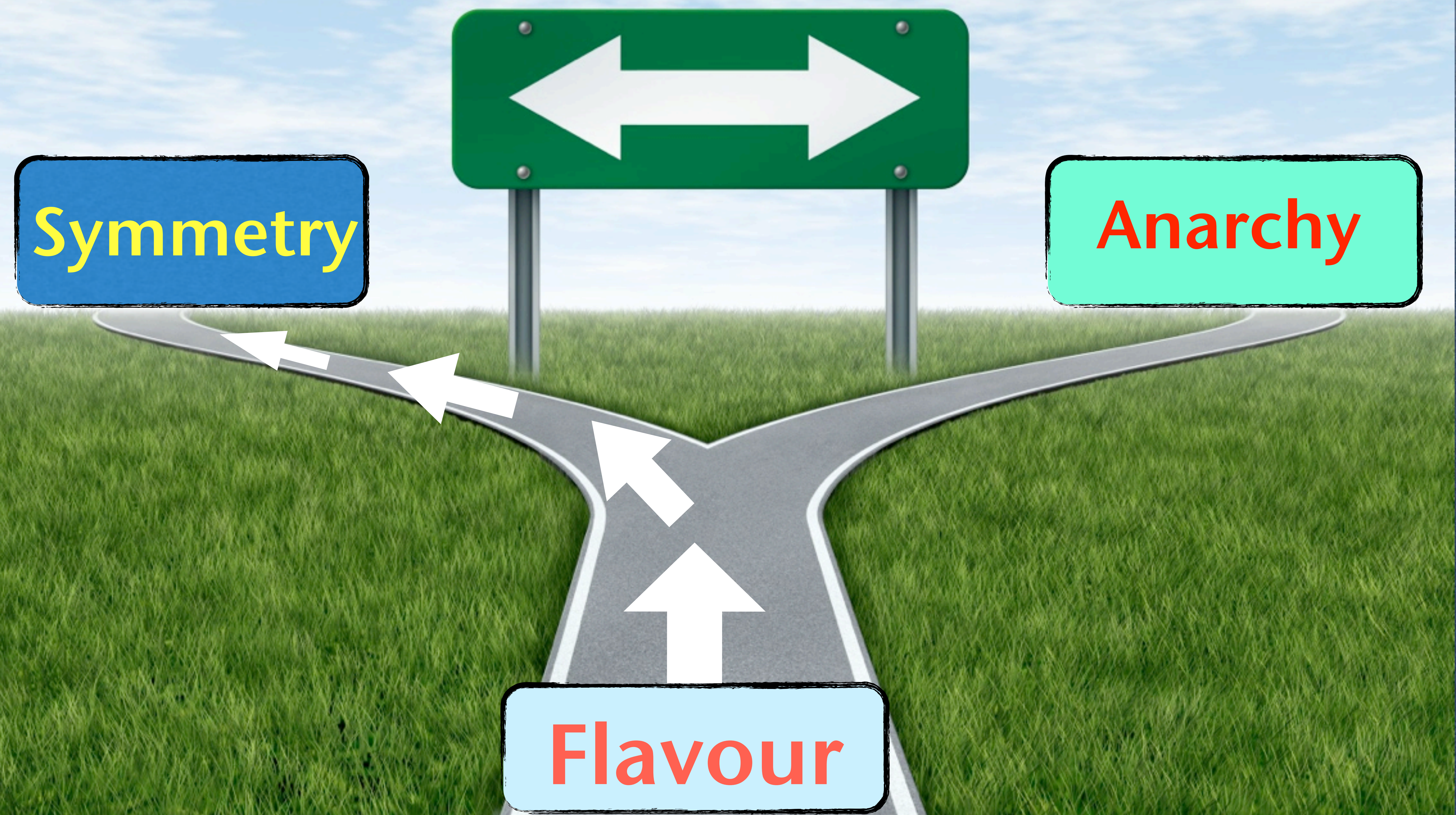
Neutrino Mass Models

Lecture 3: Flavour Models

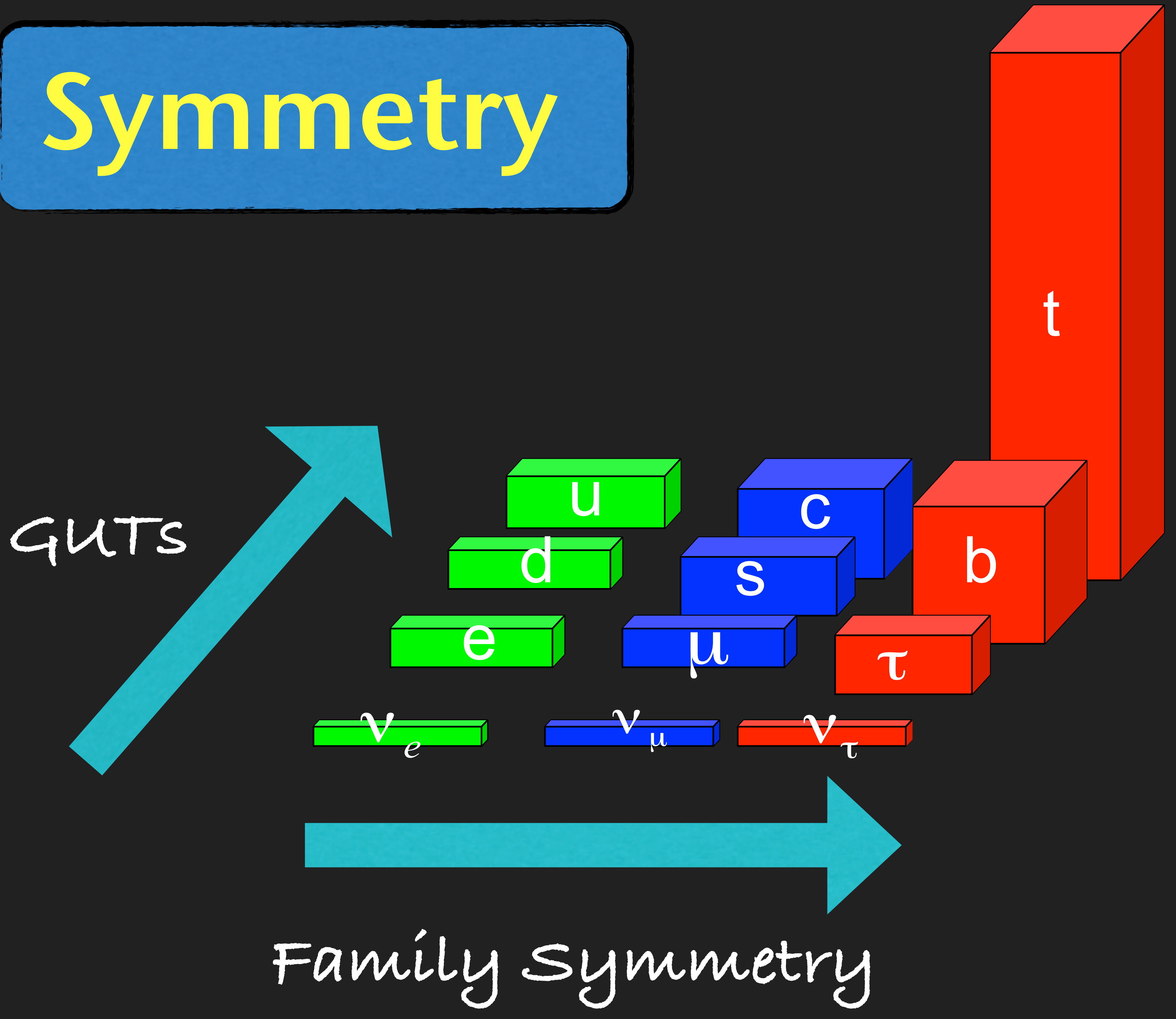
Steve King, St.Andrews,
Scotland, 10-22 August, 2014

International Neutrino Summer School 2014 (INSS 2014)
70th Scottish Universities
Summer School in Physics (SUSSP70)

Towards a Theory of Flavour

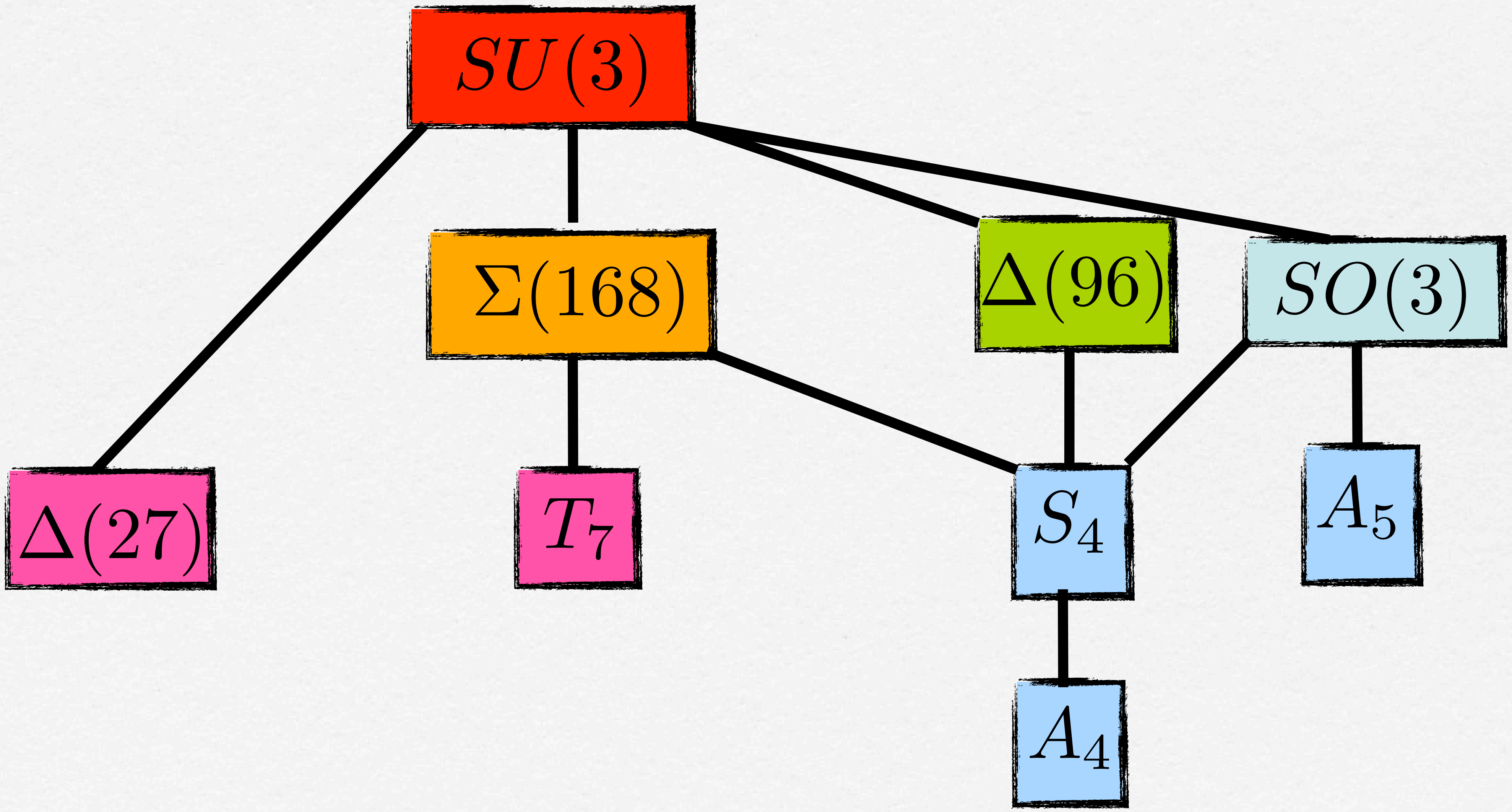


Symmetry



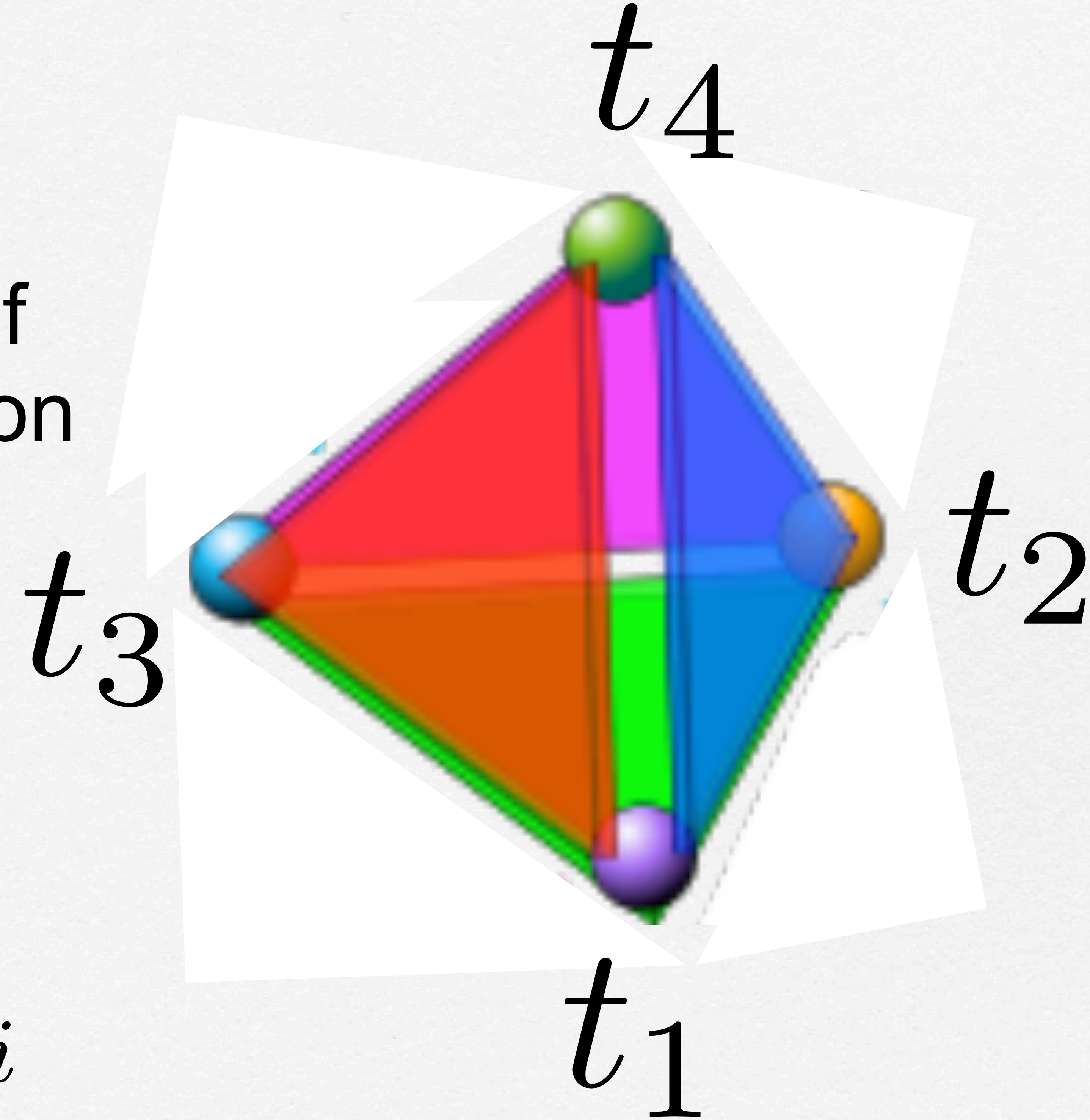
Example of a commutative group

Family Symmetry (non-Abelian)



A_4

Symmetry of the tetrahedron



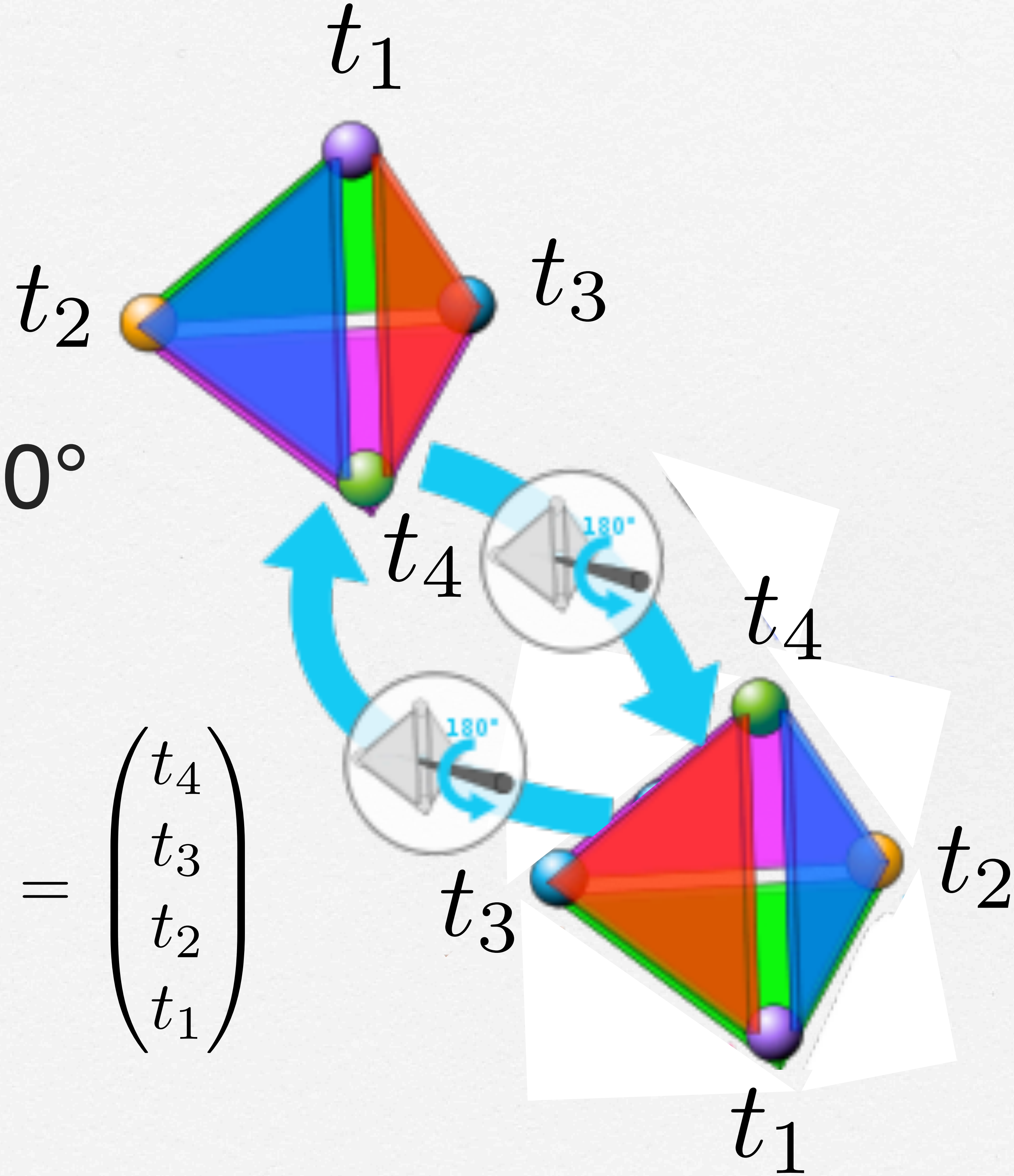
Vertices labelled by t_i

A_4

- rotation by 180°

$$S$$

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{pmatrix} = \begin{pmatrix} t_4 \\ t_3 \\ t_2 \\ t_1 \end{pmatrix}$$

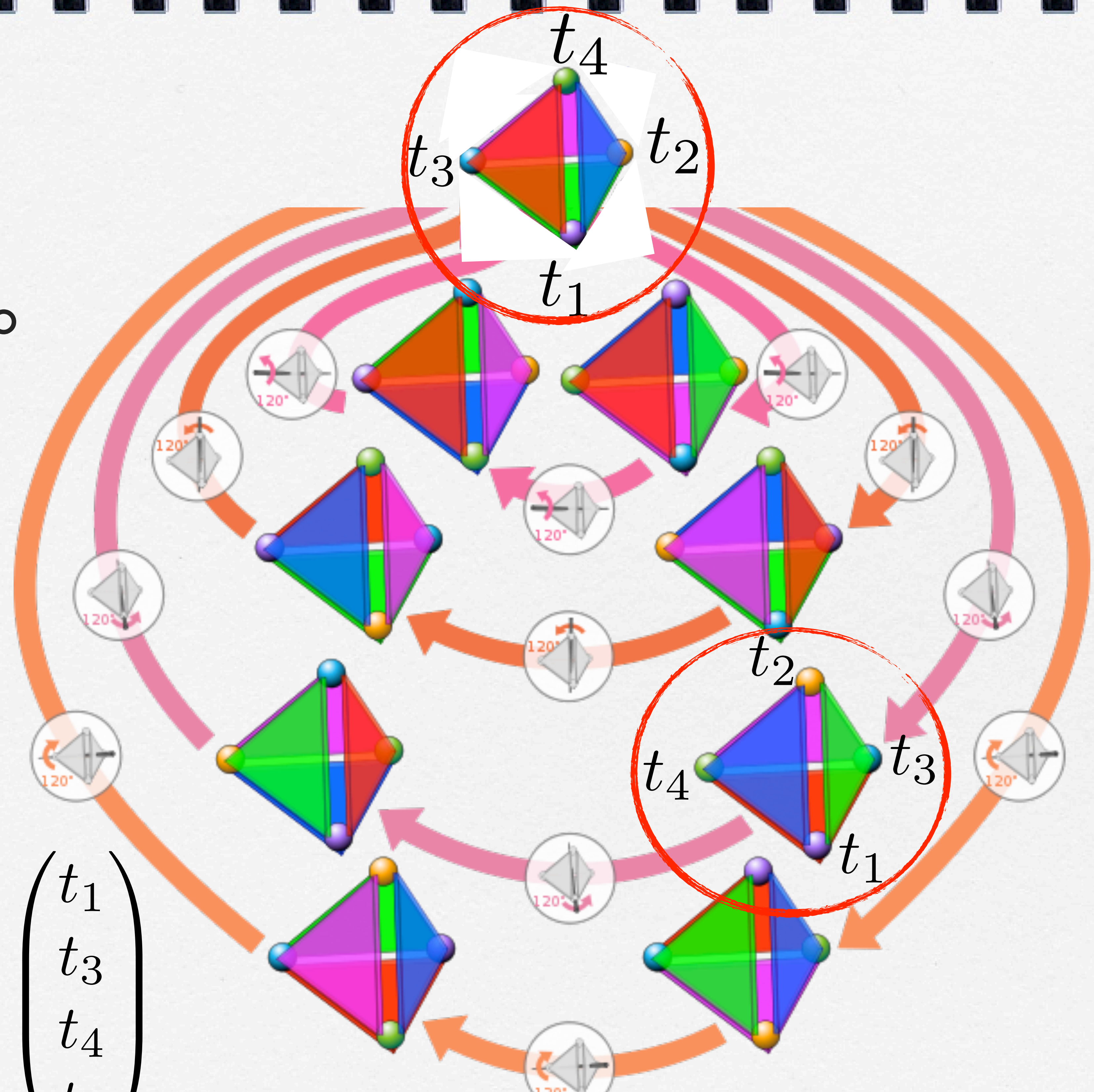


A_4

- rotation by 120° anti-clockwise (seen from a vertex)

$$T$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{pmatrix} = \begin{pmatrix} t_1 \\ t_3 \\ t_4 \\ t_2 \end{pmatrix}$$



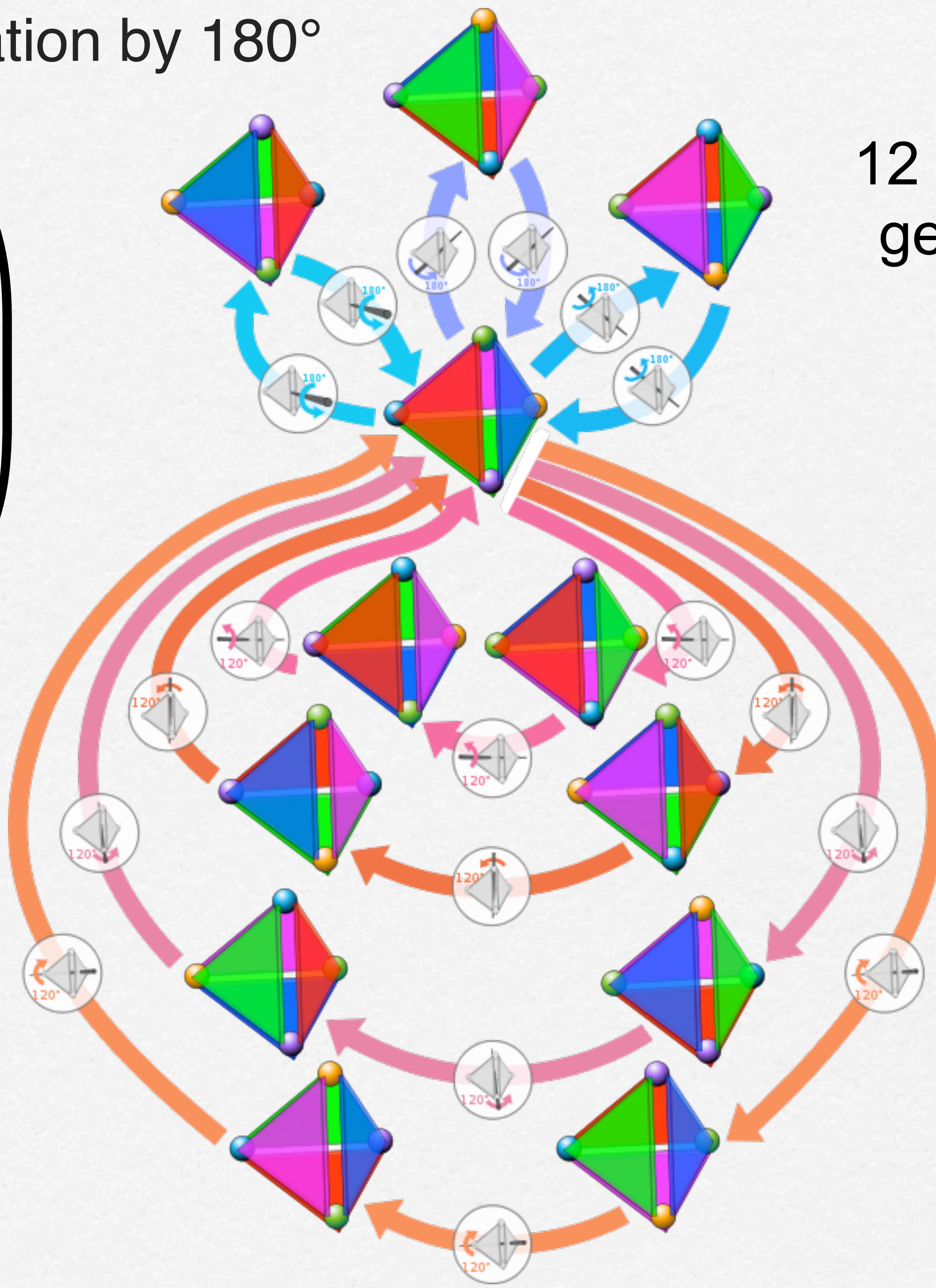
A_4

- 4 × rotation by 120° clockwise (seen from a vertex) T-type rotations
- 4 × rotation by 120° anti-clockwise (ditto) T-type rotations
- 3 × rotation by 180° S-type rotations

$$S = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Diagonal

$$S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$



12 rotations (“group elements”) generated by products of S,T (“generators”)

$$S^2 = T^3 = I$$
$$(ST)^3 = I$$

Block diagonal (rotate about first vertex)

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Since S,T are block diagonal, the 4 dimensional matrix of vertex transformations is equivalent to a triplet plus singlet

$$4 \rightarrow 3 \oplus 1$$

A_4 Family Symmetry

Lepton doublets of $SU(2)_L$ form triplets of A_4

$$L = (L_1, L_2, L_3) \sim 3$$

Higgs which break family symmetry called "flavons"

$$\phi = (\phi_1, \phi_2, \phi_3) \sim 3$$

Neutrino Yukawa couplings involve $L.\phi \sim 1$

$$\overline{\nu_R}(L.\phi)H = \overline{\nu_R}(L_1\phi_1 + L_2\phi_2 + L_3\phi_3)H$$

"Flavon" VEVs with various "vacuum alignments" control the Yukawa couplings

A_4 Vacuum alignments

Symmetry preserving eigenvectors of group elements

$$\langle\phi\rangle = \begin{pmatrix}0\\0\\v\end{pmatrix} \begin{pmatrix}0\\v\\0\end{pmatrix} \begin{pmatrix}v\\0\\0\end{pmatrix} \begin{pmatrix}\pm v\\\pm v\\\pm v\end{pmatrix}$$

Direct

Orthogonal alignments

$$\langle\phi\rangle = \begin{pmatrix}0\\e\\e\end{pmatrix} \perp \begin{pmatrix}v\\v\\-v\end{pmatrix}, \begin{pmatrix}v\\0\\0\end{pmatrix}$$

Orthogonal alignments

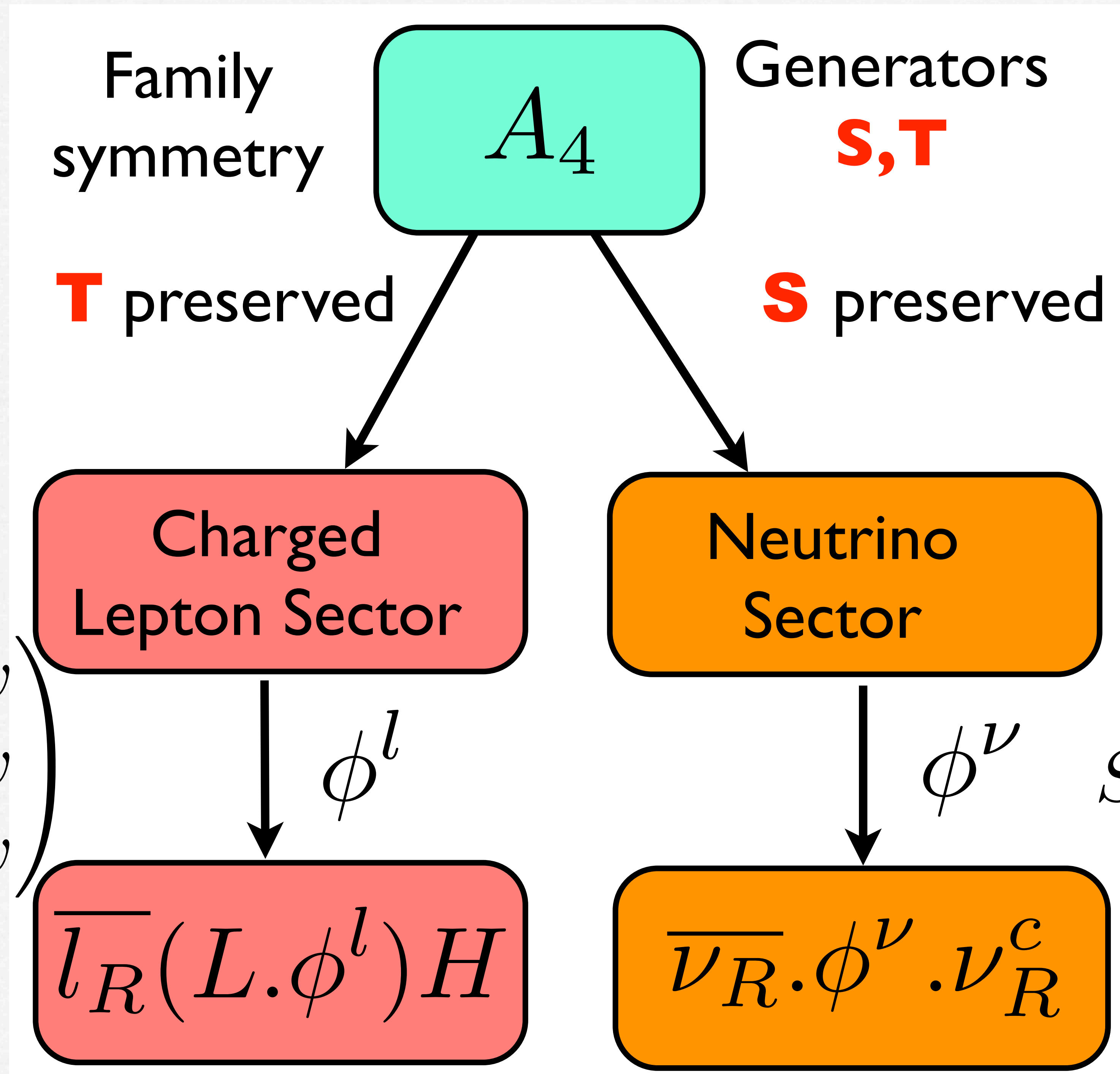
$$\langle\phi\rangle = \begin{pmatrix}2v\\-v\\v\end{pmatrix} \perp \begin{pmatrix}v\\v\\-v\end{pmatrix}, \begin{pmatrix}0\\e\\e\end{pmatrix}$$

Indirect

CSD(n)

$$\langle\phi\rangle = \begin{pmatrix}a\\na\\(n-2)a\end{pmatrix} \perp \begin{pmatrix}2v\\-v\\v\end{pmatrix}$$

Direct Models



$$T\begin{pmatrix} v \\ v \\ v \end{pmatrix} = \begin{pmatrix} v \\ v \\ v \end{pmatrix}$$

$$S\begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix}$$

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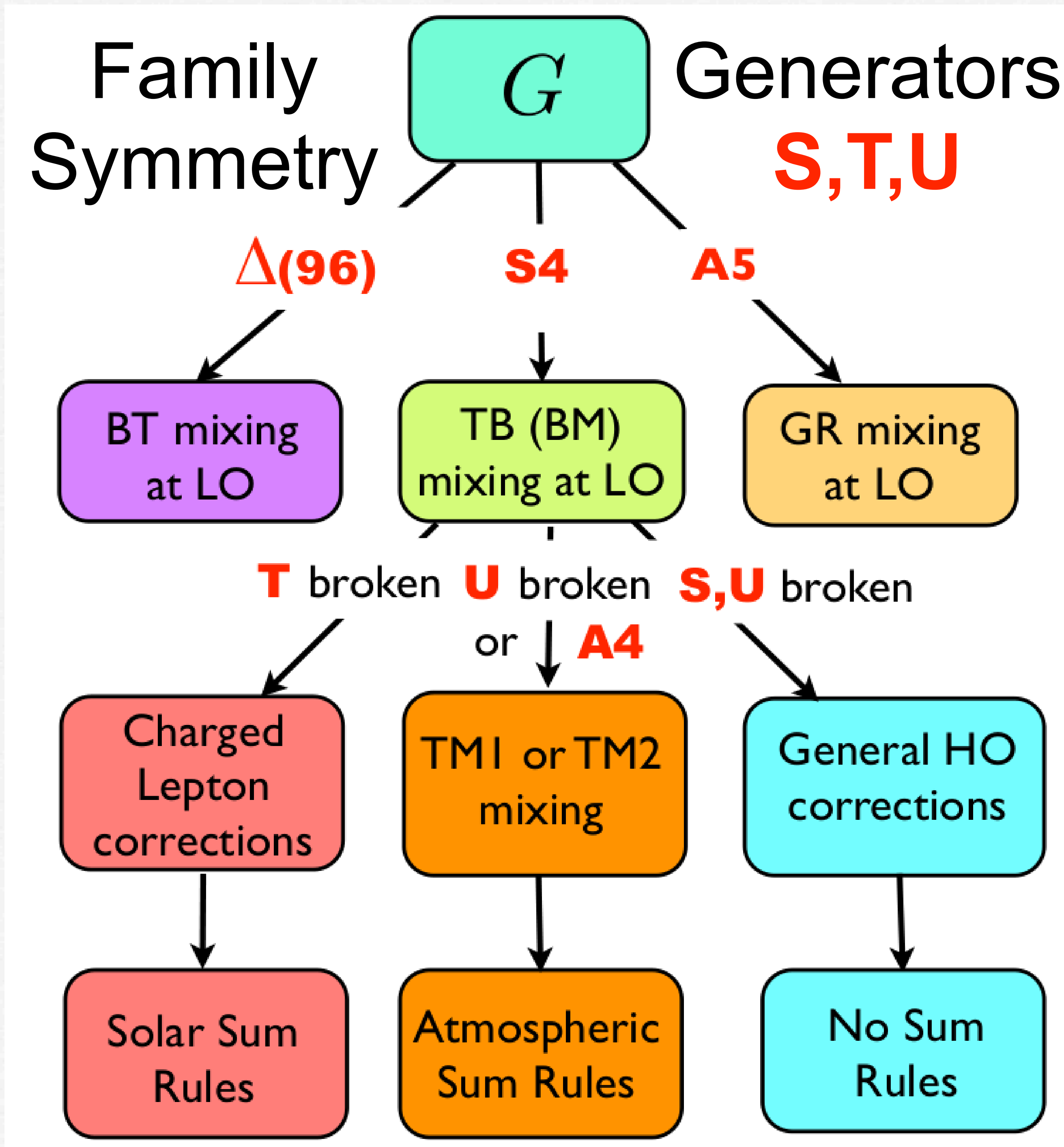
1402.4271

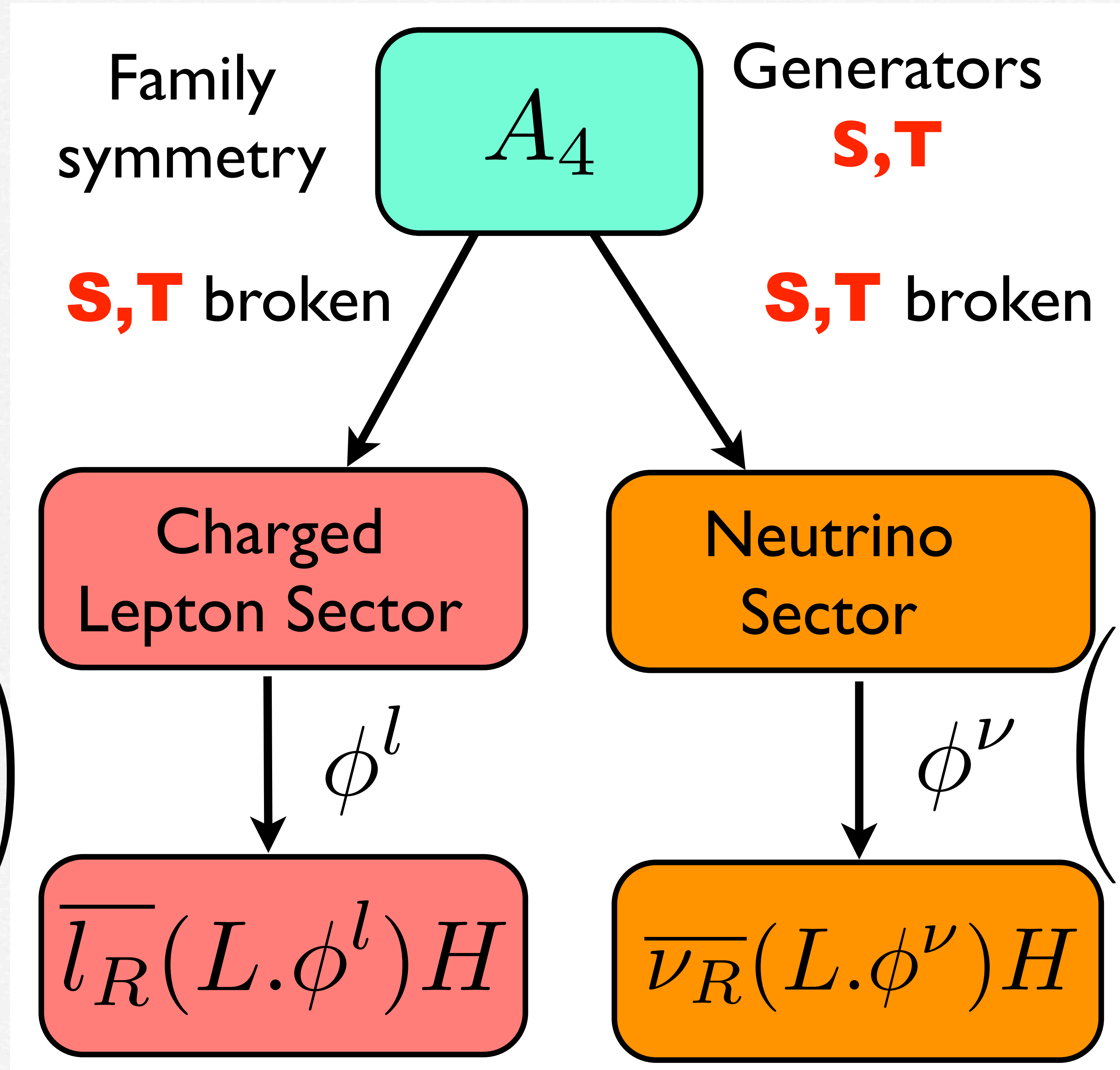
Direct Model Building



TB = tri-bimaximal
BM = bimaximal
GR = golden ratio
BT = bi-trimaximal
TM = trimaximal

	θ_{13}	θ_{23}	θ_{12}
TB	0°	45°	35.3°
BM	0°	45°	45°
GR	0°	45°	31.7°
BT	12.2°	36.2°	36.2°
TM	$\neq 0^\circ$	$\neq 45^\circ$	35.3°

Indirect Models



Diagonal

$$\begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix}\begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}$$

CSD(n)

$$\begin{pmatrix} 0 \\ e \\ e \end{pmatrix}\begin{pmatrix} a \\ na \\ (n-2)a \end{pmatrix}$$

Indirect Models (cont'd)

$$\overline{l_R}(L.\phi^l)H$$

$$\overline{e_R}(L.\phi^e)H + \overline{\mu_R}(L.\phi^\mu)H + \overline{\tau_R}(L.\phi^\tau)H$$

$$\langle\phi^e\rangle \propto \begin{pmatrix}1\\0\\0\end{pmatrix} \quad \langle\phi^\mu\rangle \propto \begin{pmatrix}0\\1\\0\end{pmatrix} \quad \langle\phi^\tau\rangle \propto \begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$m^l_{RL} = \begin{pmatrix}m_e & 0 & 0\\ 0 & m_\mu & 0\\ 0 & 0 & m_\tau\end{pmatrix}$$

$$\overline{\nu_R}(L.\phi^\nu)H$$

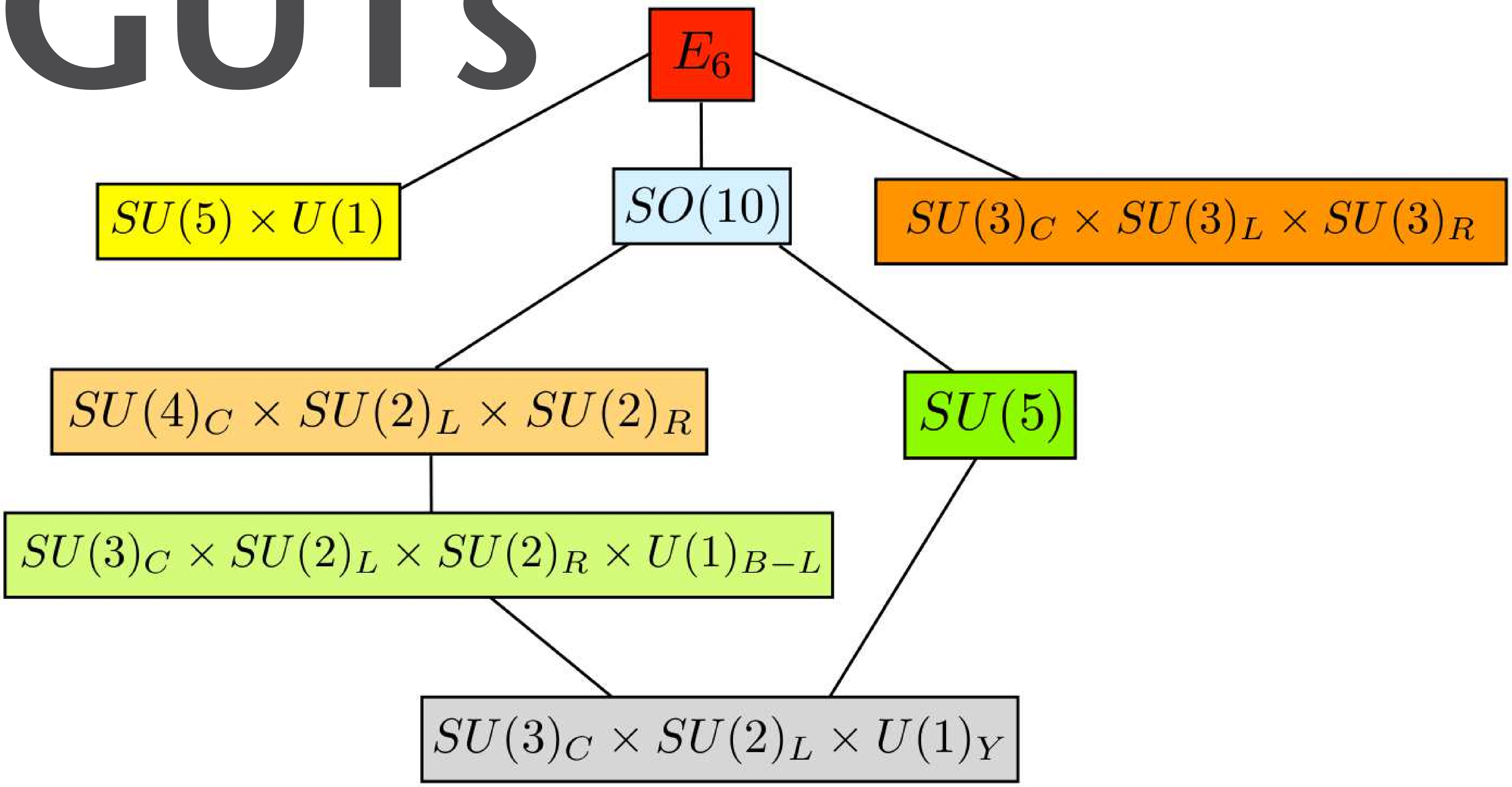
$$\overline{\nu_R^{\rm sol}}(L.\phi^{\rm sol})H + \overline{\nu_R^{\rm atm}}(L.\phi^{\rm atm})H$$

$$\langle\phi^{\rm sol}\rangle = \begin{pmatrix}a\\ na\\ (n-2)a\end{pmatrix} \quad \langle\phi^{\rm atm}\rangle = \begin{pmatrix}0\\ e\\ e\end{pmatrix}$$

$$m^D_{RL} = \begin{pmatrix}a & na & (n-2)a\\ 0 & e & e\end{pmatrix}$$

CSD(n)

GUTs

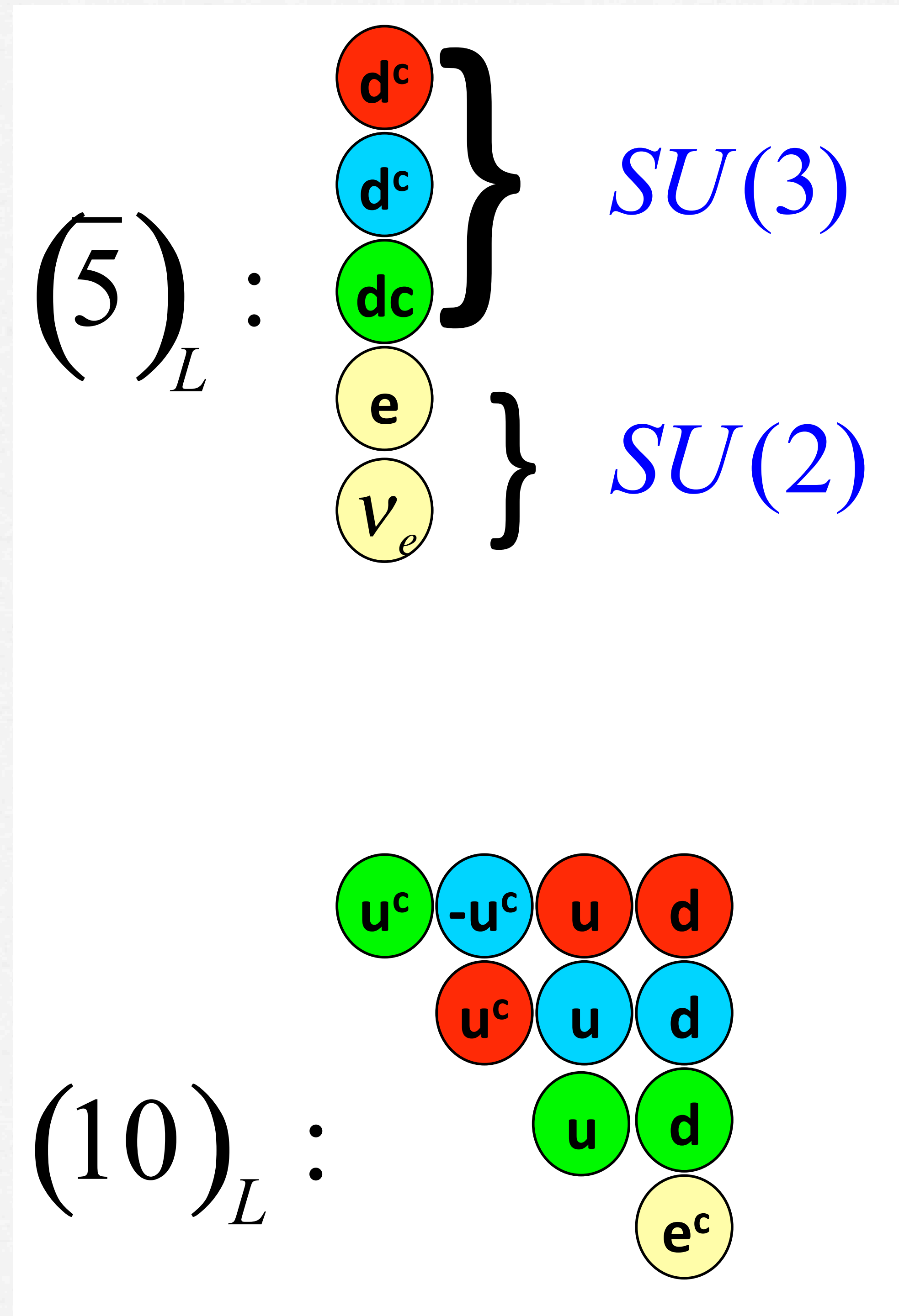


E_6
$SU(5) \times U(1)$
$SO(10)$
$SU(3)_C \times SU(3)_L \times SU(3)_R$
$SU(4)_C \times SU(2)_L \times SU(2)_R$
$SU(5)$
$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
$SU(3)_C \times SU(2)_L \times U(1)_Y$

SU(5) GUT

Right-handed neutrino is a singlet

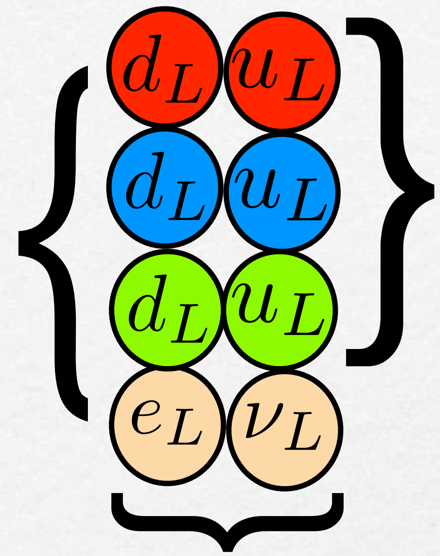
$(1)_L : \nu^c$



Pati-Salam

$$SU(4)_C \times SU(2)_L \times SU(2)_R$$

$(4, 2, 1)_L :$ $SU(4)_C$

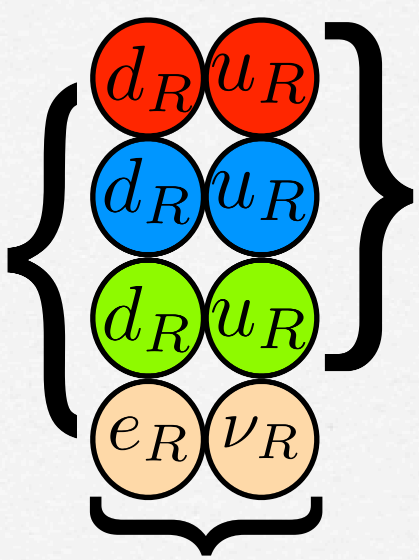


$SU(3)_C$

$SU(2)_L$

"Lepton number as the fourth colour"

$(4, 1, 2)_R :$ $SU(4)_C$



$SU(3)_C$

$SU(2)_R$

Right-handed neutrino is predicted

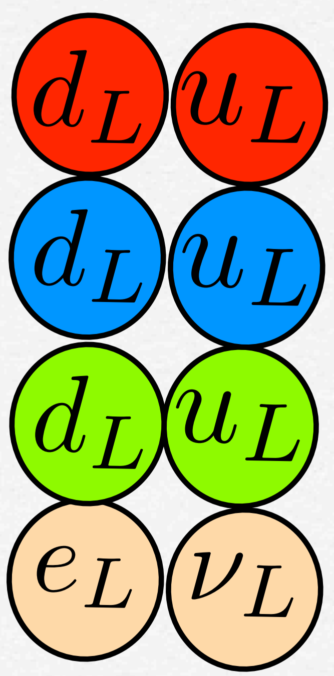
1406.7005

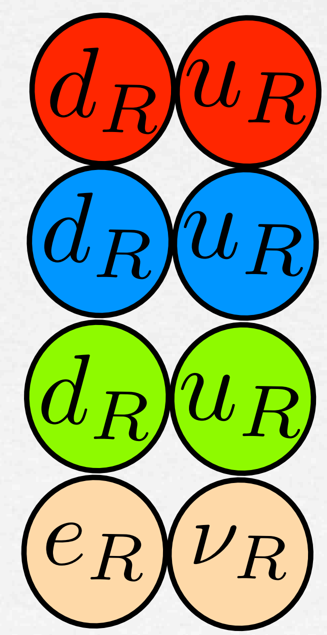
A to Z of Flavour with Pati-Salam

$$A_4 \times Z_5 \times SU(4)_C \times SU(2)_L \times SU(2)_R$$

Left-handed quarks and leptons triplets of A_4

Right-handed quarks and leptons distinguished by Z_5





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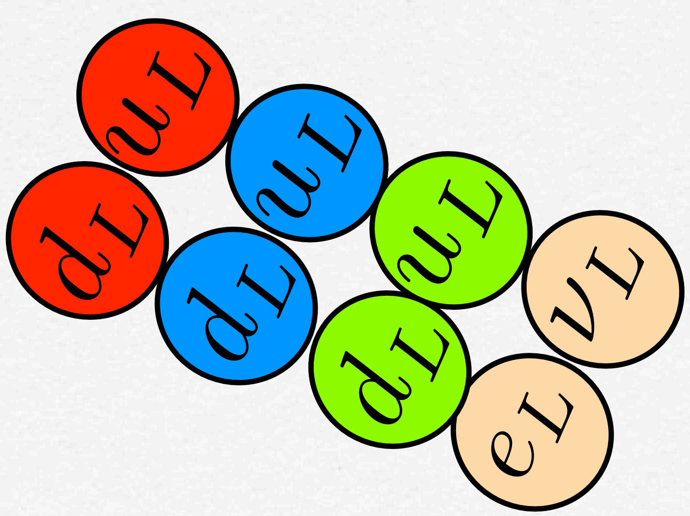

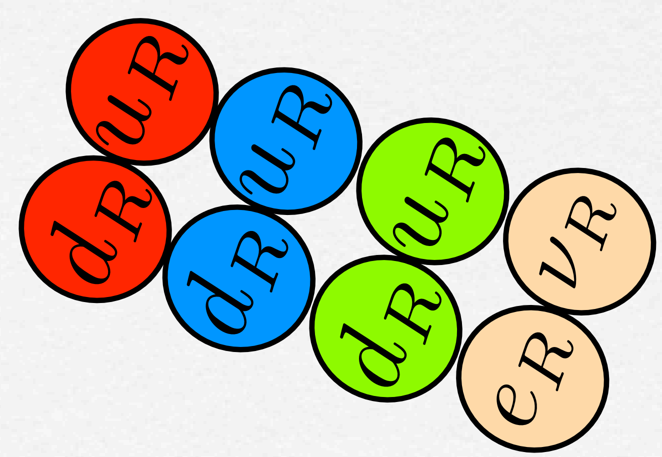

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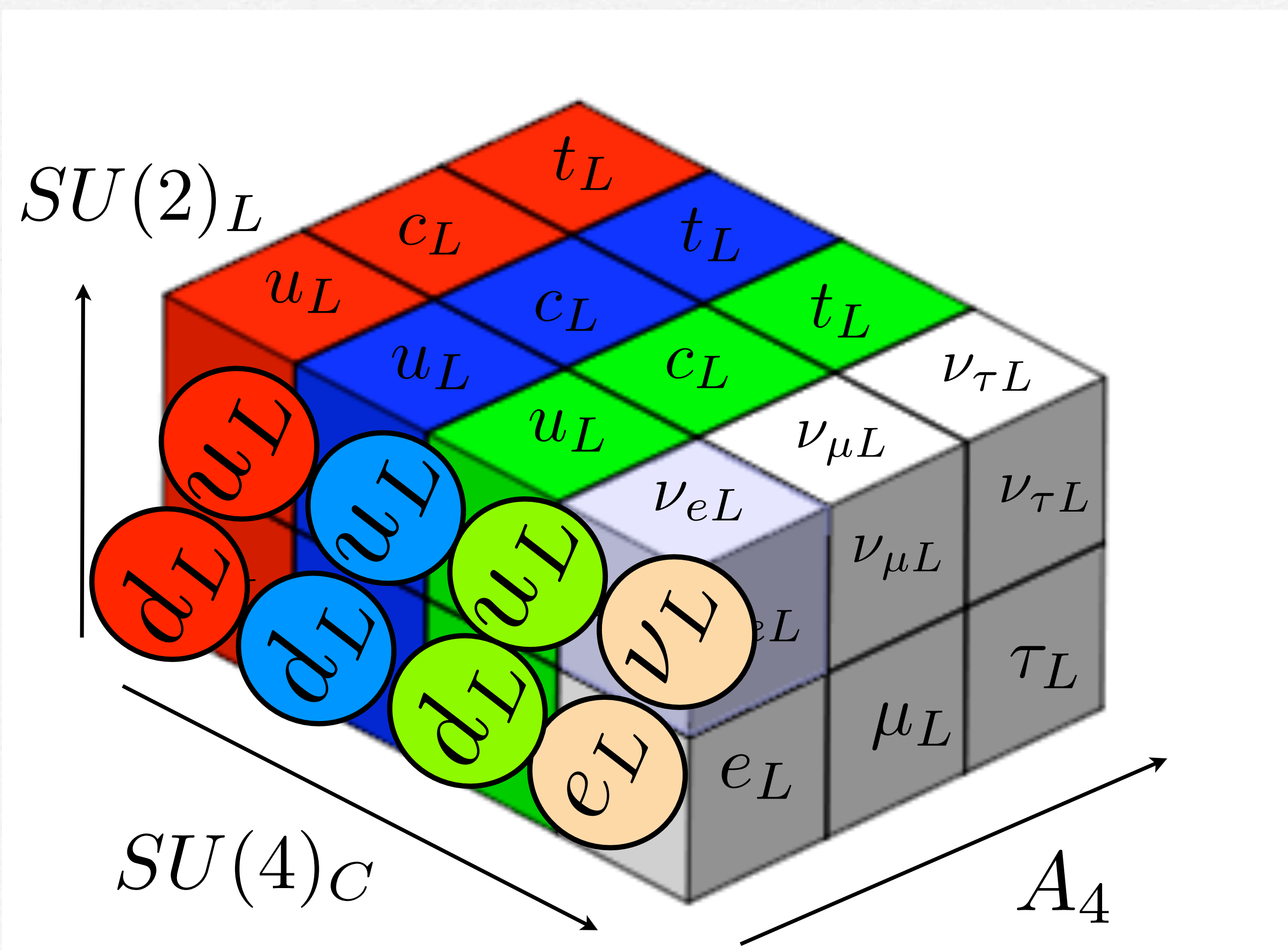

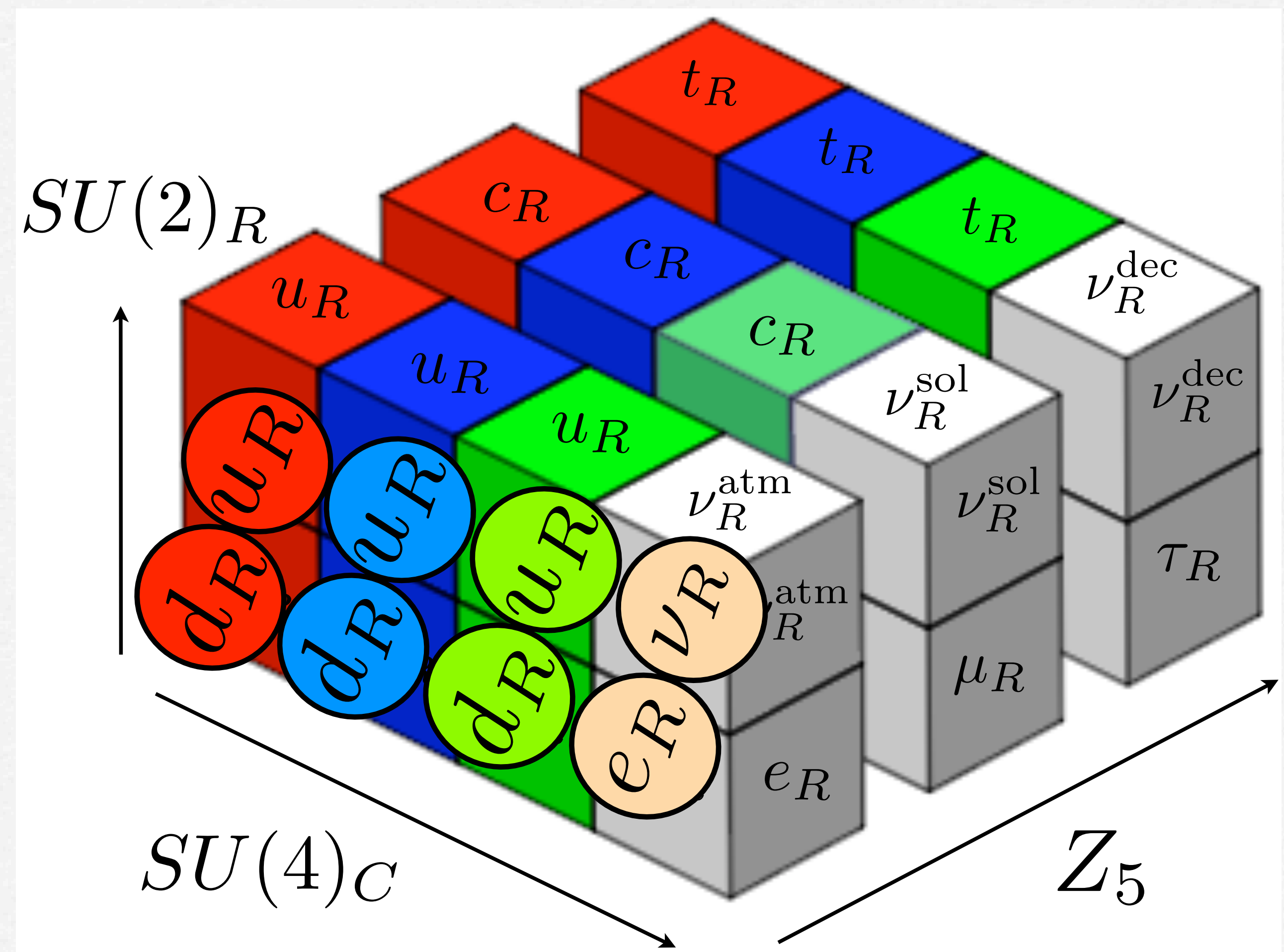

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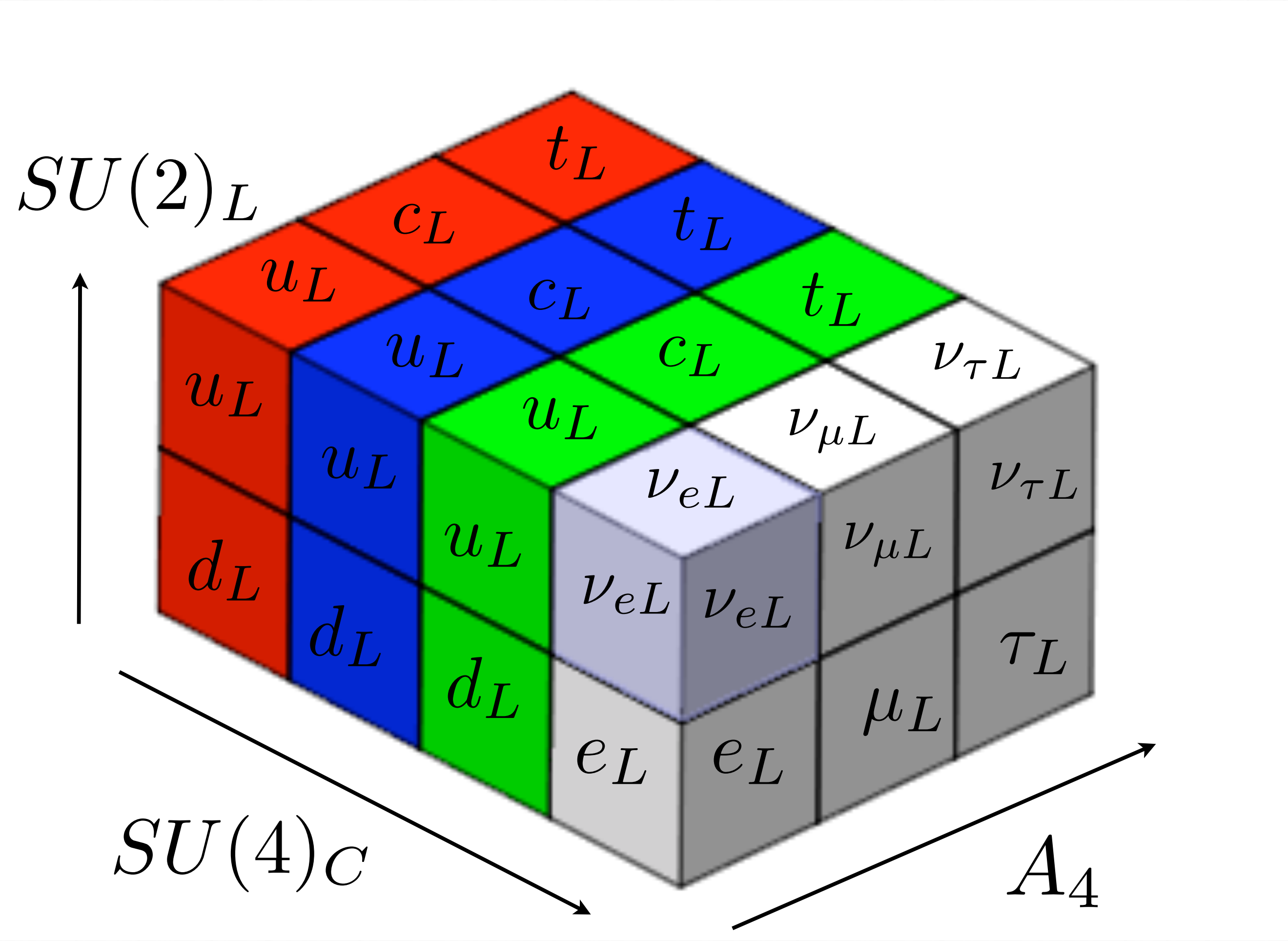

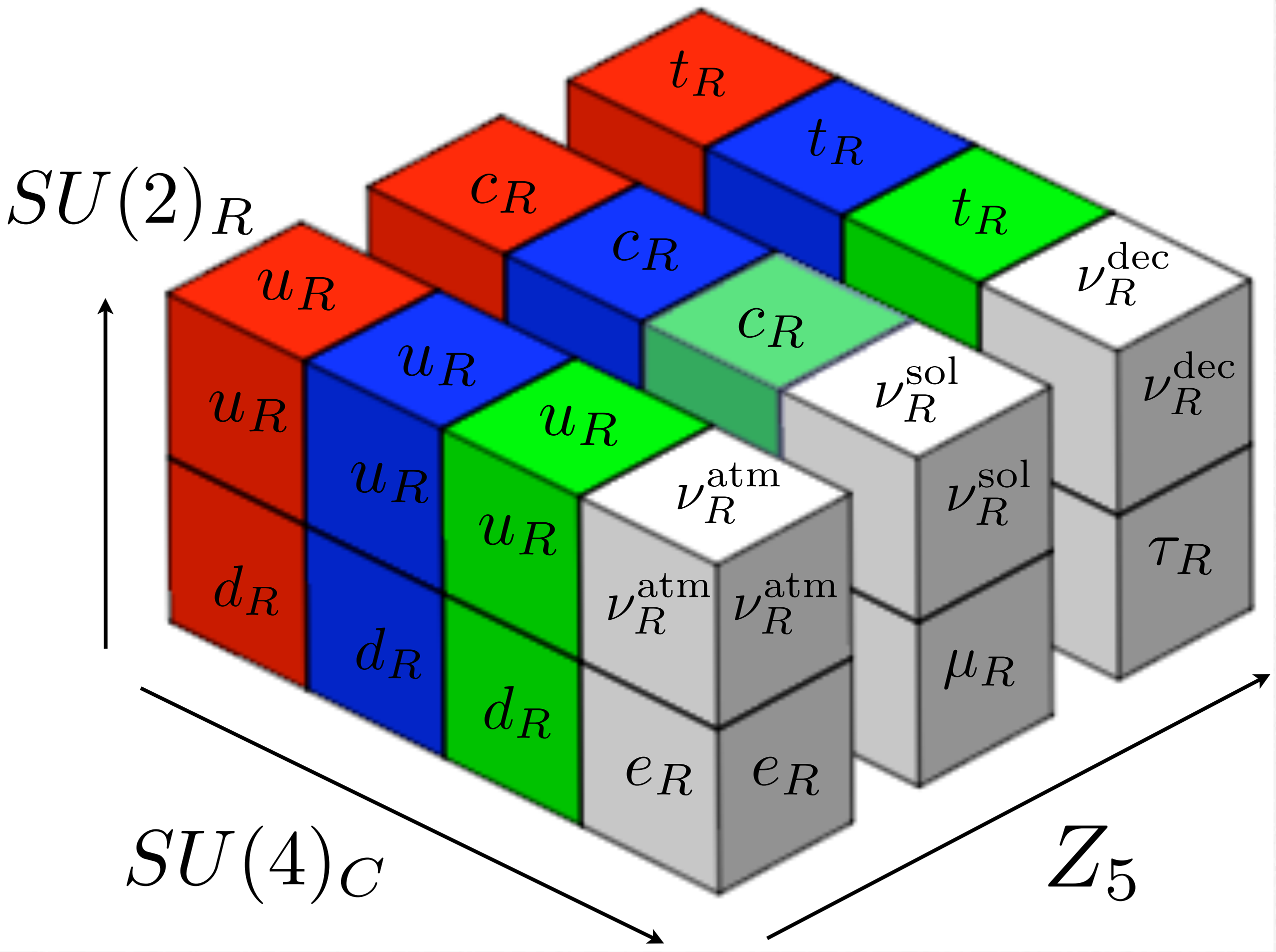

name	field	$SU(4)_C \times SU(2)_L \times SU(2)_R$	A_4	Z_5	R
Quarks	F	$(4, 2, 1)$	3	1	1
and leptons	$F^c_{1,2,3}$	$(\overline{4}, 1, 2)$	1	α,α^3,1	1

name	field	$SU(4)_C \times SU(2)_L \times SU(2)_R$	A_4	Z_5	R
Quarks	F	$(4, 2, 1)$	3	1	1
and leptons	$F^c_{1,2,3}$	$(\overline{4}, 1, 2)$	1	α,α^3,1	1
PS Higgs	$\overline{H^c}$, H^c	$(4, 1, 2)$, $(\overline{4}, 1, 2)$	1	1	0

name	field	$SU(4)_C \times SU(2)_L \times SU(2)_R$	A_4	Z_5	R
Quarks	F	$(4, 2, 1)$	3	1	1
and leptons	$F^c_{1,2,3}$	$(\bar{4}, 1, 2)$	1	$\alpha,\alpha^3,1$	1
PS Higgs	$\overline{H^c}$, H^c	$(4, 1, 2)$, $(\bar{4}, 1, 2)$	1	1	0
A_4 triplet	$\phi^u_{1,2}$	$(1, 1, 1)$	3	α^4, α^2	0
flavons	$\phi^d_{1,2}$	$(1, 1, 1)$	3	α^3, α	0

name	field	$SU(4)_C \times SU(2)_L \times SU(2)_R$	A_4	Z_5	R
Quarks	F	$(4,2,1)$	3	1	1
and leptons	$F^c_{1,2,3}$	$(\bar{4},1,2)$	1	α,α^3,1	1
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A_4 triplet	$\phi^u_{1,2}$	$(1,1,1)$	3	α^4, α^2	0
flavons	$\phi^d_{1,2}$	$(1,1,1)$	3	α^3, α	0
Higgs bidoublets	h_3	$(1,2,2)$	3	1	0
	h_u	$(1,2,2)$	$1''$	α	0
	h_d,h^d_{15}	$(1,2,2)$, $(15,2,2)$	$1'$	α^3,α^4	0
	h^u_{15}	$(15,2,2)$	1	α	0

name	field	$SU(4)_C \times SU(2)_L \times SU(2)_R$	A_4	Z_5	R
Quarks	F	$(4, 2, 1)$	3	1	1
and leptons	$F^c_{1,2,3}$	$(\bar{4}, 1, 2)$	1	α,α^3,1	1
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A_4 triplet	$\phi^u_{1,2}$	$(1, 1, 1)$	3	α^4, α^2	0
flavons	$\phi^d_{1,2}$	$(1, 1, 1)$	3	α^3, α	0
	h_3	$(1, 2, 2)$	3	1	0
Higgs	h_u	$(1, 2, 2)$	$1''$	α	0
bidoublets	h_d,h^d_{15}	$(1, 2, 2)$, $(15, 2, 2)$	$1'$	α^3,α^4	0
	h^u_{15}	$(15, 2, 2)$	1	α	0
Dynamical	Σ_u	$(1, 1, 1)$	$1''$	α	0
masses	Σ_d,Σ^d_{15}	$(1, 1, 1)$, $(15, 1, 1)$	$1'$	α^3,α^2	0

name	field	$SU(4)_C \times SU(2)_L \times SU(2)_R$	A_4	Z_5	R
Quarks	F	$(4,2,1)$	3	1	1
and leptons	$F^c_{1,2,3}$	$(\bar{4},1,2)$	1	α,α^3,1	1
PS Higgs	$\overline{H^c}$, H^c	$(4,1,2)$, $(\bar{4},1,2)$	1	1	0
A_4 triplet	$\phi^u_{1,2}$	$(1,1,1)$	3	α^4, α^2	0
flavons	$\phi^d_{1,2}$	$(1,1,1)$	3	α^3, α	0
	h_3	$(1,2,2)$	3	1	0
Higgs	h_u	$(1,2,2)$	$1''$	α	0
bidoublets	h_d,h^d_{15}	$(1,2,2)$, $(15,2,2)$	$1'$	α^3,α^4	0
	h^u_{15}	$(15,2,2)$	1	α	0
Dynamical	Σ_u	$(1,1,1)$	$1''$	α	0
masses	Σ_d,Σ^d_{15}	$(1,1,1)$, $(15,1,1)$	$1'$	α^3,α^2	0
Majoron	ξ	$(1,1,1)$	1	α^4	0

name	field	$SU(4)_C \times SU(2)_L \times SU(2)_R$	A_4	Z_5	R
Quarks	F	$(4,2,1)$	3	1	1
and leptons	$F^c_{1,2,3}$	$(\bar{4},1,2)$	1	α,α^3,1	1
PS Higgs	$\overline{H^c}$, H^c	$(4,1,2)$, $(\bar{4},1,2)$	1	1	0
A_4 triplet	$\phi^u_{1,2}$	$(1,1,1)$	3	α^4,α^2	0
flavons	$\phi^d_{1,2}$	$(1,1,1)$	3	α^3,α	0
Higgs bidoublets	h_3	$(1,2,2)$	3	1	0
	h_u	$(1,2,2)$	$1''$	α	0
	h_d,h^d_{15}	$(1,2,2)$, $(15,2,2)$	$1'$	α^3,α^4	0
	h^u_{15}	$(15,2,2)$	1	α	0
Dynamical	Σ_u	$(1,1,1)$	$1''$	α	0
masses	Σ_d,Σ^d_{15}	$(1,1,1)$, $(15,1,1)$	$1'$	α^3,α^2	0
Majoron	ξ	$(1,1,1)$	1	α^4	0
Fermion Messengers	$X_{F''_{1,3}}$	$(4,2,1)$	$1''$	α,α^3	1
	$X_{F'_{1,3}}$	$(4,2,1)$	$1'$	α,α^3	1
	$X_{\overline{F_i}}$	$(\bar{4},2,1)$	1	α^i	1
	X_{ξ_i}	$(1,1,1)$	1	α^i	1

name	field	$SU(4)_C \times SU(2)_L \times SU(2)_R$	A_4	Z_5	R
Quarks	F	$(4,2,1)$	3	1	1
and leptons	$F^c_{1,2,3}$	$(\overline{4},1,2)$	1	α,α^3,1	1
PS Higgs	$\overline{H^c}$, H^c	$(4,1,2)$, $(\overline{4},1,2)$	1	1	0
A_4 triplet	$\phi^u_{1,2}$	$(1,1,1)$	3	α^4,α^2	0
flavons	$\phi^d_{1,2}$	$(1,1,1)$	3	α^3,α	0
	h_3	$(1,2,2)$	3	1	0
Higgs	h_u	$(1,2,2)$	$1''$	α	0
bidoublets	h_d,h^d_{15}	$(1,2,2)$, $(15,2,2)$	$1'$	α^3,α^4	0
	h^u_{15}	$(15,2,2)$	1	α	0
Dynamical	Σ_u	$(1,1,1)$	$1''$	α	0
masses	Σ_d,Σ^d_{15}	$(1,1,1)$, $(15,1,1)$	$1'$	α^3,α^2	0
Majoron	ξ	$(1,1,1)$	1	α^4	0
	$X_{F''_{1,3}}$	$(4,2,1)$	$1''$	α,α^3	1
Fermion	$X_{F'_{1,3}}$	$(4,2,1)$	$1'$	α,α^3	1
Messengers	$X_{\overline{F_i}}$	$(\overline{4},2,1)$	1	α^i	1
	X_{ξ_i}	$(1,1,1)$	1	α^i	1

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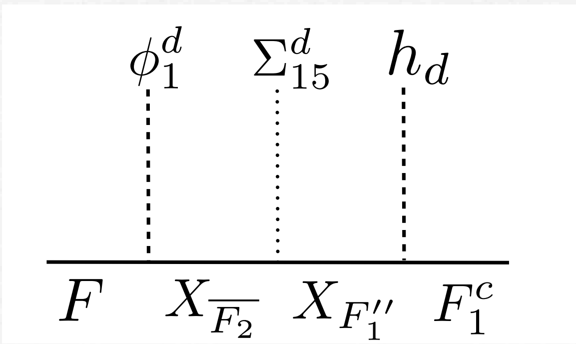
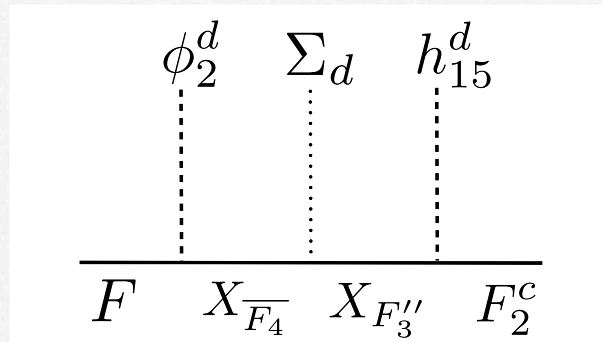
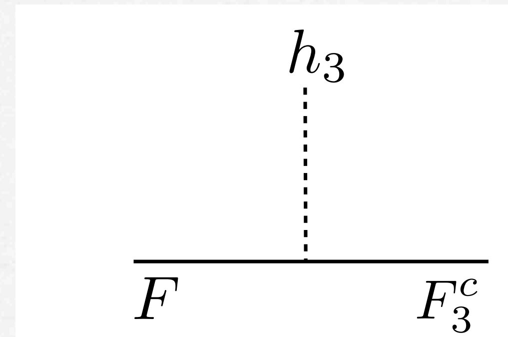
Yukawa operators







$$F.\frac{\phi_1^d}{\Sigma_{15}^d} h_d F_1^c + F.\frac{\phi_2^d}{\Sigma_d} h_{15}^d F_2^c + F.h_3 F_3^c$$

Down type quarks and charged leptons

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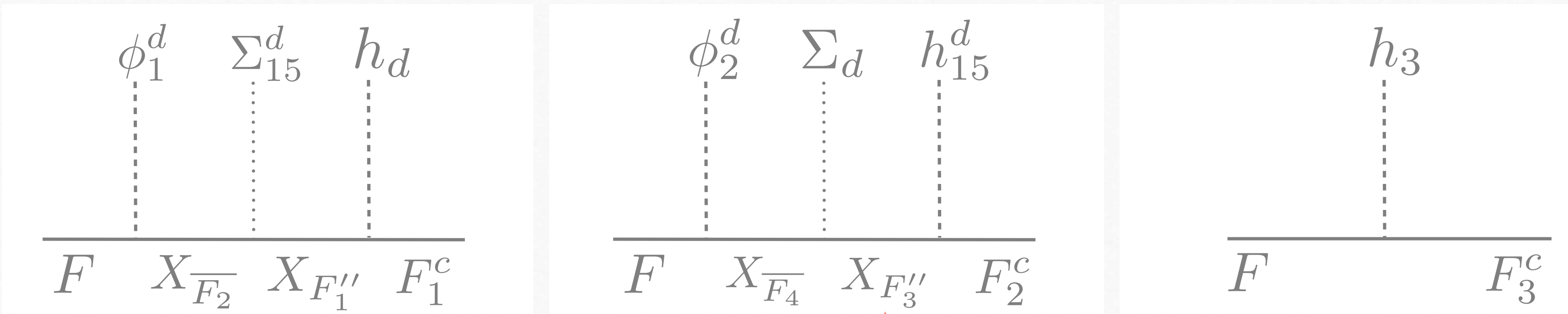
Yukawa operators

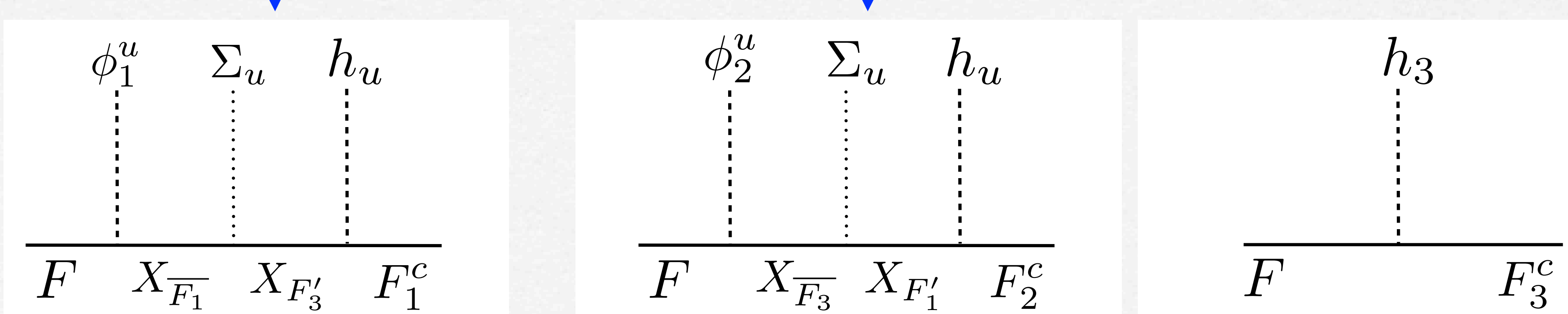


$$F.\frac{\phi_1^d}{\Sigma_{15}^d} h_d F_1^c + F.\frac{\phi_2^d}{\Sigma_d} h_{15}^d F_2^c + F.h_3 F_3^c$$

Down type quarks and charged leptons

$$F.\frac{\phi_1^u}{\Sigma_u} h_u F_1^c + F.\frac{\phi_2^u}{\Sigma_u} h_u F_2^c + F.h_3 F_3^c$$

up type quarks and neutrinos



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Yukawa operators (cont'd)

Third family renormalisable

$$F.h_3F_3^c \to Q_3H_uu_3^c + Q_3H_dd_3^c + L_3H_u\nu_3^c + L_3H_de_3^c,$$

Yukawa operators (cont'd)

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Third family renormalisable

$$F.h_3F^c_3 \to Q_3H_uu^c_3 + Q_3H_dd^c_3 + L_3H_u\nu^c_3 + L_3H_de^c_3,$$

First and second family involve flavons

$$F.\phi^u_ih_uF^c_i \to \quad Q.\langle\phi^u_i\rangle H_uu^c_i + L.\langle\phi^u_i\rangle H_u\nu^c_i,$$

$$F.\phi^d_ih_dF^c_i \to \quad Q.\langle\phi^d_i\rangle H_dd^c_i + L.\langle\phi^d_i\rangle H_de^c_i,$$

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Yukawa operators (cont'd)

Third family renormalisable

$$F.h_3F_3^c \to Q_3H_uu_3^c + Q_3H_dd_3^c + L_3H_u\nu_3^c + L_3H_de_3^c,$$

First and second family involve flavons

$$F.\phi_i^u h_u F_i^c \to Q.\langle\phi_i^u\rangle H_u u_i^c + L.\langle\phi_i^u\rangle H_u \nu_i^c,$$

$$F.\phi_i^d h_d F_i^c \to Q.\langle\phi_i^d\rangle H_d d_i^c + L.\langle\phi_i^d\rangle H_d e_i^c,$$

CSD4 vacuum alignment

$$\langle\phi_1^u\rangle = \frac{V_1^u}{\sqrt{2}} e^{im\pi/5} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \langle\phi_2^u\rangle = \frac{V_2^u}{\sqrt{21}} e^{im\pi/5} \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$$

"$\langle\phi_{\mathrm{atm}}\rangle$" "$\langle\phi_{\mathrm{sol}}\rangle$"

$$\langle\phi_1^d\rangle = V_1^d e^{in\pi/5} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \langle\phi_2^d\rangle = V_2^d e^{in\pi/5} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

"$\langle\phi_e\rangle$" "$\langle\phi_\mu\rangle$"

1406.7005

Yukawa operators (cont'd)

Third family renormalisable

$$F.h_3F_3^c \to Q_3H_uu_3^c + Q_3H_dd_3^c + L_3H_u\nu_3^c + L_3H_de_3^c,$$

First and second family involve flavons

$$F.\phi_i^u h_u F_i^c \to Q.\langle\phi_i^u\rangle H_u u_i^c + L.\langle\phi_i^u\rangle H_u \nu_i^c,$$

$$F.\phi_i^d h_d F_i^c \to Q.\langle\phi_i^d\rangle H_d d_i^c + L.\langle\phi_i^d\rangle H_d e_i^c,$$

Flavons form first two columns of Yukawa matrices

CSD4 Vacuum alignment

$$\langle\phi_1^u\rangle = \frac{V_1^u}{\sqrt{2}}e^{im\pi/5}\begin{pmatrix}0\\1\\1\end{pmatrix}, \quad \langle\phi_2^u\rangle = \frac{V_2^u}{\sqrt{21}}e^{im\pi/5}\begin{pmatrix}1\\4\\2\end{pmatrix}$$

"$\langle\phi_{\rm atm}\rangle$" "$\langle\phi_{\rm sol}\rangle$"

$$\Rightarrow \quad Y^u = Y^\nu = \begin{pmatrix}0 & b & 0\\ a & 4b & 0\\ a & 2b & c\end{pmatrix}$$

$$\langle\phi_1^d\rangle = V_1^d e^{in\pi/5}\begin{pmatrix}1\\0\\0\end{pmatrix}, \quad \langle\phi_2^d\rangle = V_2^d e^{in\pi/5}\begin{pmatrix}0\\1\\0\end{pmatrix}$$

"$\langle\phi_e\rangle$" "$\langle\phi_\mu\rangle$"

$$\Rightarrow \quad Y^d \sim Y^e \sim \begin{pmatrix}y_d^0 & 0 & 0\\ 0 & y_s^0 & 0\\ 0 & 0 & y_b^0\end{pmatrix}$$

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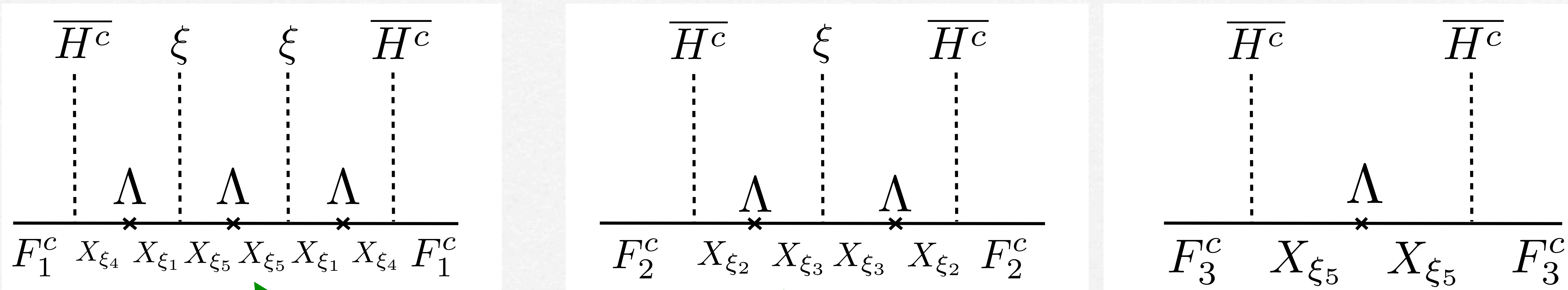
Majorana operators



$$\frac{\xi^2}{\Lambda^2}\frac{\overline{H^cH^c}}{\Lambda}F_1^cF_1^c + \frac{\xi}{\Lambda}\frac{\overline{H^cH^c}}{\Lambda}F_2^cF_2^c + \frac{\xi}{\Lambda}\frac{\overline{H^cH^c}}{\Lambda}F_1^cF_3^c + \frac{\overline{H^cH^c}}{\Lambda}F_3^cF_3^c$$

$$M_R = \begin{pmatrix} M_1 & 0 & M_{13} \\ 0 & M_2 & 0 \\ M_{13} & 0 & M_3 \end{pmatrix}$$

$$M_1 : M_2 : M_3 \sim \tilde{\xi}^2 : \tilde{\xi} : 1 \quad M_{13}^2/M_3 \sim \tilde{\xi}^2$$

$$M_3 \sim \frac{\langle \overline{H^c}\rangle^2}{\Lambda} \sim 5.10^{15}\ \text{GeV} \qquad \tilde{\xi} = \frac{\langle \xi \rangle}{\Lambda} \sim 10^{-5}$$

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Yukawa and Mass Matrices

$$Y^u = Y^\nu = \begin{pmatrix} 0 & be^{-i3\pi/5} & \epsilon c \\ ae^{-i3\pi/5} & 4be^{-i3\pi/5} & 0 \\ ae^{-i3\pi/5} & 2be^{-i3\pi/5} & c \end{pmatrix} \quad Y^d = \begin{pmatrix} y_d^0 e^{-i2\pi/5} & 0 & Ay_d^0 e^{-i2\pi/5} \\ By_d^0 e^{-i3\pi/5} & y_s^0 e^{-i2\pi/5} & Cy_d^0 e^{-i3\pi/5} \\ By_d^0 e^{-i3\pi/5} & 0 & y_b^0 + Cy_d^0 e^{-i3\pi/5} \end{pmatrix}$$

$$M_R \approx \begin{pmatrix} M_1 e^{8i\pi/5} & 0 & 0 \\ 0 & M_2 e^{4i\pi/5} & 0 \\ 0 & 0 & M_3 \end{pmatrix} \quad Y^e = \begin{pmatrix} -(y_d^0/3)e^{-i2\pi/5} & 0 & Ay_d^0 e^{-i2\pi/5} \\ By_d^0 e^{-i3\pi/5} & -4.5y_s^0 e^{-i2\pi/5} & -3Cy_d^0 e^{-i3\pi/5} \\ By_d^0 e^{-i3\pi/5} & 0 & y_b^0 - 3Cy_d^0 e^{-i3\pi/5} \end{pmatrix}$$

SO(10)-like diagonal RHN masses $M_1 : M_2 : M_3 \sim m_u^2 : m_c^2 : m_t^2$

Physical neutrino masses in a normal hierarchy (CSD)

CSD(4) + PS gives Cabibbo connection $\theta_C \approx 1/4$ or $\theta_C \approx 14^\circ$

All CP phases are fifth roots of unity due to Z_5

CKM parameters

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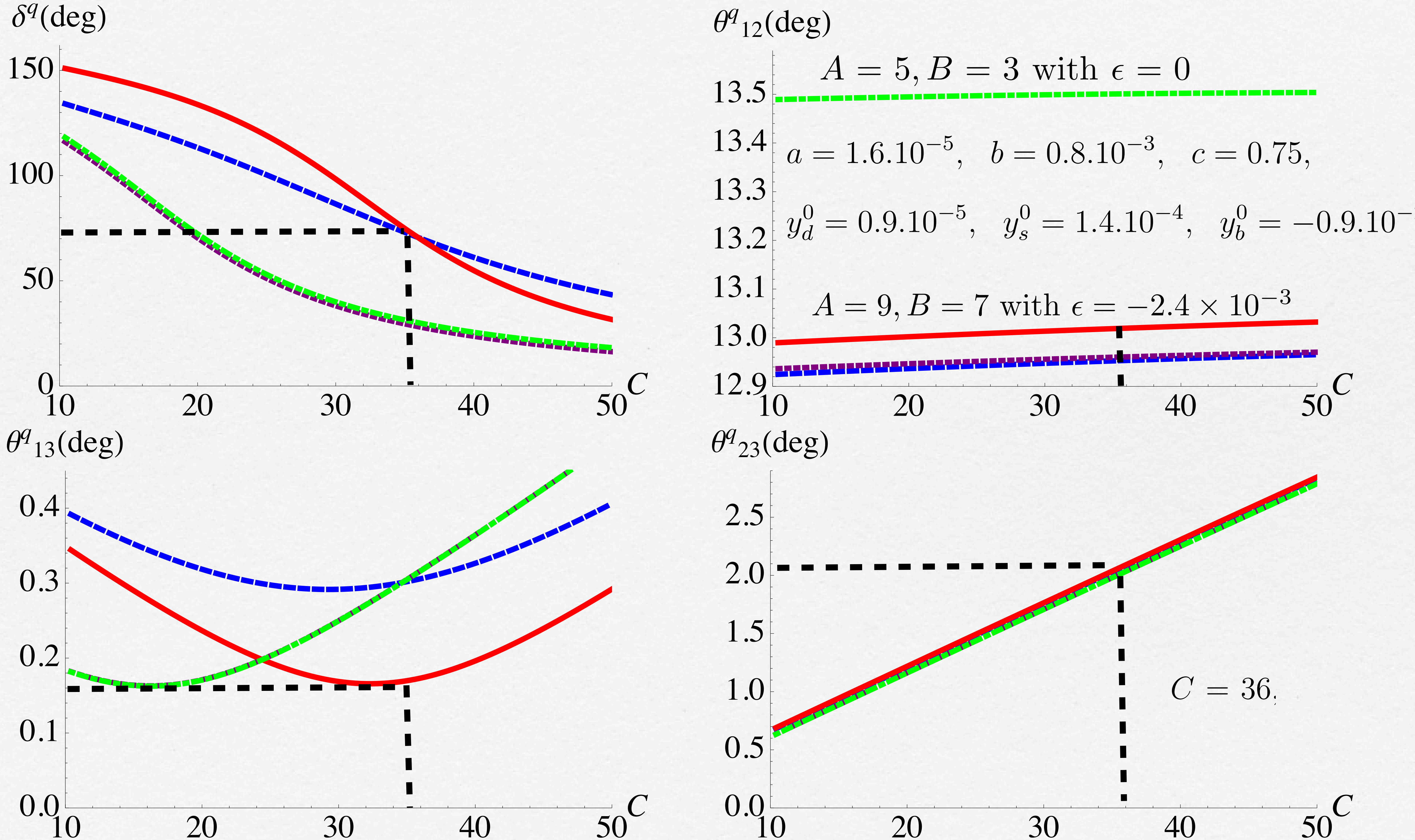

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The See-Saw mechanism

$$m^\nu = -v_u^2 Y^\nu M_{\mathrm{R}}^{-1} Y^{\nu T}$$

Neutrino mass matrix only depends on m_a, m_b, m_c

$$m^\nu = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b e^{2i\eta} \begin{pmatrix} 1 & 4 & 2 \\ 4 & 16 & 8 \\ 2 & 8 & 4 \end{pmatrix} + m_c e^{2i\eta} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

CSD(4)

$$m^\nu \sim \frac{\langle \phi_{\mathrm{atm}} \rangle \langle \phi_{\mathrm{atm}} \rangle^T}{\langle \xi \rangle^2} + \frac{\langle \phi_{\mathrm{sol}} \rangle \langle \phi_{\mathrm{sol}} \rangle^T}{\langle \xi \rangle}$$

$$\eta = 2\pi/5$$

CP phase from Z_5

Z_5 invariant potential

$$\left| \frac{\langle \xi \rangle^5}{\Lambda^3} - m^2 \right|^2 = 0 \qquad \langle \xi \rangle = |(\Lambda^3 m^2)^{1/5}| e^{-4i\pi/5}$$

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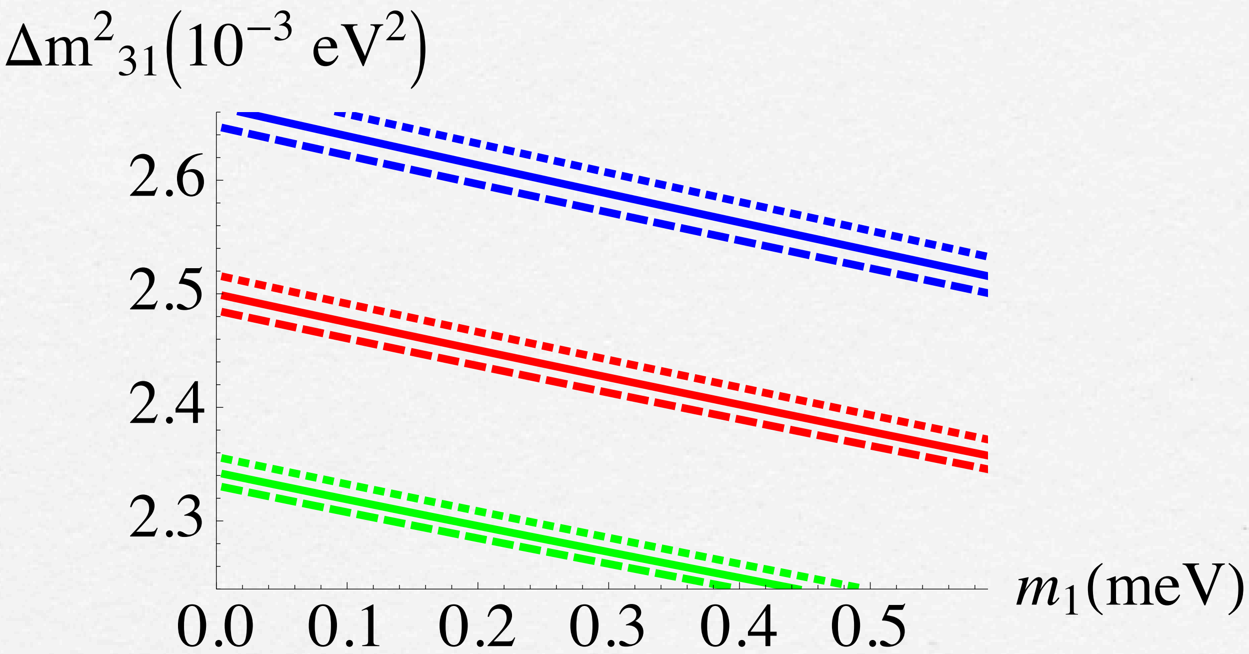
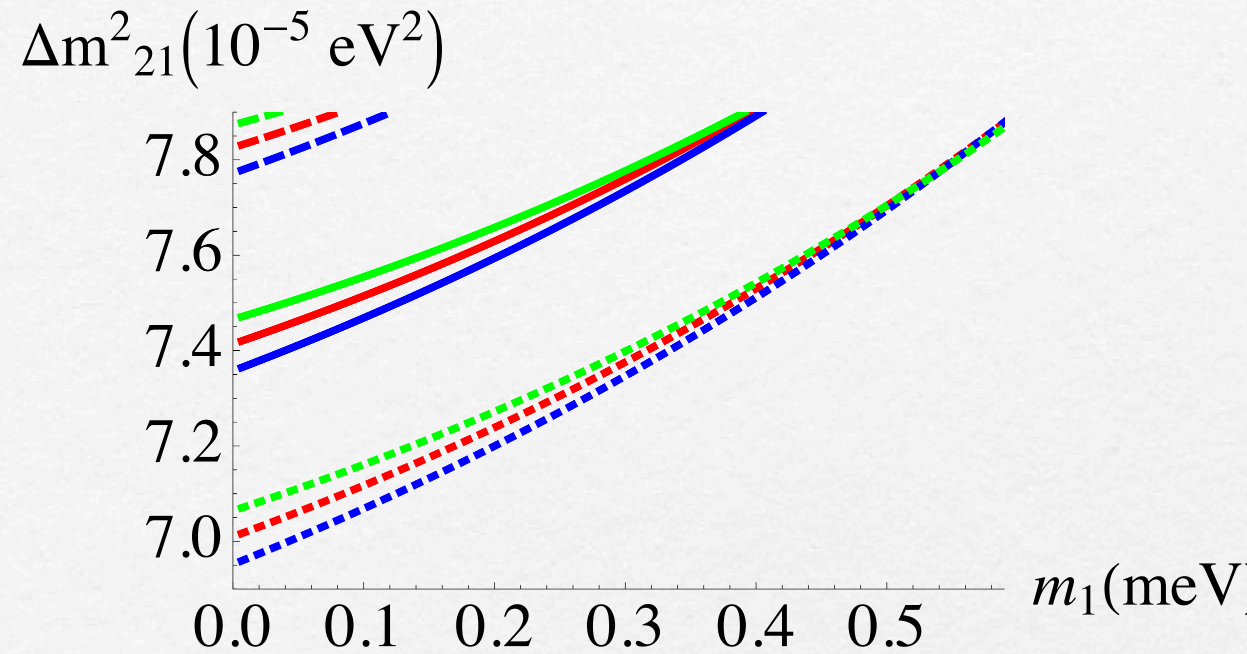
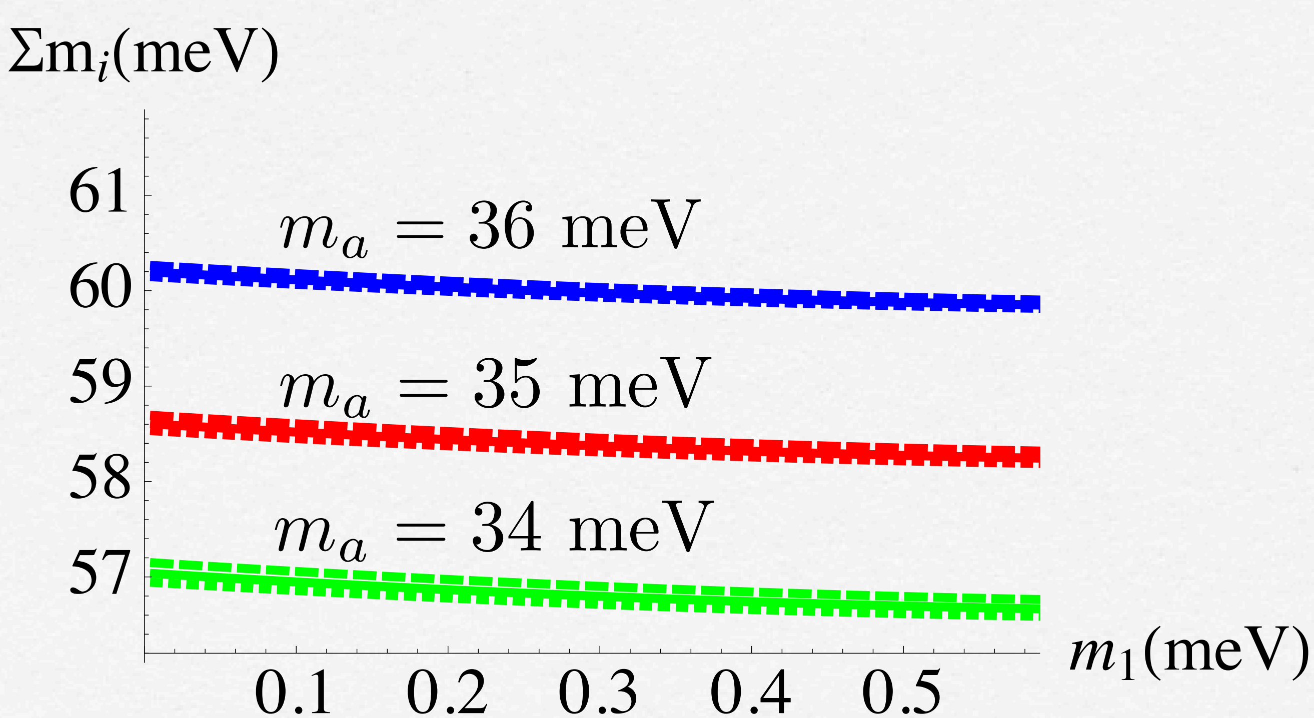
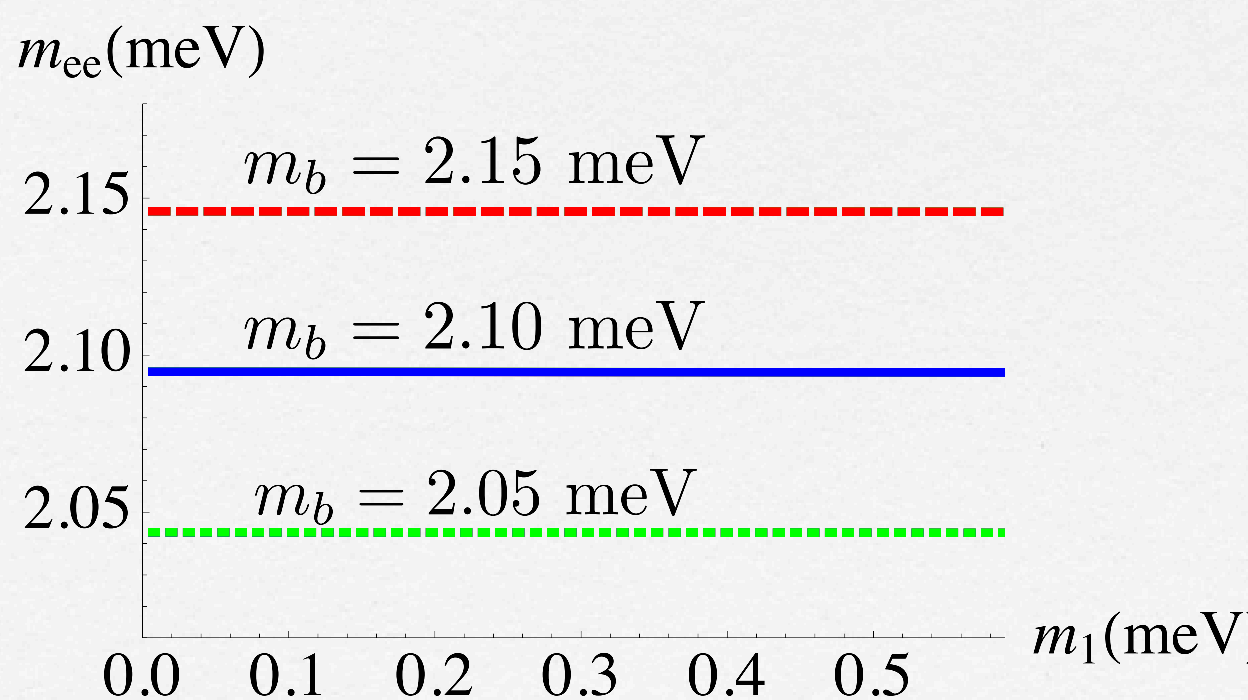
Neutrino mass variables

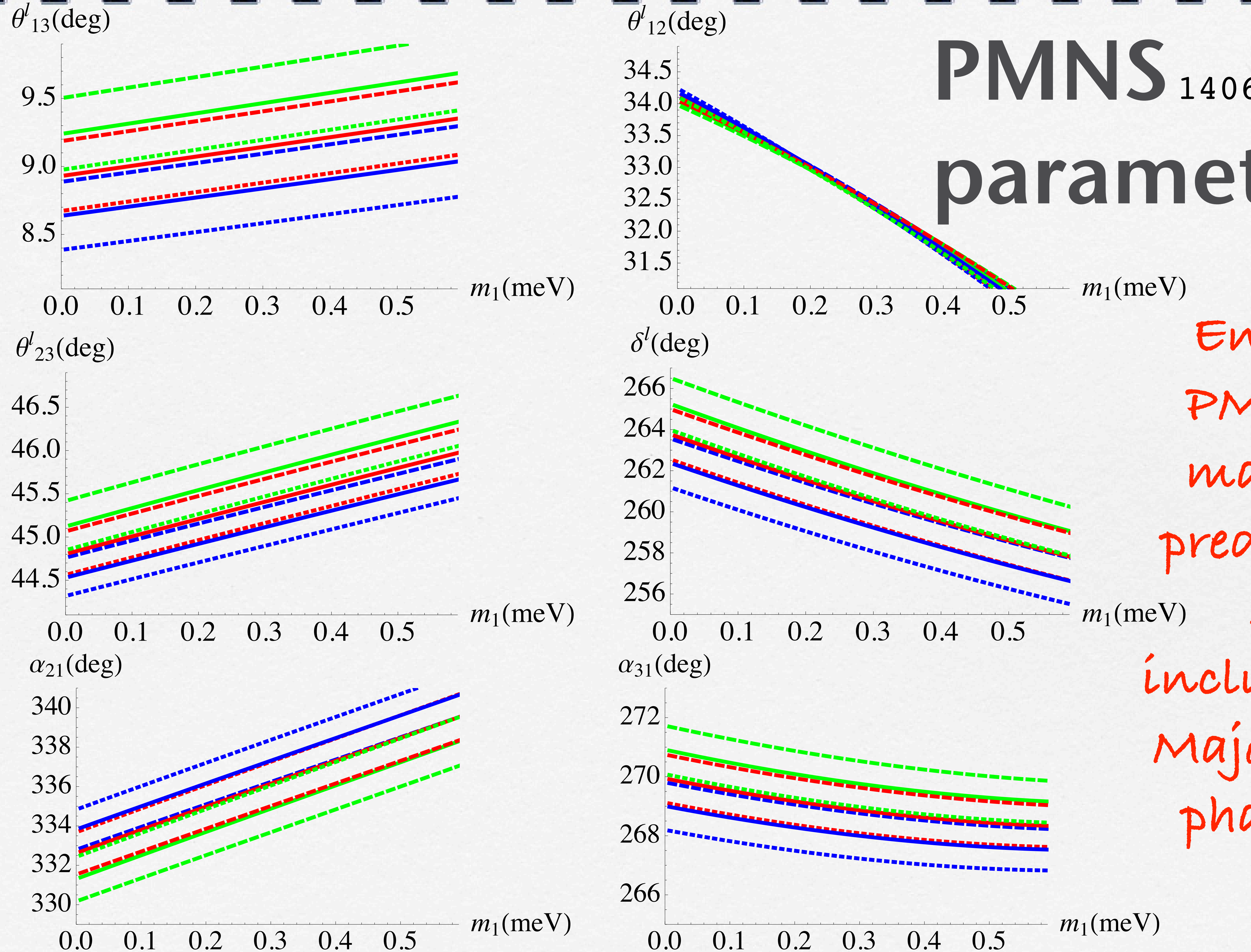








PMNS parameters 1406.7005



Entire PMNS matrix predicted … including Majorana phases!

Conclusions

- See-saw does not explain lepton mixing, we need symmetry to find a theory of flavour
- Direct models: either a small family symmetry with large correction or a large family group with a small correction
- Indirect models: allow small family symmetry groups such as A_4 if broken by new alignments as in CSD(n)
- GUT embeddings of indirect models, unifying quarks and leptons, is a highly predictive framework
- Considered a $A_4 x Z_5$ Pati-Salam model with PMNS predicted (plus normal hierarchy, Cabibbo angle)

Extra material

- A4 group theory
- Pati-Salam Breaking
- $\Delta(6n^2)$

A_4

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

Multiplying S and T we generate 12 group elements

$$a_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \; a_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \; a_3 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \; a_4 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$b_1 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \; b_2 = \begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}, \; b_3 = \begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}, \; b_4 = \begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$c_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \; c_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}, \; c_3 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}, \; c_4 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}$$

With eigenvectors

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \begin{pmatrix} \pm\frac{1}{\sqrt{3}} \\ \pm\frac{1}{\sqrt{3}} \\ \pm\frac{1}{\sqrt{3}} \end{pmatrix}$$

c.f. Trimaximal mixing

A_4 Non-trivial singlets 1’,1’’

Since $T^3 = 1$ it may be rep by cube roots of unity

hence two additional one dimensional reps $1', \ 1''$

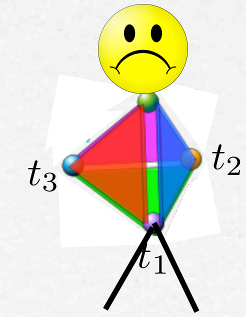
	S	T
$1'$	1	ω
$1''$	1	ω^2

$$S^2 = T^3 = 1$$

$$(ST)^3 = 1$$

$$\omega = e^{i2\pi/3}$$

These additional singlets have no geometric interpretation in terms of a tetrahedron



A_4 Clebsch Gordan coefficients

Irreducible reps

$1,\ 1',\ 1'',\ 3$

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$1 \otimes 1 = 1 \quad 1' \otimes 1'' = 1 \quad 1' \otimes 1' = 1'' \quad 1'' \otimes 1'' = 1'$$

$$\begin{aligned}
(ab)_1 &= a_1b_1 + a_2b_2 + a_3b_3\,; & 3 \otimes 3 = 1 \\
(ab)_{1'} &= a_1b_1 + \omega a_2b_2 + \omega^2 a_3b_3\,; & \oplus\, 1' \\
(ab)_{1''} &= a_1b_1 + \omega^2 a_2b_2 + \omega a_3b_3\,; & \oplus\, 1'' \\
(ab)_{3_1} &= (a_2b_3, a_3b_1, a_1b_2)\,; & \oplus\, 3_1 \\
(ab)_{3_2} &= (a_3b_2, a_1b_3, a_2b_1)\,, & \oplus\, 3_2
\end{aligned}$$

where $\omega^3 = 1,\ a = (a_1, a_2, a_3)$ and $b = (b_1, b_2, b_3)$

Pati-Salam breaking

$$SU(4)_C \times SU(2)_L \times SU(2)_R$$
$$\rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$H^c = (\bar{4}, 1, 2) = (u^c_H, d^c_H, \nu^c_H, e^c_H)$$
$$\overline{H^c} = (4, 1, 2) = (\bar{u}^c_H, \bar{d}^c_H, \bar{\nu}^c_H, \bar{e}^c_H)$$

$$\langle H^c \rangle = \langle \nu^c_H \rangle = \langle \overline{H^c} \rangle = \langle \bar{\nu}^c_H \rangle \sim 2 \times 10^{16}\ \mathrm{GeV}$$

$$F_i = (4, 2, 1)_i = \begin{pmatrix} u & u & u & \nu \\ d & d & d & e \end{pmatrix}_i \rightarrow (Q_i, L_i),$$

$$F^c_i = (\bar{4}, 1, 2)_i = \begin{pmatrix} u^c & u^c & u^c & \nu^c \\ d^c & d^c & d^c & e^c \end{pmatrix}_i \rightarrow (u^c_i, d^c_i, \nu^c_i, e^c_i)$$

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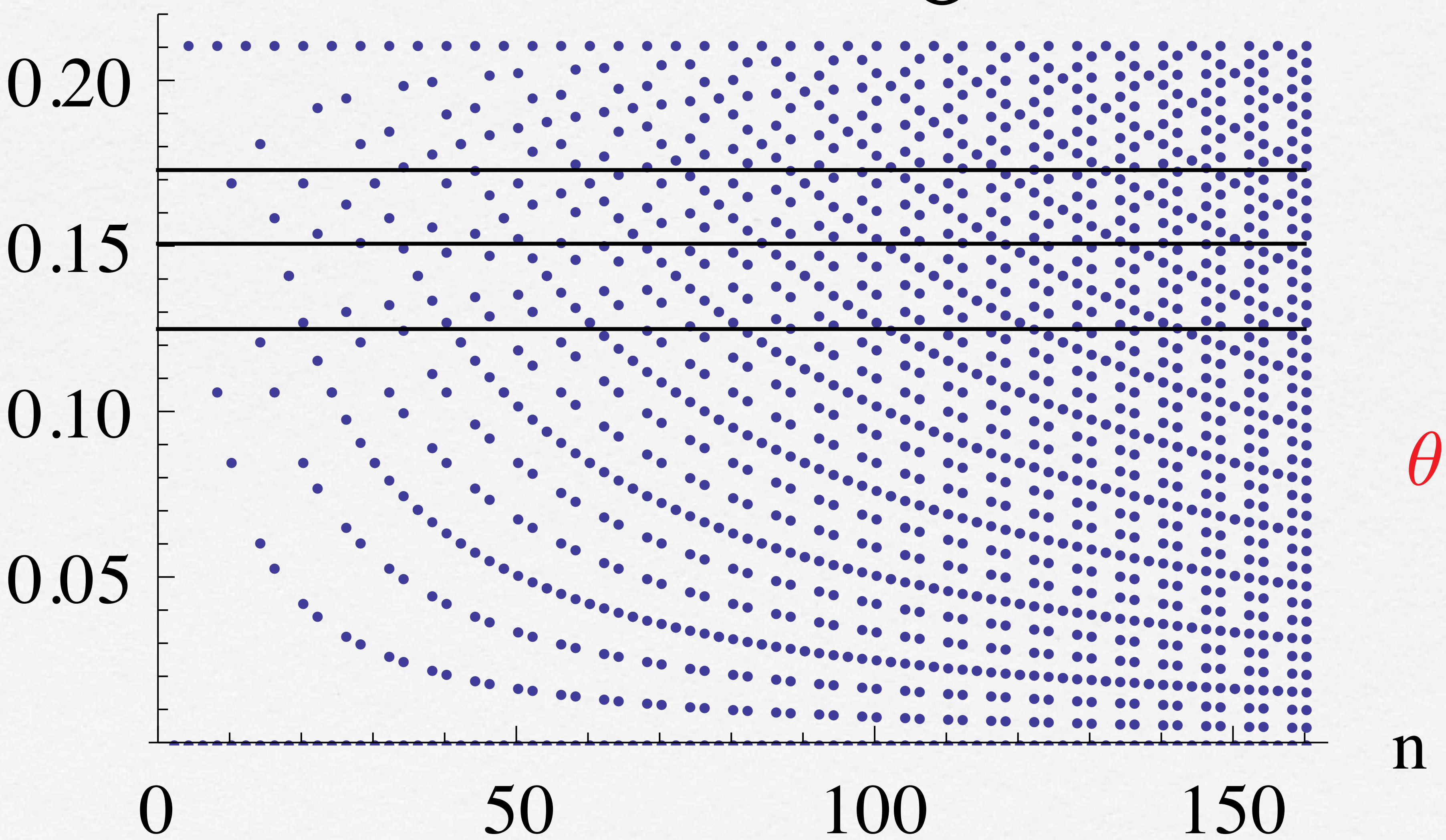
$\Delta(6n^2)$

$$\vartheta = \pi\gamma'/n$$
$$\gamma' = 1,\ldots,n/2,$$

$$V = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos(\vartheta) & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\sin(\vartheta) \\ -\sqrt{\frac{2}{3}}\sin\left(\frac{\pi}{6}+\vartheta\right) & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\cos\left(\frac{\pi}{6}+\vartheta\right) \\ \sqrt{\frac{2}{3}}\sin\left(\frac{\pi}{6}-\vartheta\right) & -\frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\cos\left(\frac{\pi}{6}-\vartheta\right) \end{pmatrix}$$

TM2 mixing

$|V_{13}|$



Predictions:

$\delta = 0, \pi$

$\theta_{23} = 45^\circ \mp \theta_{13}/\sqrt{2}.$

Atmospheric sum rule