Overview of the SM

Lecture I: the SM tapestry

- Particles as Quantum Fields
- Particle zoo vs symmetry
- Gauge invariance and particle interactions
- The origin of mass: Spontaneous Symmetry breaking
- The flavour of the SM

Lecture II: the SM swiss watch

- Observables and field correlation functions
- How we calculate ? Perturbation Theory and beyond
- Precision tests of the SM (LEP-TEVATRON-B factories)

Lecture III: the open-ended SM

- The SM at the LHC: Higgs physics
- Open questions

The puzzle in the 60's

- Particles with different names in the same gauge SU(2) multiplet
- Parity violation: L, R different charges, but fermions massive
- Three of the gauge fields not massless ✔
- Weak interactions mix quark generations

The SU(2)xU(1) symmetry is hidden

Fermion masses

Dirac fermion of mass m:

$$-\mathcal{L}_m^{\rm Dirac} = m\bar{\psi}\psi = m(\overline{\psi_L + \psi_R})(\psi_L + \psi_R) = m(\overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L)$$

Breaks SU(2)xU(1) gauge invariance!

But we can have other invariants with the conjugated scalar doublet:

$$\tilde{\phi} \equiv \sigma^2\phi^*, \quad \tilde{\phi} : (1, 2, -\frac{1}{2}), \quad \langle\tilde{\phi}\rangle = \begin{pmatrix} \frac{v}{\sqrt{2}} \\ 0 \end{pmatrix}$$

$$\mathcal{L}_{SM} \supset -Y_d \bar{q}_L \phi d_R - Y_u \bar{q}_L \tilde{\phi} u_R - Y_l \bar{l}_L \phi l_R$$
$$\rightarrow -m_d \bar{d}_L d_R - m_u \bar{u}_L u_R - m_l \bar{l}_l l_R + O(h)$$

Exercise: check that the charge assignment of the tilde-field is correct

$$\tilde{\phi} \equiv \sigma^2 \phi^*, \quad \tilde{\phi} : (1, 2, -\frac{1}{2}), \quad \langle \tilde{\phi} \rangle = \begin{pmatrix} \frac{v}{\sqrt{2}} \\ 0 \end{pmatrix}$$

Flavour mixing

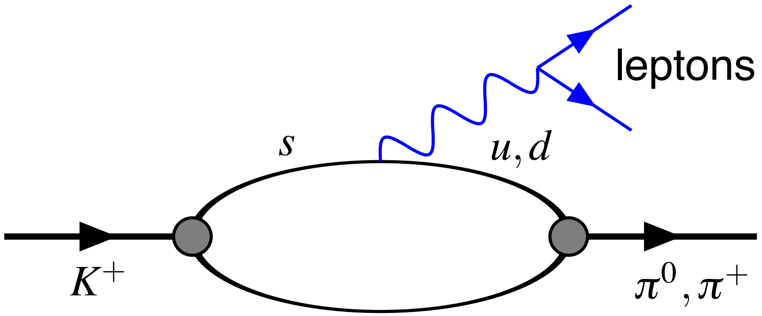
No mixing between different families as it stands…but it turns out there are three families, why can't these Yukawa interactions mix families ?

$(1,2)_{-\frac{1}{2}}$	$(3,2)_{\frac{1}{6}}$	$(1,1)_{-1}$	$(3,1)_{\frac{2}{3}}$	$(3,1)_{-\frac{1}{3}}$
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} u^i \\ d^i \end{pmatrix}_L$	e_R	u^i_R	d^i_R
$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} c^i \\ s^i \end{pmatrix}_L$	μ_R	c^i_R	s^i_R
$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$\begin{pmatrix} t^i \\ b^i \end{pmatrix}_L$	τ_R	t^i_R	b^i_R

Quark mixing

There is flavour changing in charged currents: s -> u, but very suppressed in neutral currents

$$Br(K^+ \to \pi^0 e^+ \nu_e) \simeq 5\% \qquad Br(K^+ \to \pi^+ e^+ e^-) \simeq 3 \times 10^{-7}$$



How to explain mixing in CC without that in NC ?

Glashow-Illiopoulos-Maiani mechanism

Quark mixing: Cabbibo-Kobayashi-Maskawa

The Yukawa couplings are generic matrices in flavour space:

basis where CC and NC diagonal $\neq$ mass eigenbasis

$$\mathcal{L}_{SM} \supset -(\bar{d}_L, \bar{s}_L, \bar{b}_L) \underbrace{m_d}_{3\times 3} \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} - (\bar{u}_L, \bar{c}_L, \bar{t}_L) \underbrace{m_u}_{3\times 3} \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} - (\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L) \underbrace{m_u}_{3\times 3} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$$

$$m_i = U^\dagger_{Li} \mathrm{Diag}(m_i) V_{Ri}$$

$$u'_L = U_{Lu} u_L, \;\; d'_L = U_{Ld} d_L, \;\; l'_L = U_{Ll} l_L, \;\; u'_R = V_{Ru} u_R, \;\; d'_R = V_{Rd} d_R, \;\; l'_R = V_{Rl} l_R$$

$$\mathcal{L}^{CC}_{SM} \supset -\frac{g}{\sqrt{2}} \bar{u}'_{Li} \underbrace{(U_{Lu} U^\dagger_{Ld})_{ij}}_{CKM} \gamma_\mu W^+_\mu d'_{Lj} + h.c.$$

Quark mixing: Cabbibo-Kobayashi-Maskawa

The Yukawa couplings are generic matrices in flavour space:

basis where CC and NC diagonal ≠ mass eigenbasis

$$\mathcal{L}_{SM} \supset -(\bar{d}_L, \bar{s}_L, \bar{b}_L) \underbrace{m_d}_{3\times 3} \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} - (\bar{u}_L, \bar{c}_L, \bar{t}_L) \underbrace{m_u}_{3\times 3} \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} - (\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L) \underbrace{m_u}_{3\times 3} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$$

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$$\mathcal{L}^{NC}_{SM} \supset -\frac{g}{\cos\theta_W} \bar{d}'_{Li} \left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W \right) \underbrace{(U_{Ld} U^\dagger_{Ld})_{ij}}_{\delta_{ij}} \gamma_\mu Z_\mu d'_{Lj}$$

$$-\frac{g}{\cos\theta_W} \bar{u}'_{Li} \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W \right) \underbrace{(U_{Lu} U^\dagger_{Lu})_{ij}}_{\delta_{ij}} \gamma_\mu Z_\mu u'_{Lj}$$

Quark mixing: Cabbibo-Kobayashi-Maskawa

Neutral currents diagonal in the mass eigenbasis: only quarks in the same family can exchange a Z boson

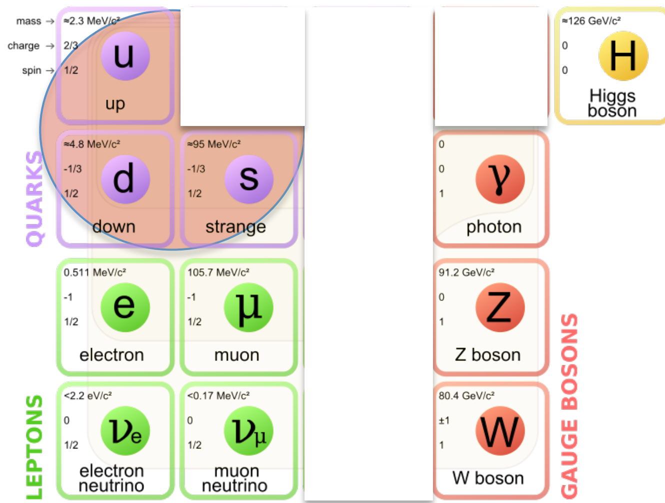
Charged currents not diagonal: CKM 3x3 unitary matrix

$$|V|_{\mathrm{CKM}} = \begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.0065 & (3.51 \pm 0.15) \times 10^{-3} \\ 0.2252 \pm 0.00065 & 0.97344 \pm 0.00016 & (41.2^{+1.1}_{-5}) \times 10^{-3} \\ (8.67^{+0.29}_{-0.31}) \times 10^{-3} & (40.4^{+1.1}_{-0.5}) \times 10^{-3} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix}$$

PDG

GIM

It was quite of a challenge to come up with this when only 1.5 quark family was known: u, d, s -> prediction of the charm !



Exercise: draw diagrams that can mediate this process in the Fermi approximation (W integrated out)

$$Br(K^+ \to \pi^+ e^+ e^-) \simeq 3 \times 10^{-7}$$

CKM Parametrization

Not all entries are independent: how many physical parameters are there ?

3 Euler angles and 1 complex phase : $s_{12} \simeq$ Cabbibo angle

$$V_{CKM} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix}.$$

Since $s_{12} >> s_{23} >> s_{13}$: Wolfenstein parametrization

$$V = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\,\lambda^3\,(\rho - i\,\eta) \\ -\lambda & 1 - \lambda^2/2 & A\,\lambda^2 \\ A\,\lambda^3\,(1 - \rho - i\,\eta) & -A\,\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Counting parameters

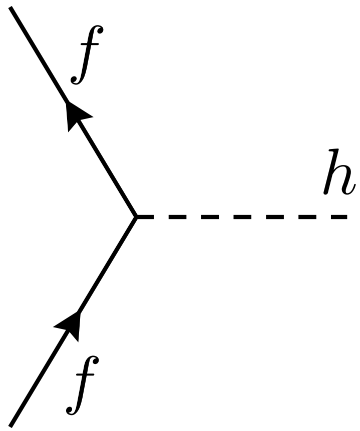
\# physical parameters = # parameters in Yukawas
- \# parameters in field redefinitions
+ \# parameters of exact symmetries

		Field Redef.	Symmetries	Physical
	Y_u , Y_d	$U_{qL}(3)xU_{dR}(3)xU_{uR}(3)$	$U(1)_B$	
Moduli	2×3^2	3 x 3	0	9
Phases	2×3^2	3 x 6	1	1

Moduli = 9 = 6 masses + 3 angles

Higgs-fermion couplings

$$(1 + h/v)\left(-m_d \bar{d}_L d_R - m_u \bar{u}_L u_R - m_l \bar{l}_L l_R\right) + h.c.$$



$$-i\,\frac{g}{2}\,\frac{m_f}{m_W}$$

Higgs couplings to fermions do not change flavour!

CP violation

Charge conjugation: particle <-> antiparticle (without changing helicity)

C: $$\Psi \to i\gamma_2 \Psi^* = i\gamma_2\gamma_0 \bar{\Psi}^T$$

This is not a symmetry of the chiral SM

C: $$\bar{\Psi} i\gamma_\mu \partial_\mu P_L \Psi \to_C \bar{\Psi} i\gamma_\mu \partial_\mu P_R \Psi$$

The combination CP is a good symmetry except if there are phases in the mixing matrix!

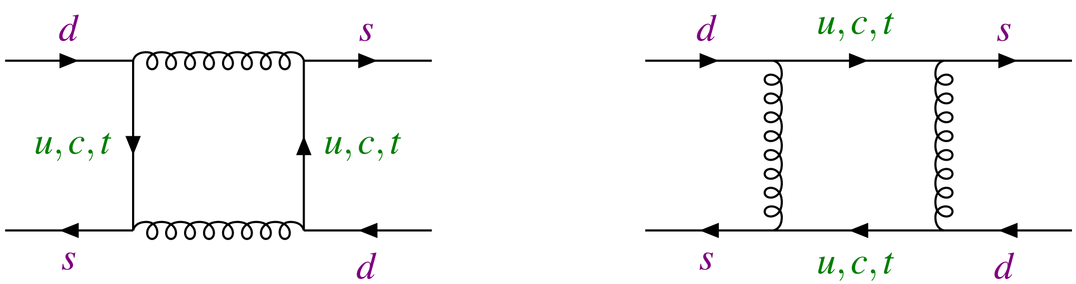
$$W^+_\mu \bar{u}_i \gamma_\mu (1-\gamma_5) V_{ij} d_j + W^-_\mu \bar{d}_j \gamma_\mu (1-\gamma_5) V^*_{ij} u_i \to_{CP} W^+_\mu \bar{u}_i \gamma_\mu (1-\gamma_5) V^*_{ij} d_j + W^-_\mu \bar{d}_j \gamma_\mu (1-\gamma_5) V_{ij} u_i$$

CP violation

CP violation was discovered in the kaon sector

Cronin, Fitch 1964

$$|K_0\rangle = \bar{d}s \leftrightarrow_{CP} |\bar{K}_0\rangle = \bar{s}d$$



If there was no CP violation, the mass eigenstates would be CP eigenstates:

$$|K_{1,2}\rangle = \frac{1}{\sqrt{2}}(|K_0\rangle \pm |K_0^\dagger\rangle), CP = \pm 1$$

$$K_{1,2} \to \pi^+\pi^-(CP = +1), \pi^0\pi^+\pi^-(CP = -1)$$

CP violation

$$|K_1\rangle = |K_S\rangle, |K_2\rangle = |K_L\rangle$$

$$\tau_{K_S} \simeq 0.9 \times 10^{-10} s, c\tau \simeq 2.7cm$$
$$\tau_{K_L} \simeq 5.2 \times 10^{-8} s, c\tau \simeq 15.5m$$

The CP forbidden decay $K_L \to \pi\pi$ was measured !

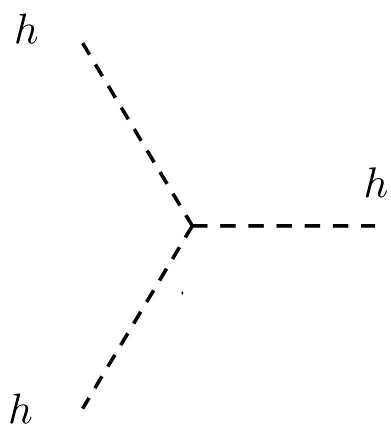
Exercise: would there be phases if there were two families ?

Third family was conjectured based on this....

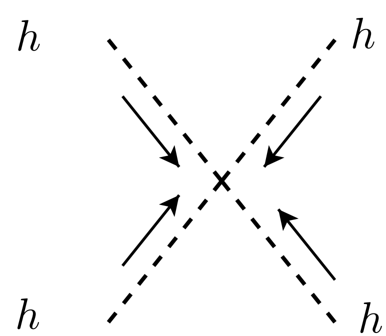
Kobayashi, Maskawa

Higgs self-couplings

From the Higgs potential:



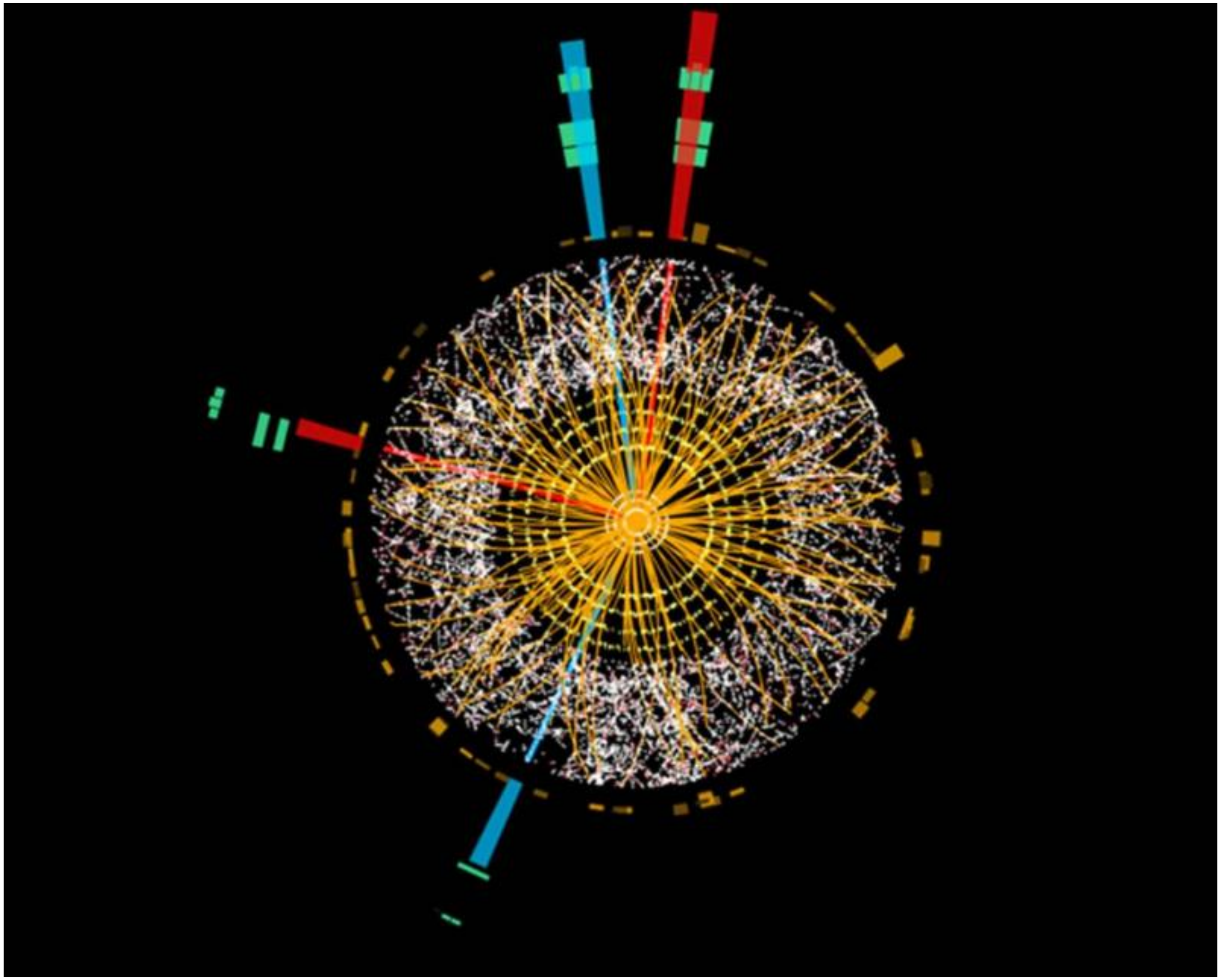
$$-\frac{3}{2}\,i\,g\,\frac{m_h^2}{m_W^2}$$



$$-\frac{3}{4}\,i\,g^2\,\frac{m_h^2}{m_W^2}$$

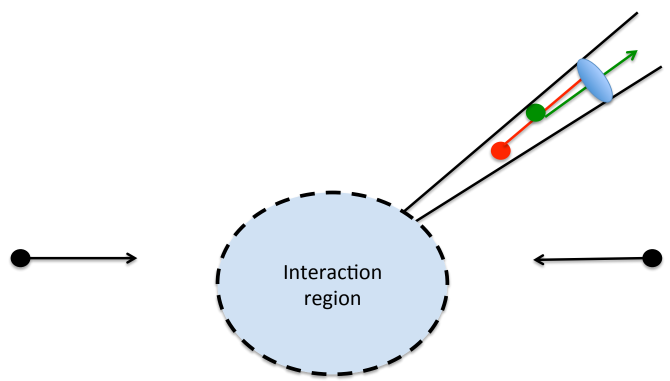
Lecture II: the SM swiss watch

- Observables and field correlation functions
- How we calculate ?
- Precision tests of the SM (LEP-TEVATRON-B factories)



QFT in a nutshell

What we need to compute ? x-sections

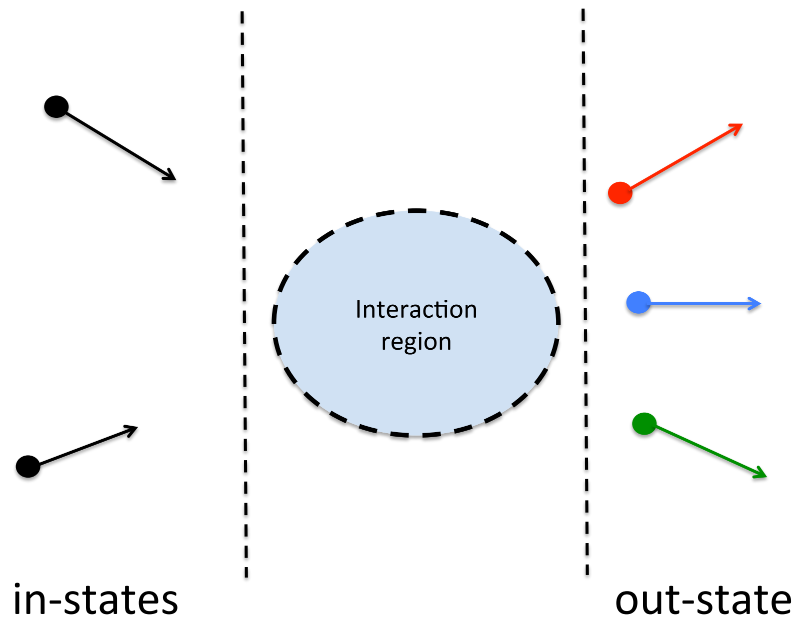


$$\frac{dN}{d\Omega dt}\Big|_{\text{scattered}} = \frac{dN}{dS dt}\Big|_{\text{incident}} \times \frac{d\sigma}{d\Omega}$$

$$\sigma = \frac{1}{2E_{\mathbf{p}_1} 2E_{\mathbf{p}_2} |\mathbf{v}_{12}|} \int \prod_f \frac{d^3\mathbf{q}_f}{(2\pi)^3 2E_{\mathbf{q}_f}} |\mathcal{M}(i \to f)|^2 (2\pi)^4 \delta^{(4)}\left(\sum_i p_i - \sum_f q_f\right)$$

QFT in a nutshell

The observables: x-sections



$$|\mathbf{p}_1, \mathbf{p}_2; in\rangle \qquad |\mathbf{q}_1, \ldots, \mathbf{q}_n; out\rangle$$

Amplitude $= \langle \mathbf{q}_1, \ldots, \mathbf{q}_n; out|\mathbf{p}_1, \mathbf{p}_2; in\rangle = \langle \mathbf{q}_1, \ldots, \mathbf{q}_n|\hat{S}|\mathbf{p}_1, \mathbf{p}_2\rangle$

S: time evolution operator $\quad \hat{S} = \hat{1} + i\hat{T}, \;\; \hat{S}^\dagger = \hat{S}^{-1}$

$$\langle \mathbf{q}_1, \ldots, \mathbf{q}_n|\hat{T}|\mathbf{p}_1, \mathbf{p}_2\rangle \equiv (2\pi)^4 \delta^{(4)}(\textstyle\sum_i p_i - \sum_f q_f) i\mathcal{M}(\mathbf{p}_1, \mathbf{p}_2; \mathbf{q}_1, .., \mathbf{q}_n)$$

QFT in a nutshell

How do we compute ? LSZ reduction formulae:

S-matrix elements <-> T-ordered correlation functions of field operators

For the (real) scalar case:

$$\prod_{i=1}^{2}\int d^4x_i e^{-ip_i\cdot x_i}\prod_{j=1}^{k}\int d^4y_j e^{iq_j\cdot y_i}\langle 0|T\left(\hat\phi(x_1)...\hat\phi(x_2)\hat\phi(y_1)....\hat\phi(y_k)\right)|0\rangle$$

$$\simeq_{p_i^0\to E_{\vec p_i},q_j^0\to E_{\vec q_j}}\prod_{i=1}^{2}\left(\frac{i\sqrt{Z}}{p_i^2-m^2+i\epsilon}\right)\prod_{j=1}^{k}\left(\frac{i\sqrt{Z}}{q_j^2-m^2+i\epsilon}\right)\langle\vec q_1,....,\vec q_n;out|\vec p_1,\vec p_2;in\rangle$$

In the path integral formulation

$$\langle 0|T\left(\hat\phi(x_1)\hat\phi(x_2)\hat\phi(y_1)....\hat\phi(y_k)\right)|0\rangle=\frac{\int\mathcal{D}\phi\ \phi(x_1)\phi(x_2)\ldots\phi(y_k)\ e^{iS[\phi]}}{\int\mathcal{D}\phi\ e^{iS[\phi]}}$$

$$S[\phi]=\int d^4x\ \mathcal{L}(\phi)$$

QFT in a nutshell

Method 1: Perturbation Theory

(Taylor expansion in coefficients of non-quadratic terms in Lagrangian)

$$S[\phi] = S_0[\phi] + S_{int}[\phi] = \int d^4x\ \mathcal{L}_0[\phi] + \int d^4x\ \mathcal{L}_{int}[\phi]$$

$$\langle 0|T\left(\hat\phi(x_1)\hat\phi(x_2)\hat\phi(y_1)....\hat\phi(y_k)\right)|0\rangle = \frac{\int \mathcal{D}\phi\ \phi(x_1)\phi(x_2)\ldots\phi(y_k)\ \sum_n \frac{i^n}{n!}S_{int}[\phi]^n\ e^{iS_0[\phi]}}{\int \mathcal{D}\phi\ \sum_n \frac{i^n}{n!}S_{int}[\phi]^n e^{iS_0[\phi]}}$$

- All integrals are Gaussian because S_0 is quadratic in the fields

$$\int d\mathbf{x}\ x_i x_j e^{-\frac{1}{2}\mathbf{x}^T A\mathbf{x}} / \int d\mathbf{x}\ e^{-\frac{1}{2}\mathbf{x}^T A\mathbf{x}} = (A^{-1})_{ij}$$

- Can be represented by a sum of connected Feynman diagrams

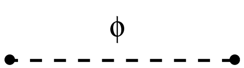
Wick's theorem: all fields paired-up in propagators

QFT in a nutshell

Method 1: Perturbation Theory

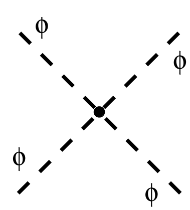
$$\mathcal{L}(\phi) = \underbrace{\frac{1}{2}\partial_\mu\phi\partial_\mu\phi - \frac{m^2}{2}\phi^2}_{\mathcal{L}_0} - \underbrace{\frac{\lambda}{4!}\phi^4}_{\mathcal{L}_{int}}$$

Free propagator(s)



$$\frac{i}{p^2 - m^2 + i\epsilon}$$

Vertices

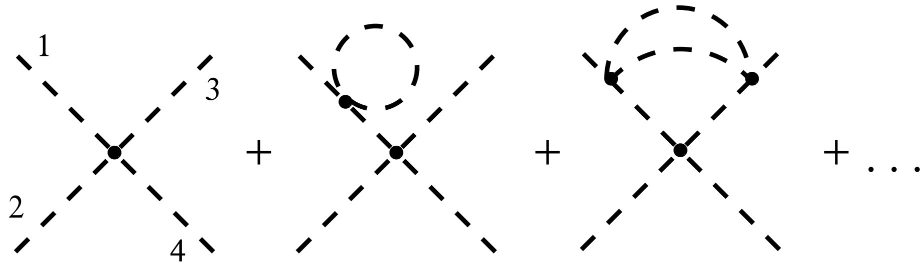


$$-i\lambda(2\pi)^4\delta(\sum_i p_i)$$

QFT in a nutshell

Method 1: Perturbation Theory

$$\mathcal{M}(1+2 \to 3+4) =$$



- Draw the external points x_1, x_2 and the vertices z_i
  (from each external point emerges one line from each vertex 4 lines emerge)

- Pair-up all the lines and link them via a propagator in such a way that the diagram remains connected

- Calculate the contribution of each diagram:

$$\int \prod_I \frac{d^4p}{(2\pi)^4} \prod_I (\text{Internal propagators}) \times \prod_V (\text{couplings vertex}) \times \delta(\sum_i p_i)$$

- Multiply by the symmetric factor (this is the hardest part)

QFT in a nutshell

Generic diagram is divergent (L loops, I internal propagators, V vertices N external lines):

$$\propto \int \prod_{l=1}^{L} d^4 k_l \prod_{i=1}^{I} \frac{1}{r_i(k_l;\ \underbrace{p_1, p_2, q_1, \ldots q_n}_{external\ momenta}\ )^2 + m^2} \quad \propto \Lambda^{\omega}$$

Superficial UV divergence $\qquad \omega = 4L - 2I$

$$4V = N + 2I \qquad\qquad L = I - V + 1$$

$$\boxed{\omega = 4 - N}$$

Only divergences in diagrams with 2 or 4 external legs !

Can be reabsorbed in a redefinition of the field normalization, mass and coupling: the theory is renormalizable!

QFT in a nutshell

Imagine we had an interaction Lagrangian of the form

$$\mathcal{L}_{int} \supset g\phi^{N_\phi} \qquad [g] = 4 - N_\phi$$

At each new vertex there are N_ϕ fields

$$\omega = 4 - N - [g]V$$

If [g] < 0, diagrams with arbitrary N become divergent for large enough V: the theory is non-renormalizable

Only interactions with at most d=4 can be added!!

The SM is renormalizable

t'Hooft, Veltman 73

- Gauge symmetry principle is essential

- Any modification of the relations between the couplings would destroy this property

- Generically the renormalization procedure involves a scale dependence of the renormalized couplings:

Q^2 is a physical scale s,t,u,m^2

$$\lambda_R(\mu) = \lambda + C\lambda^2 \log\left(\frac{\Lambda^2}{Q^2}\right)\Big|_{Q^2=\mu^2}$$

Running couplings

Physics quantities do not depend on this scale, but we can improve the convergence of the series by setting it appropriately

$$\mu^2 \sim p_{\text{ext}}^2$$

The change of the couplings with the renormalization scale is called running and it is defined in terms of beta-functions:

$$\beta(g) \equiv \mu \frac{\partial g(\mu)}{\partial \mu} \qquad \beta(g) \equiv \beta_0 \frac{g^3}{16\pi^2} + \beta_1 \frac{g^5}{(16\pi^2)^2} + \ldots$$

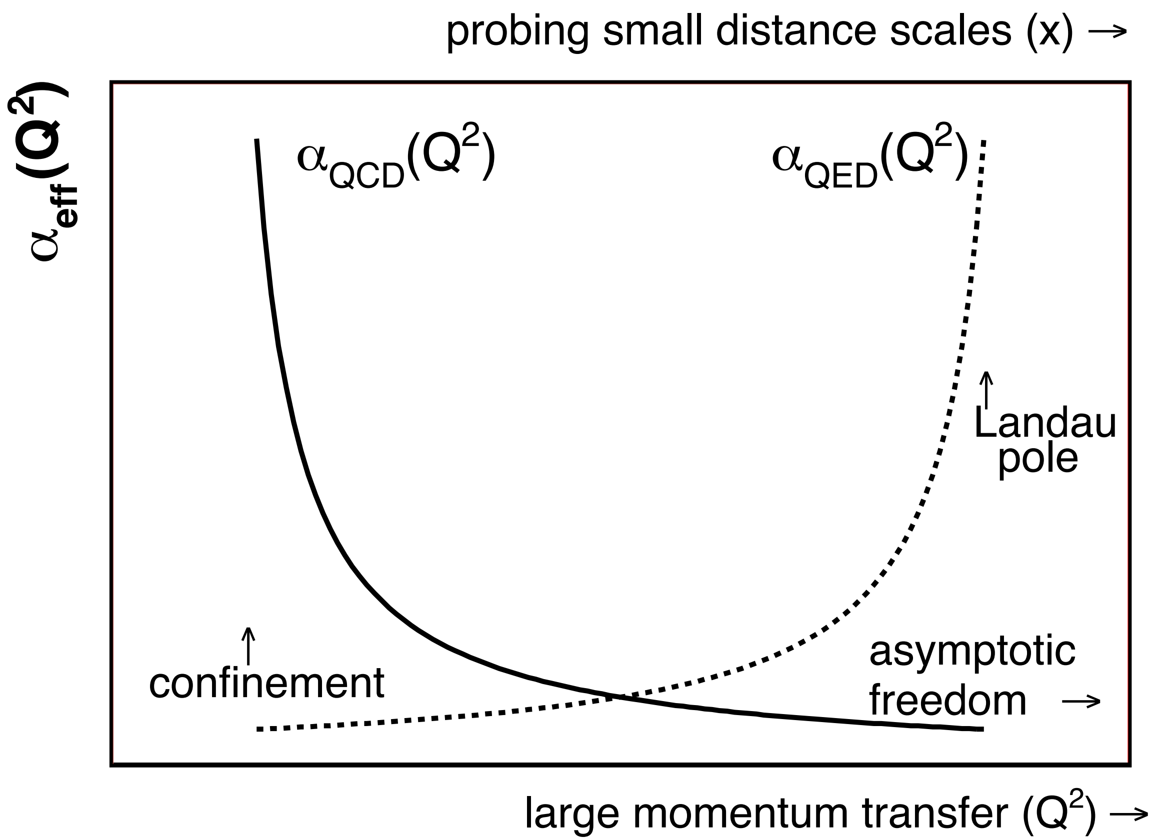
Partial resummation of the perturbative series

$$g^2(\mu) = \frac{g^2(\Lambda)}{1 - \frac{\beta_0}{16\pi^2} \log(\frac{\mu^2}{\Lambda^2})}$$

Running couplings

Very different behaviour depending on the sign of β_0

$$g^2(\mu) = \frac{g^2(\Lambda)}{1 - \frac{\beta_0}{16\pi^2} g^2(\Lambda) \log(\frac{\mu^2}{\Lambda^2})}$$

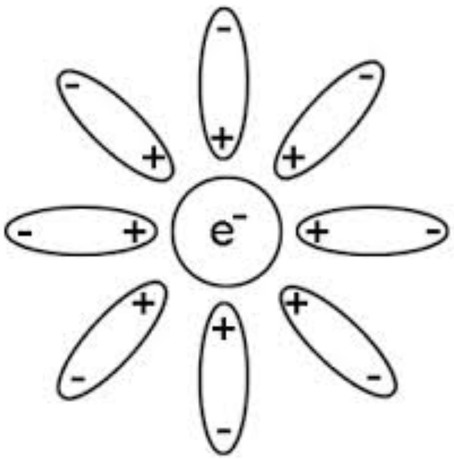


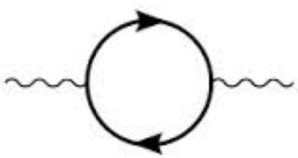
Running couplings

Very different behaviour depending on the sign of β_0

$$g^2(\mu) = \frac{g^2(\Lambda)}{1 - \frac{\beta_0}{16\pi^2} g^2(\Lambda) \log(\frac{\mu^2}{\Lambda^2})}$$

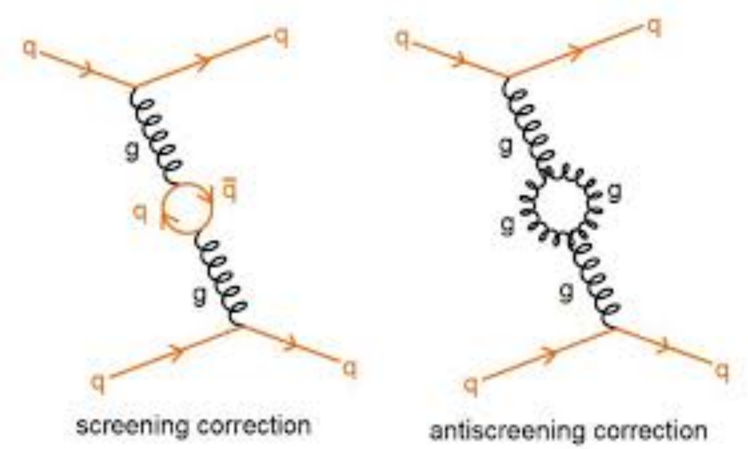
$\beta_0 > 0$





QED

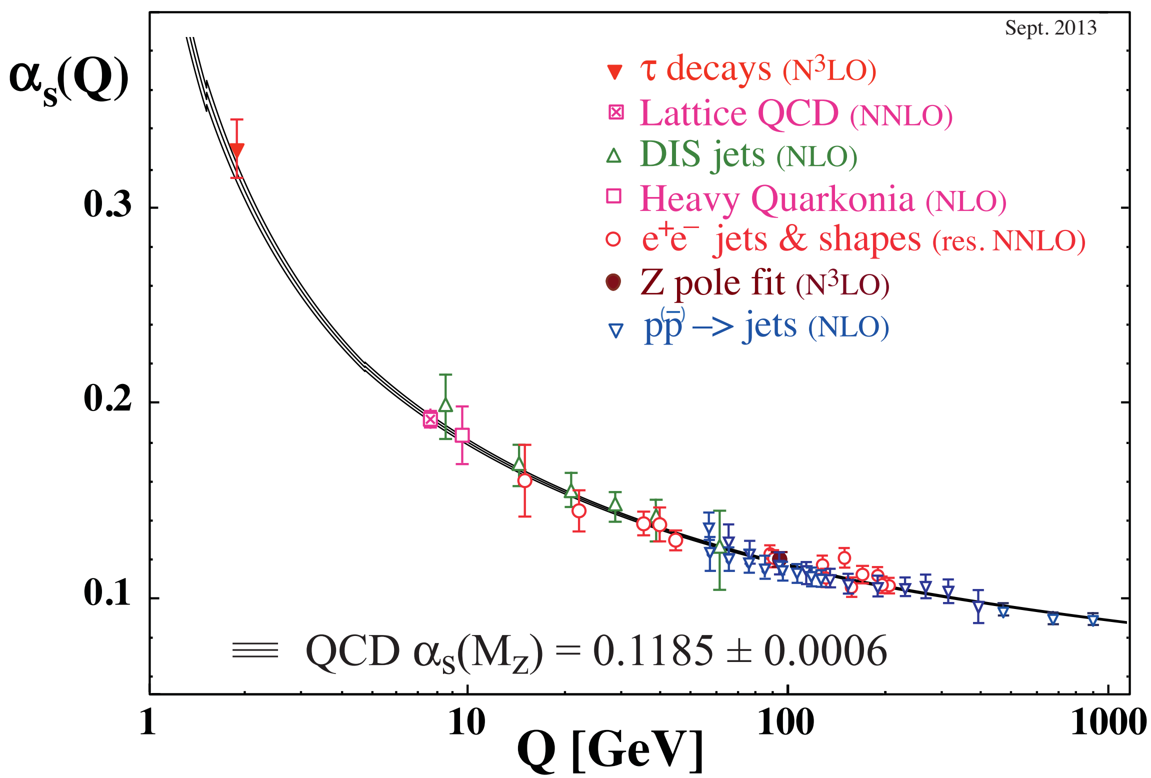
$\beta_0 < 0$



QCD

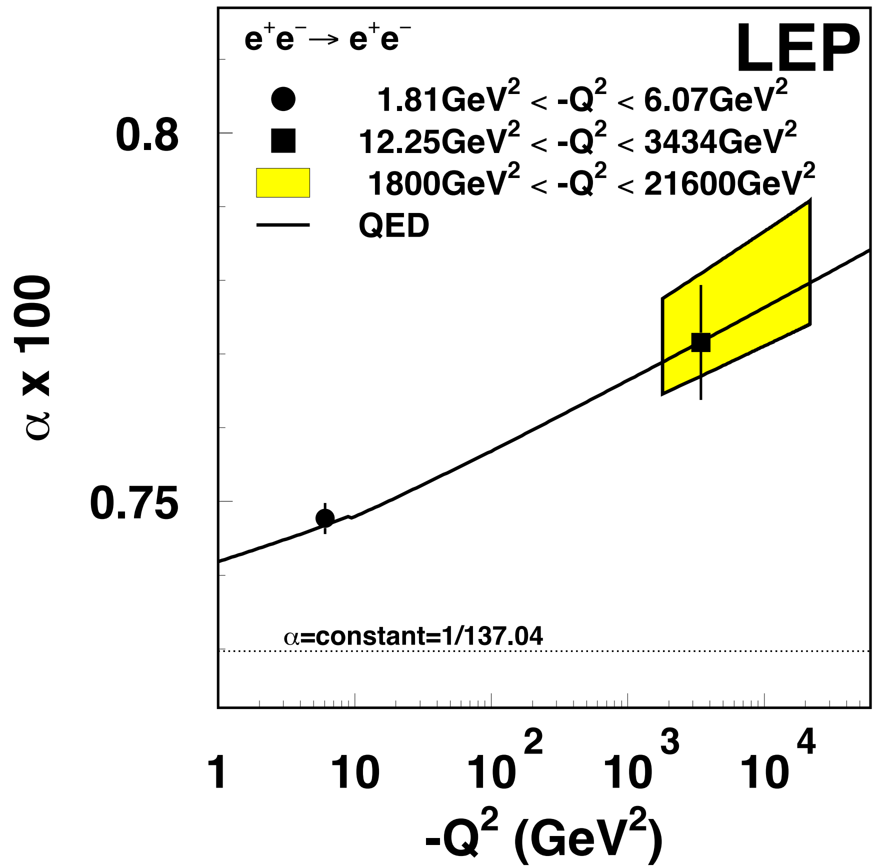
Asymptotic freedom

QCD coupling: $$\beta_0 = -\left(11 - \frac{2}{3}N_q\right)$$



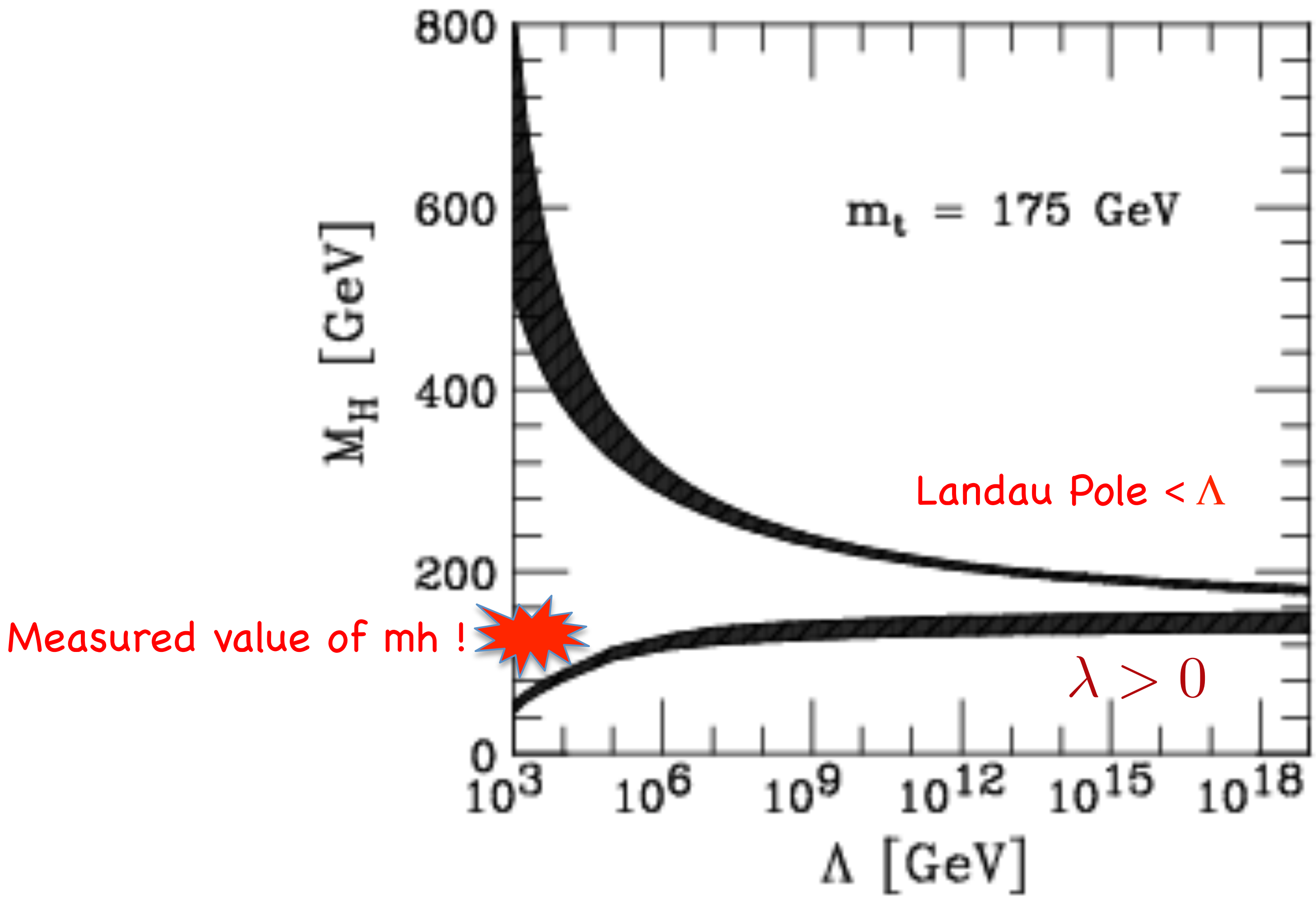
Triviality & Landau Poles

QED: $\beta_0 = \frac{4}{3}$ $\qquad g^2(\mu) = \frac{g^2(\Lambda)}{1 - \frac{\beta_0}{16\pi^2} g^2(\Lambda) \log(\frac{\mu^2}{\Lambda^2})}$



Triviality vs Stability

Higgs self-coupling $$16\pi^2\mu\frac{d\lambda}{d\mu} = 12\lambda^2 + 12\lambda g_t^2 - 12g_t^4 + \ldots$$



Precision QCD ?

Asymptotic freedom was a fundamental step in confirming QCD experimentally via deep inelastic experiments, but made it impossible to use perturbation theory to understand the rich hadron spectrum

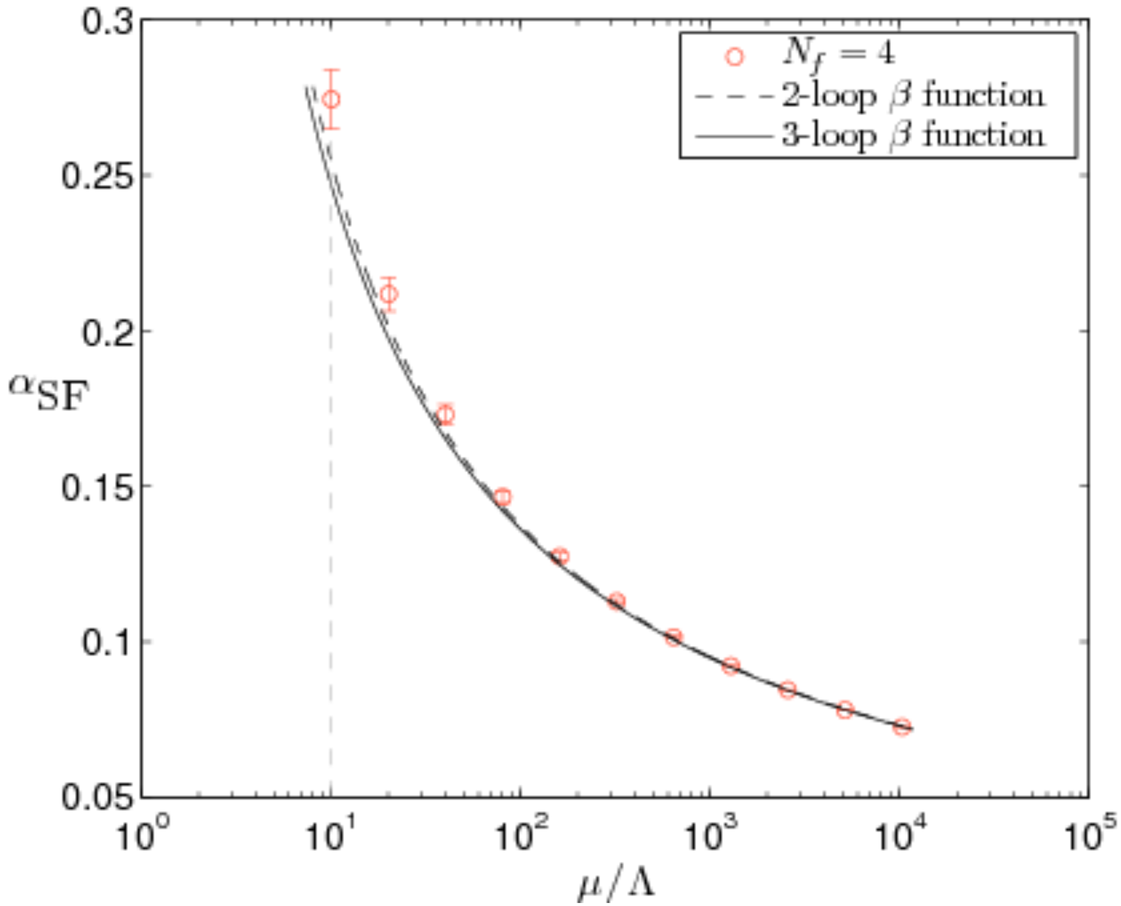
- **Method 2: Lattice field theory**

After discretizing space-time and performing a Wick rotation, correlation functions can be computed via Montecarlo methods

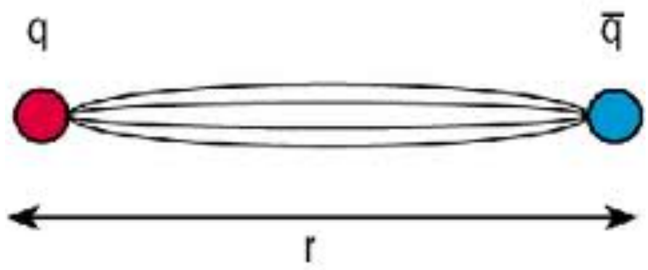
$$\frac{\int \mathcal{D}\phi\ \phi(x_1)\phi(x_2)\ldots\phi(y_k)\ e^{iS[\phi]}}{\int \mathcal{D}\phi\ e^{iS[\phi]}} \to^{\text{Wick rotation}} \frac{\int \mathcal{D}\phi\ \phi(x_1)\phi(x_2)\ldots\phi(y_k)\ e^{-S[\phi]}}{\int \mathcal{D}\phi\ e^{-S[\phi]}}$$

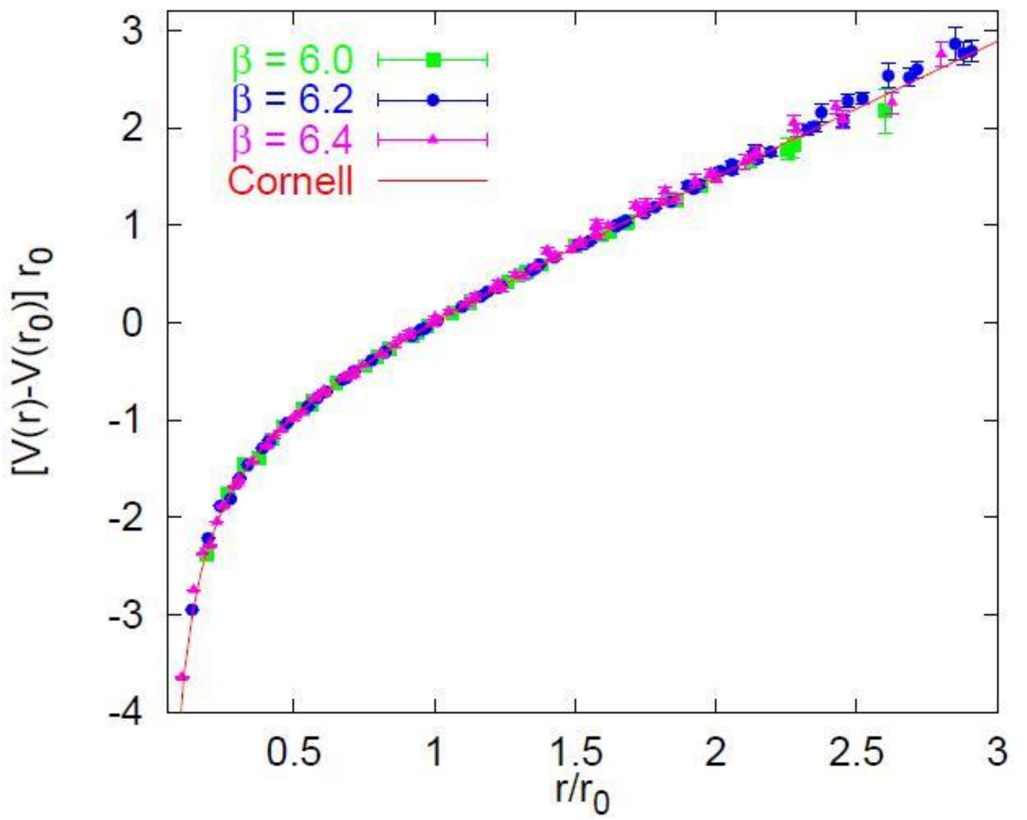
Lattice QCD is QCD

Running coupling beyond perturbation theory:



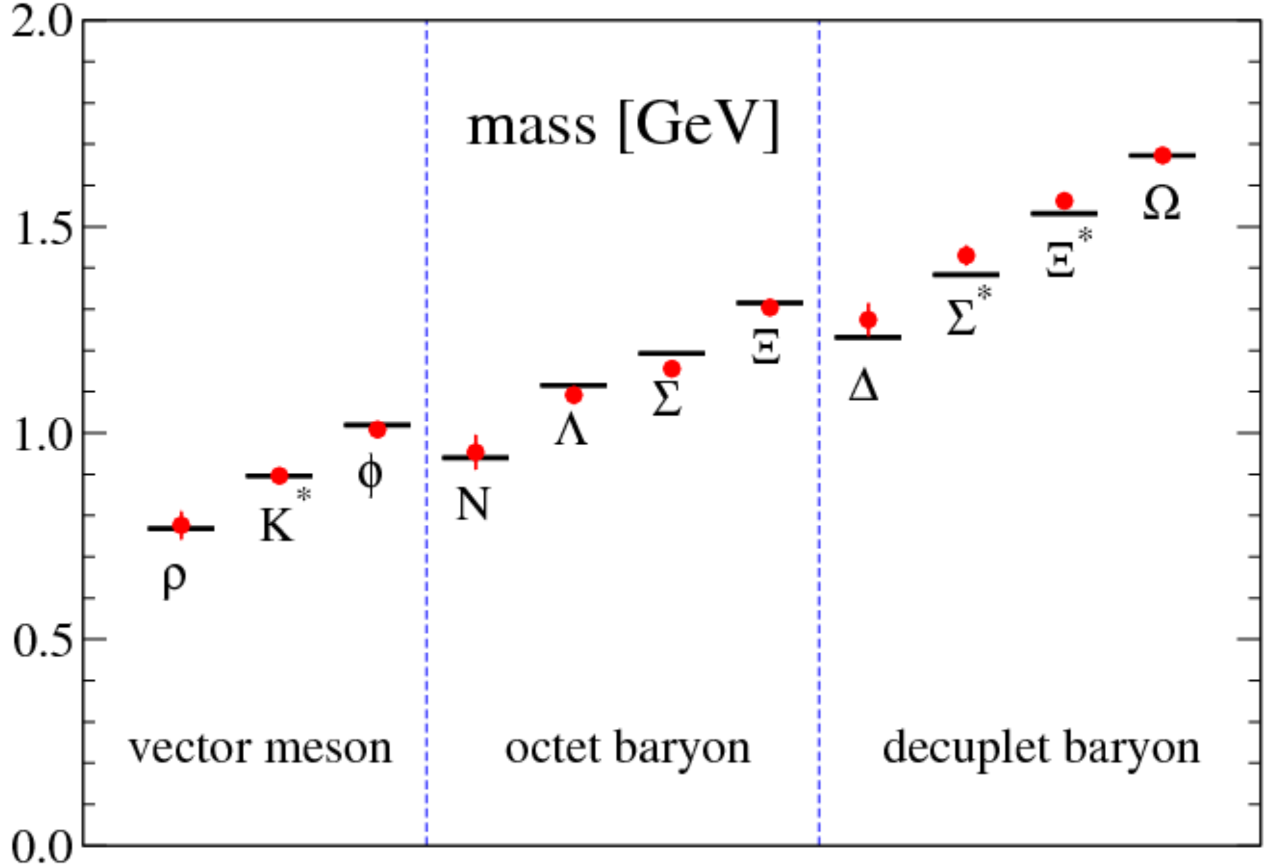
Confinement





The potential between static charges grows linearly with distance: quark confinement

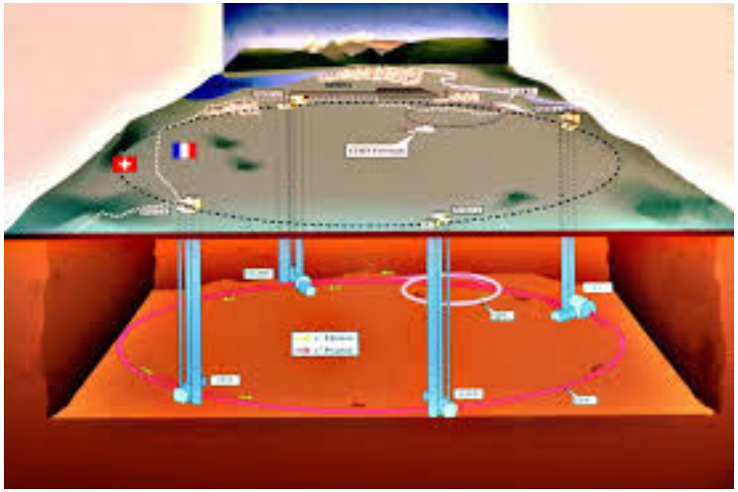
Light Hadron spectrum from lattice QCD



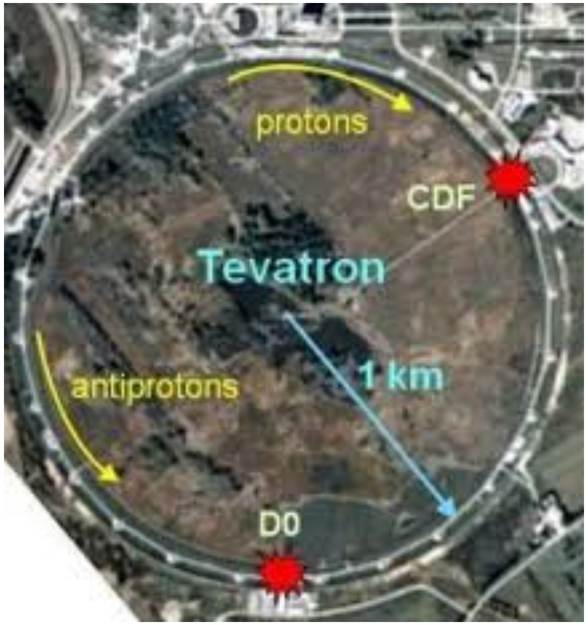
EW precision tests

In the 90's LEP/SLD and Tevatron tested the SM at few per mille level!

e+e- colliders LEP1 (s=90GeV), LEP2 (200GeV)



p pbar collider s=2 TeV



EW precision tests

Need to fix the free parameters of the SM: g_s, g, g', v, M_h

$\alpha^{-1} = 137.035999074(44)$ -> (g-2)e

$G_F = 1.1663787(6) \times 10^{-5}\mathrm{GeV}$ -> Muon lifetime

$M_Z = 91.1876(21)\mathrm{GeV}$ -> Z-pole mass (LEP)

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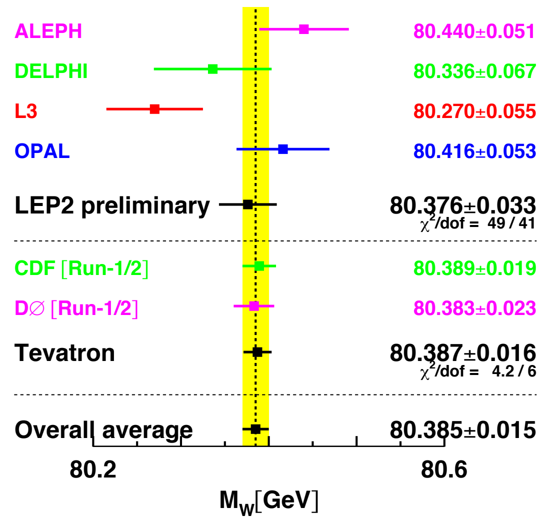
$$\left.\begin{aligned} M_W^2 \sin^2\theta_W &= \frac{\pi\alpha}{\sqrt{2}G_F} \\ \sin^2\theta_W &= 1 - \frac{M_W^2}{M_Z^2} \end{aligned}\right\} \rightarrow M_W = 80.938\ \mathrm{GeV}, \sin^2\theta_W = 0.212$$

$$|M_W(exp) - M_W(tree)| = 553(15)MeV$$



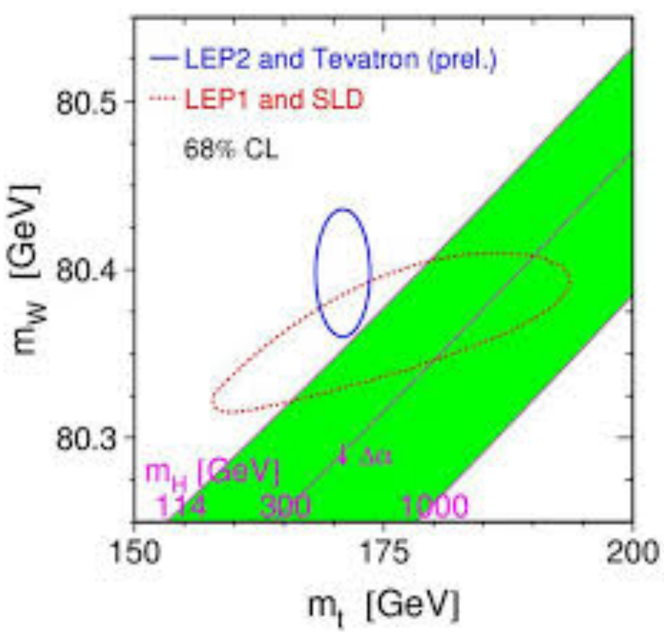
Tree level relations get modified at higher orders !

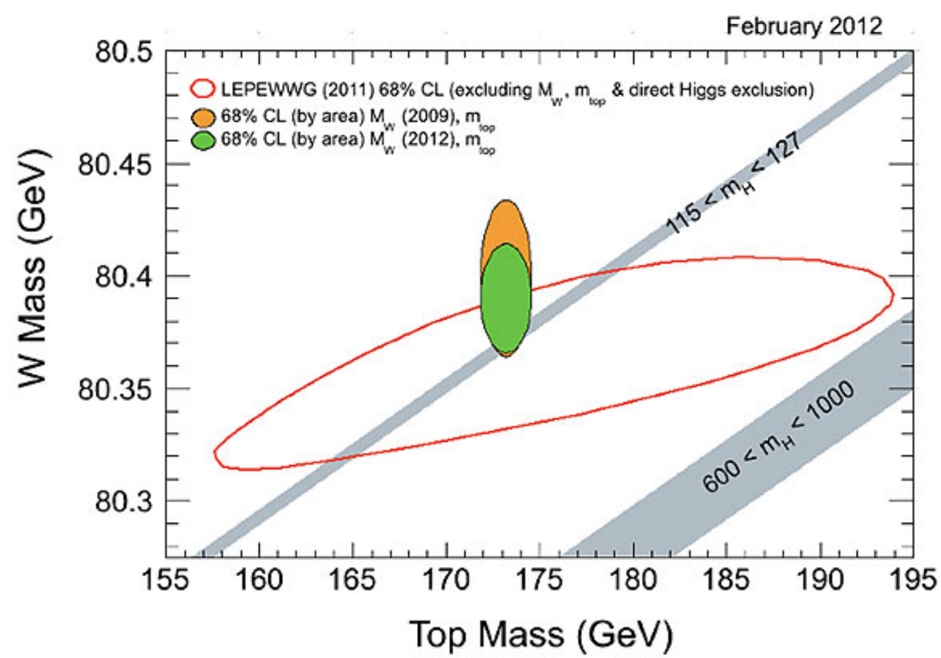
EW precision tests

Need one loop corrections: also virtual effects of heavy particles enter

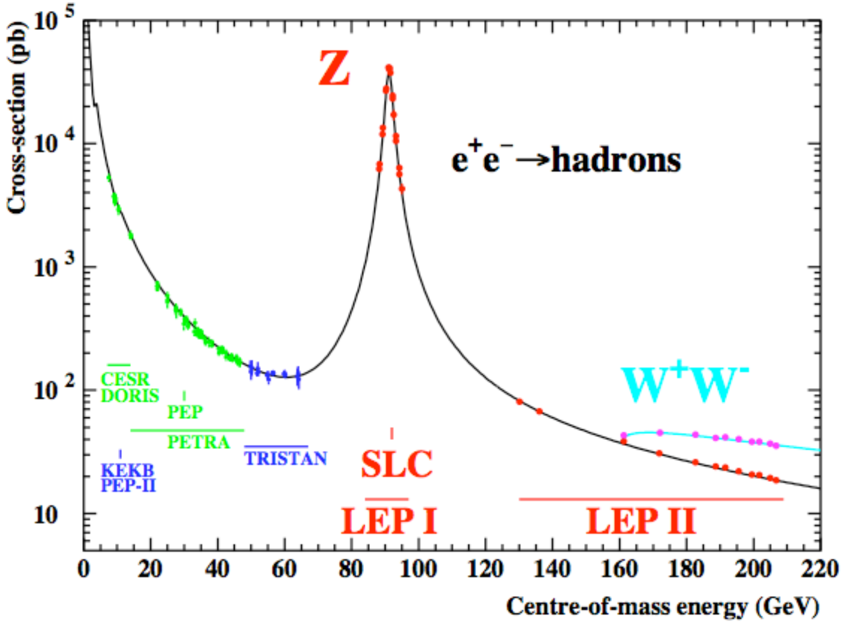
$$G_F = \frac{\pi\alpha}{\sqrt{2}M_W^2 \sin^2\theta_W}(1+\Delta r)$$

$$\Delta r = -\frac{3G_F m_t^2}{8\sqrt{2}\pi^2}\frac{\cos^2\theta_W}{\sin^2\theta_W} + \frac{11 G_F M_W^2}{24\sqrt{2}\pi^2}\log\frac{M_h^2}{M_W^2}$$

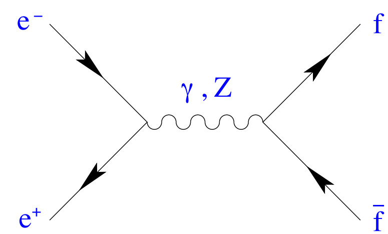


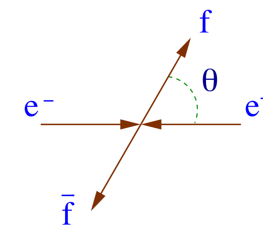


LEP



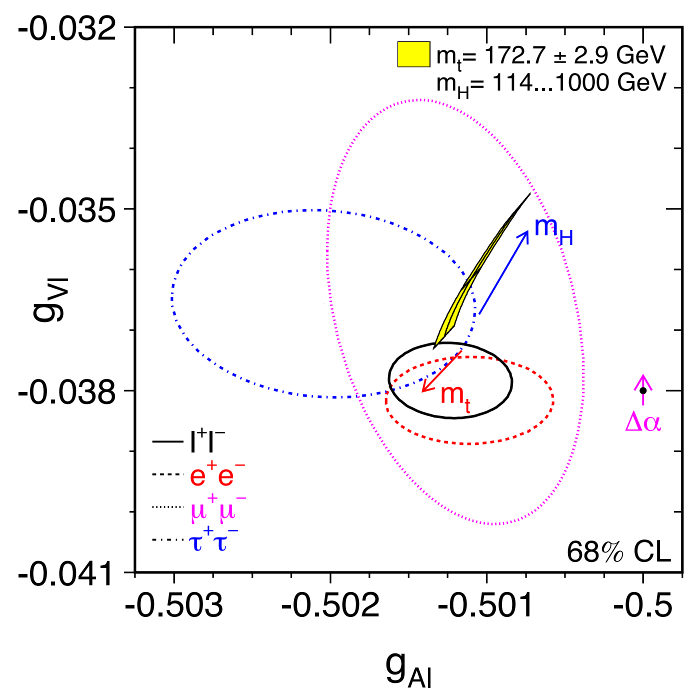
10 million Z's





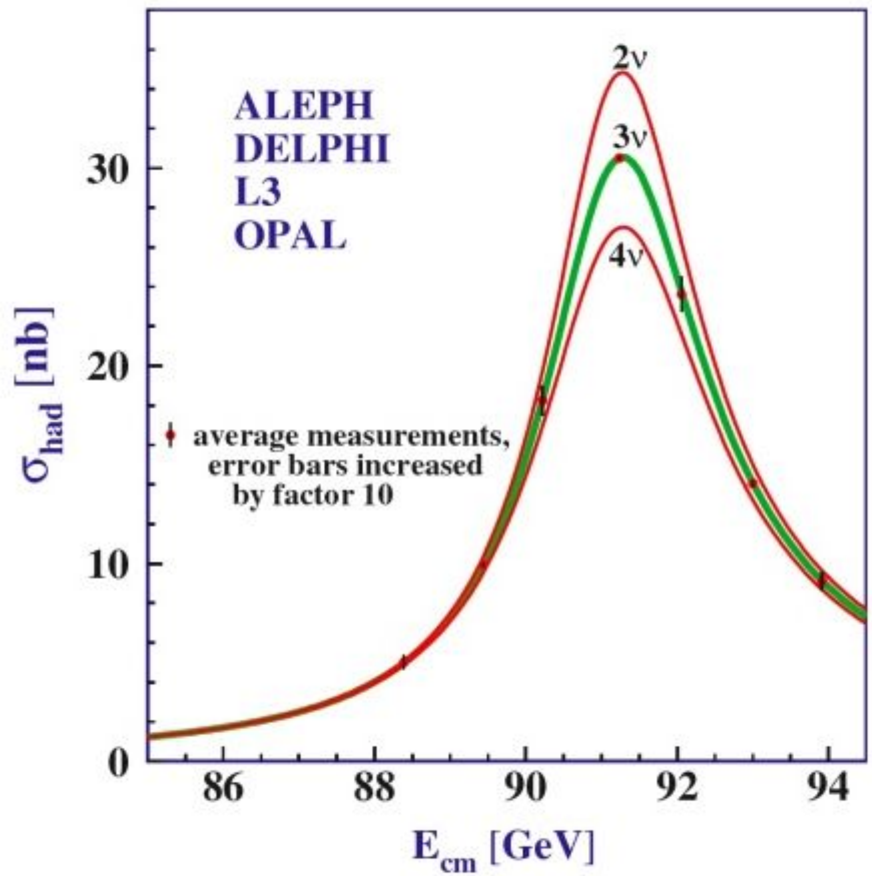
$$BR(Z \to f\bar{f}) = \frac{\Gamma_f}{\Gamma_Z}$$

$$M_Z, \Gamma_Z, \sigma_f(M_Z) = \frac{12\pi}{M_Z^2}\frac{\Gamma_e \Gamma_f}{\Gamma_Z^2}$$



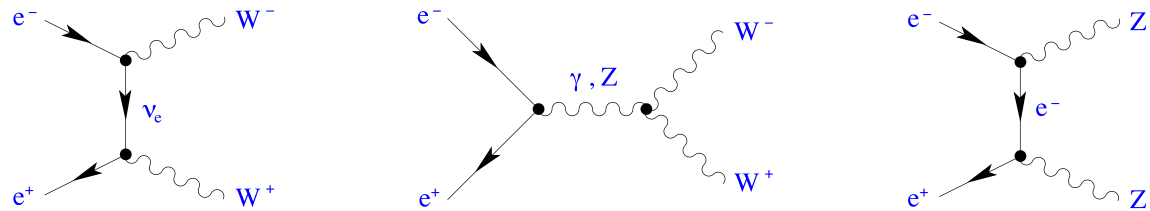
LEP: 3 flavours/families

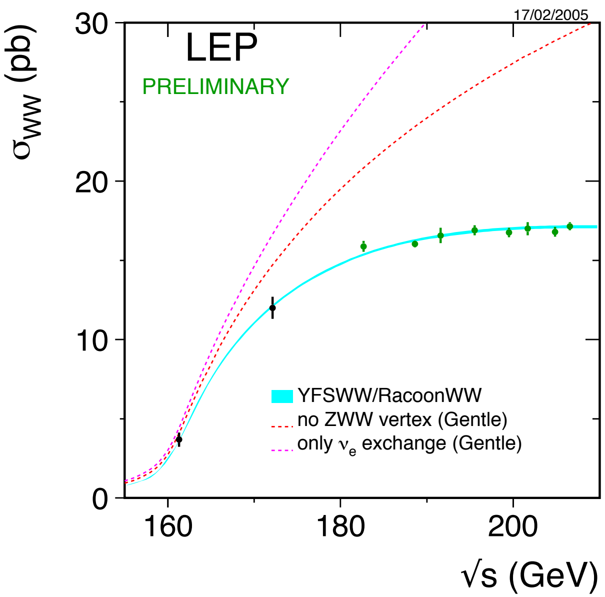
$$N_\nu = \frac{\Gamma_{\rm inv}}{\Gamma_{\nu\bar{\nu}}} = 2.984 \pm 0.008$$

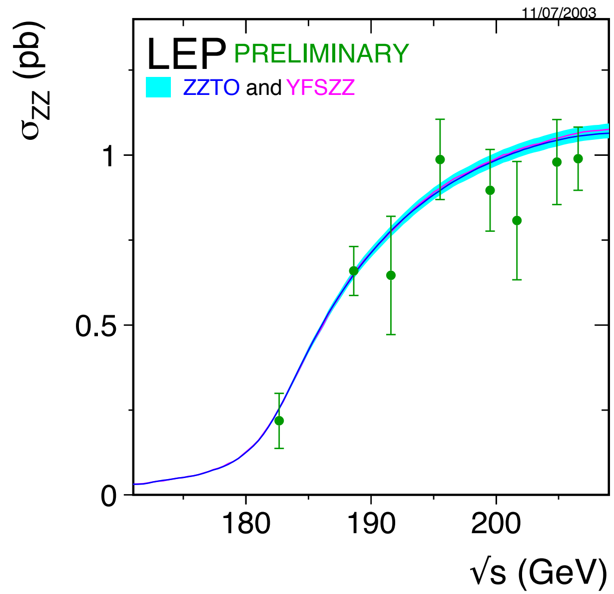


Thanks to the lightest of neutrinos we know that no new heavier families will show up

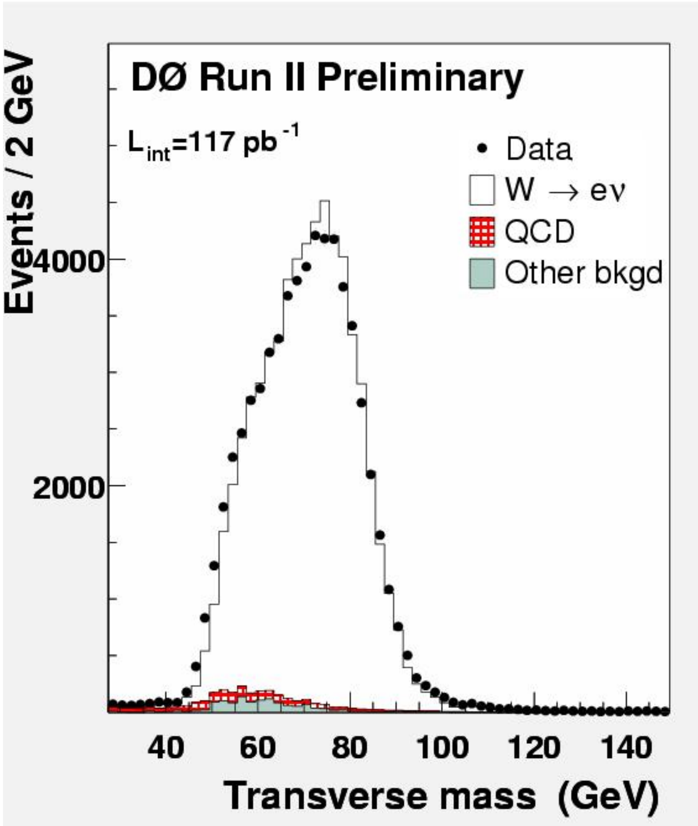
LEP: gauge boson selfcouplings



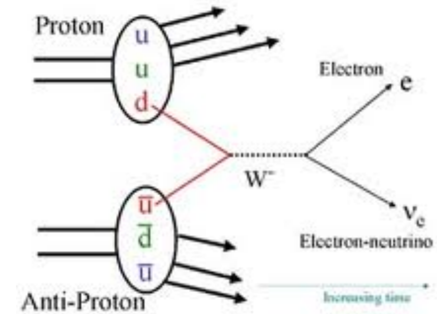


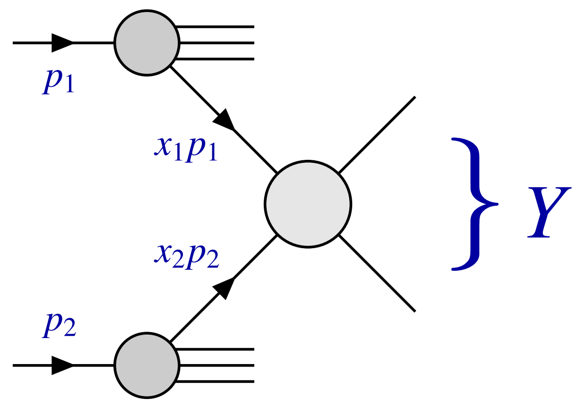


TEVATRON



O(10) million W's

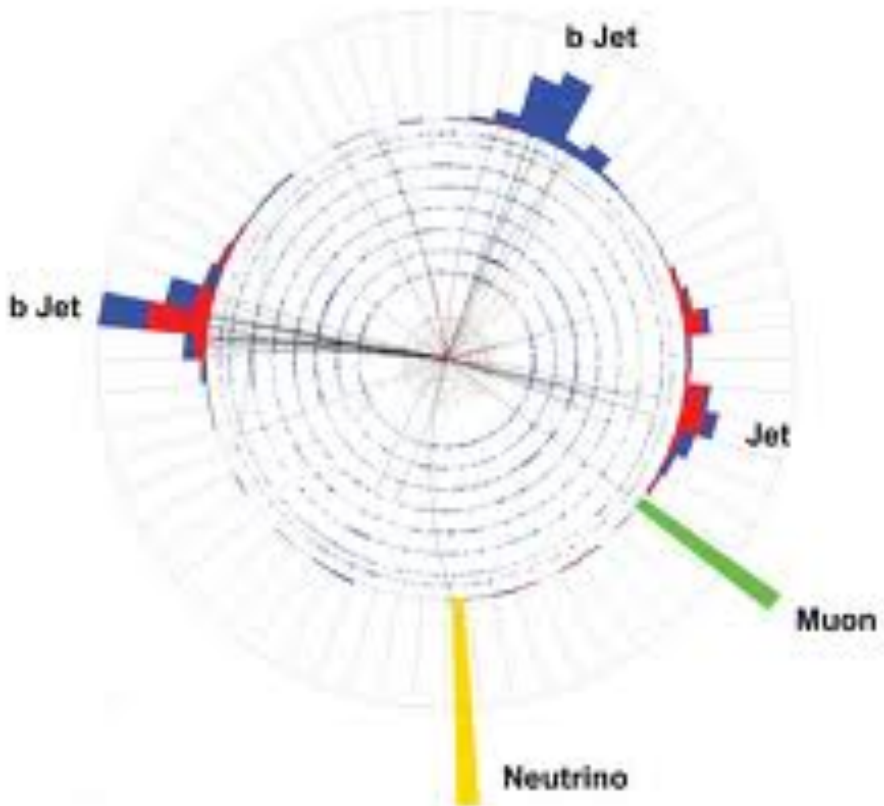


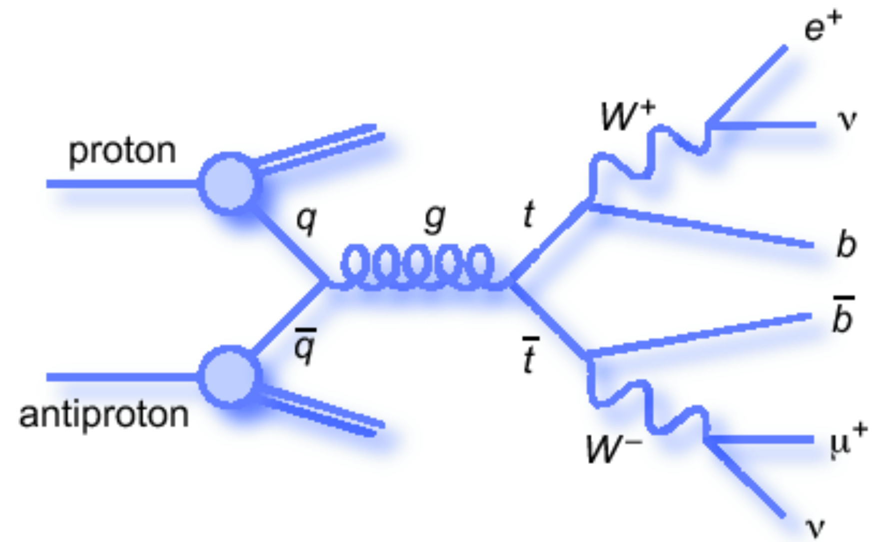


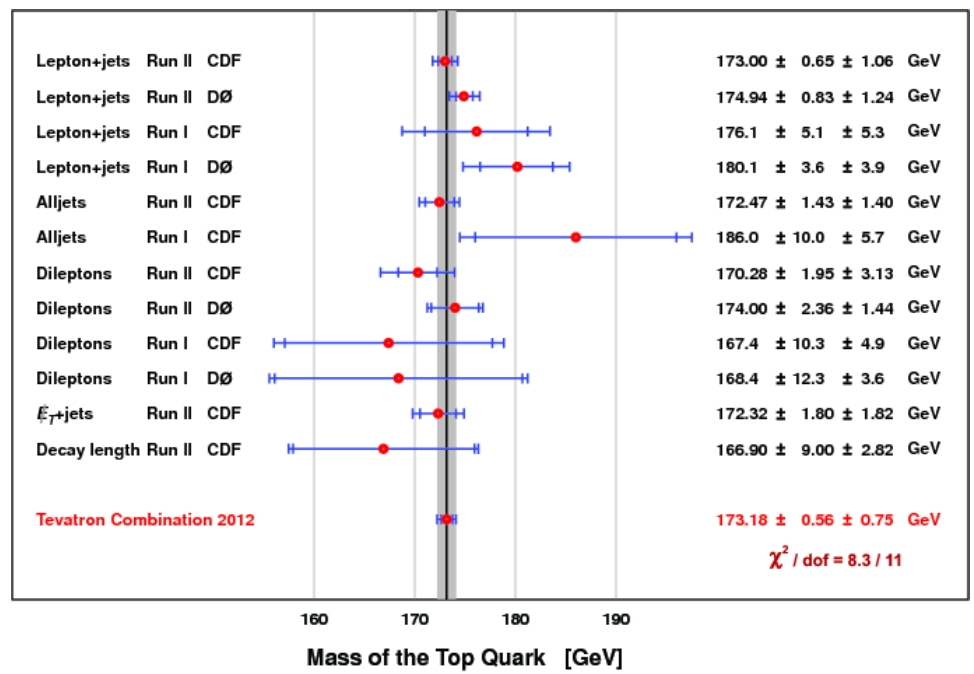
$$\sigma(h_1 + h_2 \to Y + X) = \int_{x_1} \int_{x_2} \sum_{f_1, f_2} f_{p_1}(x_1) f_{p_2}(x_2) \sigma(f_1 + f_2 \to Y)$$

$$M_W, \Gamma_W, BR's$$

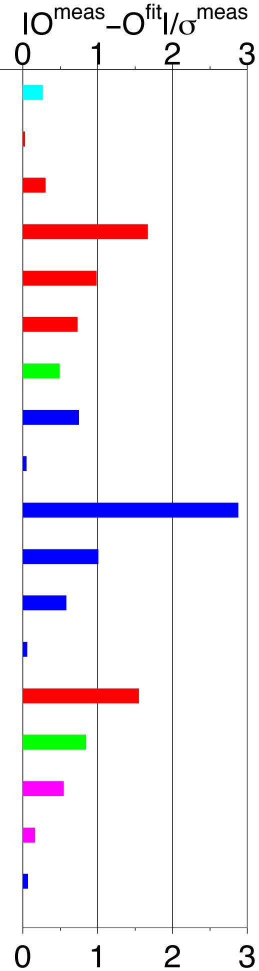
TEVATRON: top quark







	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02750 ± 0.00033	0.02759
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ^0_{had} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1038
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.385 ± 0.015	80.377
Γ_W [GeV]	2.085 ± 0.042	2.092
m_t [GeV]	173.20 ± 0.90	173.26

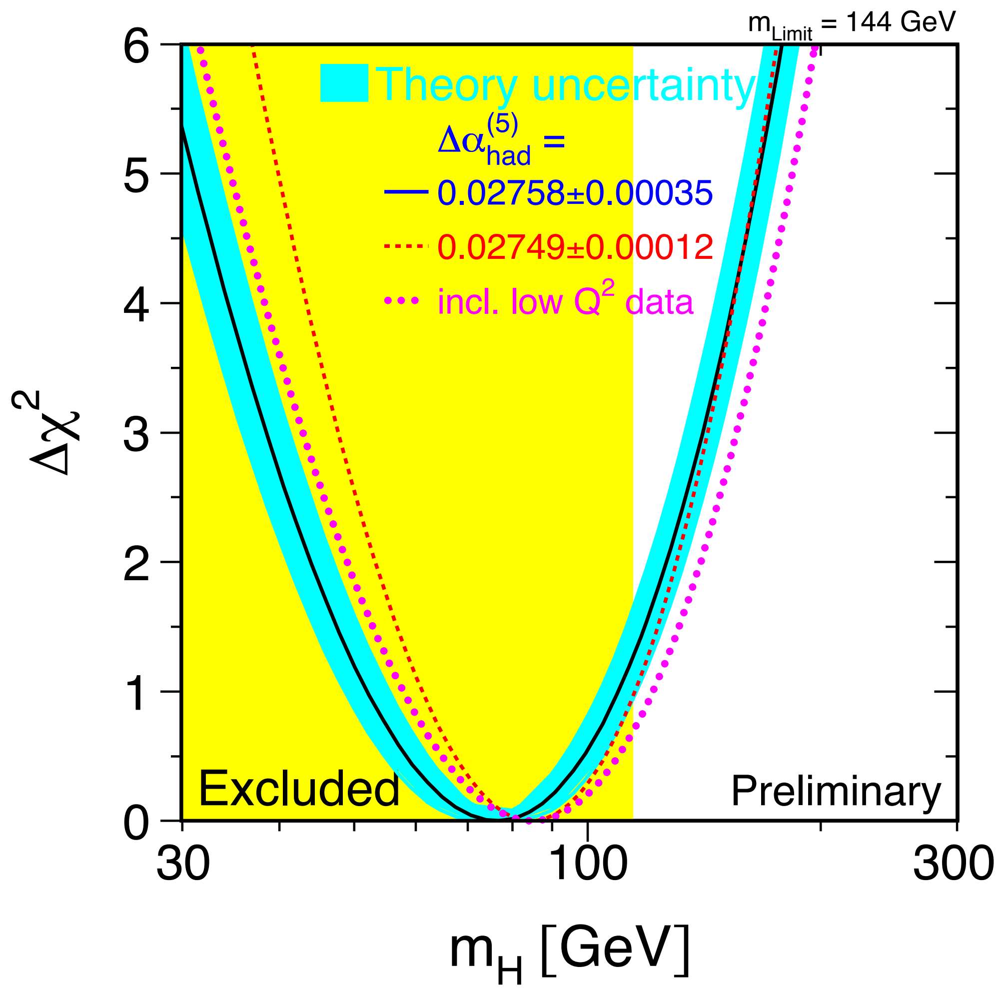


March 2012

$$\mathcal{A}^f = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$

$$\mathcal{A}^f_{FB} = \frac{\int_0^1 d\cos\theta \frac{d\sigma_f}{d\cos\theta} - \int_{-1}^0 d\cos\theta \frac{d\sigma_f}{d\cos\theta}}{\int_{-1}^1 d\cos\theta \frac{d\sigma_f}{d\cos\theta}}$$

Higgs mass before the discovery



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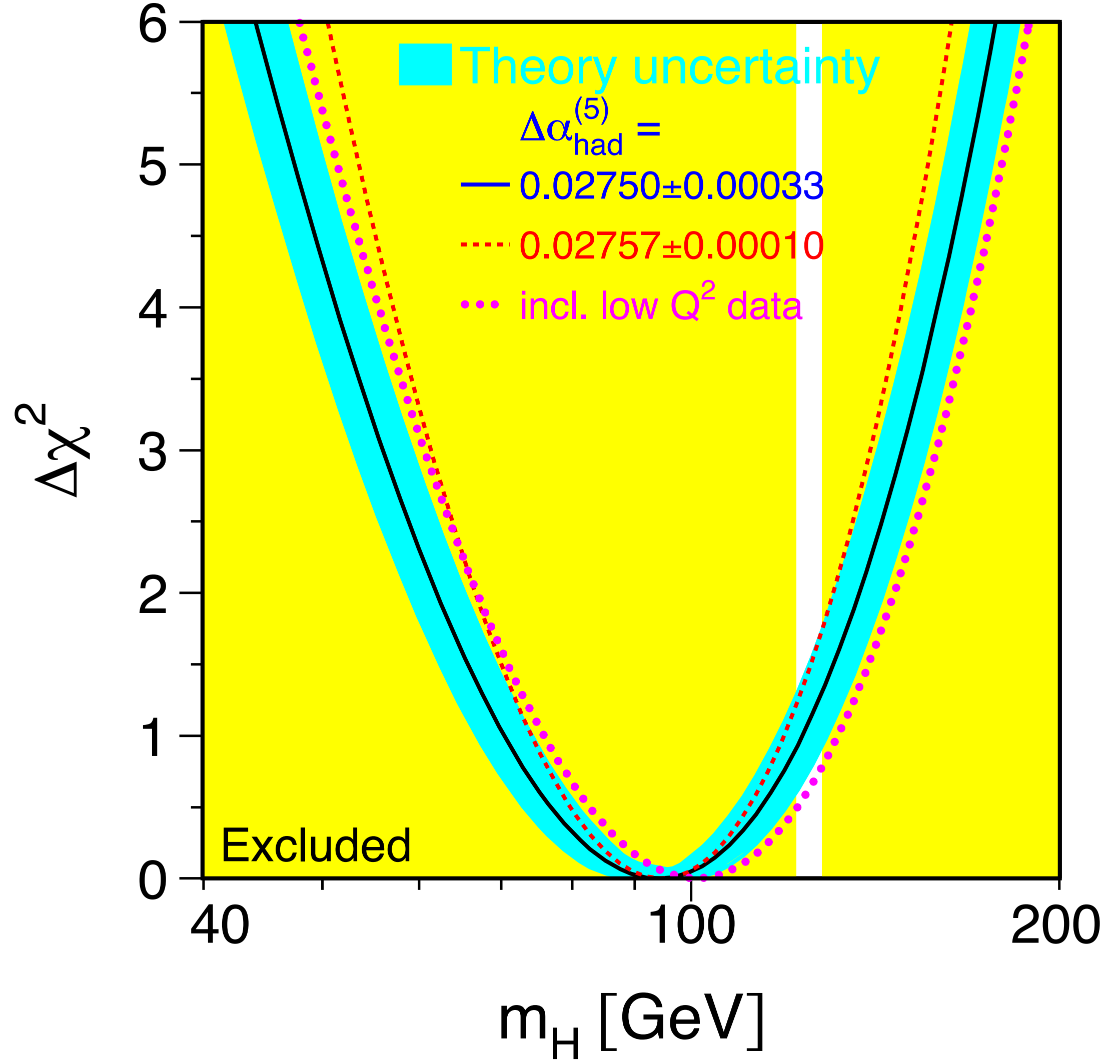


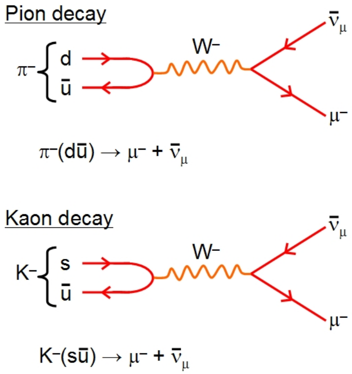
Flavour Precision Physics

$$
|V_{CKM}| = \begin{pmatrix}
\begin{array}{l}|V_{ud}| \\ \text{Nuclear } \beta \text{ decay}\end{array} & \begin{array}{l}|V_{us}| \\ K \to \pi l \nu,\ K, \pi \to l \nu\end{array} & \begin{array}{l}|V_{ub}|^2 \\ B \to \pi l \nu\end{array} \\
\begin{array}{l}V_{cd}| \\ D \to \pi l \nu,\ \nu d \to c X\end{array} & \begin{array}{l}|V_{cs}| \\ D \to K l \nu,\ W^+ \to c \bar{s}\end{array} & \begin{array}{l}|V_{cb}|^2 \\ B \to D l \nu, b \to c l \nu\end{array} \\
\begin{array}{l}|V_{td}| \\ \text{loops}\end{array} & \begin{array}{l}|V_{ts}| \\ \text{loops}\end{array} & \begin{array}{l}|V_{tb}| \\ p \bar{p} \to t b + X\end{array}
\end{pmatrix}
$$

Extract precision physics from hadronic observables is a major achievement!

Flavour Precision Physics

One example: a precise determination of |Vus|/|Vud| comes from comparing K, pi leptonic decays:



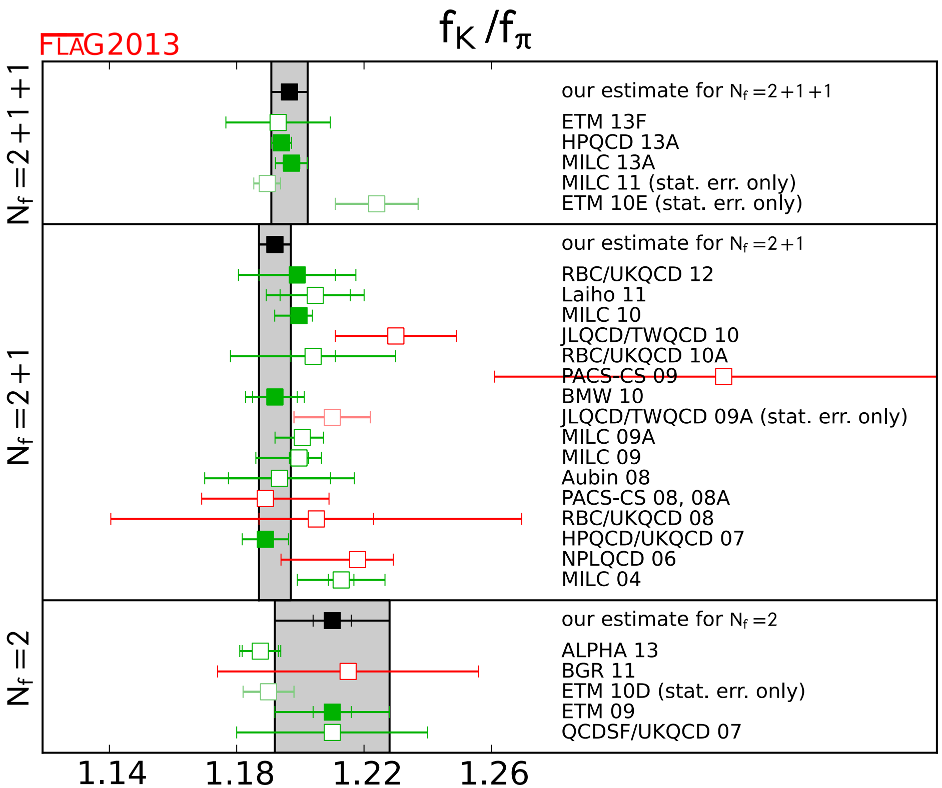
$$\mathcal{A}(M \to \mu\nu) \propto G_F \langle \mu\nu | \bar{\mu}\gamma_\mu(1-\gamma_5)\nu|0\rangle \underbrace{\langle 0|\bar{q}\gamma_\mu(1-\gamma_5)q|M(q)\rangle}_{i f_M q_\mu}$$

$$\frac{\Gamma(K\to\mu\nu)}{\Gamma(\pi\to\mu\nu)} = \frac{|V_{us}|^2}{|V_{ud}|^2}\frac{f_K^2}{f_\pi^2}\frac{m_K(1-m_l^2/m_K^2)^2}{m_\pi(1-m_l^2/m_\pi^2)^2}(1+\delta_{EM})$$

requires a non-perturbative evaluation

Extract precision physics from hadronic observables is a major achievement!

Flavour Precision Physics



FLAG WG

Percent level non-perturbative determination

Extract precision physics from hadronic observables is a major achievement!

Flavour Precision Physics

Phases of CKM: only one for three families.

We can formulate the criterium for CP violation in the quark sector in terms of a basis independent invariant:

$$\mathrm{Im}\left\{\det[Y_u Y_u^\dagger, Y_d Y_d^\dagger]\right\} \neq 0$$

Jarkskog 85

$$\mathrm{Im}[V_{ij}V_{ik}^* V_{lk} V_{lj}^*] = \mathcal{J} \sum_{m,n} \epsilon_{ilm}\epsilon_{jkn}$$

In terms of the usual parametrization:

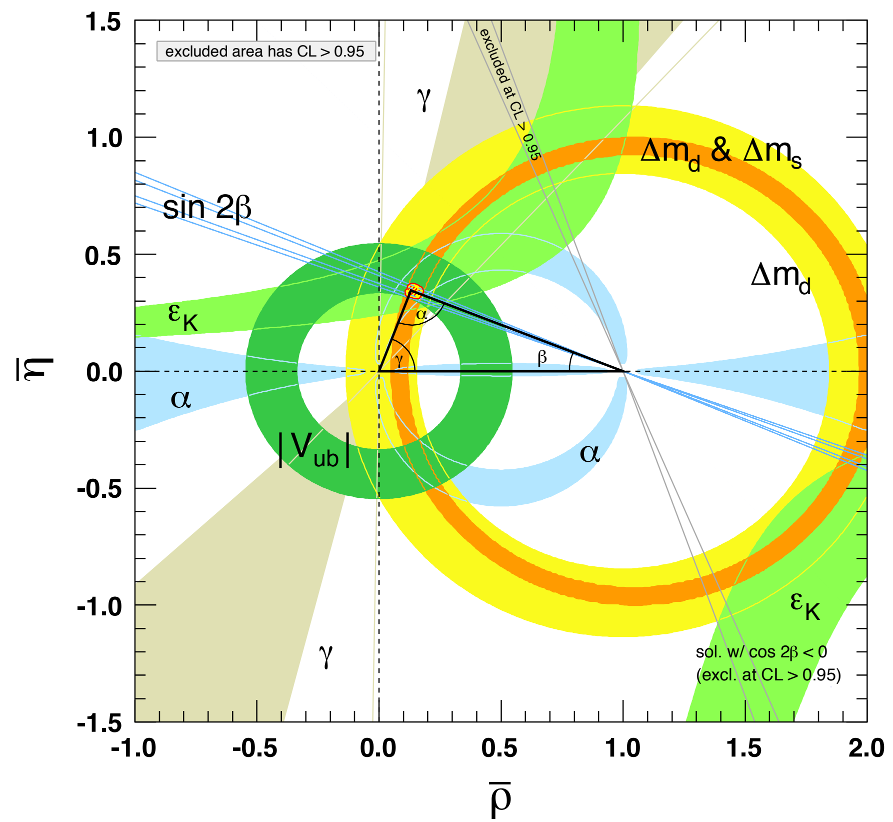
$$\mathcal{J} = c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13}\sin\delta$$

Flavour Precision Physics

The unitarity triangles: all have the same area J, but sides are different

$$\begin{aligned} \mathbf{V}_{ud}^*\mathbf{V}_{us} + \mathbf{V}_{cd}^*\mathbf{V}_{cs} + \mathbf{V}_{td}^*\mathbf{V}_{ts} &= 0\,, \\ \mathbf{V}_{us}^*\mathbf{V}_{ub} + \mathbf{V}_{cs}^*\mathbf{V}_{cb} + \mathbf{V}_{ts}^*\mathbf{V}_{tb} &= 0\,, \\ \mathbf{V}_{ub}^*\mathbf{V}_{ud} + \mathbf{V}_{cb}^*\mathbf{V}_{cd} + \mathbf{V}_{tb}^*\mathbf{V}_{td} &= 0\,. \end{aligned}$$

The last one has larger area/sides:
CP violation more significant in B sector



Angles:

$$\beta = \phi_1 = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right),$$

$$\alpha = \phi_2 = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right),$$

$$\gamma = \phi_3 = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right).$$