UNIVERSITY OF Southampton
School of Physics and Astronomy

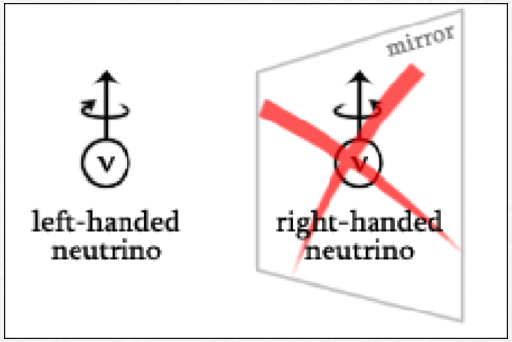
Neutrino Mass Models

Lecture 2: Neutrino Mass

Steve King, St.Andrews, Scotland, 10-22 August, 2014

International Neutrino Summer School 2014 (INSS 2014)
70th Scottish Universities
Summer School in Physics (SUSSP70)

Why neutrino masses are zero in the Standard Model



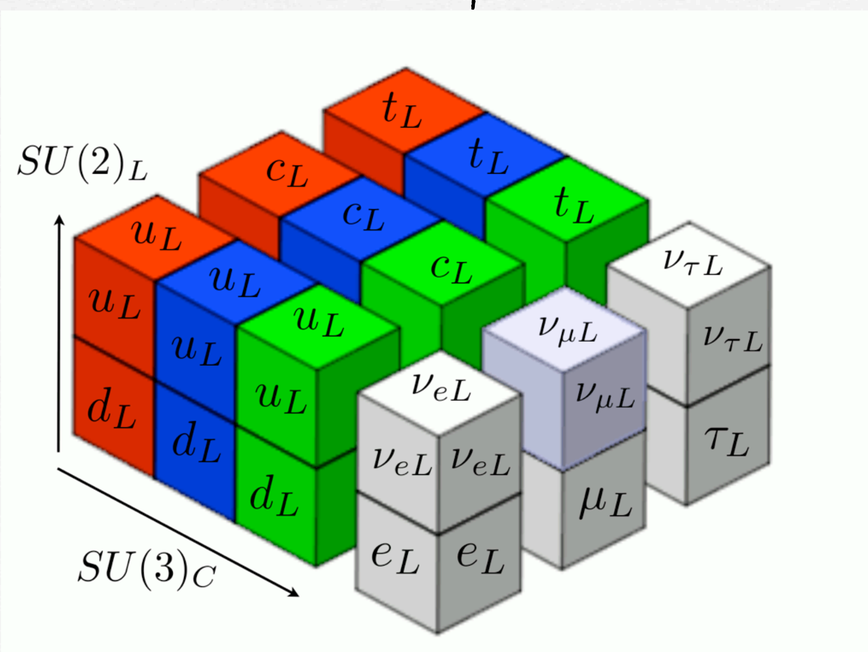
1. There are no right-handed neutrinos
2. There are no Higgs triplets of $SU(2)_L$
3. There are no non-renormalizable terms

- So neutrinos are massless, with ν_e, ν_μ, ν_τ distinguished by separate lepton numbers L_e, L_μ, L_τ
- Neutrinos and anti-neutrinos are distinguished by the total conserved lepton number $L = L_e + L_\mu + L_\tau$
- To generate neutrino mass we must relax 1 and/or 2 and/or 3 e.g. add right-handed neutrinos

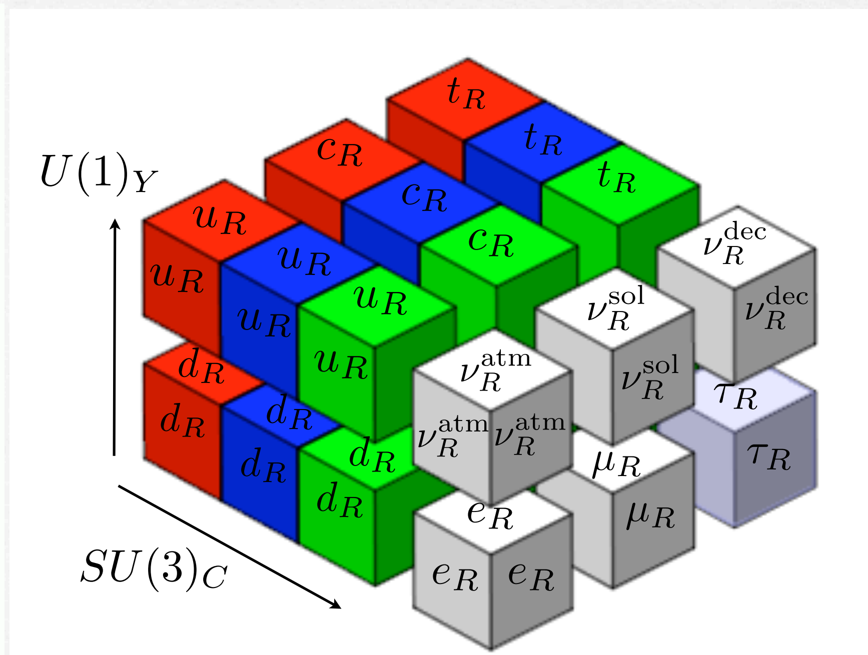
Standard Model of Quarks and Leptons with right-handed neutrinos

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

Left-handed quarks and leptons

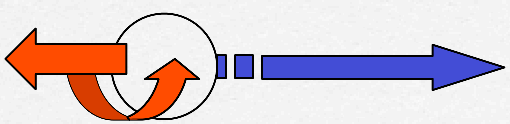


Right-handed quarks and leptons



The Electron Mass

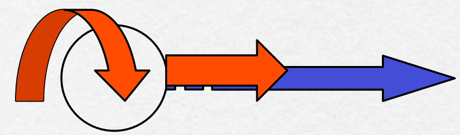
e_L^-



Dirac

$$m_e \overline{e_L^-}\, e_R^-$$

e_R^-



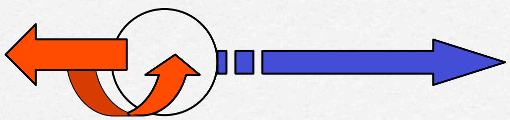
Left-handed electron

Right-handed electron

Neutrino Mass

Left-handed neutrino

ν_L

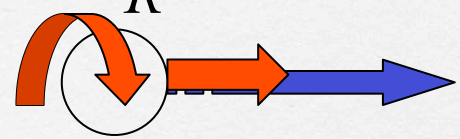


Dirac

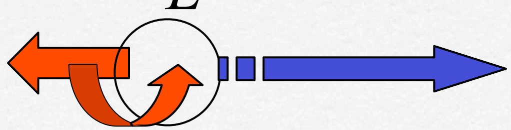
$$m^{\nu}_{LR}\overline{\nu}_L \nu_R$$

Right-handed neutrino

ν_R



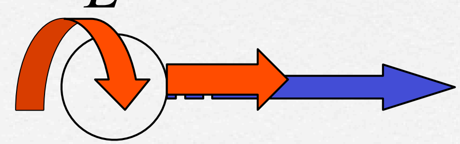
ν_L



Majorana

$$m^{\nu}_{LL}\overline{\nu}_L \nu^c_L$$

ν^c_L



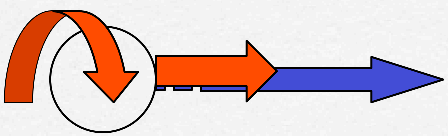
Left-handed neutrino ←-------- CP --------→ Right-handed antineutrino

Right-handed neutrino mass

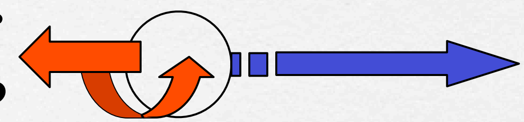
ν_R

Majorana

ν_R^c



$$M_{RR}\overline{\nu}_R \nu_R^c$$



Right-handed neutrino ←-------- CP --------→ Left-handed antineutrino

Summary of Neutrino Masses

Majorana masses

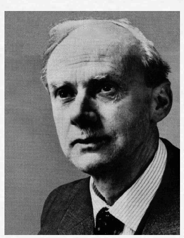
CP conjugate

$$m_{LL}\overline{\nu}_L \nu_L^c$$

$$M_{RR}\overline{\nu}_R \nu_R^c$$

Violates L

Violates L_e, L_μ, L_τ



$$m_{LR}\overline{\nu}_L \nu_R$$

Conserves L

Violates L_e, L_μ, L_τ

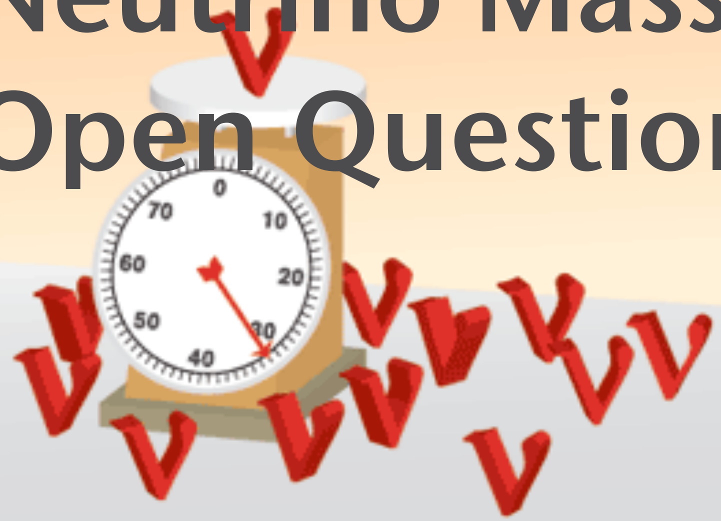
Dirac mass

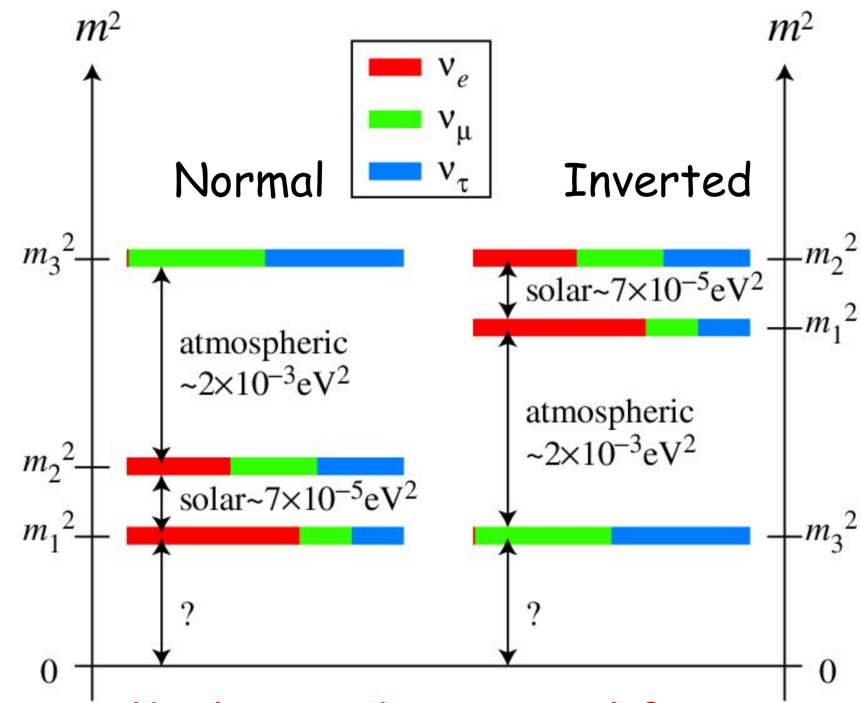
Majorana Mass Matrices

	Type A (zero in 11)	Type B (non-zero 11)
Normal Hierarchy	$m_{LL}^{HI} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \frac{m}{2}$	$\begin{array}{c} \\ \bar{\nu}_{eL} \\ \bar{\nu}_{\mu L} \\ \bar{\nu}_{\tau L} \end{array} \begin{array}{c} \nu^c_{eL} \quad \nu^c_{\mu L} \quad \nu^c_{\tau L} \\ \begin{pmatrix} . & . & . \\ . & . & . \\ . & . & . \end{pmatrix} \end{array}$
Inverted hierarchy	$m_{LL}^{IH(A)} \approx \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \frac{m}{\sqrt{2}}$	$m_{LL}^{IH(B)} \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{pmatrix} m$
Degenerate	$m_{LL}^{DEG(A)} \approx \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} m$	$m_{LL}^{DEG(B1)} \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} m$ $m_{LL}^{DEG(B2)} \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} m$

Pseudo-Dirac

Neutrino Mass
Open Questions





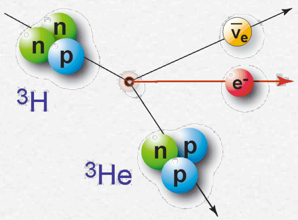
Absolute neutrino mass scale?

- ☐ What is the mass squared ordering (normal or inverted) ?
- ☐ What is the neutrino mass scale (mass of lightest neutrino)?
- ☐ What is the nature of neutrino mass (i.e. Dirac or Majorana)?

How we can learn about neutrino mass

$\beta\beta_{0\nu}$	Δm^2_{13}	KATRIN	Conclusion
yes	> 0	yes	Degenerate, Majorana
yes	> 0	No	Degenerate, Majorana
			or normal, Majorana with heavy particle contribution
yes	< 0	no	Inverted, Majorana
yes	< 0	yes	Degenerate, Majorana
no	> 0	no	Normal, Dirac or Majorana
no	< 0	no	Dirac
no	< 0	yes	Dirac
no	> 0	yes	Dirac

Tritium beta decay



$$|m_{\nu_e}|^2 = \sum_i |U_{ei}|^2 |m_i|^2$$

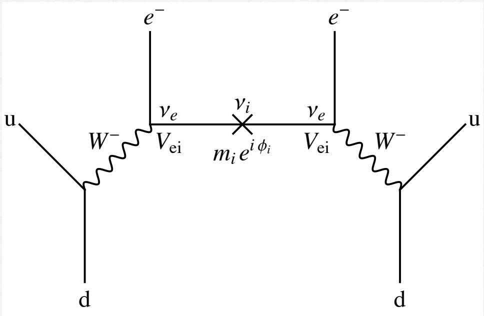
Present Mainz < 2.2 eV

KATRIN ~0.35eV

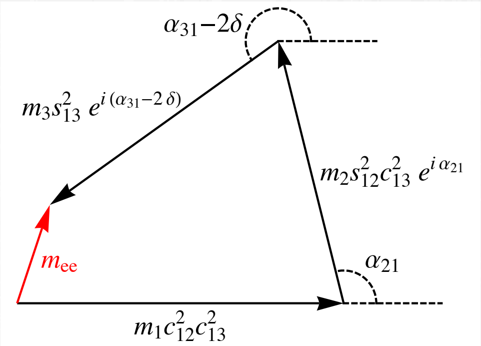
Neutrinoless double beta decay

Majorana only (no signal if Dirac)

$$|m_{ee}| = \left|\sum_i U_{ei}^2 m_i\right|$$



$$|m_{ee}|_{\rm PDG} = |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{i\alpha_{21}} + m_3 s_{13}^2 e^{i(\alpha_{31}-2\delta)}|$$



Neutrino Mass Sum Rules

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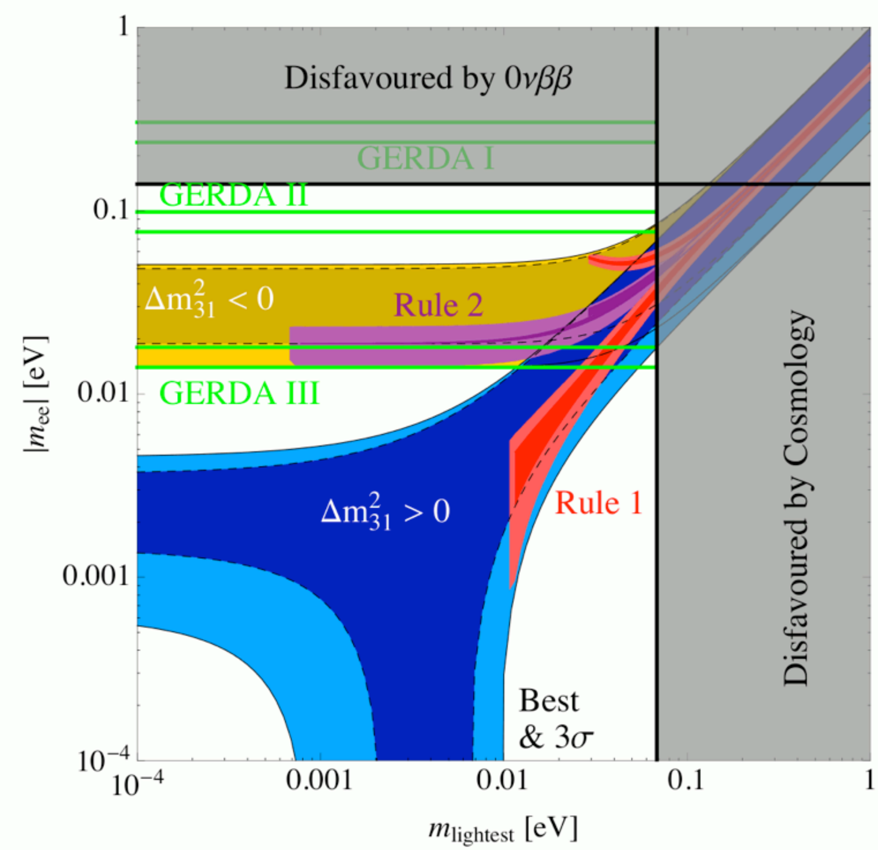
Rule 1

$$\frac{1}{m_1} + \frac{1}{m_2} = \frac{1}{m_3}$$

Rule 2

$$m_1 + m_2 = m_3$$

Give restricted regions



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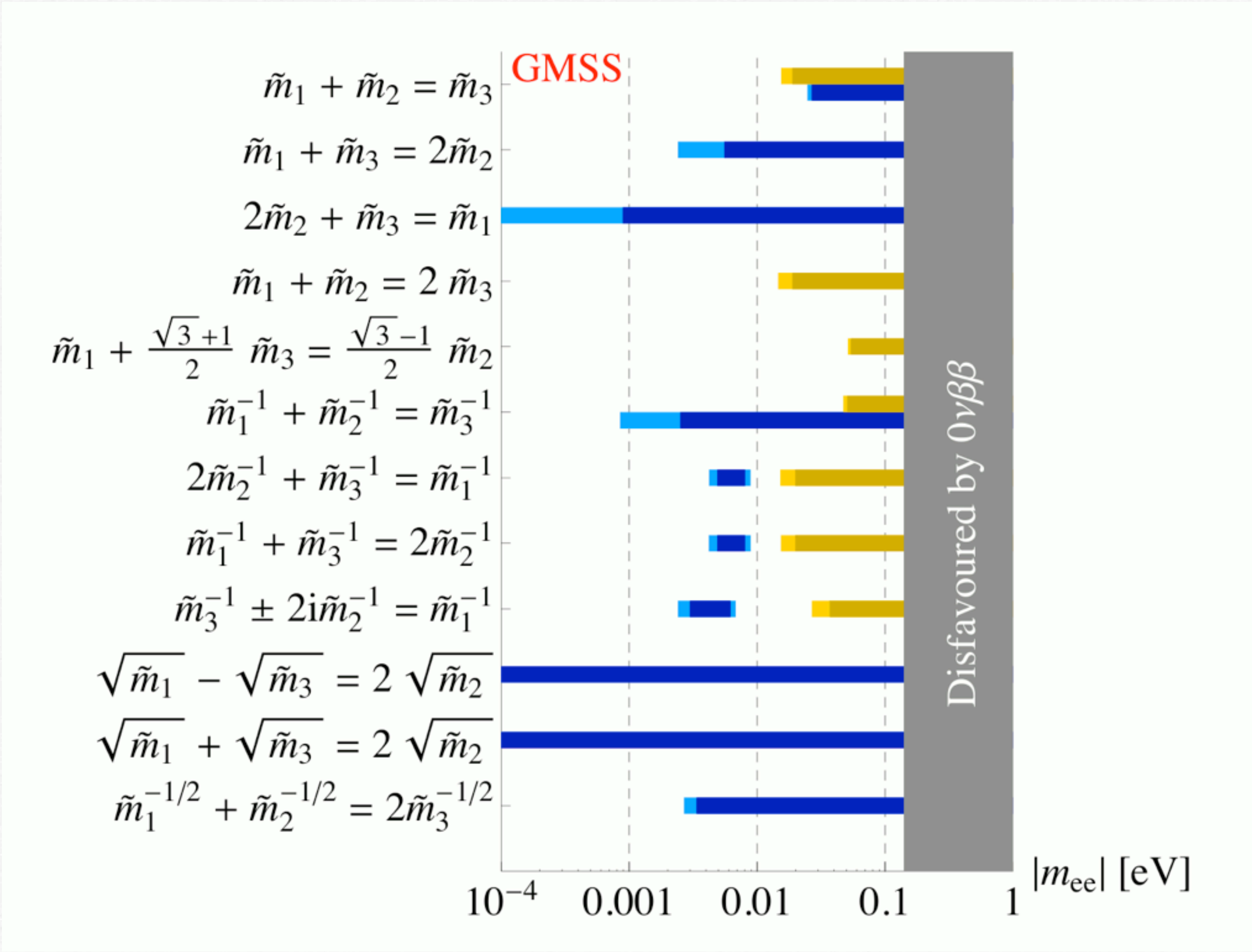
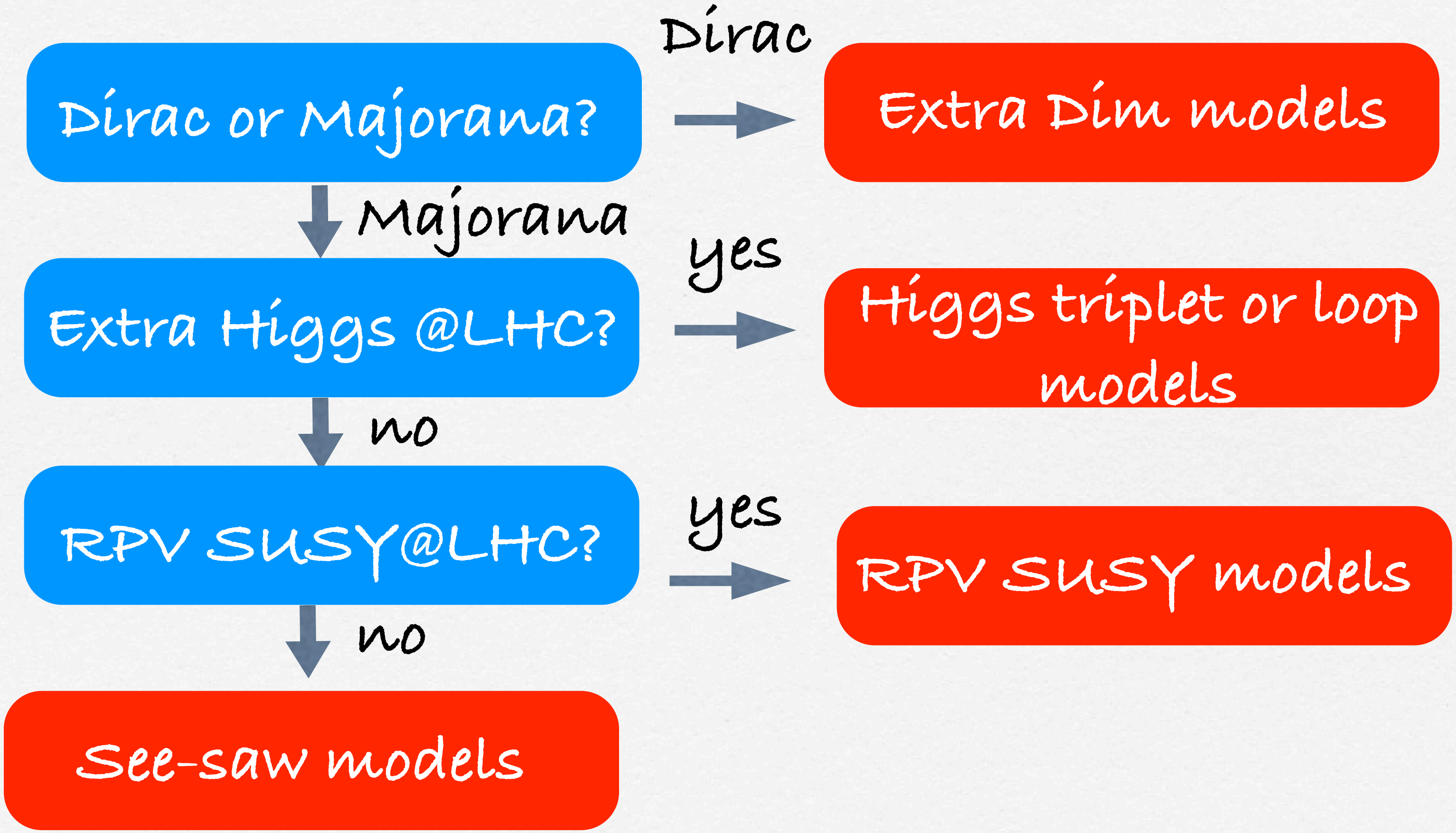
Predictions of mass sum rules



Neutrino mass model roadmap



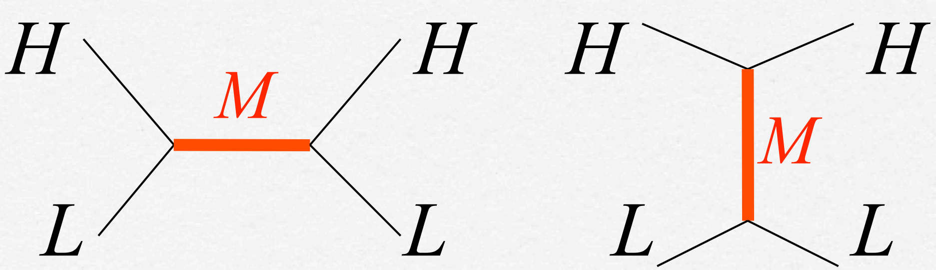
Majorana Neutrino Mass

Renormalisable $\Delta L = 2$ operator $\lambda_\nu LL\Delta$ where Δ is light Higgs triplet with VEV < 8GeV from ρ parameter

Non-renormalisable $\Delta L = 2$ operator $\frac{\lambda_\nu}{M} LLHH = \frac{\lambda_\nu}{M} \langle H^0 \rangle^2 \overline{\nu}_{eL} \nu^c_{eL}$ Weinberg

This is nice because it gives naturally small Majorana neutrino masses $m_{LL} \sim \langle H^0 \rangle^2 / M$ where M is some high energy scale

The high mass scale can be associated with some heavy particle of mass M being exchanged (can be singlet or triplet)



See-saw mechanisms

See-saw mechanism

P. Minkowski; T. Yanagida;
M. Gell- Mann, P. Ramond and R. Slansky;

Possible type II contribution

Dirac matrix

$$\begin{pmatrix} \overline{\nu_L} & \overline{\nu_R^c} \end{pmatrix} \begin{pmatrix} 0 & m_{LR} \\ m_{LR}^T & M_{RR} \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}$$

$$m^\nu = m_{LR}.\frac{1}{M_{RR}}.m_{LR}^T$$

Light Majorana matrix

Heavy Majorana matrix

- ❑ Neutrinos are light because RH neutrinos are heavy
- ❑ Allows large neutrino mixing

Example: two right-handed neutrinos

Tutorial Problem 3

$$m^D_{RL} = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \qquad M_{RR} = \begin{pmatrix} M_{\rm sol} & 0 \\ 0 & M_{\rm atm} \end{pmatrix}$$

$$m^\nu = (m^D_{RL})^T M^{-1}_{RR} m^D_{RL} = \begin{pmatrix} \frac{a^2}{M_{\rm sol}} + \frac{d^2}{M_{\rm atm}} & \frac{ab}{M_{\rm sol}} + \frac{de}{M_{\rm atm}} & \frac{ac}{M_{\rm sol}} + \frac{df}{M_{\rm atm}} \\ \frac{ab}{M_{\rm sol}} + \frac{de}{M_{\rm atm}} & \frac{b^2}{M_{\rm sol}} + \frac{e^2}{M_{\rm atm}} & \frac{bc}{M_{\rm sol}} + \frac{ef}{M_{\rm atm}} \\ \frac{ac}{M_{\rm sol}} + \frac{df}{M_{\rm atm}} & \frac{bc}{M_{\rm sol}} + \frac{ef}{M_{\rm atm}} & \frac{c^2}{M_{\rm sol}} + \frac{f^2}{M_{\rm atm}} \end{pmatrix}$$

- Determinant vanishes → massless neutrino $m_1=0$ and normal hierarchy

  Tutorial Problem 3 (e)

- Choosing $d=0$, $e=f$, $b=a$, $c=-a$ gives TB mixing

- Other forms of constrained sequential dominance (CSD) are possible 1304.6264

CSD(n)

1304.6264

$$m^D_{RL} = \begin{pmatrix} a & na & (n-2)a \\ 0 & e & e \end{pmatrix}$$

Justified in A4 models (see lecture 3)

$$m^\nu = (m^D_{RL})^T M^{-1}_{RR} m^D_{RL} = \begin{pmatrix} \frac{a^2}{M_{\rm sol}} & \frac{na^2}{M_{\rm sol}} & \frac{(n-2)a^2}{M_{\rm sol}} \\ \frac{na^2}{M_{\rm sol}} & \frac{n^2a^2}{M_{\rm sol}} + \frac{e^2}{M_{\rm atm}} & \frac{n(n-2)a^2}{M_{\rm sol}} + \frac{e^2}{M_{\rm atm}} \\ \frac{(n-2)a^2}{M_{\rm sol}} & \frac{n(n-2)a^2}{M_{\rm sol}} + \frac{e^2}{M_{\rm atm}} & \frac{(n-2)^2a^2}{M_{\rm sol}} + \frac{e^2}{M_{\rm atm}} \end{pmatrix}.$$

Note that $m^\nu \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ since $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \perp \begin{pmatrix} 0 \\ e \\ e \end{pmatrix}, \begin{pmatrix} a \\ na \\ (n-2)a \end{pmatrix}$

Since $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ is an eigenvector with zero eigenvalue

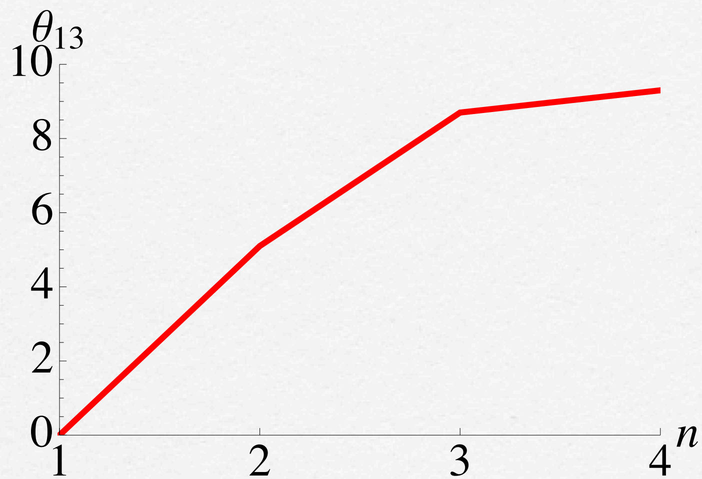
it is first column of the MNS matrix (i.e. TM1 mixing)

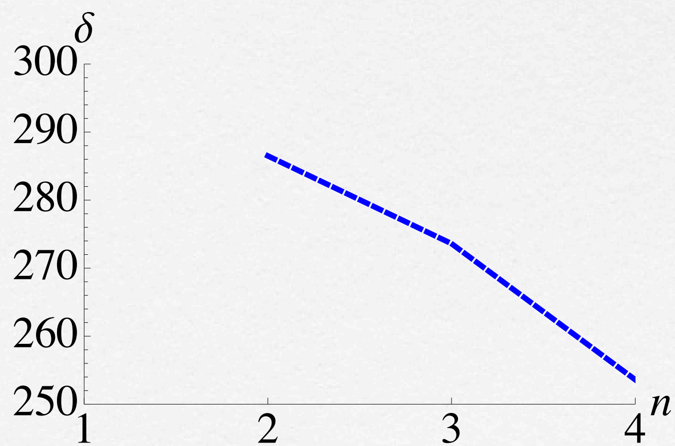
CSD(n)

1304.6264

$$m^D_{RL} = \begin{pmatrix} a & na & (n-2)a \\ 0 & e & e \end{pmatrix} \qquad \arg\left(\frac{a}{e}\right) = \phi_n$$

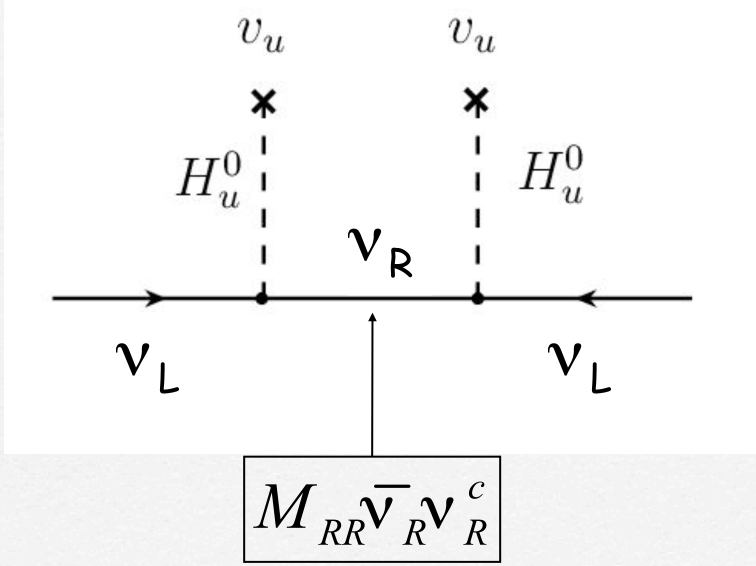
$$\phi_2 = \pi/4, \quad \phi_3 = \pi/3, \quad \phi_4 = 2\pi/5$$





Type I see-saw mechanism

P. Minkowski (1977), Gell-Mann, Glashow, Mohapatra, Ramond, Senjanovic, Slanski, Yanagida (1979/1980), Schechter and Valle (1980)...

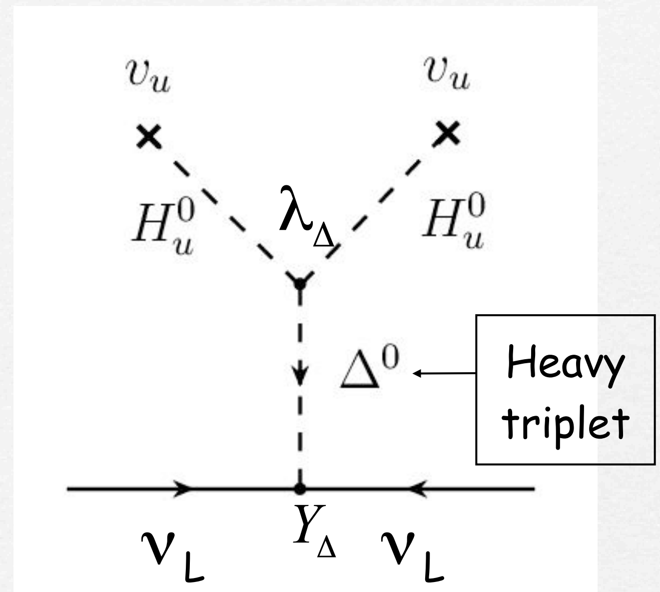


$$m_{LL}^{I} \approx -m_{LR} M_{RR}^{-1} m_{LR}^{T}$$

Type I

Type II see-saw mechanism (SUSY)

Lazarides, Magg, Mohapatra, Senjanovic, Shafi, Wetterich, Schechter and Valle...



$$m_{LL}^{II} \approx \lambda_\Delta Y_\Delta \frac{v_u^2}{M_\Delta}$$

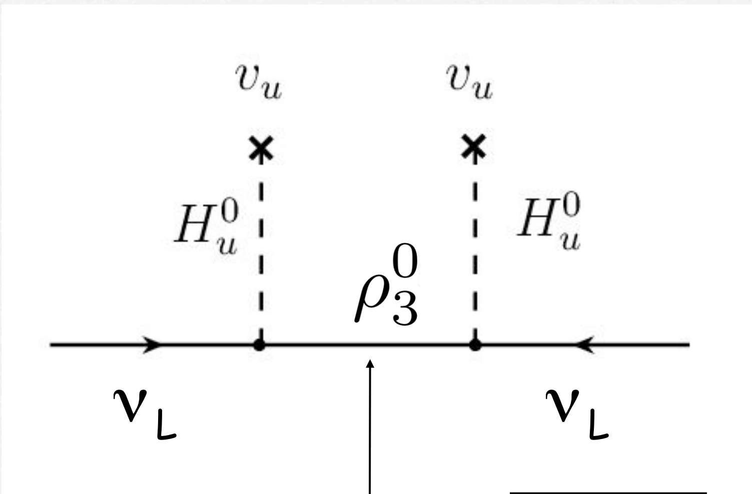
Type II

Type III see-saw mechanism

Foot, Lew, He, Joshi; Ma…

Supersymmetric adjoint SU(5)

Perez et al; Cooper, SFK, Luhn,…



$SU(2)_L$ fermion triplet

$$M_\rho \rho\rho$$

$$m_{LL}^{III} \approx -m_{LR} M_\rho^{-1} m_{LR}^T$$

Type III

See-saw mechanisms with extra singlets S

Inverse see-saw

Wyler, Wolferstein; Mohapatra, Valle

$$\begin{pmatrix} 0 & M_D & 0 \\ M_D^T & 0 & M \\ 0 & M^T & \mu \end{pmatrix} \qquad M_\nu = M_D M^{T^{-1}} \mu M^{-1} M_D^T$$

M ≈ TeV ➔ LHC

Linear see-saw

Malinsky, Romao, Valle

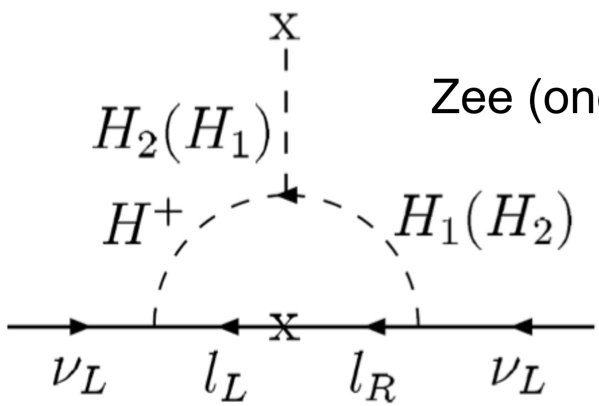
$$\begin{pmatrix} 0 & M_D & M_L \\ M_D^T & 0 & M \\ M_L^T & M^T & 0 \end{pmatrix}$$

$$M_\nu = M_D (M_L M^{-1})^T + (M_L M^{-1}) M_D{}^T$$

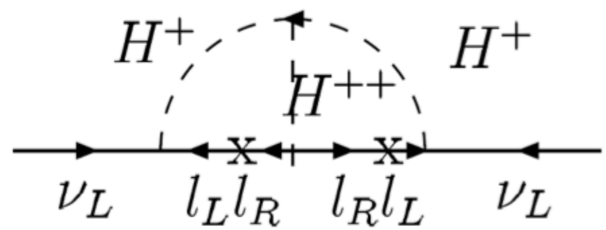
LFV predictions

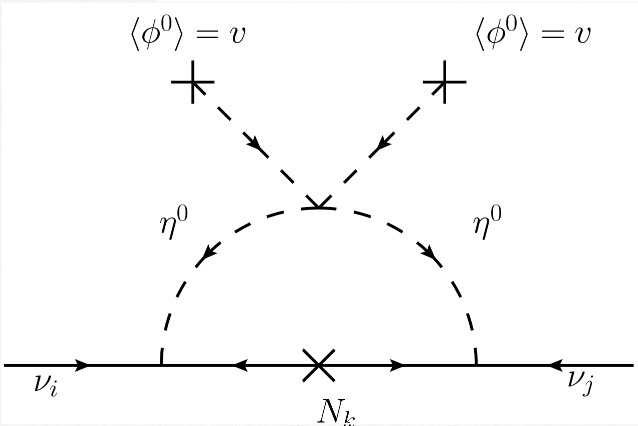
Loop Models of Neutrino Mass

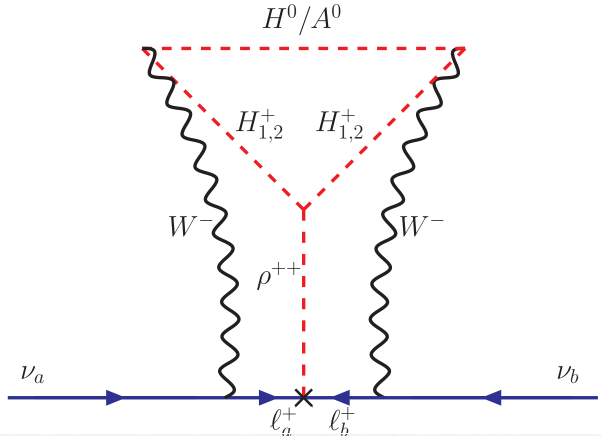
Zee (one loop)

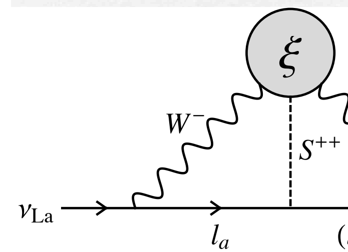


Babu (two loop)









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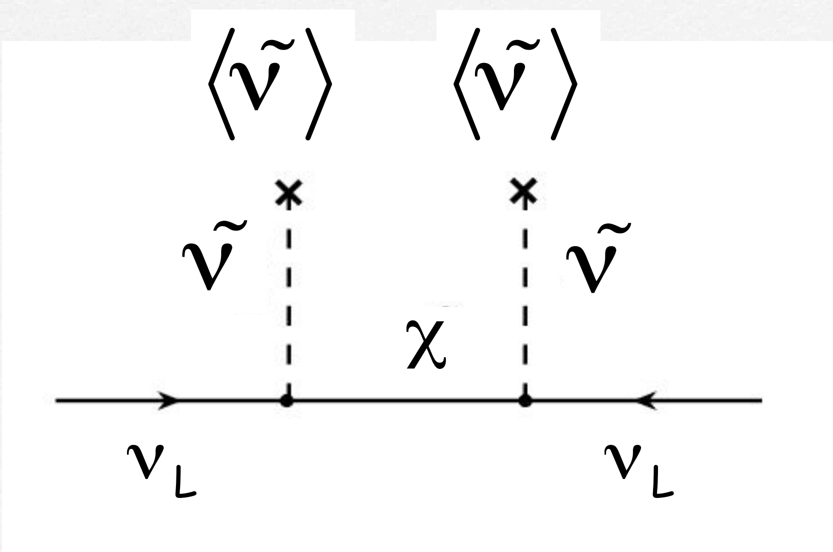
Scotogenic model

Cocktail model

Doubly charged singlet scalar

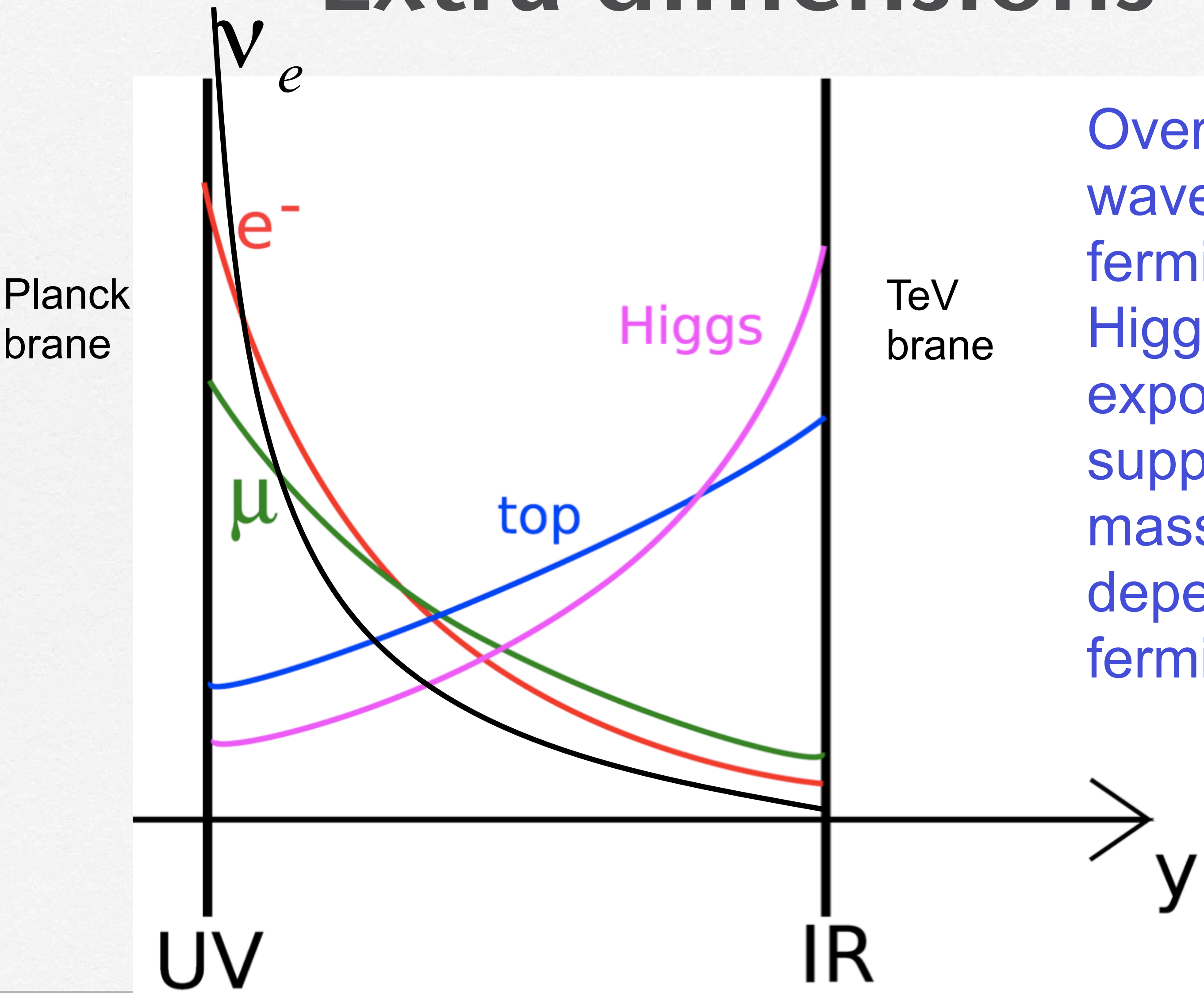
R-Parity Violating SUSY

- Majorana masses can be generated via RPV SUSY
- Scalar partners of lepton doublets (slepton doublets) have same quantum numbers as Higgs doublets
- If R-parity is violated then sneutrinos may get (small) VEVs inducing a mixing between neutrinos and neutralinos χ



$$m_{LL}^{\nu} \approx \frac{\langle \tilde{\nu} \rangle^2}{M_\chi} \approx \frac{MeV^2}{TeV} \approx eV$$

Extra dimensions



Overlap wavefunction of fermions with Higgs gives exponentially suppressed Dirac masses, depending on the fermion profiles

Conclusions

- Neutrino masses may be Dirac or Majorana, hierarchical or degenerate, normal or inverted hierarchy
- Neutrinoless DBD, Katrin, LBL experiments will decide
- Type I see-saw with CSD(n) predicts normal mass hierarchy and precise mixing angles
- Other types of see-saw possible, some at low scale
- Other mass mechanisms include Loops (Majorana), RPV SUSY (Majorana), Extra dimensions (Dirac)
- The Origin of Neutrino Mass is unknown but type I see-saw most attractive if no new physics at LHC

Tutorial Questions

2. Consider a *Dirac neutrino* mass model involving *one* right-handed neutrino $\nu_R^{\rm atm}$ with Yukawa couplings [4],

$$\overline{\nu_R^{\rm atm}}(dL_e + eL_\mu + fL_\tau)H, \tag{7}$$

where $L_e = (\nu_e, e)_L$, etc., H is the Higgs doublet and d, e, f are real Yukawa couplings.

(a) When the Higgs gets a VEV in its first component, explain why this model leads to *one massive Dirac neutrino*, together with *two massless neutrinos.*

(b) If we interpret the massive neutrino as the *atmospheric neutrino*, show that left-handed component can be parametrized in terms of two angles θ_{13} and θ_{23} as

$$\nu_L^{\rm atm} = s_{13}\nu_{eL} + s_{23}c_{13}\nu_{\mu L} + c_{23}c_{13}\nu_{\tau L}. \tag{8}$$

where $\nu_L^{\rm atm}$ is *correctly normalised* ($s_{13} = \sin\theta_{13}$, etc.). Then, by comparing the above parametrisation of $\nu_L^{\rm atm}$ to the third column of the PMNS matrix (with zero CP phase), explain why θ_{13} is the reactor angle and θ_{23} is the atmospheric angle.

$$\overline{\nu_R^{\rm atm}}(dL_e + eL_\mu + fL_\tau)H, \tag{7}$$

$$\nu_L^{\rm atm} = s_{13}\nu_{eL} + s_{23}c_{13}\nu_{\mu L} + c_{23}c_{13}\nu_{\tau L}. \tag{8}$$

(c) Using Eqs.7 and 8, find expressions for the sine of the reactor angle $\sin\theta_{13}$ and the tangent of the atmospheric angle $\tan\theta_{23}$ in terms of the Yukawa couplings d, e, f.

(d) If the solar neutrino is identified as one of the massless neutrinos, explain why the solar angle θ_{12} is not well defined in this model.

3. Consider a *see-saw* neutrino model involving *two* right-handed neutrinos $\nu_R^{\rm sol}$ and $\nu_R^{\rm atm}$ with Yukawa couplings [5],

$$\overline{\nu_R^{\rm sol}}(aL_e + bL_\mu + cL_\tau)H + \overline{\nu_R^{\rm atm}}(dL_e + eL_\mu + fL_\tau)H, \qquad (9)$$

and heavy right-handed Majorana masses,

$$M_{\rm sol}\overline{\nu_R^{\rm sol}}(\nu_R^{\rm sol})^c + M_{\rm atm}\overline{\nu_R^{\rm atm}}(\nu_R^{\rm atm})^c. \qquad (10)$$

(a) After the Higgs gets a VEV in its first component, write down the Dirac mass matrix m_{RL}^D.

(b) Write down the (diagonal) right-handed neutrino heavy Majorana mass matrix M_{RR}.

(c) Using the see-saw formula, $m^\nu = (m_{RL}^D)^T M_{RR}^{-1} m_{RL}^D$, calculate the light effective left-handed Majorana neutrino mass matrix m^ν (i.e. the physical neutrino mass matrix).

(d) Assuming that the determinant of m^ν vanishes (which you may if you wish check by explicit calculation) what is the physical implication of this?

(e) Imposing the constraints $d = 0$ and $e = f$, with $a = b = -c$ known as "constrained sequential dominance" [6], show that the resulting physical neutrino mass matrix m^ν is diagonalised by the tri-bimaximal mixing matrix, $U_{\rm TB}^T m^\nu U_{\rm TB}$. What is the physical interpretation of this result if the charged lepton mass matrix is diagonal?

(f) If the charged lepton mixing matrix has a Cabibbo-like mixing angle [1],

$$U_e = \begin{pmatrix} c_{12}^e & s_{12}^e e^{-i\delta_{12}^e} & 0 \\ -s_{12}^e e^{i\delta_{12}^e} & c_{12}^e & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{11}$$

calculate the (1,3), (3,1) and (3,3) elements of PMNS matrix $U = U_e U_{\rm TB}$ (you don't need to calculate the whole matrix). Comparing the absolute value of the (1,3) element to that of the standard parameterisation of the PMS matrix, find s_{13} in terms of s_{12}^e and show that choosing $\theta_{12}^e = \theta_C \approx 13^\circ$ (the Cabibbo angle) gives a reasonable value for the reactor angle [7]. Comparing the absolute value of the (3,1) and (3,3) elements to that of the standard parameterisation of the PMS matrix, find relations between PMNS parameters. By combining and expanding these relations show that they lead to the approximate "solar sum rule",

$$\theta_{12} - 35^\circ \approx \theta_{13} \cos\delta, \tag{12}$$

[**Hint**: take the sine of both sides of the Eq.12, assuming $\sin\theta_{13} \approx \theta_{13}$ as well as $\sin 35^\circ \approx 1/\sqrt{3}$.] Discuss the resulting prediction for the CP phase δ [7].