

$$-\frac{g}{\cos\theta_W} \bar{u}'_{Li} \left(\frac{1}{2} - \frac{2}{3} \sin^2\theta_W \right) \underbrace{(U_{Lu} U^\dagger_{Lu})_{ij}}_{\delta_{ij}} \gamma_\mu Z_\mu u'_{Lj}$$

Quark mixing: Cabbibo-Kobayashi-Maskawa

Neutral currents diagonal also in the mass eigenbasis: only quarks in the same family can exchange a Z boson

It was quite of a challenge to come up with this when only 1.5 quark family was known: u, d, s -> prediction of the charm !

Charged currents not diagonal: CKM 3x3 unitary matrix

$$|V|_{\mathrm{CKM}} = \begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.0065 & (3.51 \pm 0.15) \times 10^{-3} \\ 0.2252 \pm 0.00065 & 0.97344 \pm 0.00016 & (41.2^{+1.1}_{-5}) \times 10^{-3} \\ (8.67^{+0.29}_{-0.31}) \times 10^{-3} & (40.4^{+1.1}_{-0.5}) \times 10^{-3} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix}$$

PDG

CKM Parametrization

Not all entries are independent: how many physical parameters are there ?

3 Euler angles and 1 complex phase : s_{12} <-> Cabbibo angle

$$V_{CKM} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix}.$$

Since $s_{12} >> s_{23} >> s_{13}$: Wolfenstein parametrization

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Counting parameters

physical parameters = # parameters in Yukawas
- # parameters in field redefinitions
+ # parameters of exact symmetries

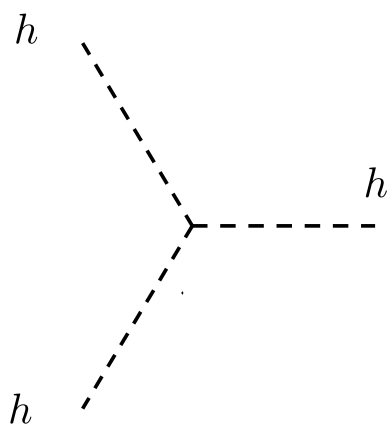
		Field Redef.	Symmetries	Physical
	Y_u , Y_d	$U_{qL}(n) x U_{dR}(n) x U_{uR}(n)$	$U(1)_B$	
Moduli	2×3^2	3 x 3	0	9
Phases	2×3^2	3 x 6	1	1

Moduli = 9 = 6 masses + 3 angles

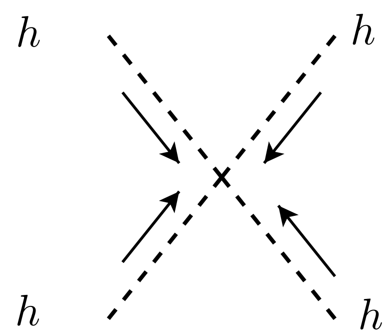
Exercise: repeat the counting including the lepton Yukawa
can there be mixing in the lepton sector ?

Higgs self-couplings

From the Higgs potential:



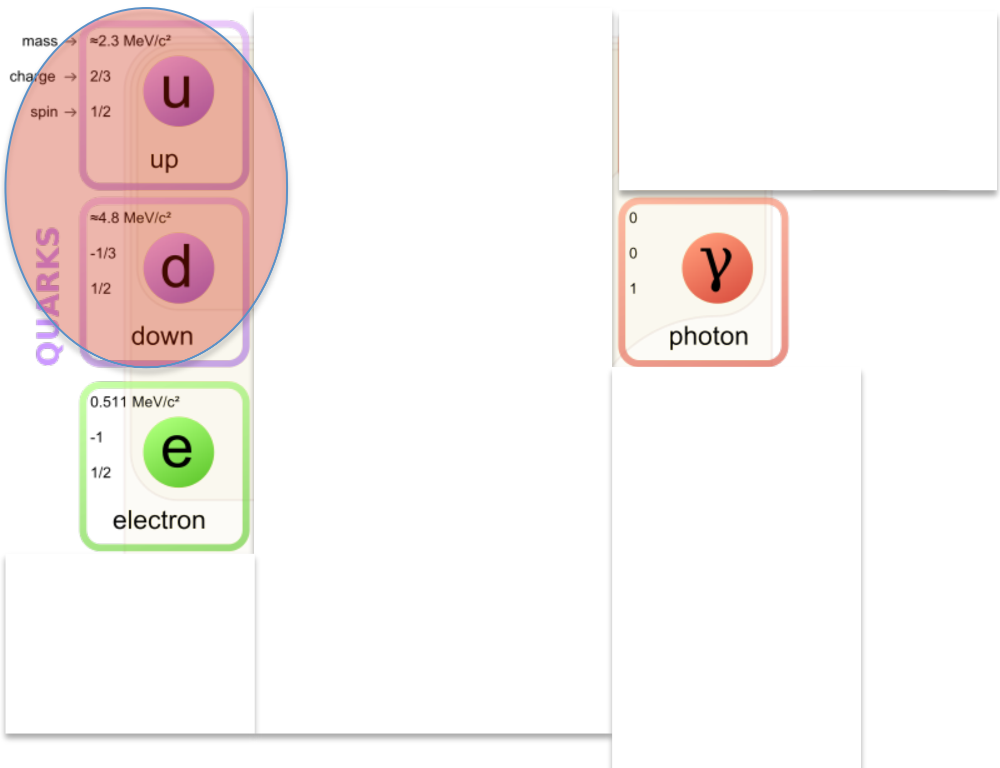
$$-\frac{3}{2}\, i\, g \frac{m_h^2}{m_W^2}$$



$$-\frac{3}{4}\, i\, g^2 \frac{m_h^2}{m_W^2}$$

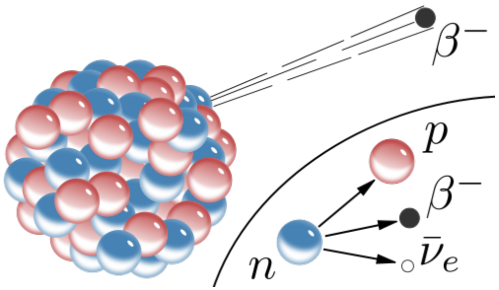
“Threads in a tapestry”…

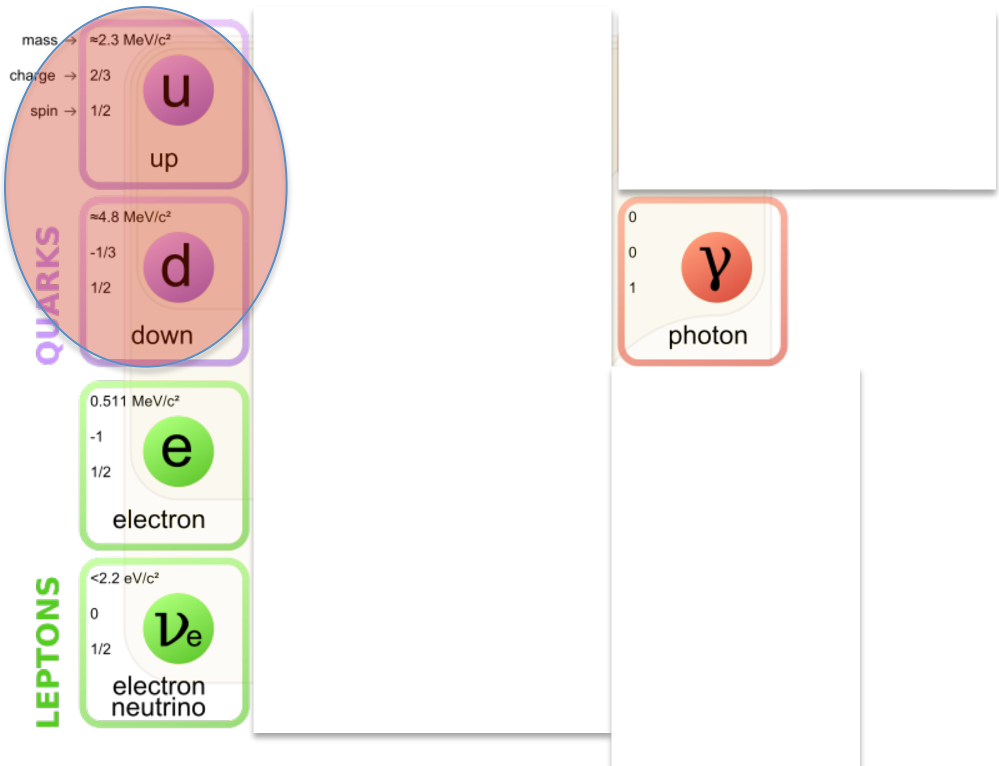
several Nobel-prize winning milestones



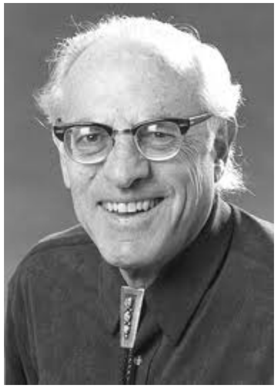
At the beginning there were just electrons, nuclei, electromagnetism (and gravity)...

1930 Neutrinos "showed up"



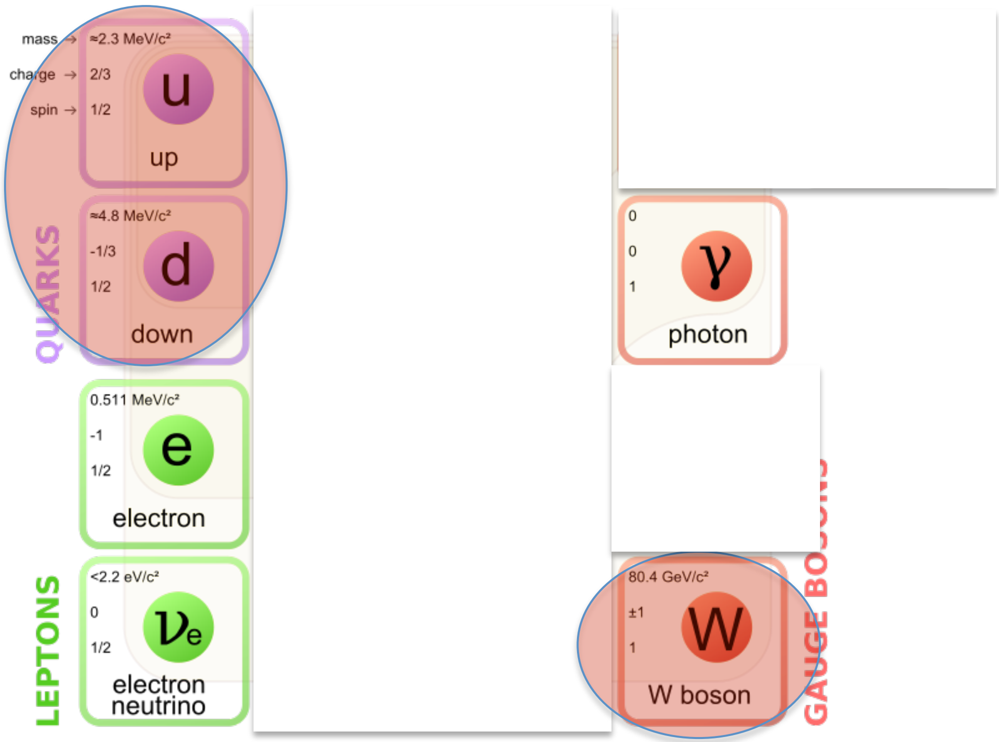


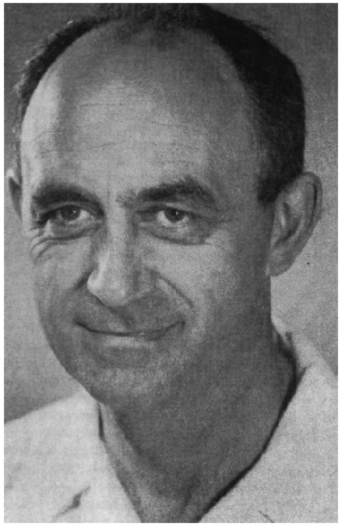
Pauli (Nobel 1945)



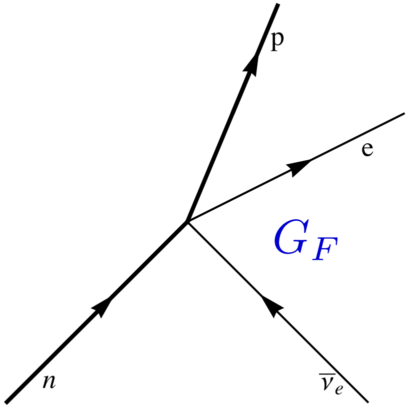
Reines (Nobel 1995)

1934: Theory of beta decay

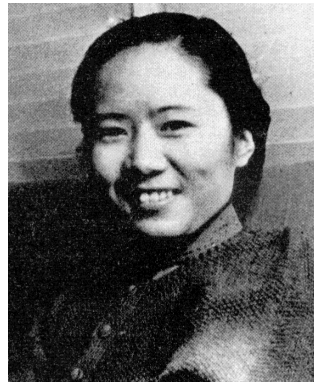




E. Fermi
(Nobel 1938)



1956: Parity violation in β decay



C-S Wu

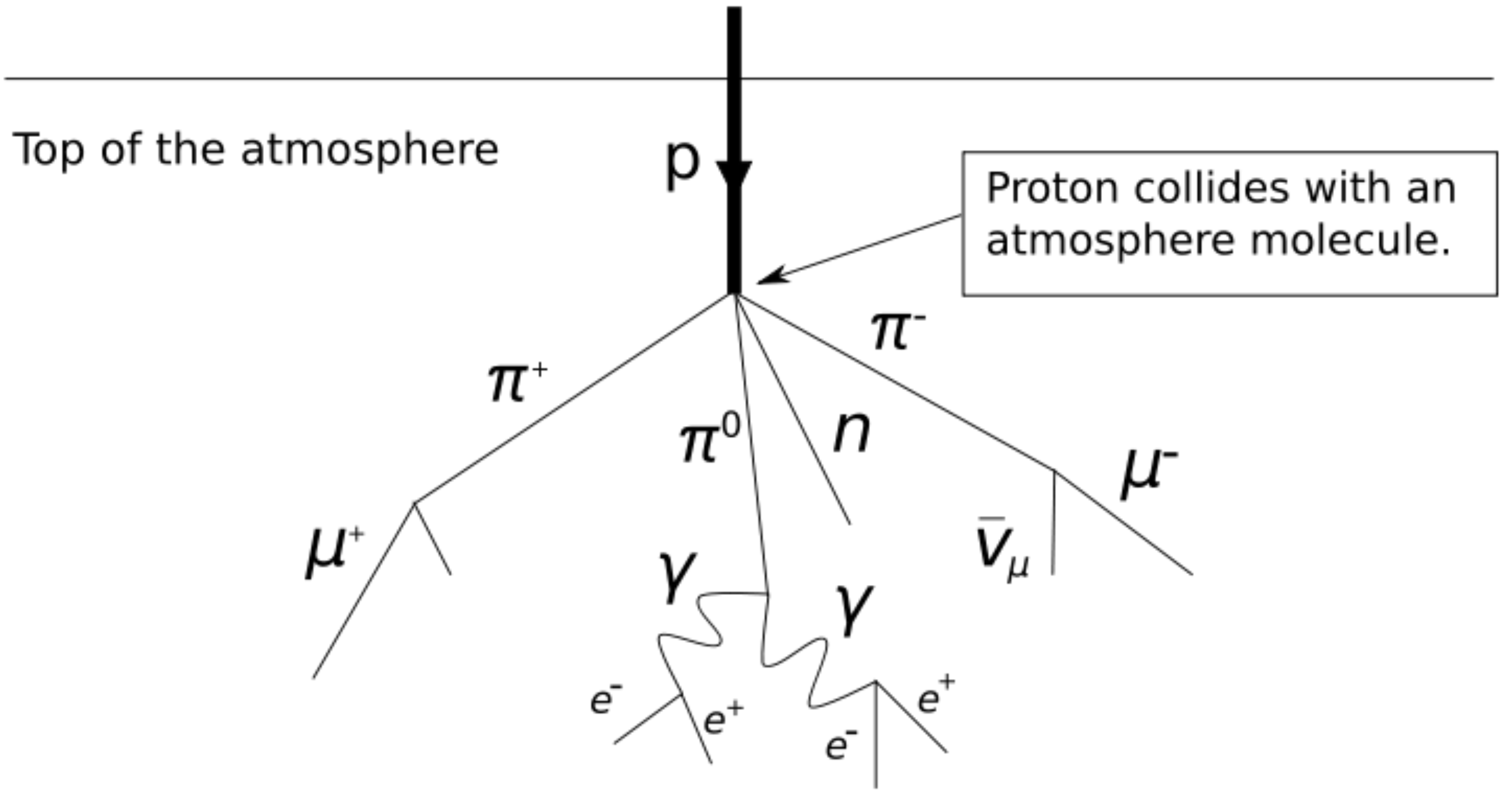
Yang, Lee
(Nobel 1957)

V-A structure of weak interactions: only left-handed fields involved in beta-processes

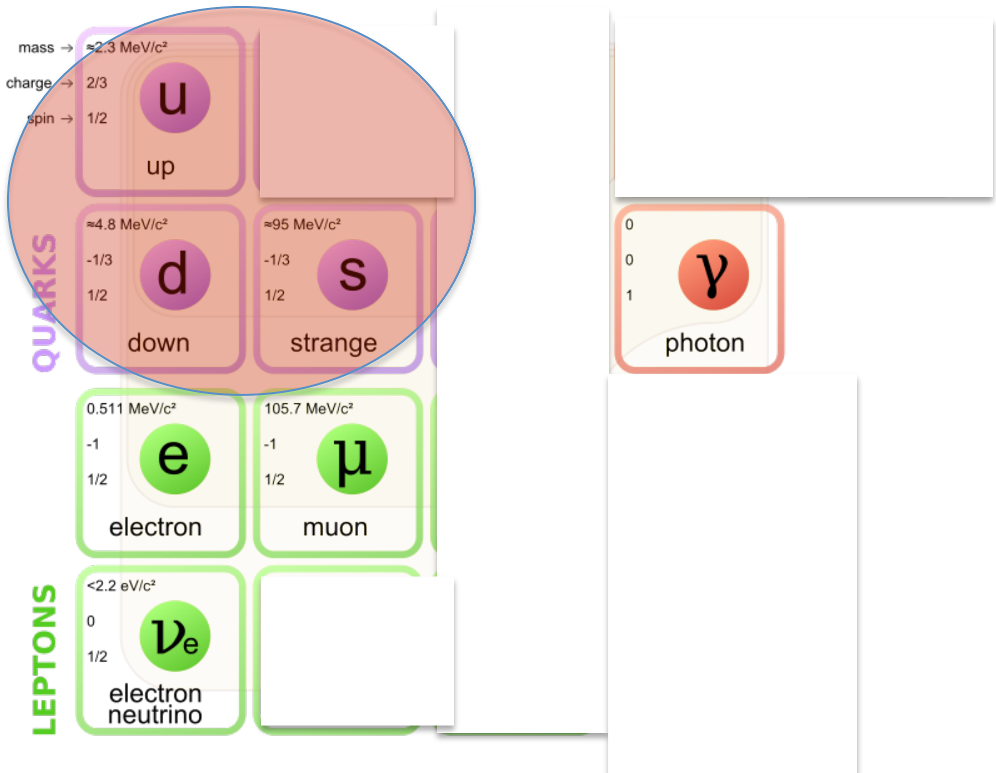
Gell-Mann, Feynman, Sudarshan, Marshak 1958

Cosmic Particle parade starts

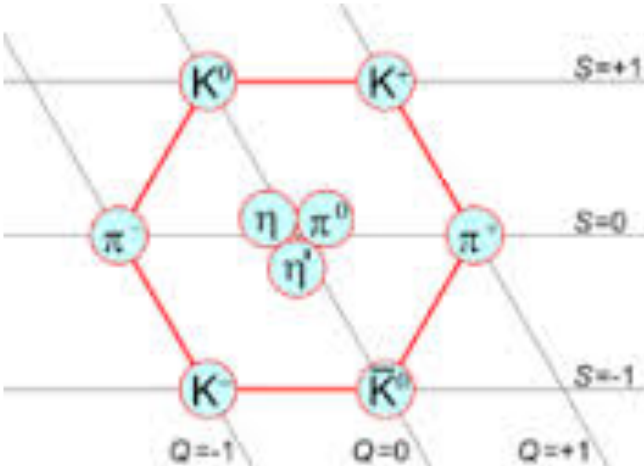
New-looking particles start to show up in Nature-given fixed-target experiment: cosmic rays

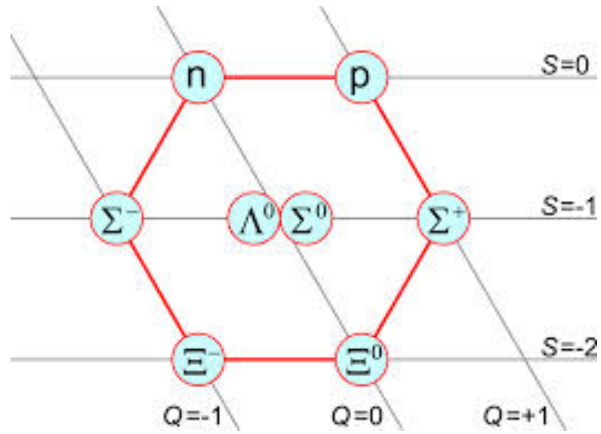


Some elementary

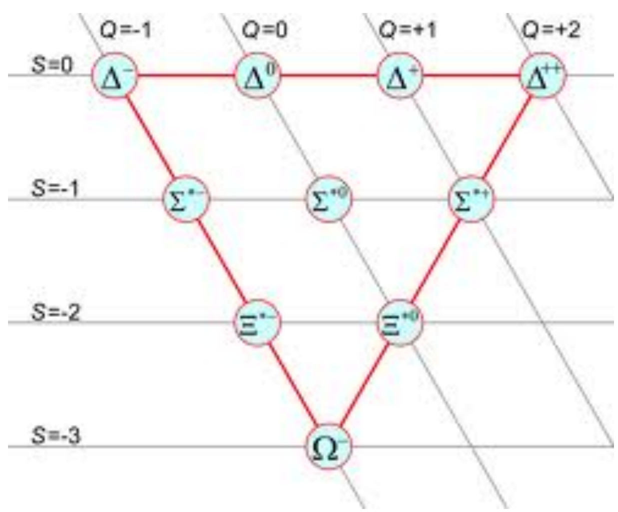


Many more not, but new symmetries start to be evident



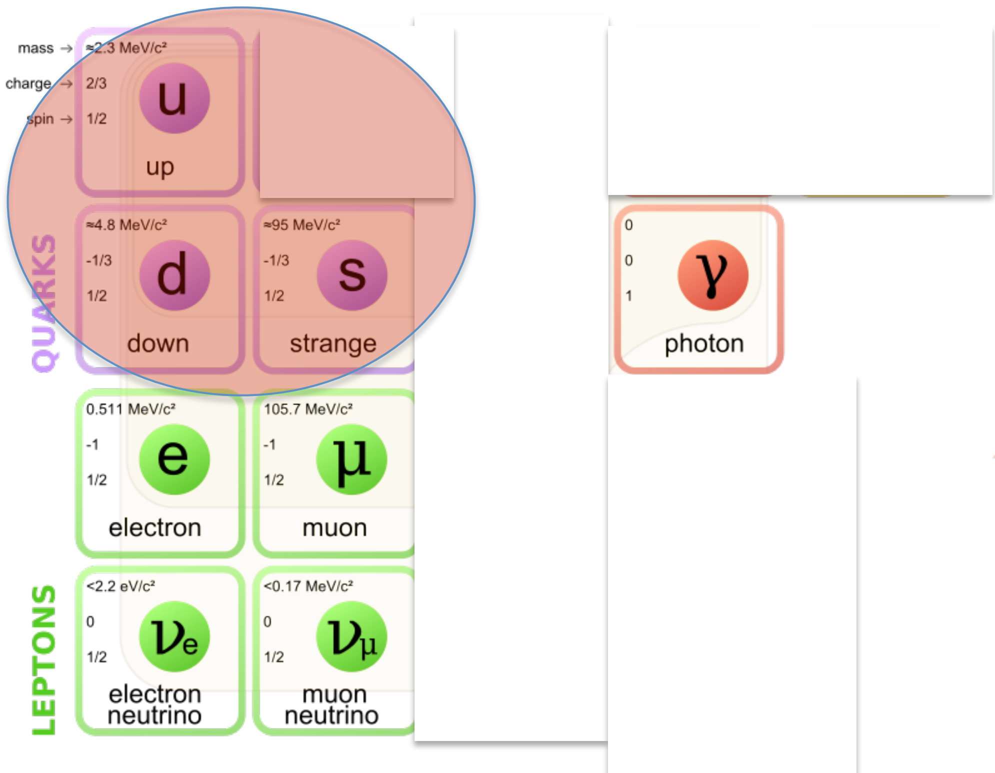


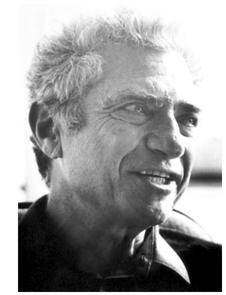
Gell-Mann
(Nobel 1969)



Quark model

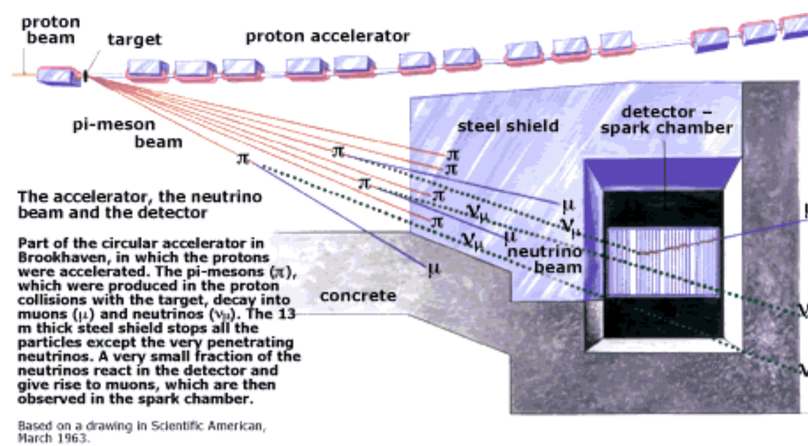
Accelerator Particle Parade





Lederman Schwartz Steinberger

Nobel 1988

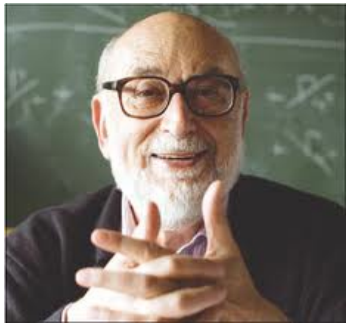


SSB and BEH mechanism

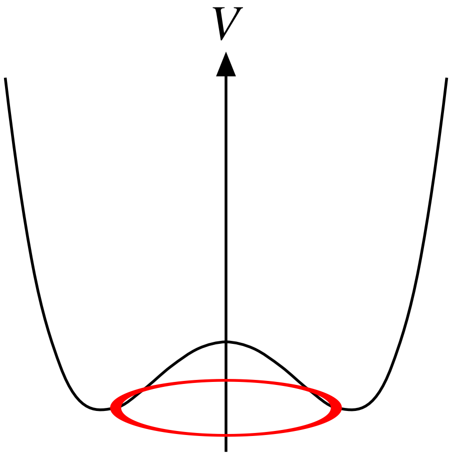
1960 Nambu-Goldstone spontaneous symmetry breaking,
 Nambu-Goldstone bosons

Nambu
Nobel 2008

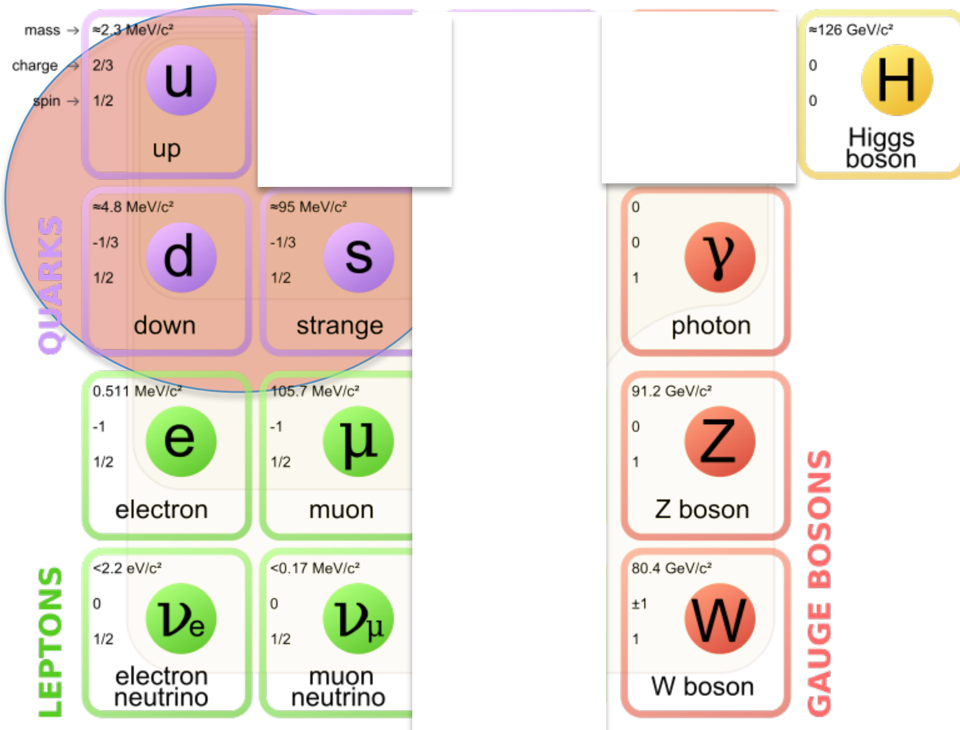
1964 Englert-Brout-Higgs et al
Massive gauge fields from goldstone bosons



Englert, Higgs
Nobel 2013



1967 Glashow,Weinberg,Salam



SU(2) x U(1)

Glashow , Weinberg, Salam
Nobel 1979

T'Hooft, Veltman
Nobel 1999

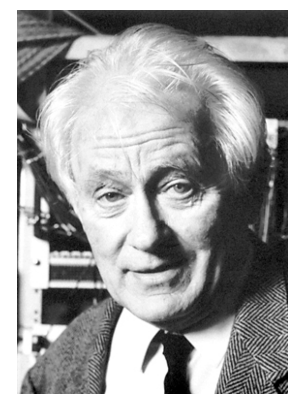
1971 it is renormalizable

Weak mediators appeared

1973 Detection of neutral currents in Gargamelle

1981 W, Z were directly observed UA1, UA2

Rubbia, Van der Meer
Nobel 1984



Charpak
Nobel 1992

Weak mediators appeared

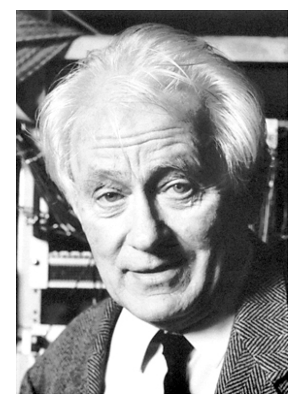
1973 Detection of neutral currents in Gargamelle

1981 W, Z were directly observed UA1, UA2


Accelerator Physicist

Rubbia, Van der Meer
Nobel 1984

Detector Physicist



Charpak
Nobel 1992

Quark model -> SU(3) color

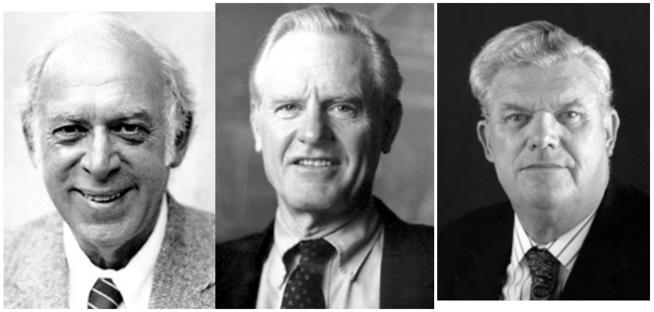
1954 Non-abelian gauge theories: Yang-Mills

1973 Asymptotic freedom

Politzer, Gross, Wilczek
Nobel 2004

1974 DIS experiments

1974 Lattice QCD: confinement

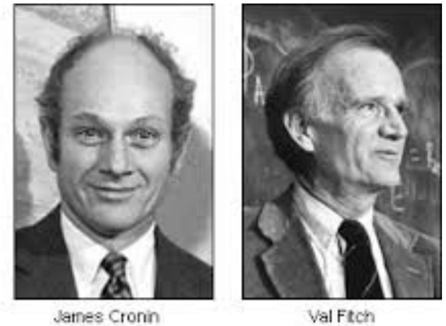


Friedman, Kendall, Taylor
Nobel 1990

Family structure

1963 Cabbibo (also Gell-Mann Levy): the first mixing angle

1964 CP violation discovered



Cronin.Fitch
Nobel 1980

1970 GIM: no FCNC prediction of charm
1973 Kobayashi-Maskawa predict 3 families to explain CP
1974 charm detected !

Richter, Ting Nobel 1976

Kobayashi, Maskawa
Nobel 2008

Family structure

M. Perl
Nobel 1995

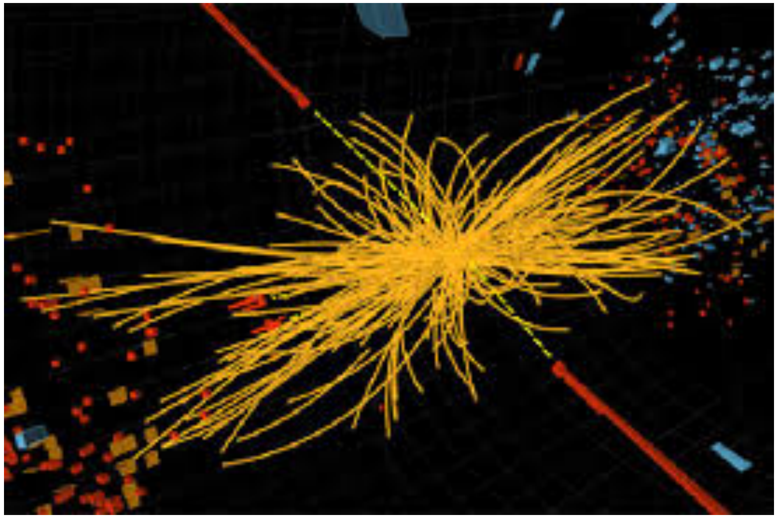
1974-78 third family shows up (tau lepton)
Perl et al

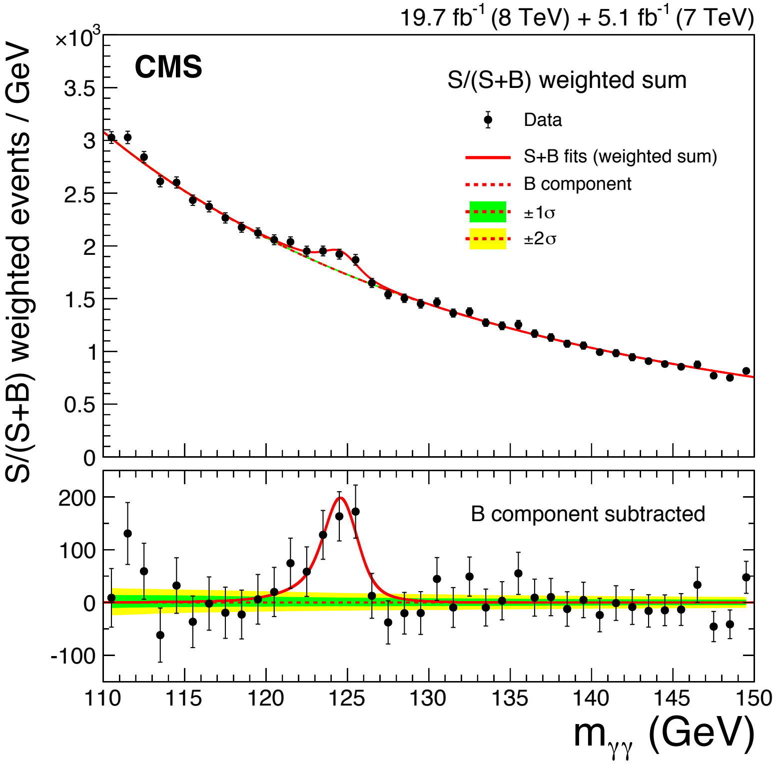
1977 b-quark Lederman et al

1995 top quark Tevatron (D0, CDF) appears after many years of being chased through its quantum effects

2000 ν_τ

and...the scottish particle





Is this the end of the particle parade ?

SM Nobel tapestry