SCALAR GAUGE FIELDS

Eduardo Guendelman, BGU

15TH PATRAS WORKSHOP,

CERN , GENEVA, JULY 1. 2014

Gauging a Globlal Symmetry

gauge symmetries, for simplicity we will consider a complex scalar field, ϕ, with a $U(1)$ symmetry which has the Lagrange density

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi^* - V(\phi) \ , \tag{1}$$

where $V(\phi)$ is some scalar self interaction potential. A typical example is to have a Higgs, symmetry breaking potential of the form $V(\phi) = -m^2|\phi|^2 + \lambda|\phi|^4$ where $|\phi|^2 = \phi^*\phi$. Now the Lagrangian density in (1) has the global gauge symmetry $\phi \rightarrow e^{ie\Lambda}\phi$ where e is the electric charge of the scalar field and Λ is a global phase parameter. If one lets the phase parameter become space-time dependent (i.e. $\Lambda \rightarrow \Lambda(x_\mu)$) one can still maintain this new introducing a four-vector gauge field A_μ

covariant derivatives, D_μ, of the form

$$\partial_\mu \to D_\mu = \partial_\mu + ieA_\mu \ ,$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ .$$

is invariant under $A_\mu \to A_\mu - \partial_\mu \Lambda$

THIS STANDARD MINIMAL COUPLING

leads to the following Lagrangian

$$\mathcal{L} = D_\mu\phi(D^\mu\phi)^* - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \ ,$$

with D_μ and $F_{\mu\nu}$ defined above. This Lagrangian represents a complex, charged scalar ϕ coupled to a vector gauge boson, A_μ. It respects the local gauge transformation

$$\phi \to e^{ie\Lambda}\phi \quad ; \quad A_\mu \to A_\mu - \partial_\mu\Lambda \ .$$

SCALAR GAUGE FIELDS

introducing a real, scalar $B(x_\mu)$ and two types of covariant derivatives

$$D_\mu^A = \partial_\mu + ieA_\mu \quad ; \quad D_\mu^B = \partial_\mu + ie\partial_\mu B \ .$$

gauge transformation

$$\phi \rightarrow e^{ie\Lambda}\phi \quad ; \quad A_\mu \rightarrow A_\mu - \partial_\mu \Lambda \quad ; \quad B \rightarrow B - \Lambda$$

$$\mathcal{L} = c_1 D_\mu^A \phi (D^{A\mu}\phi)^* + c_2 D_\mu^B \phi (D^{B\mu}\phi)^* + c_3 D_\mu^A \phi (D^{B\mu}\phi)^* + c_4 D_\mu^B \phi (D^{A\mu}\phi)^* - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + c_5 (A_\mu - \partial_\mu B)(A^\mu - \partial^\mu B)$$

where c_i's are constants

For i= 1, 2, 5 these constants are real, for i= 3 and 4 we must have,

$$c_3 = c_4^*$$

we require that $(c_1 + c_2 + c_3 + c_4) = (c_1 + c_2 + Re[c_3 + c_4]) = 1$

$$c_3 = a + ib \text{ and } c_4 = a - ib$$

$$\begin{aligned}
\mathcal{L} &= \partial_\mu \phi \partial^\mu \phi^* - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + c_5 A_\mu A^\mu \\
&\quad + c_5 \partial_\mu B \partial^\mu B - 2 c_5 A_\mu \partial^\mu B \\
&\quad + ie[\phi \partial_\mu \phi^* - \phi^* \partial_\mu \phi] \left((c_1 + a) A^\mu + (c_2 + a) \partial_\mu B \right) \\
&\quad + e^2 \phi \phi^* \left(c_1 A_\mu A^\mu + c_2 \partial_\mu B \partial^\mu B + 2a \partial_\mu B A^\mu \right) \\
&\quad - eb \partial_\mu (\phi^* \phi) (A^\mu - \partial^\mu B) \; .
\end{aligned}$$

$-eb\partial_\mu(\phi^*\phi)(A^\mu - \partial^\mu B)$ will lead to C and CP violation

Indeed, all the other terms in the lagrangian, with the exception of this one are invariant under the C transformation

$$\phi \to \phi^* \quad ; \quad A_\mu \to -A_\mu \quad ; \quad B \to -B$$

the currents by a particle and its associated antiparticle will not be exactly opposite

Parity on the other hand is a symmetry therefore CP is also violated

The parity transformation is

$$A_0(x^i,t) \to A_0(-x^i,t) \quad ; \quad A_i(x^i,t) \to -A_i(-x^i,t)$$

$$\phi(x^i,t) \to \phi(-x^i,t) \quad ; \quad B(x^i,t) \to B(-x^i,t)\ .$$

the Lagrangian violates CP as well as C.

CURRENT SHOWING C VIOLATION AND MASS GENERATION FOR VECTOR FIELD

$$\partial_\nu F^{\mu\nu} = ie(c_1 + a)[\phi\partial^\mu\phi^* - \phi^*\partial^\mu\phi] - eb\partial^\mu[\phi^*\phi] + 2[c_1 e^2(\phi^*\phi) + c_5]A^\mu + 2[ae^2(\phi^*\phi) - c_5]\partial^\mu B$$

PARTICLE CONTENT AND THE GENERALIZED UNITARY GAUGE

Let us recall how the unitary gauge works:

scalar field as an amplitude and phase – $\phi(x) = \rho(x)e^{i\theta(x)}$

If $\phi(x)$ develops a VEV due to the form of the potential

unitary gauge $\phi \to e^{ie\Lambda(x)}\phi(x)$ with $\Lambda = -\theta(x)/e$.

this way one removes the field $\theta(x)$

With the introduction of the scalar gauge field, $B(x)$

one no longer can gauge away *both* $\theta(x)$ and $B(x)$

What is the basic feature of the unitary gauge, what is it good for?

the unitary gauge eliminates cross terms like $A_\mu \partial^\mu \theta$

In the present case the cross terms

we wish to eliminate by a generalized unitary gauge are

$$\mathcal{L}_{cross} = -2c_5 A_\mu \partial^\mu B +$$

$$ie(c_1 + a)[\phi \partial_\mu \phi^* - \phi^* \partial_\mu \phi] A^\mu + 2ae^2 \partial_\mu B A^\mu \phi^* \phi$$

$$\rho(x) \approx \rho_0$$

So then we get,

$$\mathcal{L}_{cross} = 2A_\mu \partial^\mu \left(-c_5 B + ec_1 \rho_0^2 \theta + ae\rho_0^2 \theta + ae^2 \rho_0^2 B\right)$$

Defining $F(x) = -c_5 B + c_1 e \rho_0^2 \theta + ae\rho_0^2 \theta + ae^2 \rho_0^2 B$,

a gauge transformation (i.e. $\theta \to \theta + e\Lambda$, $B \to B - \Lambda$) we can take some initial non-zero value $F = F_0$, and always arrive at a gauge $F = 0$. is possible by choosing the gauge function as $\Lambda = -F_0/(c_5 + c_1 e^2 \rho_0^2)$.

In this Physical Gauge F=0

we can solve the θ field in terms of the B field as

$$\theta = \frac{c_5 - ae^2\rho_0^2}{e\rho_0^2(c_1 + a)} B \ .$$

After replacing θ in terms of B,

the kinetic term for B takes the form

$$\left(c_5 + c_2 e^2 \rho_0^2 + 2\frac{(c_2 + a)(c_5 - ae^2\rho_0^2)}{\rho_0^2(c_1 + a)} \right) \partial_\mu B \partial^\mu B$$

Define a canonically normalized Field

in the canonically normalized form $\frac{1}{2}\partial_\mu \bar{B}\partial^\mu \bar{B}$, one should define $\bar{B}$ as

$$\bar{B} = \left[\sqrt{2\left(c_2\rho_0^2 e^2 + c_5 + 2\frac{(c_2+a)(c_5 - ae^2\rho_0^2)}{\rho_0^2(c_1+a)}\right)}\right] B = f_B B \ ,$$

there is a remaining Goldstone boson like particle.

one can add non-derivative, potential terms which give mass

Non derivative interactions for the B field

$$V(e^{ieB}\phi) = -m^2\phi\phi^* +$$

$$\lambda(\phi\phi^*)^2 + \lambda_1 e^{ieB}\phi + \lambda_1^* e^{-ieB}\phi^* + \lambda_2 e^{i2eB}\phi^2 + \lambda_2^* e^{-i2eB}(\phi^*)^2$$

$$+\lambda_3 e^{i3eB}\phi^3 + \lambda_3^* e^{-i3eB}(\phi^*)^3 + \lambda_4 e^{i4eB}\phi^4 + \lambda_4^* e^{-i4eB}(\phi^*)^4 \ .$$

Specialize to the special case

$$V(e^{ieB}\phi) = -m^2\phi\phi^* + \lambda(\phi\phi^*)^2 + \lambda_1 e^{ieB}\phi + \lambda_1^* e^{-ieB}\phi^*$$

$$V(\bar{B},\rho) = -m^2\rho^2 + \lambda\rho^4 + (\lambda_1 e^{iK\bar{B}} + \lambda_1^* e^{-iK\bar{B}})\rho\ .$$

$$K = \frac{c_5 + \rho_0^2 e^2 c_1}{f_B \rho_0^2 (c_1 e + ae)}$$

Can obtain a sine Gordon equation. Assuming the conventional (first two terms in V) dominate and defining

$\lambda_1 = \alpha_1 e^{i\omega_1}$ we see that near the vacuum value $\phi \approx \sqrt{m^2/2\lambda}$

$$V(\bar{B}) = -\frac{m^4}{4\lambda} + 2\alpha_1\rho_0 \cos(K\bar{B} + \omega_1) \ .$$

Interaction strength of the B particles

From looking at the covariant derivative that uses the B field, we would think the interaction goes as e, but the true strength of the interaction is only found when we re-express B In terms of the canonically normalize field.

The strength of the interaction of $\bar{B}$ ·

is not e (as is the case with $B(x)$) but rather e/f_B

If f_B is big the coupling of the B particles will be reduced.

$\text{VEV}, \rho_0, \text{ and/or } c_2, c_5 \text{ are large}$

gives us such a situation and therefore a suppression of the interaction of these B particles. So this is a mechanism by means of which these particles can be made of weak strength. Also, as we have seen, these particles can acquire a mass.

FINAL THOUGHT:

Could these WISPS, the B particles be good Dark Matter Candidates? .The can be weakly interacting and massive, They look similar to the axion, except they are scalar particles rather than pseudo scalars.

Based on

- **Scalar gauge fields**
  Eduardo I. Guendelman, Douglas Singleton.

JHEP 1405 (2014) 096

. ArXiv:1402.7334 **[hep-th]**