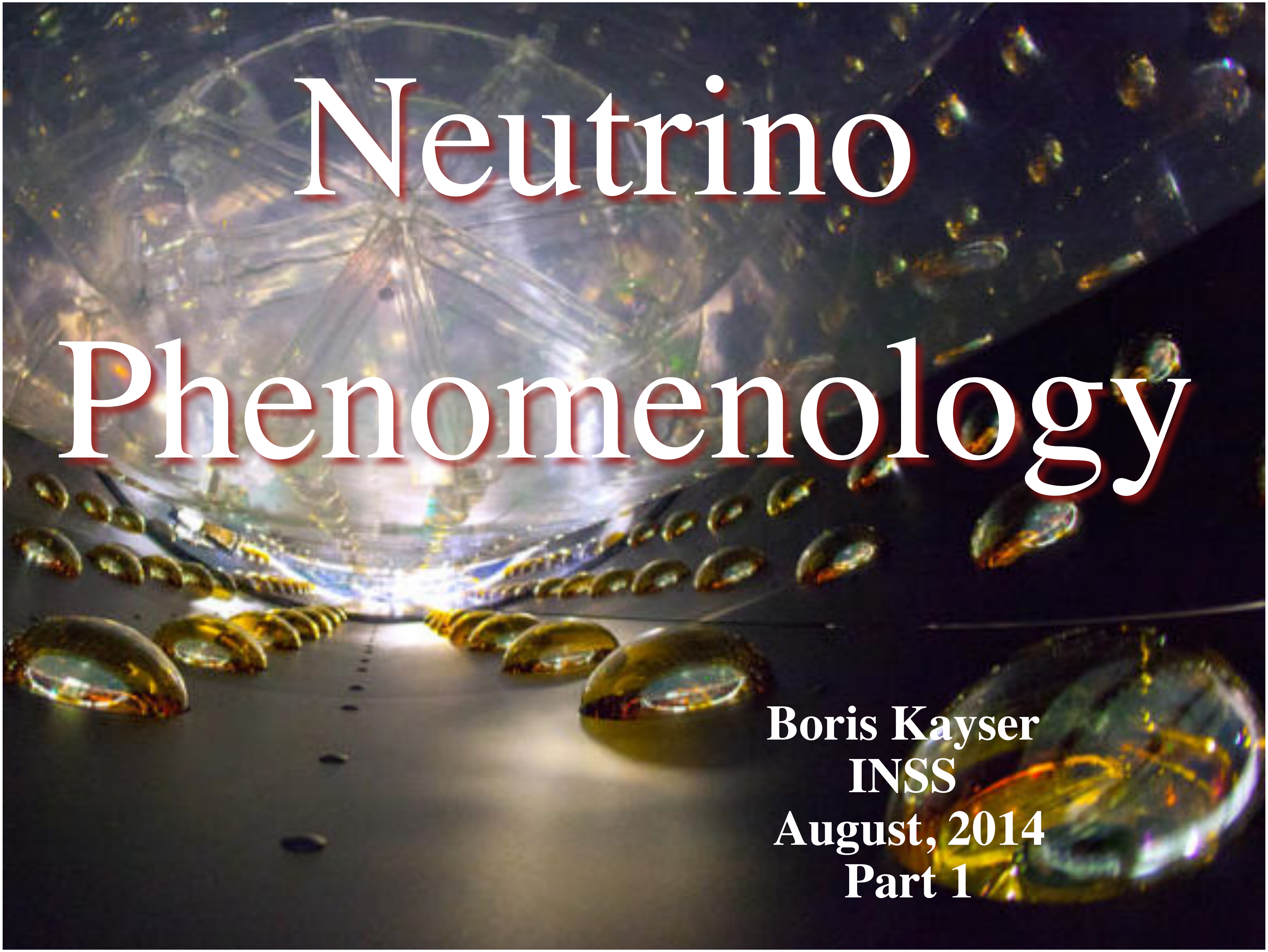

Neutrino Phenomenology

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INSS

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Part 1

What Are Neutrinos Good For?

Energy generation in the sun starts with the reaction —

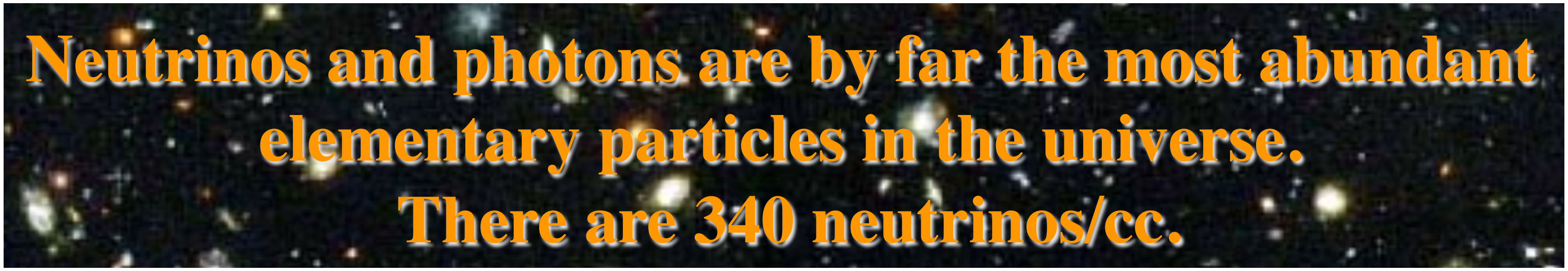
$$p + p \rightarrow d + e^{+} + \nu$$

Spin: $\frac{1}{2} \quad \frac{1}{2} \qquad 1 \quad \frac{1}{2} \qquad \frac{1}{2}$

Without the neutrino, angular momentum would not be conserved.

Uh, oh ……

The Neutrinos



The neutrinos are spin – 1/2, electrically neutral, leptons.

The only known forces they experience are the weak force and gravity.

This means that their interactions with other matter have very low strength.
Thus, neutrinos are difficult to detect and study.

Their weak interactions are successfully described by the Standard Model.

The Neutrino Revolution
(1998 – …)

Neutrinos have nonzero masses!

Leptons mix!

The Origin of Neutrino Mass

The fundamental constituents of matter
are the ***quarks***, the ***charged leptons***,
and the ***neutrinos***.

Most theorists strongly suspect that the origin of the neutrino masses is different from the origin of the quark and charged lepton masses.

The Standard-Model ***Higgs field*** may still be involved,
but not in the same way
as for the quarks and charged leptons.

More later ………

The discovery of neutrino mass
and leptonic mixing
comes from the observation of
neutrino flavor change
(neutrino oscillation).

The Physics of Neutrino Oscillation

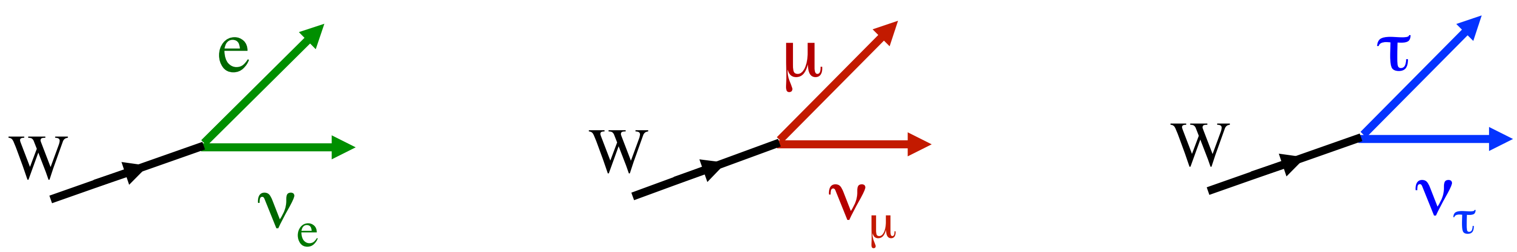
— Preliminaries

The Neutrino Flavors

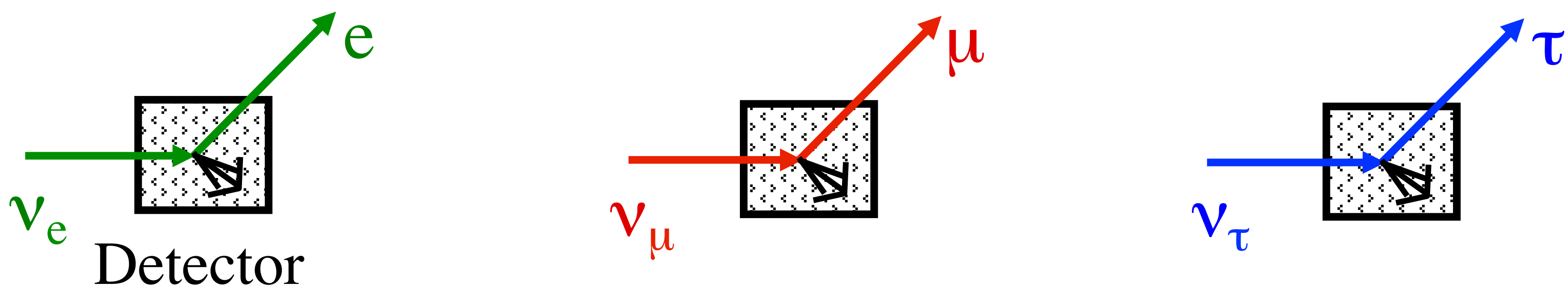
There are three flavors of charged leptons: e , μ , τ

There are three known flavors of neutrinos: ν_e, ν_μ, ν_τ

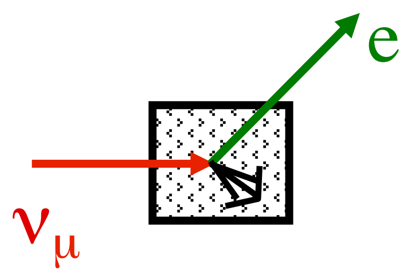
We ***define*** the neutrinos of specific flavor, ν_e, ν_μ, ν_τ, by W boson decays:



As far as we know, when a neutrino of given flavor interacts and turns into a charged lepton, that charged lepton will always be of the same flavor as the neutrino.



but not

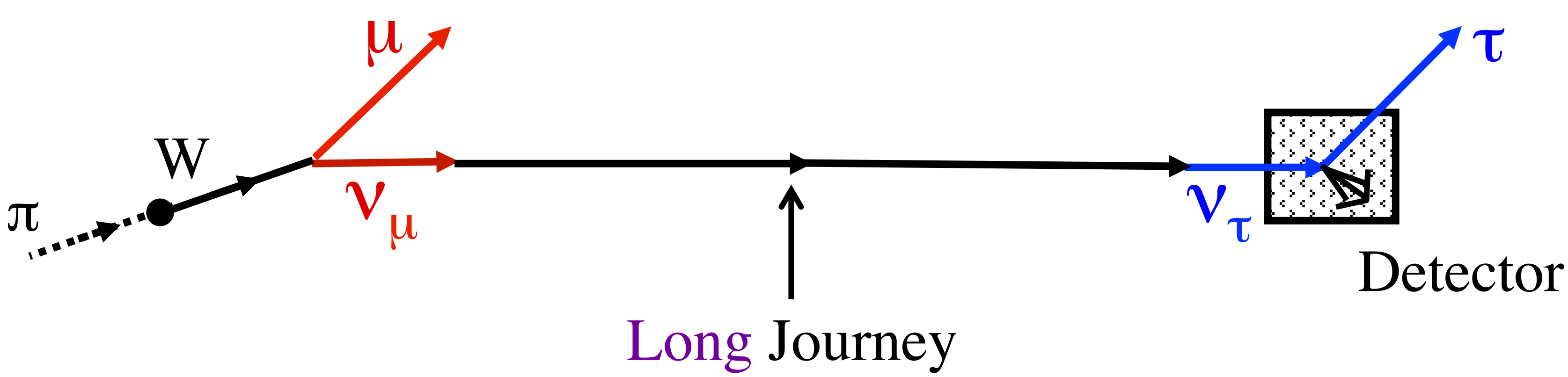


Lederman
Schartz
Steinberger

The weak interaction couples the neutrino of a given flavor only to the charged lepton of the same flavor.

Neutrino Flavor Change ("Oscillation")

If neutrinos have masses, and leptons mix, we can have —



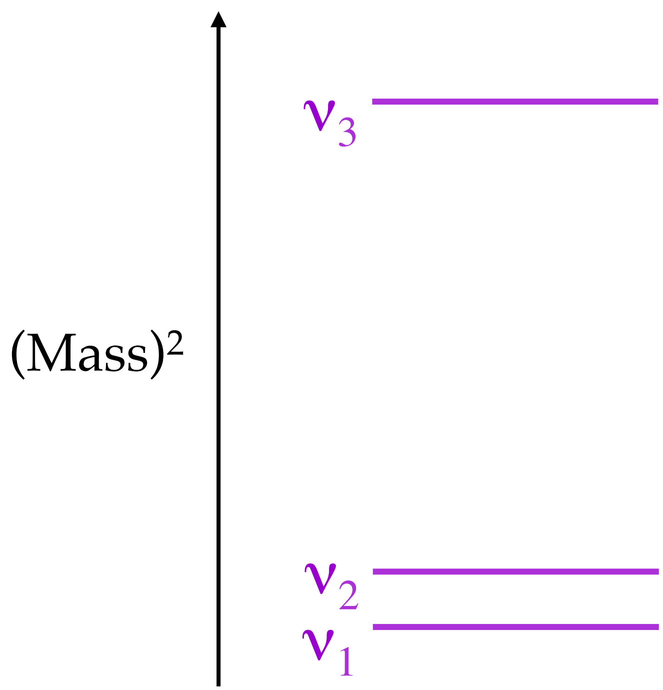
Give a ν time to change character, and you can have

for example: $\nu_\mu \longrightarrow \nu_\tau$

The last 16 years have brought us compelling evidence that such flavor changes actually occur.

Flavor Change Requires ***Neutrino Masses***

There must be some spectrum of neutrino mass eigenstates ν_i:



Mass (ν_i) $\equiv m_i$

Flavor Change Requires ***Leptonic Mixing***

The neutrinos $\nu_{e,\mu,\tau}$ of definite flavor

$(W \rightarrow e\nu_e$ or $\mu\nu_\mu$ or $\tau\nu_\tau)$

must be superpositions of the mass eigenstates:

$$|\nu_\alpha> = \sum_i U^*_{\alpha i} \, |\nu_i> \quad .$$

Neutrino of flavor
$\alpha = e, \mu$, or τ

Neutrino of definite mass m_i

"PMNS" Leptonic Mixing Matrix

Notation: ℓ denotes a charged lepton. $\ell_e \equiv e$, $\ell_\mu \equiv \mu$, $\ell_\tau \equiv \tau$.

Since the only charged lepton ν_α couples to is ℓ_α,
the 3 ν_α must be orthogonal.

To make up 3 orthogonal ν_α, we must have at least 3 ν_i.
Unless some ν_i masses are degenerate,
all ν_i will be orthogonal.

Then —

$$\delta_{\alpha\beta} = \langle \nu_\alpha | \nu_\beta \rangle = \left\langle \sum_i U^*_{\alpha i} \nu_i \middle| \sum_j U^*_{\beta j} \nu_j \right\rangle$$

$$= \sum_{i,j} U_{\alpha i} U^*_{\beta j} \langle \nu_i | \nu_j \rangle = \sum_i U_{\alpha i} U^*_{\beta i}$$

If there are ***only*** 3 ν_i, U is unitary.

Leptonic mixing is easily incorporated into the Standard Model (SM) description of the $\ell\nu W$ interaction.

For this interaction, we then have —

Semi-weak coupling

Left-handed

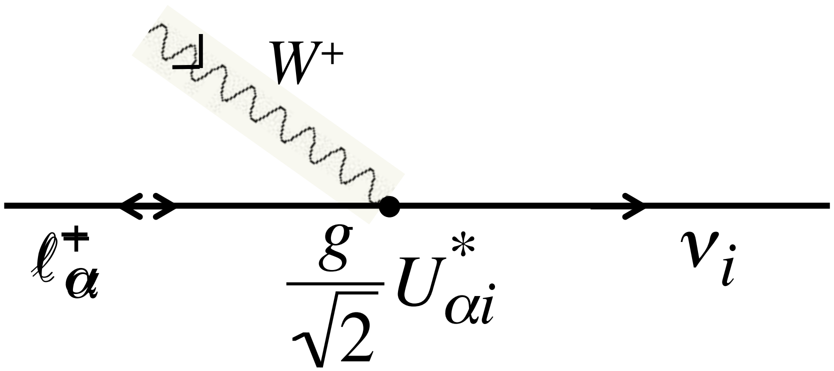
$$\mathcal{L}_{SM} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \left(\overline{\ell}_{L\alpha} \gamma^{\lambda} \nu_{L\alpha} W_{\lambda}^{-} + \overline{\nu}_{L\alpha} \gamma^{\lambda} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

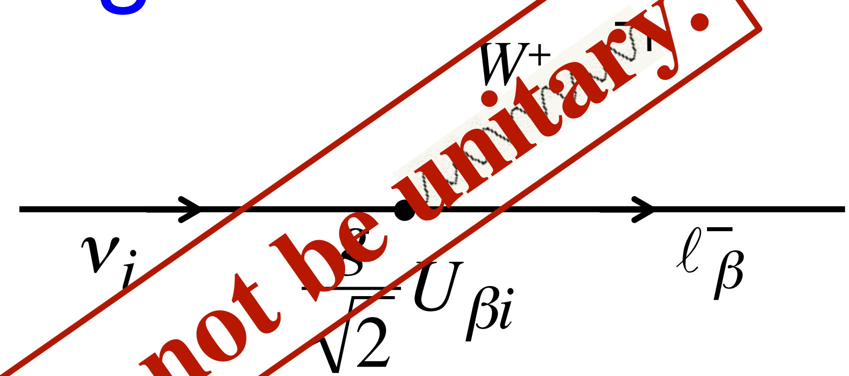
$$= -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} \left(\overline{\ell}_{L\alpha} \gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}^{-} + \overline{\nu}_{Li} \gamma^{\lambda} U_{\alpha i}^{*} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

Taking mixing into account

The SM interaction conserves the Lepton Number L, defined by $L(\nu) = L(\ell^{-}) = -L(\overline{\nu}) = -L(\ell^{+}) = 1$.

The Meaning of *U*





Caution: The 3 x 3 *U* may not be unitary.

$$U = \begin{array}{c} \\ e \\ \mu \\ \tau \end{array} \!\!\! \begin{array}{c} \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \end{array}$$

The e row of U: The linear combination of neutrino mass eigenstates that couples to e.

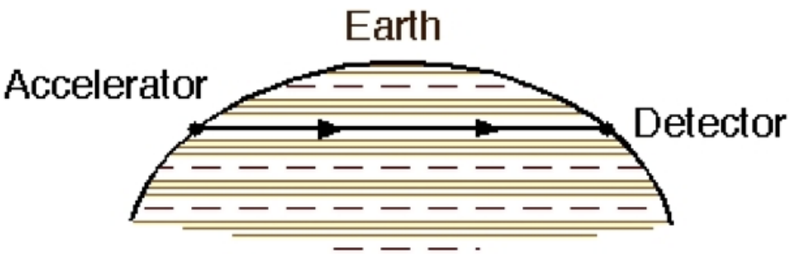
The ν_1 column of U: The linear combination of charged-lepton mass eigenstates that couples to ν_1 .

Slides on

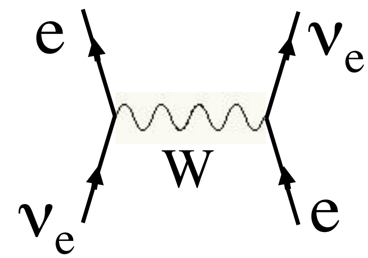
The Physics of Neutrino Oscillation

go here.

Neutrino Flavor Change In Matter



involves



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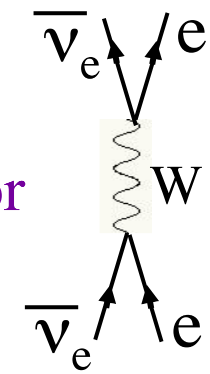


Coherent forward scattering via this
W-exchange interaction leads to
an extra interaction potential energy —

$$V_W = \begin{cases} +\sqrt{2}G_F N_e, & \nu_e \\ -\sqrt{2}G_F N_e, & \overline{\nu}_e \end{cases}$$

Fermi constant — Electron density

This raises the effective mass of ν_e, and lowers that of $\overline{\nu}_e$.

The fractional importance of matter effects on an oscillation involving a vacuum splitting Δm^2 is —

$$\underbrace{[\sqrt{2}G_F N_e]}_{\text{Interaction energy}} \,/\, \underbrace{[\Delta m^2/2E]}_{\text{Vacuum energy}} \equiv x \,.$$

The matter effect —

— Grows with neutrino energy E

— Is sensitive to Sign(Δm^2)

— Reverses when ν is replaced by $\bar{\nu}$

This last is a "fake CP violation", but the matter effect is negligible when $x \ll 1$.

Evidence For Flavor Change

Neutrinos	Evidence of Flavor Change
Solar	Compelling
Reactor (Long-Baseline)	Compelling
Atmospheric	Compelling
Accelerator (Long-Baseline)	Compelling
Accelerator & Reactor (Short-Baseline)	"Interesting"

KamLAND Evidence for Oscillatory Behavior

The KamLAND detector studies $\overline{\nu}_e$ produced by Japanese nuclear power reactors ~ 180 km away.

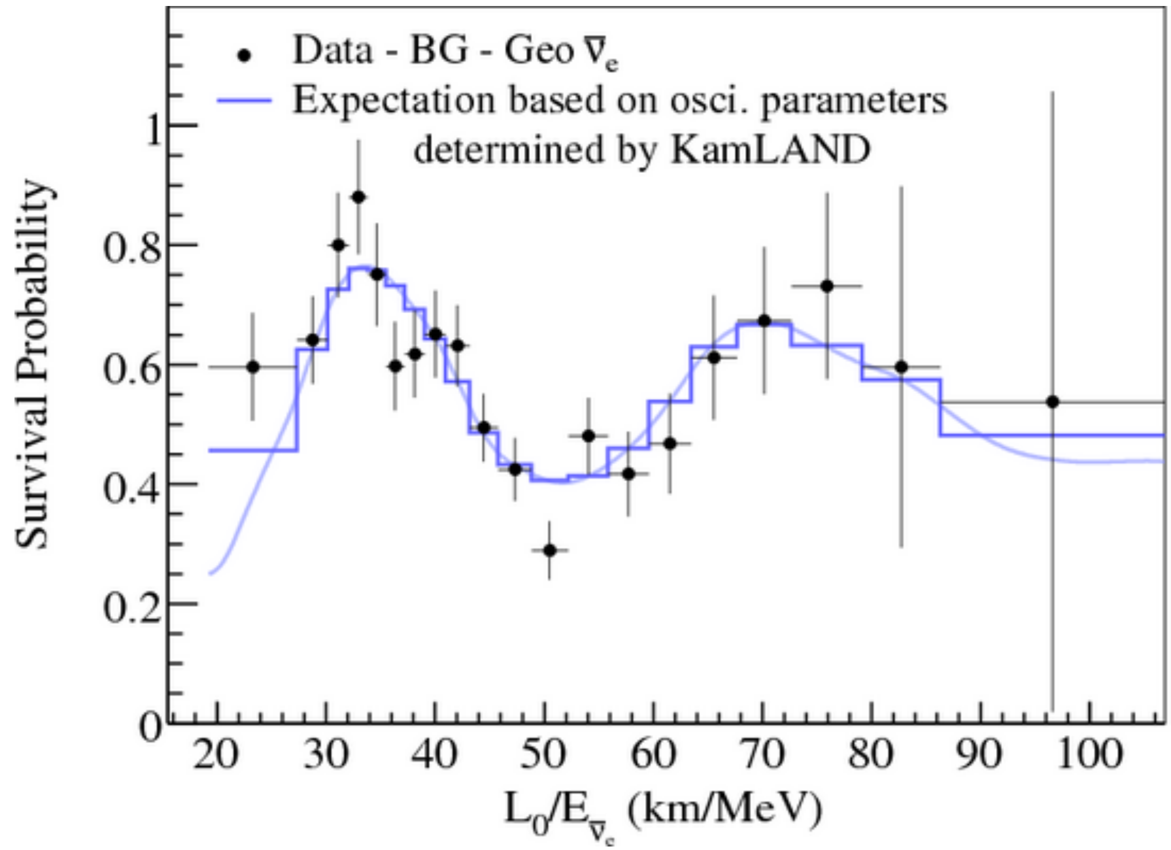
For KamLAND, $x_{Matter} < 10^{-2}$. Matter effects are negligible.

The $\bar{\nu}_e$ survival probability, $P(\bar{\nu}_e \to \bar{\nu}_e)$, should oscillate as a function of L/E following the vacuum oscillation formula.

In the two-neutrino approximation, we expect —

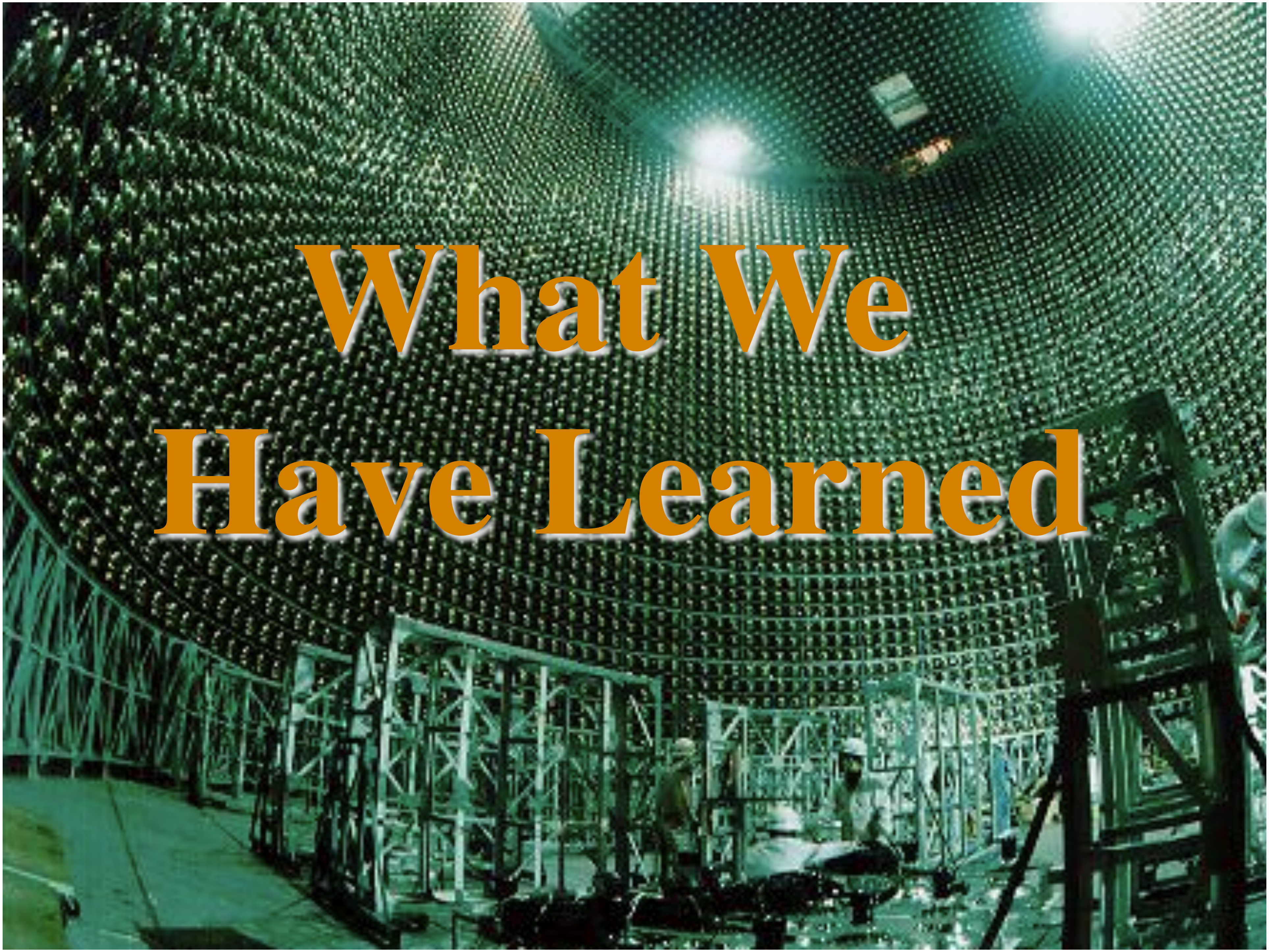
$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta \sin^2\left[1.27\Delta m^2\left(eV^2\right)\frac{L(km)}{E(GeV)}\right].$$

Survival probability $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ of reactor $\bar{\nu}_e$

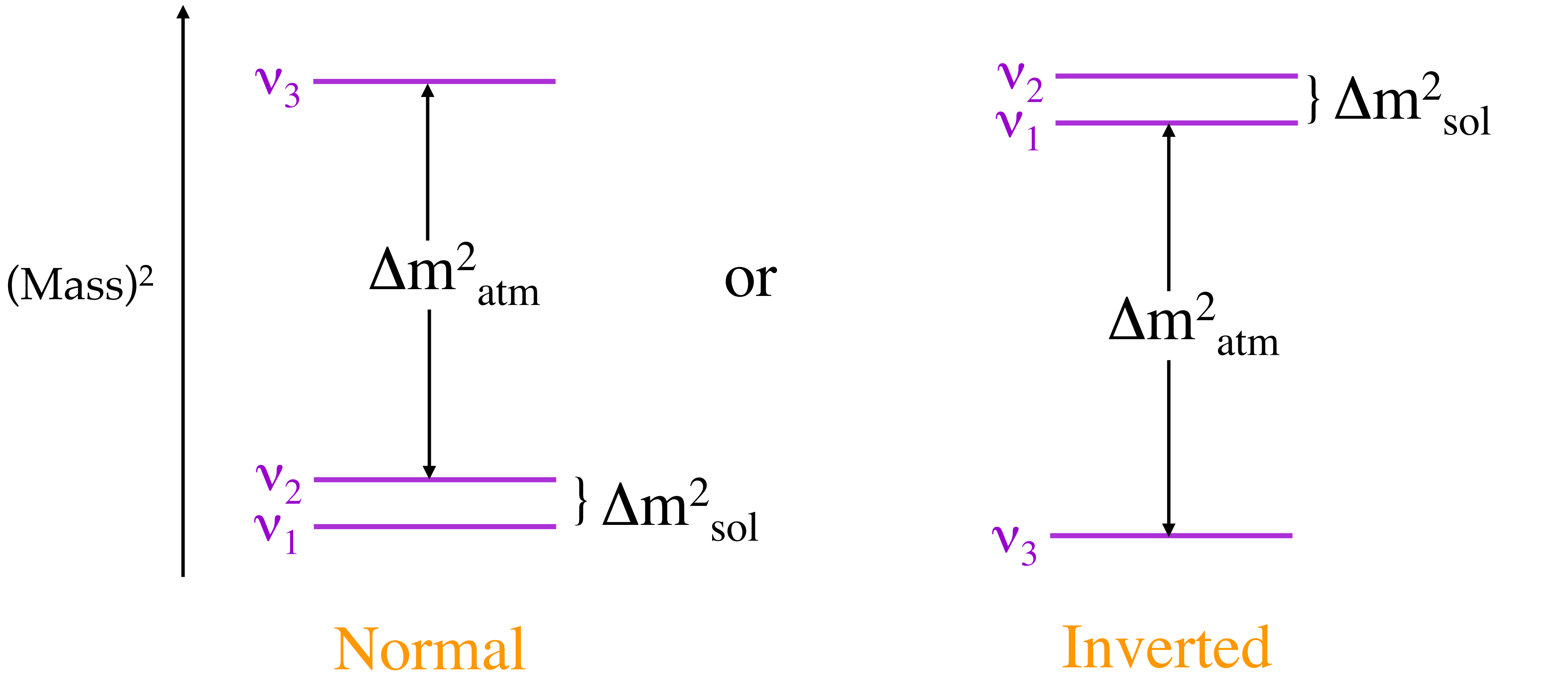


L_0 = 180 km is a flux-weighted average travel distance.

$P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ actually oscillates!

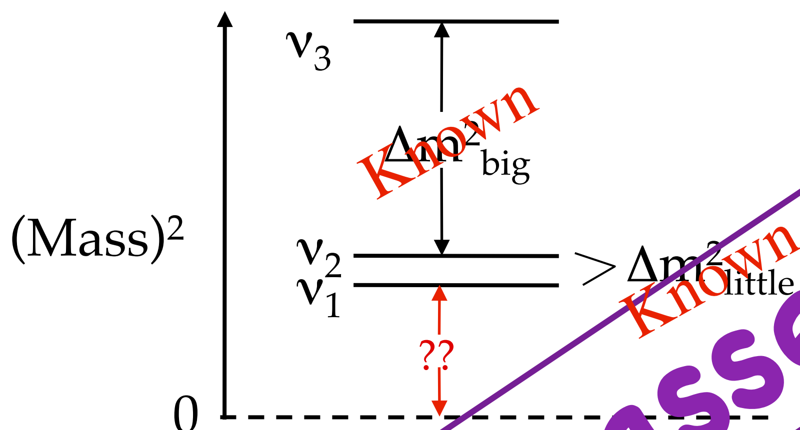

What We
Have Learned

The (Mass)^2 Spectrum



$\Delta m^2_{sol} \cong 7.5 \times 10^{-5}\ \mathrm{eV}^2$, $\quad \Delta m^2_{atm} \cong 2.4 \times 10^{-3}\ \mathrm{eV}^2$

Constraints On the Absolute Scale of Neutrino Mass



How far above zero is the whole pattern?

Neutrino masses are tiny

Cosmology, *under certain assumptions* ➡ $\sum m(\nu_i) < 0.23$ eV

All i

Tritium beta decay ➡ $\sqrt{0.69m^2(\nu_1)+0.29m^2(\nu_2)+0.02m^2(\nu_3)} < 2$ eV

Oscillation ➡ Mass[Heaviest ν_i] $> \sqrt{\Delta m^2_{big}} > 0.04$ eV

What Tritium β Decay Measures

Tritium decay: $\quad {}^3H \rightarrow {}^3He + e^- + \overline{\nu_i}\ ;\ i = 1, 2, \text{or } 3$

There are 3 distinct final states.

The amplitudes for the production of these 3 distinct final states contribute *incoherently*.

$$BR\left({}^3H \rightarrow {}^3He + e^- + \overline{\nu_i}\right) \propto \left|U_{ei}\right|^2$$

In ${}^3H \rightarrow {}^3He + e^- + \overline{\nu_i}$, the bigger m_i is, the smaller the maximum electron energy is.

There are 3 separate thresholds in the β energy spectrum.

The β energy spectrum is modified according to —

$$(E_0 - E)^2 \Theta[E_0 - E] \Rightarrow \sum_i |U_{ei}|^2 (E_0 - E)\sqrt{(E_0 - E)^2 - m_i^2}\,\Theta[(E_0 - m_i) - E]$$

Maximum β energy when there is no neutrino mass

β energy

Present experimental energy resolution is insufficient to separate the thresholds.

Measurements of the spectrum bound the average neutrino mass —

$$\langle m_\beta \rangle = \sqrt{\sum_i |U_{ei}|^2 m_i^2}$$

$$\boxed{\text{Presently: } \langle m_\beta \rangle < 2\text{ eV}}$$

Mainz & Troitzk

Leptonic Mixing

Mixing means that —

$$|\nu_\alpha> = \sum_i U^*_{\alpha i}\, |\nu_i> \quad .$$

Neutrino of flavor α = e, μ, or τ

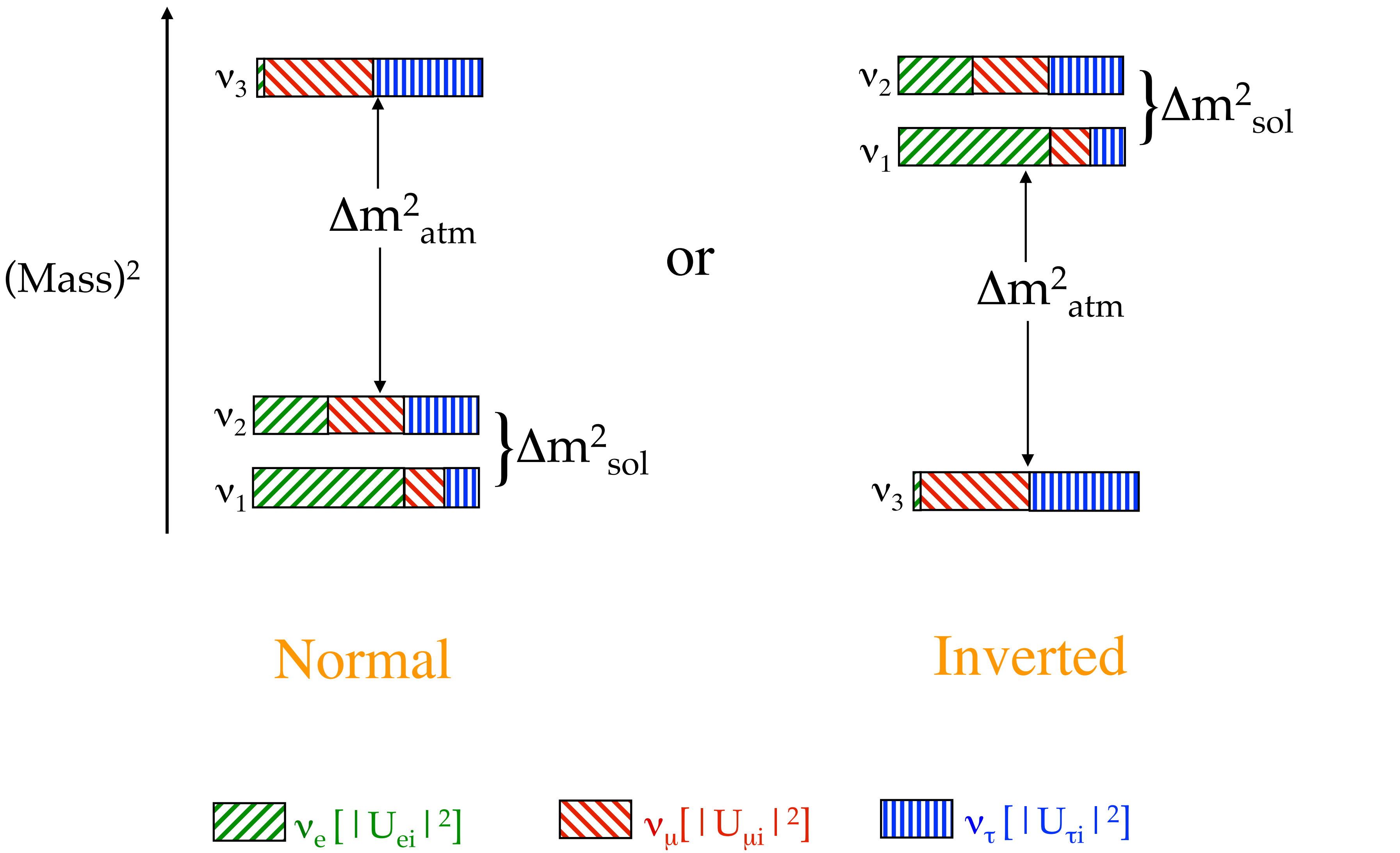
Neutrino of definite mass m_i

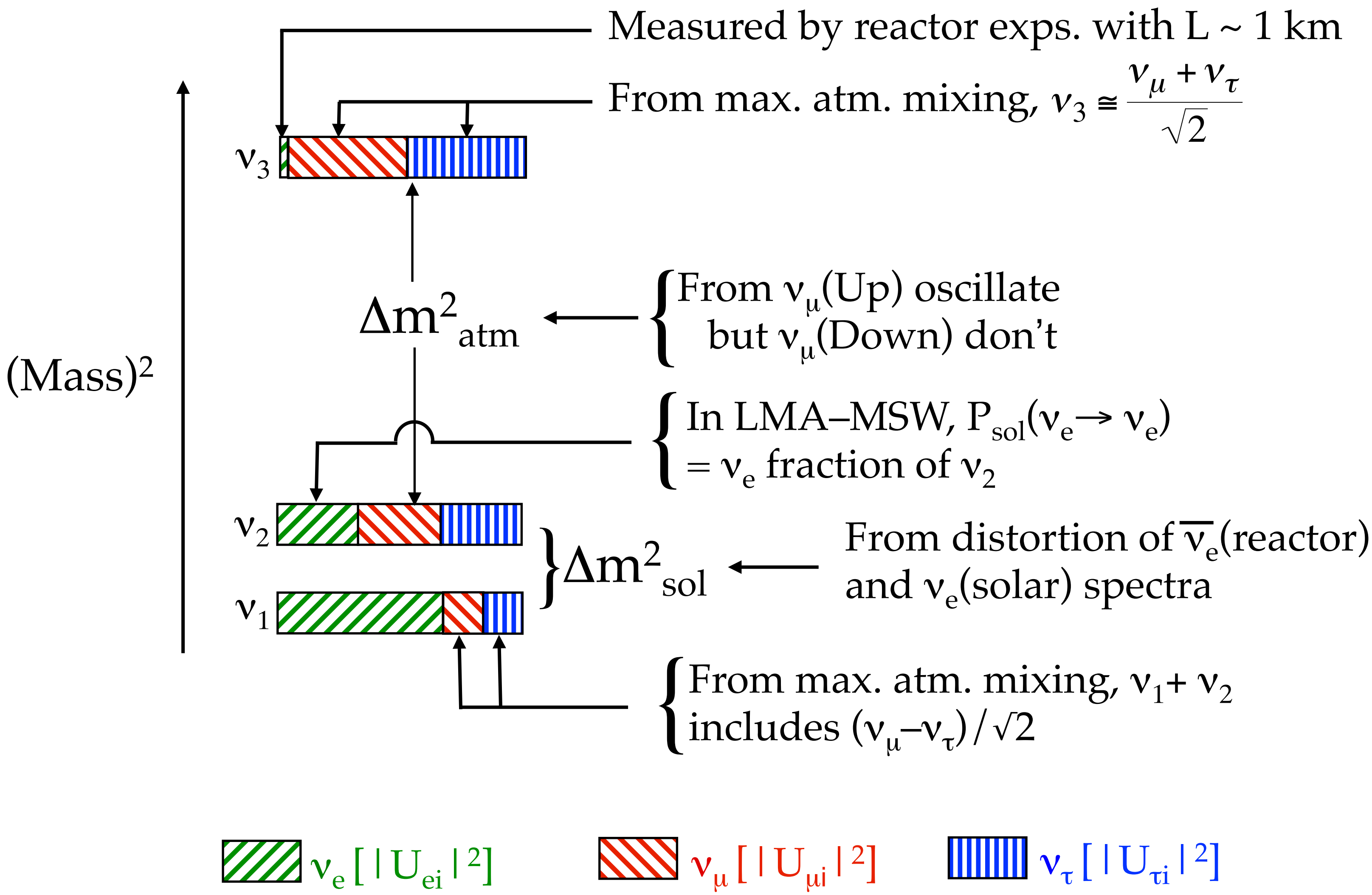
Inversely, $|\nu_i> = \sum_\alpha U_{\alpha i}\, |\nu_\alpha>$.

Flavor-α fraction of $\nu_i = |U_{\alpha i}|^2$.

When a ν_i interacts and produces a charged lepton, the probability that this charged lepton will be of flavor α is $|U_{\alpha i}|^2$.

Experimentally, the flavor fractions are —

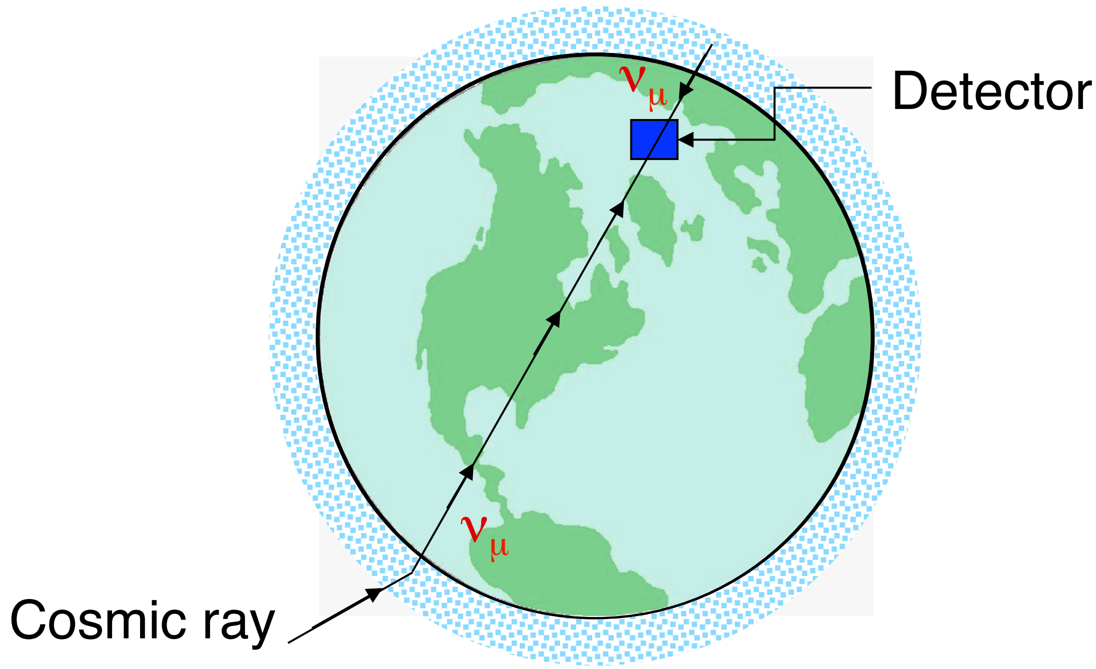




Measured by reactor exps. with L ~ 1 km
From max. atm. mixing, $\nu_3 \cong \frac{\nu_\mu + \nu_\tau}{\sqrt{2}}$
ν_3
Δm^2_{atm}
From ν_μ(Up) oscillate but ν_μ(Down) don't
(Mass)^2
In LMA–MSW, $P_{sol}(\nu_e \rightarrow \nu_e)$ = ν_e fraction of ν_2
ν_2
Δm^2_{sol}
From distortion of $\overline{\nu}_e$(reactor) and ν_e(solar) spectra
ν_1
From max. atm. mixing, $\nu_1 + \nu_2$ includes $(\nu_\mu - \nu_\tau)/\sqrt{2}$
$\nu_e\ [\,|U_{ei}|^2]$
$\nu_\mu\ [\,|U_{\mu i}|^2]$
$\nu_\tau\ [\,|U_{\tau i}|^2]$

Observations We Will Use

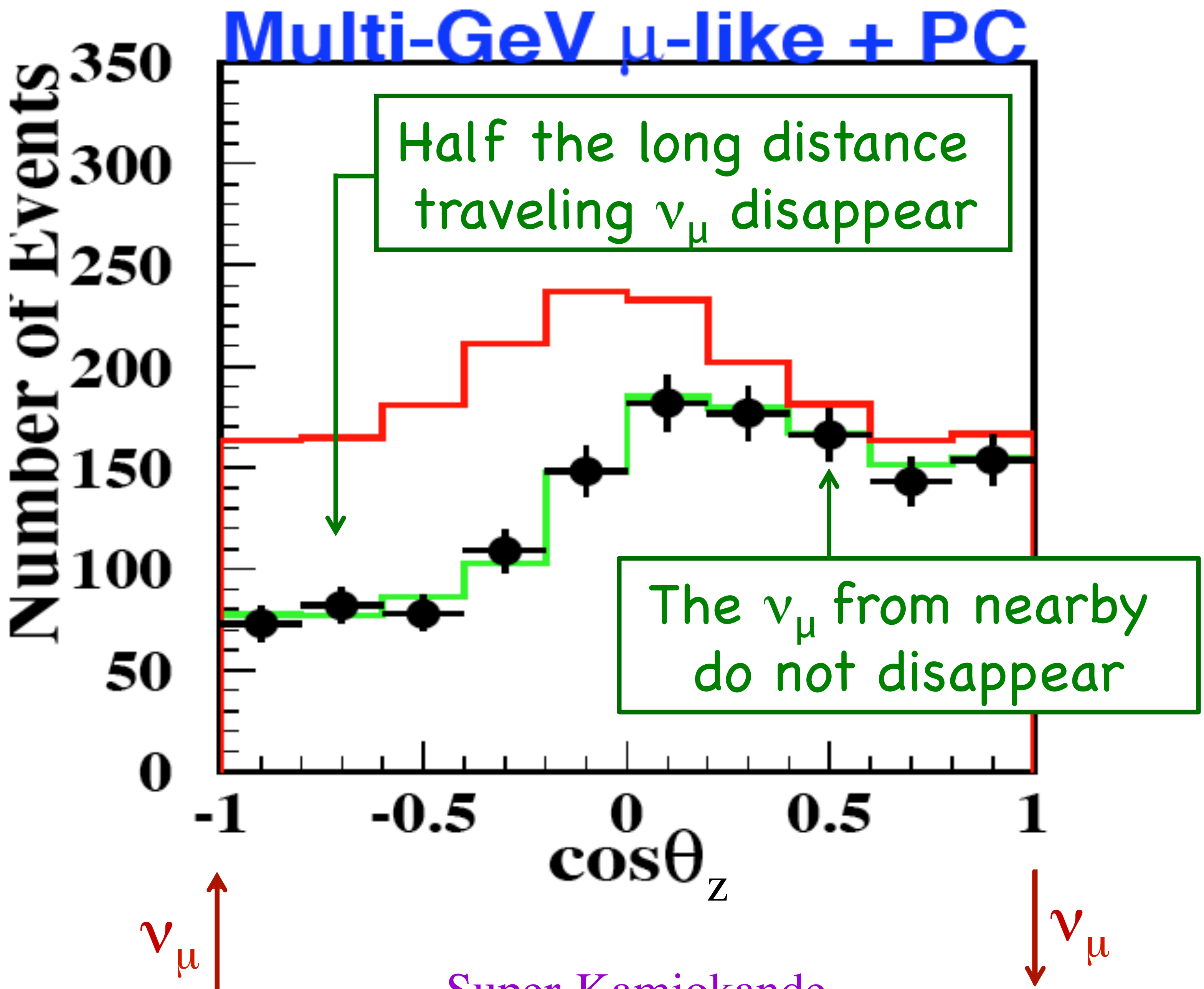
The Disappearance of Atmospheric ν_μ



Isotropy of the $\gtrsim$ 2 GeV cosmic rays + Gauss' Law + No ν_μ disappearance

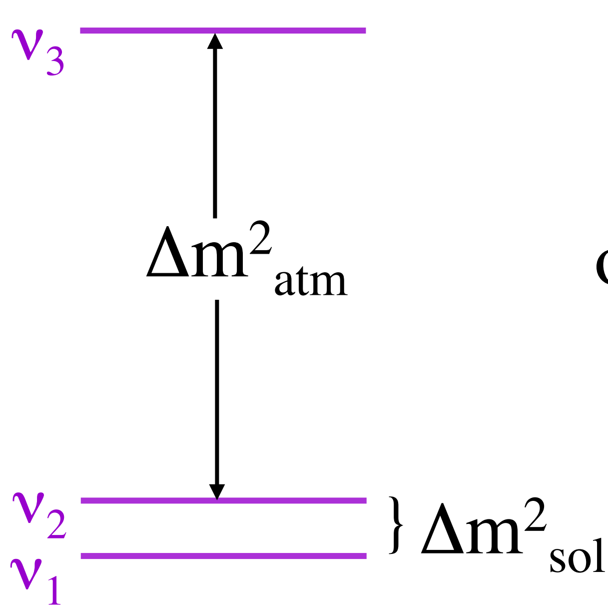
$$\Rightarrow \quad \frac{\phi_{\nu_\mu}(\mathrm{Up})}{\phi_{\nu_\mu}(\mathrm{Down})} = 1 \; .$$

But Super-Kamiokande finds for $E_\nu > 1.3$ GeV —



Super-Kamiokande

At $E_\nu > 1.3$ GeV, in —



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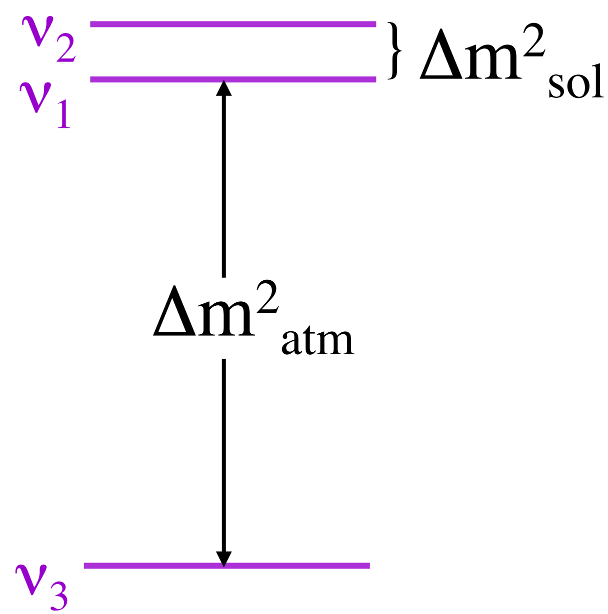


the solar splitting is largely invisible. Then—

$$\underbrace{P\left(\nu_\mu \to \nu_\mu\right)}_{\frac{1}{2}} \cong 1 - \underbrace{4\left|U_{\mu 3}\right|^2\left(1-\left|U_{\mu 3}\right|^2\right)}_{1} \underbrace{\sin^2\left[1.27\Delta m^2_{\text{atm}} \frac{L(\text{km})}{E(\text{GeV})}\right]}_{\frac{1}{2}}$$

$$1 \Rightarrow \left|U_{\mu 3}\right|^2 = \frac{1}{2}$$

At large *L*/*E*

Reactor – Neutrino Experiments and $|U_{e3}|^2 = \sin^2\theta_{13}$

Reactor $\bar{\nu}_e$ have $E \sim 3$ MeV, so if $L \sim 1.5$ km,

$$\sin^2\left[1.27\Delta m^2\left(\mathrm{eV}^2\right)\frac{L(\mathrm{km})}{E(\mathrm{GeV})}\right] \text{ will be sensitive to —}$$

$$\Delta m^2 = \Delta m^2_{\mathrm{atm}} = 2.4\times10^{-3}\mathrm{eV}^2 \approx \frac{1}{400}\mathrm{eV}^2$$

but not to —

$$\Delta m^2 = \Delta m^2_{\mathrm{sol}} = 7.5\times10^{-5}\mathrm{eV}^2 \approx \frac{1}{13{,}000}\mathrm{eV}^2\,.$$

Then —

$$P\left(\bar{\nu}_e \to \bar{\nu}_e\right) \cong 1-4\left|U_{e3}\right|^2\left(1-\left|U_{e3}\right|^2\right)\sin^2\left[1.27\Delta m^2_{\mathrm{atm}}\frac{L(\mathrm{km})}{E(\mathrm{GeV})}\right]$$

Measurements by the Daya Bay, RENO,
and Double CHOOZ reactor neutrino experiments,
and by the T2K accelerator neutrino experiment

$\Rightarrow$ $|U_{e3}|^2 \cong 0.02$

The Change of Flavor of Solar ν_e

Nuclear reactions in the core of the sun produce ν_e. Only ν_e.

The Sudbury Neutrino Observatory (SNO) measured, for the high-energy part of the solar neutrino flux:

$$\nu_{sol}\, d \rightarrow e\, p\, p \;\Rightarrow \phi_{\nu_e}$$

$$\nu_{sol}\, d \rightarrow \nu\, n\, p \;\Rightarrow \phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau} \qquad (\nu \text{ remains a } \nu)$$

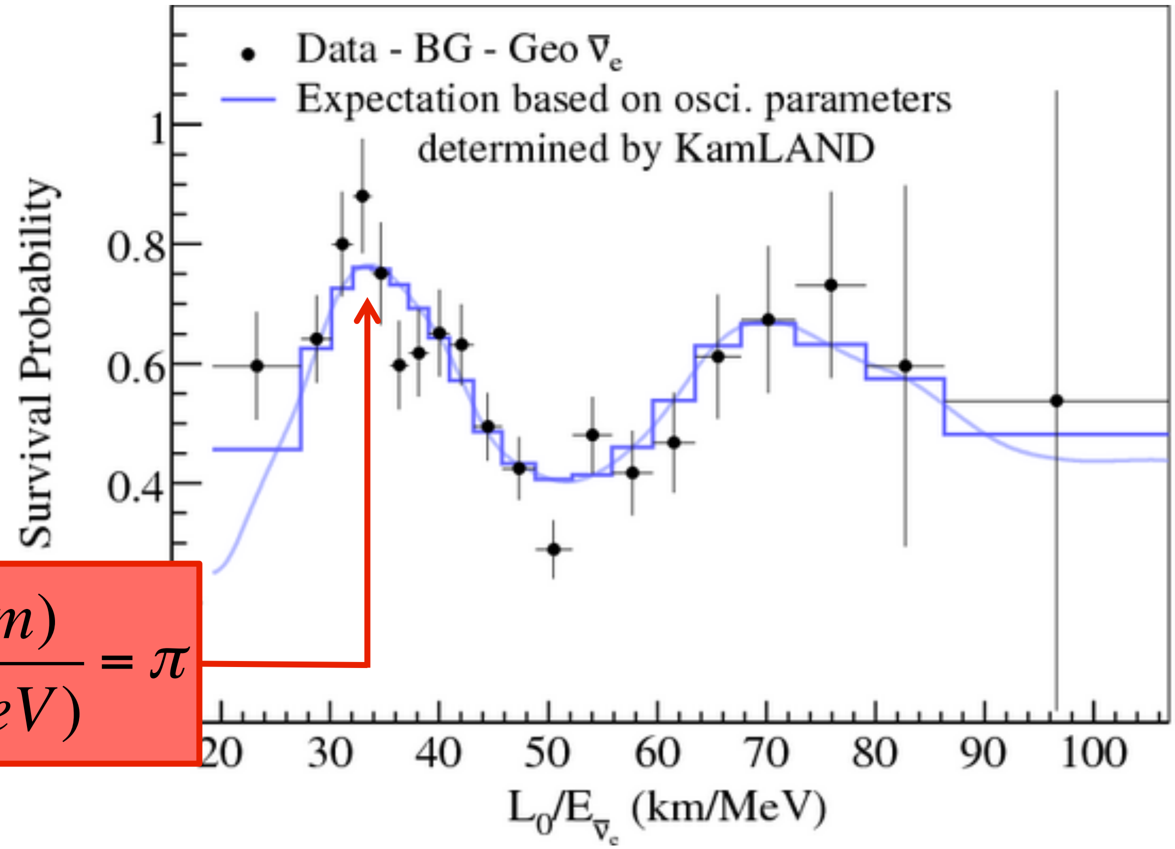
From the two reactions,

$$\frac{\phi_{\nu_e}}{\phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau}} = 0.301 \pm 0.033$$

For solar neutrinos, $P(\nu_e \rightarrow \nu_e) = 0.3$.

The Behavior of Reactor $\overline{\nu}_e$ In KamLAND

Survival probability $P(\overline{\nu}_e \rightarrow \overline{\nu}_e)$ of reactor $\overline{\nu}_e$



$$1.27\Delta m^2_{\text{sol}}\left(eV^2\right)\frac{L(km)}{E(GeV)} = \pi$$

$$P\left(\overline{\nu}_e \rightarrow \overline{\nu}_e\right) = 1 - \sin^2 2\theta \sin^2\left[1.27\Delta m^2_{\text{sol}}\left(eV^2\right)\frac{L(km)}{E(GeV)}\right]$$

The 3 X 3 Unitary Mixing Matrix

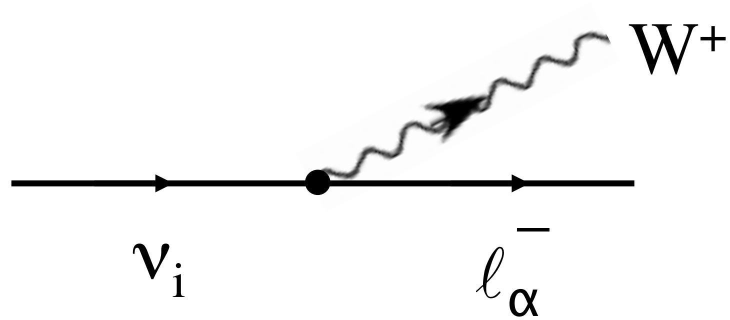
Caution: We are *assuming* the mixing matrix U to be 3 x 3 and unitary.

$$\mathcal{L}_{SM} = -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} \left(\overline{\ell}_{L\alpha} \gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}^{-} + \overline{\nu}_{Li} \gamma^{\lambda} U_{\alpha i}^{*} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

$$(CP)\left(\overline{\ell}_{L\alpha} \gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}^{-} \right)(CP)^{-1} = \overline{\nu}_{Li} \gamma^{\lambda} U_{\alpha i} \ell_{L\alpha} W_{\lambda}^{+}$$

Phases in U will lead to CP violation, unless they are removable by redefining the leptons.

$U_{\alpha i}$ describes —



$$U_{\alpha i} \sim \langle \ell_\alpha^- W^+ | H | \nu_i \rangle$$

$$\text{When } |\nu_i\rangle \rightarrow |e^{i\varphi}\nu_i\rangle, \; U_{\alpha i} \rightarrow e^{i\varphi} U_{\alpha i}, \text{ all } \alpha$$

$$\text{When } |\ell_\alpha^-\rangle \rightarrow |e^{i\varphi}\ell_\alpha^-\rangle, \; U_{\alpha i} \rightarrow e^{-i\varphi} U_{\alpha i}, \text{ all } i$$

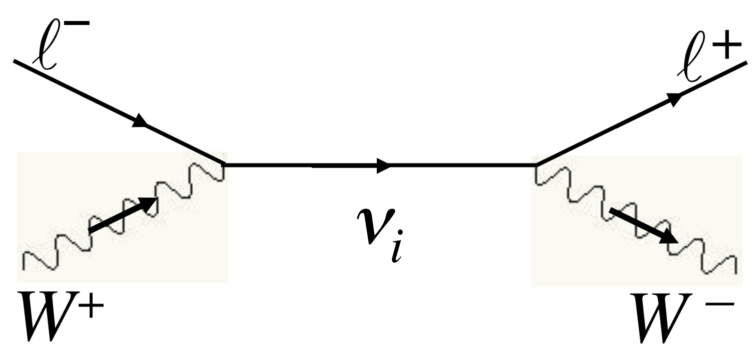
Thus, one may multiply any column, or any row, of U by a complex phase factor without changing the physics.

Some phases may be removed from U in this way.

When the Neutrino Mass Eigenstates Are Their Own Antiparticles

When this is the case, processes that
do not conserve the lepton number
$L \equiv$ #(Leptons) – #(Antileptons) can occur.

Example:



The amplitude for any such L-violating process contains an extra factor.

When we phase-redefine ν_i to remove a phase from U, that phase just moves to the extra factor.

It does not disappear from the physics.

Hence, when $\overline{\nu}_i = \nu_i$, U can contain extra physically-significant phases.

These are called Majorana phases.

How Many Mixing Angles and $\not{C}\not{P}$ Phases Does U Contain?

Real parameters before constraints:	18
Unitarity constraints — $\sum_i U_{\alpha i}{}^* U_{\beta i} = \delta_{\alpha\beta}$	
Each row is a vector of length unity:	– 3
Each two rows are orthogonal vectors:	– 6
Rephase the three ℓ_α:	– 3
Rephase two ν_i , if $\overline{\nu}_i \neq \nu_i$:	– 2
Total physically-significant parameters:	4
Additional (Majorana) $\not{C}\not{P}$ phases if $\overline{\nu}_i = \nu_i$:	2

How Many Of The Parameters Are Mixing Angles?

The *mixing angles* are the parameters in U when it is *real*.

U is then a three-dimensional rotation matrix.

Everyone knows such a matrix is described in terms of 3 angles.

Thus, U contains 3 mixing angles.

Summary

Mixing angles	$\cancel{CP}$ phases if $\bar{\nu}_i \neq \nu_i$	$\cancel{CP}$ phases if $\bar{\nu}_i = \nu_i$
3	1	3

The Lepton Mixing Matrix U

$$U = \overset{\text{Atmospheric}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}} \times \overset{\text{Reactor (L ~ 1 km)}}{\begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix}} \times \overset{\text{Solar}}{\begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$c_{ij} \equiv \cos\theta_{ij} \qquad s_{ij} \equiv \sin\theta_{ij}$$

$$\times \underbrace{\begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Does not affect oscillation}}$$

$$\overbrace{\theta_{12} \approx 33°,\ \theta_{23} \approx 40\text{-}52°,}^{\textit{Note big mixing!}}\ \theta_{13} \approx 8\text{-}9° \leftarrow \textit{Not very small!}$$

The phases violate CP. δ would lead to $P(\overline{\nu}_\alpha \rightarrow \overline{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$.

$\uparrow$ ~~CP~~

But note the crucial role of $s_{13} \equiv \sin\theta_{13}$.

We know essentially nothing about the phases. Only hints.

The Majorana ~~CP~~ Phases

The phase α_i is associated with neutrino mass eigenstate ν_i:

$$U_{\alpha i} = U^0{}_{\alpha i} \exp(i\alpha_i/2) \text{ for all flavors } \alpha.$$

$$\text{Amp}(\nu_\alpha \rightarrow \nu_\beta) = \sum_i U_{\alpha i}{}^* \exp(-\, i m_i{}^2 L/2E)\, U_{\beta i}$$

is insensitive to the Majorana phases α_i .

Only the phase δ can cause CP violation in neutrino oscillation.

There Is Nothing Special About θ_{13}

All mixing angles must be nonzero for $\cancel{CP}$ in oscillation.

For example —

$$P(\bar{\nu}_\mu \to \bar{\nu}_e) - P(\nu_\mu \to \nu_e) = 2\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin\delta$$
$$\times \sin\left(\Delta m^2{}_{31}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{32}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{21}\frac{L}{4E}\right)$$

In the factored form of U, one can put δ next to θ_{12} instead of θ_{13}.