

Astroparticles and Extra Dimensions

A.Nicolaidis

Theoretical Physics Department
Aristotle University of Thessaloniki

The race to unification in Physics

Newton (17th century)

unification of terrestrial gravity and celestial mechanics into Universal Gravitation

The race to unification in Physics

Newton (17th century)

unification of terrestrial gravity and celestial mechanics into Universal Gravitation

Maxwell(19th century)

unification of electricity and magnetism into Electromagnetism

The race to unification in Physics

Newton (17th century)

unification of terrestrial gravity and celestial mechanics into Universal Gravitation

Maxwell(19th century)

unification of electricity and magnetism into Electromagnetism

Einstein(1905-1916)

unification of gravity and space-time geometry into General Relativity

The race to unification in Physics

Heisenberg, Schrödinger (1920s)

unification of particle and wave behavior of matter through the advent of Quantum Mechanics

The race to unification in Physics

Heisenberg, Schrödinger (1920s)

unification of particle and wave behavior of matter through the advent of Quantum Mechanics

Glashow, Weinberg, Salam (late 20th century)

unification of electromagnetism and weak interactions to electroweak interactions

The race to unification in Physics

Heisenberg, Schrödinger (1920s)

unification of particle and wave behavior of matter through the advent of Quantum Mechanics

Glashow, Weinberg, Salam (late 20th century)

unification of electromagnetism and weak interactions to electroweak interactions

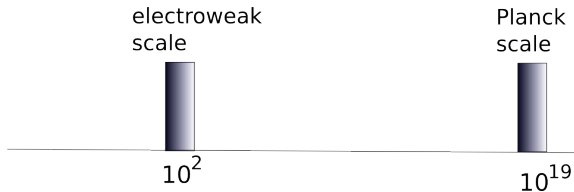
NEXT STEP??

Can we unify all forces?
(Standard Model forces + gravity)

Stumbling block

Two disparate scales coexist

$$M_W \sim 10^2 \text{ GeV} \quad M_{Pl} \sim 10^{19} \text{ GeV}$$



Quantum corrections tend to mix the scale
Hierarchy problem

A recent proposal

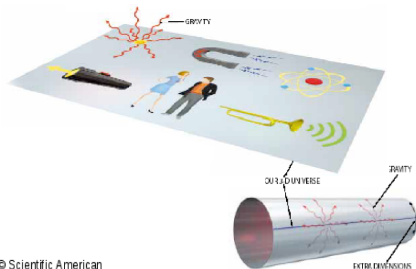
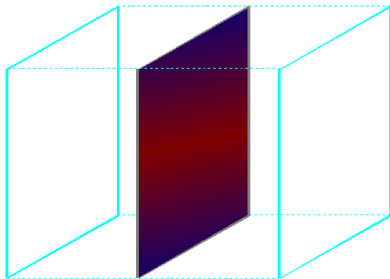
M_{Pl} is an apparent scale

Gravity lives in a higher dimensional space ($4 + \delta$)

The true scale of gravity

$M_f \simeq \text{few TeV}$

The picture



© Scientific American

The Brane World

Gravity propagates in a higher dimensional space $D = 4 + \delta$, while the SM fields live on a 4-dim brane

Extra dimensions are compact

$$y \rightarrow y + R$$

$$F_4(r) = G_{N(4)} \frac{m_1 m_2}{r^2} \quad r \gg R$$
$$G_{N(4)} = \frac{1}{M_{Pl}^2}$$

$$F_{4+\delta}(r) = G_{N(4+\delta)} \frac{m_1 m_2}{r^{\delta+2}} \quad r \ll R$$
$$G_{N(4+\delta)} = \frac{1}{M_f^{2+\delta}}$$

At $r \simeq R$

$$M_{Pl}^2 = R^\delta M_f^{2+\delta}$$

with a large volume of extra space, M_f can become as low as M_w

At $r \simeq R$

$$M_{Pl}^2 = R^\delta M_f^{2+\delta}$$

Black Hole Formation in HE Collisions

A TeV gravity will induce the formation of a black hole in particle collisions whenever $\sqrt{s} \simeq M_f$ and the impact parameter b is smaller than R_s , the Schwarzschild radius

We need Quantum Gravity to study this process since it is not available, we opt for a classical approach, where a particle is scattered in the effective curved background produced by the other

We consider a SM particle moving under the influence of a $(4 + \delta)$ -dim black hole

The effective metric

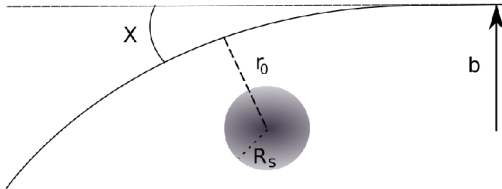
$$ds^2 = f(r)dt^2 - \frac{1}{f(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

$$f(r) = 1 - \left(\frac{R_s}{r}\right)^{1+\delta}$$

$$R_s = \frac{1}{\sqrt{\pi}M_f} \left[\frac{M}{M_f} \frac{8\Gamma\left(\frac{\delta+3}{2}\right)}{\delta+2} \right]^{\frac{1}{\delta+1}}$$

For particle collisions $M = \sqrt{s}$

A particle at impact parameter b , approaches the BH at the closest distance r_0 and escapes into infinity, deflected by an angle X



The critical parameter is

$$\varrho = \left(\frac{R_s}{r_o} \right)^{1+\delta}$$

- $\varrho \ll 1$
- $\varrho \simeq 1$

weak gravity

strong gravity

$b - r_0$ relationship

$$b^2 = \frac{r_0^2}{1 - \varrho}$$

Deflection angle χ

$$\chi = 2\Phi_0 - \pi$$

$$\Phi_0 = \int_0^1 \frac{d\omega}{[1 - \omega^2 - \varrho(1 - \omega^{3+\delta})]^{1/2}}$$

Small ϱ (Taylor expand)

$$\chi = I(\delta)\varrho$$

Agreement with available result of perturbative gravity

$$\delta = 0 \quad \chi = \frac{2R_s}{b}$$

Deflection angle X

$$X = 2\Phi_0 - \pi$$

$$\Phi_0 = \int_0^1 \frac{d\omega}{[1 - \omega^2 - \varrho(1 - \omega^{3+\delta})]^{1/2}}$$

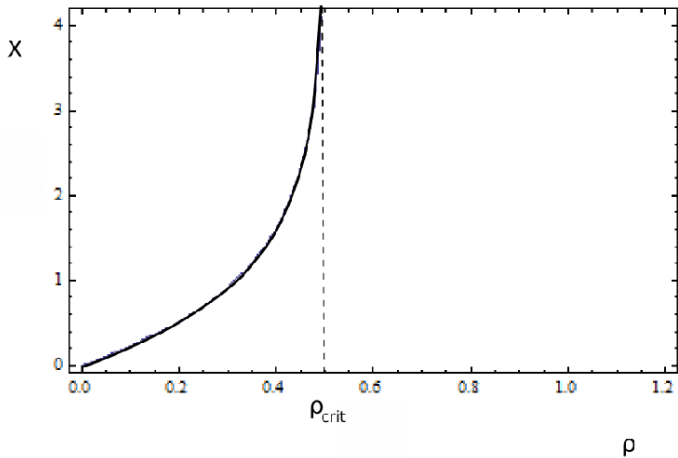
Large ϱ

With increasing ϱ , X increases and there is a critical ϱ_{crit} where X diverges logarithmically

$$X = a \ln \left[\frac{1}{1 - \frac{\varrho}{\varrho_{crit}}} \right]$$

$$a = \varrho_{crit} I(\delta)$$

$$\varrho_{crit} = \frac{2}{3+\delta}$$



What is the physical meaning of ϱ_{crit} ?

(Notice for $\delta = 0$ $r_0 = \frac{3}{2}R_s$)

ϱ_{crit} corresponds to the unstable circular orbit around the BH

As $\varrho \rightarrow \varrho_{crit}$ we witness the orbiting phenomenon

For $\varrho = \varrho_{crit}$ the particle stays in circular orbit ($X = \infty$)

For $\varrho < \varrho_{crit}$ the particle is fully absorbed by the BH

Transform into cross-section

$$\frac{d\sigma_0(X)}{d\Omega} = CR_s^2 \frac{1}{\sin X} \frac{\exp\left(-\frac{X}{a}\right)}{\left[1 - \exp\left(-\frac{X}{a}\right)\right]^{\frac{3+\delta}{1-\delta}}}$$

Different b contribute to the same X

Sum over all $X_n = 2n\pi + X$

$$\frac{d\sigma(X)}{d\Omega} = \sum_{n=0}^{\infty} \frac{d\sigma_0(X_n)}{d\Omega}$$

Small X

Cross section diverges like $(\frac{1}{X})^\gamma$

$$\gamma = \frac{4+2\delta}{1+\delta} \quad (\delta = 0, \gamma = 4)$$

Large X

Cross section diverges like

$$\frac{1}{X - \pi}$$

for any δ

The backward divergence peculiar to the BH existence (compact object with an horizon)

The backward divergence peculiar to the BH existence (compact object with an horizon)

SIGNAL FOR BH FORMATION

- Striking signatures in cosmic rays
- in neutrino telescopes

Theoretical Issue - DUALITY

Gravity at large
distances



QCD
at short distances

Gravity at short
distances



QCD
at large distances

What else can live in Extra Dimensions ?

A sterile neutrino, singlet under SM

$$N = \begin{pmatrix} N_R \\ N_L \end{pmatrix}$$

Assume for simplicity 1 extra dimension y . Each $N_{R,L}$ a KK tower

$$N(x, y) = \frac{1}{\sqrt{2\pi R}} N_0(x) + \sum_{n=1} \frac{1}{\sqrt{\pi R}} N_n(x) \cos\left(\frac{ny}{R}\right)$$

Coupling left - handed lepton (doublet) Higgs scalar - bulk neutrino

$$\frac{h}{\sqrt{M_f}} \bar{L} H N_R \delta(y)$$

⇒ mass term $m \bar{\nu}_L (N_{R_0} + \sqrt{2} \sum_{n=1} N_{R_n})$

$$m \sim \frac{h v M_f}{M_{Pl}} \sim 10^{-4} \text{eV}$$

Form

$$\Psi_R = \begin{pmatrix} N_{R_0} \\ N_{R_i} \end{pmatrix} \quad \Psi_L = \begin{pmatrix} \nu_L \\ N_{L_i} \end{pmatrix} \quad (i = 1, 2, \dots)$$

(N_{L_0} decouples)

Mass term $\bar{\Psi}_L M \Psi_R$

$$\mathbf{M} = \begin{pmatrix} m & \sqrt{2}m & \sqrt{2}m & \dots \\ 0 & \frac{1}{R} & 0 & \dots \\ 0 & 0 & \frac{1}{R} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$H = \frac{1}{2E_\nu} MM^T$$

Inside matter $H_{11} \rightarrow H_{11} + \mu$

$$\mu = \sqrt{2}E_\nu R^2 G_F N_n \simeq G_f R^2 \varrho \frac{E_\nu}{\sqrt{2}M_N}$$

With $\xi = mR$ the rescaled eigenvalues λ_n^2 of H satisfy

$$[\mu - \lambda^2 + (\lambda\pi)\cot(\lambda\pi)\xi^2] \prod_{n=1}^{\infty} (n^2 - \lambda^2) = 0$$

The corresponding eigenvectors are $B_n (e_{n_0}, e_{n_1}, e_{n_2}, \dots)$

$$e_{n_k} = \frac{k\sqrt{2}\xi}{(k^2 - \lambda_n^2)} e_{n_0}, \quad (k = 1, 2, \dots)$$

$$e_{n_0}^2 \left[\frac{1}{2} + \frac{\pi^2 \xi^2}{2} + \frac{\mu}{2\lambda_n^2} + \frac{\left(\lambda_n - \frac{\mu}{\lambda_n}\right)^2}{2\xi^2} \right] = 1$$

ξ variation

$$\xi = 0 \text{ (no mixing)} \quad \lambda_0 = \sqrt{\mu}, \quad \lambda_n = n$$

$$\text{Small } \xi \quad \lambda_n \simeq n + \frac{\xi^2}{n\pi}$$

$$\text{Large } \xi \quad \lambda_n \simeq \left(n + \frac{1}{2}\right) \left(1 - \frac{1}{\xi^2 \pi^2}\right)$$

$\nu - B_n$ admixture determined by $|e_{n0}|^2$

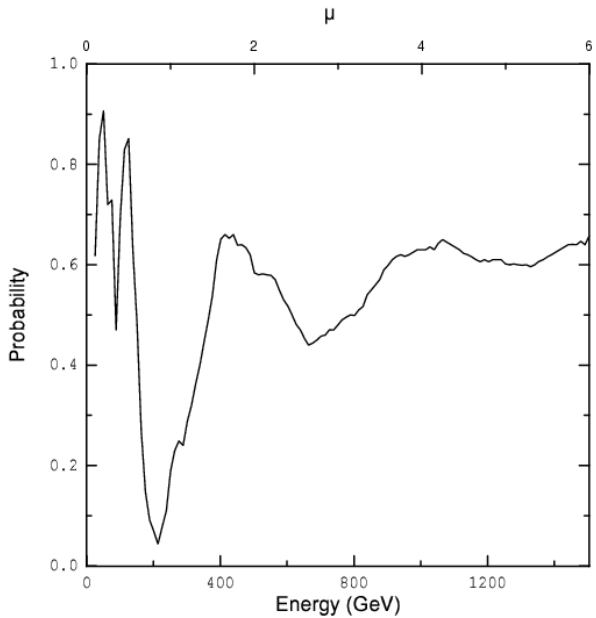
It becomes maximal when

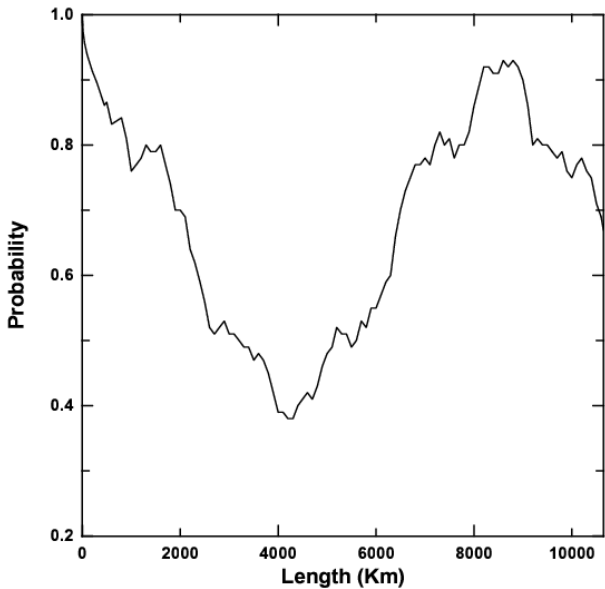
$$\mu = \lambda_n^2 \quad (\text{resonance condition})$$

For $\mu \simeq 1$ (first resonance)

$$\left(\frac{\rho}{10 \frac{gr}{cm^3}} \right) \left(\frac{E_\nu}{100 GeV} \right) \left(\frac{R}{1 \mu m} \right)^2 \simeq 1$$

Read off the radius R of the extra dimension





What else can live in Extra Dimensions

The axion

The Peccei - Quinn (PQ) solution to the strong CP problem \Rightarrow predicts a neutral, spin-zero pseudoscalar particle, the **axion**

Axion - photon oscillation

$$\mathcal{L}_{int} = \frac{1}{f_{PQ}} a F_{\mu\nu} \tilde{F}_{\mu\nu} = \frac{4}{f_{PQ}} a \vec{E} \cdot \vec{B}$$

Introduce one extra compact dimension y

The axion field $a(x^\mu, y)$, projected into the brane, looks like a collection of *KK* modes $a_n(x^\mu)$, with $m_n = \frac{n}{R}$

The coupling photon - KK axions

$$\mathcal{L}_{int} = \frac{1}{f_{PQ}} \sum_n a_n F_{\mu\nu} \tilde{F}_{\mu\nu} = \frac{4}{f_{PQ}} \sum_n a_n \vec{E} \cdot \vec{B}$$

In the presence of an external magnetic field B , the photon state A_{\parallel} parallel to the magnetic field B , the standard PQ axion a_0 and the KK axions a_n mix up

$$\mathbf{M} = \begin{pmatrix} \Delta_\gamma & \Delta_B & \Delta_B & \dots & \Delta_B \\ \Delta_B & \Delta_0 & 0 & \dots & 0 \\ \Delta_B & 0 & \Delta_1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Delta_B & 0 & 0 & & \Delta_N \end{pmatrix}$$

where

$$\Delta_\gamma = \frac{\omega_{pl}^2}{2E}, \quad \Delta_0 = \frac{m_{PQ}^2}{2E}, \quad \Delta_n = \frac{n^2}{2ER^2}, \quad \Delta_B = \frac{4B}{f_{PQ}}$$

Eigenvalues and eigenstates of the mixing matrix M

A resonance occurs $\gamma \rightarrow a_n \rightarrow \gamma$
whenever

$$\frac{n^2}{2ER^2} = \Delta_B$$

For $n = 1$ the resonance is narrow

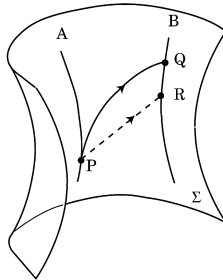
$$\left(\frac{E}{500\text{GeV}}\right) \left(\frac{R}{10^{-6}\text{cm}}\right)^2 \left(\frac{B}{10^{12}\text{G}}\right) = 1.0$$

(with $f_{PQ} = 10^{11}$ GeV)

Photons with the resonance energy, are transformed into KK axion, travel freely in the bulk space, before returning back into the brane and observed again as photon (transparency)

Axionic shortcuts

The brane is curved because of self-gravity



Then geodesics in the bulk propagate signals faster compared to the geodesics in the brane

Toy model

$$ds^2 = dt^2 - dx_1^2 - dx_2^2$$

The curved brane is

$$x_2 = A \sin kx_1$$

Bulk shortcut

$$x_2 = 0$$

$$\frac{t_\gamma - t_\alpha}{t_\gamma} \simeq \left(\frac{Ak}{2} \right)^2$$

MAGIC photons

Photons in the 0.25 – 0.6 TeV range arrive earlier compared to the 1.2 – 10 TeV photons

MAGIC photons

Photons in the 0.25 – 0.6 TeV range arrive earlier compared to the 1.2 – 10 TeV photons

Our working hypothesis:

The first photons satisfy the resonance condition and shortcut through the bulk space

The presence of extra dimensions lowers the Planck scale

The presence of extra dimensions lowers the Planck scale

Immediate phenomenological consequences

The presence of extra dimensions lowers the Planck scale

Immediate phenomenological consequences

- Gravity becomes strong at relatively low energy and we may have the formation of black holes in HE collisions

The presence of extra dimensions lowers the Planck scale

Immediate phenomenological consequences

- Gravity becomes strong at relatively low energy and we may have the formation of black holes in HE collisions
- Next to the graviton, other particles can circulate in the bulk (sterile neutrino, axion)

The presence of extra dimensions lowers the Planck scale

Immediate phenomenological consequences

- Gravity becomes strong at relatively low energy and we may have the formation of black holes in HE collisions
- Next to the graviton, other particles can circulate in the bulk (sterile neutrino, axion)
- Experimental implications

The presence of extra dimensions lowers the Planck scale

Immediate phenomenological consequences

- Gravity becomes strong at relatively low energy and we may have the formation of black holes in HE collisions
- Next to the graviton, other particles can circulate in the bulk (sterile neutrino, axion)
- Experimental implications
- Shortcuts through the bulk

The presence of extra dimensions lowers the Planck scale

Immediate phenomenological consequences

- Gravity becomes strong at relatively low energy and we may have the formation of black holes in HE collisions
- Next to the graviton, other particles can circulate in the bulk (sterile neutrino, axion)
- Experimental implications
- Shortcuts through the bulk
- Candidates for dark matter, dark energy