29. Any 2×2 special unitary matrix can be parametrized as

$$U = x_0\sigma^0 + ix^a\sigma^a, x_0^2 + \sum_a x_a^2 = 1, \tag{1}$$

where x^a are real, $\sigma^a, a = 1, 2, 3$ are the Pauli matrices and σ^0 is the identity. Show that this matrix is in $SU(2)$: $U^\dagger U = 1$ and $\det U = 1$.

Show that any doublet complex scalar can be parametrized as

$$\phi = \sqrt{\phi^\dagger\phi}\, U \begin{pmatrix} 0 \\ 1 \end{pmatrix},\ U \in SU(2). \tag{2}$$

30. Check that the kinetic terms of the Z_μ and A_μ have the canonical normalization.

31. Check that $\tilde{\phi}$ transforms as $(1, 2, -1/2)$.

32. Show that in the absence of Yukawa couplings, the SM Lagrangian has a flavour symmetry

$$U(3)_{q_L} \times U(3)_{l_L} \times U(3)_{u_R} \times U(3)_{d_R} \times U(3)_{l_R} \tag{3}$$

33. Repeat the parameter counting including the lepton Yukawa's. Can there be lepton mixing ?

34. Find one fermonic and one bosonic combination of SM fields that is a singlet under all gauge groups. Think of some hidden sector beyond the SM. Think why these combinations might be important.

Question 35: Navigating Majorana space
A practical survival kit

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a The basis

Consider the figure of merit:

$$T^{1/2}_{\beta\beta 0\nu} \propto \epsilon \sqrt{\frac{M \cdot t}{\Delta E \cdot B}} \tag{4}$$

Where:

- ϵ is the detection efficiency of the experiment.
- M is the mass of the target isotope
- t is the running time ΔE is the resolution of the region of interest (ROI). By convention, 1 FWHM at the end-point of the $\beta\beta$ spectrum $Q_{\beta\beta}$. Normally ΔE is expressed in keV. B is the background index (assumed flat in the ROI, expressed in counts per kev, per kilogram and per year). B refers to counts of background per unit of isotope mass (since we expressed M in units of isotope mass).

Caveats: When using the figure of merit, one has to be sure to compare apples with apples. Some experiments like to give resolution in terms of σ rather than FWHM (multiply by 2.35 to translate). Some experiment quote their resolution at 1 MeV or any other interesting value (but no their $Q_{\beta\beta}$). In this case, extrapolate assuming that $\Delta E/E \propto 1/\sqrt{E}$. Some experiments will give as detector efficiency only their selection efficiency and absorb the fiducial efficiency in the total exposure (this is fine, but be careful when you compare with other experiments that quote their total detection efficiency).

b Quest 1

Search the literature to find values of ϵ, ΔE, B, for GERDA, CUORE, KAMLAND-ZEN, SNO+, EXO-200, NEXT and Super-NEMO. Some useful bibliography can be found here:
https://github.com/jmalbos/pybbsens/wiki/References

c Quest 2

Using the values you have found, in Quest 1, evaluate the relative sensitivity to the period of the above experiments, assuming that they can all deploy the same exposure. Who wins and why?

d Quest 3

But can they all deploy the same exposure? If not, grade them in terms of scalability. To be specific: is there any experiment in the list that can reasonably deploy 100 tons of target? 10 ton? 1 ton? Any experiment that cannot deploy 1 ton?

Do not worry about cost, but worry about feasibility. Consider critical issues such as economy of scale. Ask yourself if the experiment if modular (e.g., you need to repeat the same module over and over to increase the target mass) or homogeneous (e.g., in a TPC, the mass increases by a factor 8 every time one doubles the size). Consider also whether, in the experiment $target = detector$, or $target \neq detector$, and how this influences scaling. Consider also availability of the isotope and enrichment technology. For example, xenon is the easiest to enrich of all the $\beta\beta$ isotopes, but tellurium is the only isotope that can be used in natural form.

To be convincing, provide your design of the various experiments at the scale you decide. For example, one Super-Nemo module has a volume of some 10 m^3 and deploys an isotope mass of the order of 5–10 kg. What would be your design for a 100 kg (1 ton, 10 ton) experiment? A second example: a xenon high pressure gas TPC at 15 bar can hold 10 ton mass in 10 m^3. More clues: liquid xenon has a density of 3 g/cm^3. What kind of TPC would you dare to build?

e Quest 4

Read the statistics papers in
https://github.com/jmalbos/pybbsens/wiki/References

f Quest 5

Download the software **pybbsens**
https://github.com/jmalbos/pybbsens/wiki
and run examples

g Quest 6

As we have seen, an experiment is described by ϵ, ΔE, and B, together with the isotope that the experiment uses. Using the **pybbsens** software and following the examples, obtain:

1. the sensitivity expected for GERDA (check with published GERDA results).
2. Ditto for EXO-200.
3. Ditto KamLAND-ZEN.

Do you get similar results? Do you the difference between your result and the published result?

h Quest 7

Compute the sensitivity as a function of the exposure of the various DBD experiments. that we are considering. Let's assume a maximum exposure of 100 ton·year.

i Quest 8

What do you conclude from 7 and previous exercises? Which one is the best experiment?

j Quest 9

But wait. Not all the above experiments will reach 100 ton·year (or will they?). Draw a point in the maximum exposure that you predict for each experiment. What do you get now?

k Quest 10

But wait. We used a given set of NME (IBM2, by default). What happens if we change sets. Repeat 7 for the various sets of NME available in **nmeset.py**

l Quest 11

What do you get? Plot the variation in sensitivity for a given exposure (say 10 ton ·year) as a function of the NME set for each experiment.

m Quest 12

Can you imagine a way to reach a background index of 10^{-5} ckky? for which technique(s)?

n Quest 13

You are the head of the World Physics Funding Agency. How many $\beta\beta 0nu$ experiments would you fund in the range of 1 ton mass? and in the range of 10 ton mass? How many isotopes do you think is wise to try?

o Quest 14

Can the inverse hierarchy be explored? And the normal hierarchy?

p Quest 15

Are neutrinos Majorana particles? How important is to answer this question?