BE condensate $\simeq$ classical field = misalignment axions

Can axions be distinguished from WIMPs using Large Scale Structure data?

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1405.1139 ,(1307.8024 with M Elmer)

1. What are relevant variables and equations?
   stress-energy tensor $T^{\mu\nu}$ and Einstein's Eqns

2. (re)discover: pressure for misalignement axion *field* different from WIMPs
   * no Bose-Einstein condensate required*
   axions could differ from WIMPs in non-lin structure formation (numerical LSS problem)

3. does gravity condense axion particles $\rightarrow$ field/evaporate field $\rightarrow$ particles?
   NO

Equations and Variables for studying axion-CDM +gravity

Suppose two CDM axion populations are classical field and distribution of cold particles (from strings). How do they evolve?

$\Rightarrow$ **consult the path integral!**

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$$\langle \phi \rangle \leftrightarrow \text{ classical field} = \text{misalignment axions } \phi_{cl}$$

$$\langle \phi(x_1)\phi(x_2) \rangle \leftrightarrow \text{ (propagator)} + \text{distribution of particles} f(x,p)$$

- get Eqns of motion for expectation values in Closed Time Path formulation

Einsteins Eqns with $T^{\mu\nu}(\phi_{cl}, f)$ + quantum corrections(λ, G_N)

$\Rightarrow$**leading order is simple:** Einsteins Eqns with $T^{\mu\nu}(\phi_{cl}, f)$. Q corr. from 2PI, CTP PI in CST?

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1. obtain $T^{\mu\nu}(\phi_{cl}, f)$ in 2nd quantised Field Theory
2. $\mathcal{O}(G_N)$: $T^{\mu\nu}{}_{;\nu} = 0$, $\nabla^2\Psi = 4\pi G_N \rho$, ($\rho \to \delta\rho$ in linear regime)
   $\simeq$ Einsteins Eqns, in U today, and inside horizon in Newtonian gauge ,
3. $\mathcal{O}(G_N^2)$: covariantly quantised GR (F rules for graviton exchange)

Dewitt

Using $T^{\mu\nu}{}_{;\nu} = 0$ vs Eqns of motion of the field ϕ

Eqns of motion for axion field cpled to gravity studied by Sikivie et al, Saikawa etal:

$$(\Box - m^2)\phi \sim G_N \phi^3 \qquad \Rightarrow \qquad i\frac{\partial n}{\partial t} \sim G_N \int \phi^4$$

Both obtained from $T^{\mu\nu}{}_{;\nu} = 0$ and Poisson Eqn ($\rightarrow$ dynamics is equivalent?)

$$\begin{aligned}
T^{\mu\nu}{}_{;\nu} &= \nabla_\nu[\nabla^\mu\phi\nabla^\nu\phi] - \nabla_\nu[g^{\mu\nu}\left(\frac{1}{2}\nabla^\alpha\phi\nabla_\alpha\phi - V(\phi)\right)] \\
&= (\nabla_\nu\nabla^\mu\phi)\nabla^\nu\phi + \nabla^\mu\phi(\nabla_\nu\nabla^\nu\phi) - g^{\mu\nu}\nabla_\nu\nabla^\alpha\phi\nabla_\alpha\phi + g^{\mu\nu}V'(\phi)\nabla_\nu\phi \\
0 &= \nabla^\mu\phi[(\nabla_\nu\nabla^\nu\phi) + V'(\phi)]
\end{aligned}$$

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1. eqns for $T_{\mu\nu} \sim \phi^2$ solvable during linear structure formation. Find $\delta \equiv \delta\rho(\vec{k}, t)/\overline{\rho}(t)$ in dust or axion field has same behaviour on LSS scales ($c_s \simeq \partial P/\partial\rho \rightarrow 0$):

Ratra, Hwang+Noh

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_N\overline{\rho}\delta + c_s^2\frac{k^2}{R^2(t)}\delta = 0$$

2. "better" handle on IR divs: ensures that long-wave-length gravitons see large objects (like MeV photons see the proton, and not quarks inside)

Calculating $T_{\mu\nu}$ for the axion particles

Use covariant (GR is covariant, but flat space!), 2nd quantised FT ($\hbar$ to simultaneously obtain classical field, particles)

1. write axion as complex (!) scalar field

$$\hat{\phi}(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2k_0}} \left\{ \hat{a}_{\vec{k}} e^{-ik\cdot x} + \hat{b}^{\dagger}_{\vec{k}} e^{ik\cdot x} \right\}$$

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2. obtain a (classical phase space?) distribution $f(X,p)$ via Wigner transform: write stress-energy tensor as a 2-pt function

$$\hat{T}_{\mu\nu}(X-\tfrac{\delta}{2}, X+\tfrac{\delta}{2}) = \partial_\mu\hat{\phi}^\dagger(X-\tfrac{\delta}{2})\partial_\nu\hat{\phi}(X+\tfrac{\delta}{2}) + \partial_\nu\hat{\phi}^\dagger(X-\tfrac{\delta}{2})\partial_\mu\hat{\phi}(X+\tfrac{\delta}{2})$$
$$- g_{\mu\nu}\left(\partial^\alpha\hat{\phi}^\dagger(X-\tfrac{\delta}{2})\partial_\alpha\hat{\phi}(X+\tfrac{\delta}{2}) - V(\hat{\phi}^\dagger(X-\tfrac{\delta}{2})\hat{\phi}(X+\tfrac{\delta}{2}))\right)$$

for $|x_1 - x_2| \sim \delta \sim 1/|\vec{p}_a| \sim$ metre (in galaxy today).

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for $|x_1 - x_2| \sim \delta \sim 1/|\vec{p}_a| \sim$ metre (in galaxy today). Then fourier transform wrt δ to get

$$\hat{T}_{\mu\nu}(X,k) = \int \frac{d^4\delta}{(2\pi)^4}e^{ik\cdot\delta}\hat{T}_{\mu\nu}(X-\delta/2, X+\delta/2)$$

so have X-dep $\hat{a}(X)$, and

$$\langle n|\hat{a}^\dagger_k(X)\hat{a}_p(X)|n\rangle = f(X,k)\delta^3(\vec{k}-\vec{p})(2\pi)^3$$

...ok provided there is separation of scales $X \gg \delta$. (δ will reappear at end of talk as an IR cut-off)

Rediscovering...stress-energy tensors

non-rel axion particles are dust, like WIMPs:

$$T_{\mu\nu} = \begin{bmatrix} \rho & \rho\vec{v} \\ \rho\vec{v} & \rho v_i v_j \\ & \end{bmatrix}$$

compare to perfect fluid: $T_{\mu\nu} = (\rho + P)U_\mu U_\nu - P g_{\mu\nu}$. $P_{int} \propto \lambda^2 \to 0$, nonrel $\Rightarrow P \ll \rho, U = (1, \vec{v}), |\vec{v}| \ll 1$

Calculating $T_{\mu\nu}$ for the axion field

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2. then the classical field can be represented as coherent state[Cohen-Tannoudji etal—the book]

$$|\phi\rangle \quad \propto \quad \exp\left\{\int \frac{d^3p}{(2\pi)^3}\tilde{\phi}(\vec{p},t)\hat{a}^\dagger_{\vec{p}}\right\}|0\rangle \quad \text{such that} \quad \langle\phi|\hat{\phi}^n(t,\vec{x})|\phi\rangle = \phi^n(t,\vec{x})$$

and get

$$\begin{aligned}\langle\phi|\hat{T}_{\mu\nu}|\phi\rangle &= T_{\mu\nu}(\phi) \\ &= \partial_\mu\phi^\dagger\partial_\nu\phi + \partial_\nu\phi^\dagger\partial_\mu\phi - g_{\mu\nu}\mathcal{L}\end{aligned}$$

Then take non-relativistic limit $\phi \to \frac{1}{\sqrt{2m}}\sigma(x)e^{i\theta(x)}e^{-imt}$

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Classical field in non-relativistic limit $\phi \to \frac{1}{\sqrt{2m}}\sigma(x)e^{i\theta(x)}e^{-imt}$

$$T_{\mu\nu} = \begin{bmatrix} \rho & \rho\vec{v} \\ \rho\vec{v} & \rho v_i v_j + \Delta T_{ij} \end{bmatrix}$$

$$\rho = m|\sigma|^2 \qquad \vec{v} = \frac{\nabla\theta}{m}$$

$$\Delta T^i_j \sim \partial_i\sigma\partial_j\sigma \ , \ \lambda\sigma^4$$

Sikivie

"extra" pressure with classical field— *not need Bose Einstein condensation!*

⋆ BE condensate described (at leading order) as a classical field. Misalignment ⋆

⋆ axions already a classical field. No need to form a BE condensate ? ⋆

Distinguishing axions vs WIMPs in structure formation?

- not during linear structure formation: pressure irrelevant — Ratra, Hwang+Noh
- ? non-linear dynamics:(black=eqns for dust)

Rindler-DallerShapiro

$$T^{\mu}{}_{\nu;\mu} = 0 \quad \Leftrightarrow \quad \begin{cases} \partial_t \rho + \nabla \cdot (\rho \vec{v}) = 0 \\ \partial_t \vec{v} + (\vec{v} \cdot \nabla)\vec{v} = -\nabla \Psi \pm \text{ extra pressures from field} \end{cases}$$

$\Rightarrow$ write an axion field DM code and compare to dust code...

- But need to know — does gravity move axions between the field and particle bath? $\Leftrightarrow$ does it condense cold axion particles/evaporate the field?

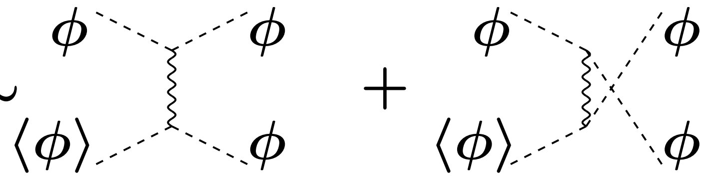
not at $\mathcal{O}(G_N)$:

$$\langle n, \phi | \hat{T}_{\mu\nu}(X) | n, \phi \rangle = T^{(\phi_c)}_{\mu\nu}(X) + T^{(part)}_{\mu\nu}(X)$$

$\Rightarrow$ at $\mathcal{O}(G_N^2)$?

Moving axions between field and bath with gravity? (in galaxy today)

at $\mathcal{O}(G_N^2)$, quantized GR ($v \sim 10^{-3}$ in cm frame) $\mathcal{M} \sim$



$$\sigma = \frac{G_N^2 m^2 \pi}{8v^4} \int \sin\theta d\theta \left(\frac{1}{\sin^2(\theta/2)} + \frac{1}{\cos^2(\theta/2)} \right)^2$$

Dewitt

IR cutoff of graviton momenta $\sim H$?

$$\sigma \sim \frac{G_N}{v^2}$$

...but this is for empty U containing two axions...

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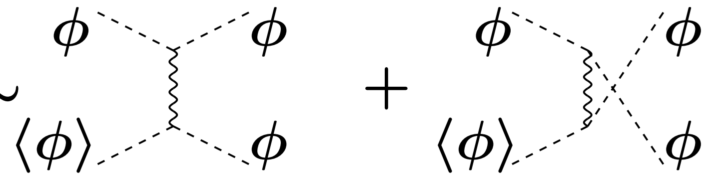

$$\sigma = \frac{G_N^2 m^2 \pi}{8 v^4} \int \sin\theta d\theta \left(\frac{1}{\sin^2(\theta/2)} + \frac{1}{\cos^2(\theta/2)} \right)^2 \to 10^4 \frac{m^2}{m_{pl}^4} \qquad (m \sim 10^{-5} eV)$$

graviton couples to $T^{\mu\nu}$! Only sees single axion when can look inside box $\delta^3 \sim 1/(mv)^3 \Rightarrow$ IR cutoff of graviton momenta $\sim mv$.

$$\text{probability} = \left| \sum \text{ indistinguisable amplitudes} \right|^2$$

graviton of 10 metre wavelength interacts coherently with all axions in 10 metre cube $\leftrightarrow T_{\mu\nu}$. (like MeV γ scatters off proton and not individual quarks inside).

To estimate rate, account for high axion occupation # (in galaxy today)

to estimate evaporation/condensation rate, must take into account high occupation number of axions:

$$\frac{\partial}{\partial t}n \;=\; \int \Pi_i \widetilde{d^3 p_i} \tilde{\delta}^4 |\mathcal{M}|^2 \Big[f_1 f_2 (1+f_3)(1+f_4) - f_3 f_4 (1+f_1)(1+f_2) \Big]$$

$[...] \sim f^3$, so rate for individual axion to evaporate/condense

$$\Gamma \sim n_\phi \sigma_G f \sim 10^{13} \left(\frac{\rho_{DM}}{\rho_c}\right)^2 \left(\frac{m}{m_{pl}}\right)^3 H_0 \ll H_0$$

is negligeable...

Summary

The QCD axion is a motivated dark matter candidate. If the PQ transition is after inflation, there are two populations: the classical "misalignment" field, and cold particles radiated by strings

to distinguish axion from WIMP CDM: direct detection, axion effects on γ propagation, maybe the extra pressures from the axion field give differences during non-linear structure formation? $\Rightarrow$*numerical galaxy formation*

1) there is a perception that axions need to be a Bose Einstein (BE) condensate, so as to differ from WIMPs

relevant question: what looks different from dust = WIMPs in $T^{\mu\nu}$?

answer: classical field = misalignment axions

(= Bose Einstein condensate)

2) there is debate as to whether gravity can put axions in a BE condensate

most discussions for the misalignmentaxions...see 1)

for the cold axion particles from strings: NO, if you believe my IR cutoff...

Advertisement!

** Beautiful papers **

Rindler-Daller + Shapiro

1. find analytic solutions representing stable rotating galactic halos formed of scalar field

2. vortices are energetically favoured, for self-interactions of opposite sign from QCD axions (?or for smaller masses?)
   (recall $V(a) = f_{\rm PQ}^2 m_a^2 [1 - \cos(a/f_{\rm PQ})] \simeq \frac{1}{2} m_a^2 a^2 - \frac{1}{4!} \frac{m_a^2}{f_{\rm PQ}^2} a^4$)

(also, today Canada is 147)

Backup

Why the axion:

gauge boson sector of QCD: input g_s,

$$-\frac{1}{4}G^A_{\mu\nu}G^{\mu\nu A} - \theta\frac{g_s^2}{32\pi^2}G^A_{\mu\nu}\widetilde{G}^{\mu\nu A} \qquad\qquad A:1..8, \quad \widetilde{G}^{\mu\nu} = \varepsilon^{\alpha\beta\mu\nu}G_{\alpha\beta}$$

neutron edm $\Rightarrow \theta \lesssim 10^{-10}$ ... *but instantons dynamically generate* $\theta \sim 1$?

How to make θ unobservable? *Aha!* There are quarks and the axial anomaly: a chiral rotn through η contributes:

$$\delta\mathcal{L} \propto \eta\partial_\mu J_5^\mu = \eta\frac{g_s^2 N}{8\pi^2}G\widetilde{G} + \eta\sum_f m_f\overline{q}_f\gamma_5 q_f$$

(N $\Leftrightarrow$ coloured fermion reps)

a chiral phase rotn moves θ onto (coloured) fermion mass matrix...still CPV

$\Rightarrow$ **solution**: add fields, such that "generalised" chiral rotns ($\equiv$ PQ sym) are a sym of classical theory.

Peccei Quinn

To build an (Invisible) axion model

ShifmanVainshteinZakharov
Srednicki NPB85

1. aim to obtain a "Peccei-Quinn" symmetry = a global symmetry of the classical Lagrangian, broken by colour anomalies ($\simeq$ some generalisation of chiral rotns)

2. for instance (SVZ), add a gauge-singlet scalar with $Q_{PQ} = 2$ and SU(2) singlet quarks $\Psi_{L,R}$ with $Q_{PQ} = \pm 1$, so

$$\mathcal{L} = \mathcal{L}_{SM} + \partial_\mu \Phi^\dagger \partial^\mu \Phi + i\overline{\Psi} \not{D} \Psi + \{\lambda \Phi \overline{\Psi} \Psi + h.c.\} + V(\Phi)$$

3. arrange to break the PQ sym spontaneously, at high scale, such that all new particles are heavy except the goldstone = axion

4. so can rotate θ to the phase of Φ...which is a dynamical field...who will get a mass and want to sit at zero.

   ...so if CDM is an oscillating axion field, the nedm oscillates at $m_a \sim 10^{10}$ s^{-1}

Review: non-thermal axion production gives Cold Dark Matter!

1. Suppose inflation before Peccei-Quinn Phase Trans.
   avoid CMB bounds on isocurvature fluctuations $\delta a/a \sim H_I/(2\pi f_{PQ})$
   Planck $\Rightarrow H_I \lesssim 10^7 \sqrt{f/10^{12}}$ GeV
   $or\ non-canonical\ kin.terms\ for\ a...$

WantzShellard
HanannHRW
FolkertsCristianoRedondo

2. then at PQPT, in each horizon, $\Phi \to f_{PQ} e^{ia/f_{PQ}}$
   * a massless, random $-\pi f_{PQ} \le a_0 \le \pi f_{PQ}$ from one horizon to the next
   * ...one string/horizon

3. QCD Phase Transition ($T \sim 200$ MeV): "tilt mexican hat"
   $$V(a) \to f_\pi^2 m_\pi^2 [1 - \cos(a/f_{PQ})] \simeq \frac{m^2}{2} a^2 - \frac{m^2}{4! f_{PQ}^2} a^4 + \dots$$
   * ... at $H < m_a$, "misaligned" axion field starts oscillating around the minimum
   * strings go away (radiate cold axion particles, $\vec{p} \sim H \lesssim 10^{-6} m_a$) Hiramatsu etal 1012.5502

PQPT after inflation $\Rightarrow$ **oscillating axion field + cold particles** redshift like CDM

Rediscovering ... linearised structure formation with axions is like WIMPs

1. initial conditions: adiabatic density fluctuations inherited from surroundings at the QCDPT

2. Einsteins Eqns and $T^{\mu}_{\ \nu;\mu} = 0$: linear perturbations $\delta \equiv \delta\rho(\vec{k}, t)/\overline{\rho}(t)$ in dust or axion field have same behaviour on LSS scales:

   Ratra, Hwang+Noh

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_N \overline{\rho}\delta + c_s^2 \frac{k^2}{R^2(t)}\delta = 0$$

   ($c_s \simeq \partial P/\partial\rho \to 0$)

   see that can *solve* eqns for $T_{\mu\nu} \sim \phi^2$ in linear growth regime (whereas non-lin eqns for ϕ).

3. (very) small scale differences....
   - there is pressure and Jeans length $\sim 1/\sqrt{H(t) m_a}$ (and funny c_s on smaller scales?)
   - if PQPT after inflation, a random from one horizon to next, so $\delta\rho_a/\rho_a \sim \mathcal{O}(1)$ on QCDPT horizon scale (5km then, 0.1 pc today)... axion "miniclusters"

     Hogan,Rees

4. (the axion field does not turn into particles by parametric resonance)

   Kolb,Singh,Srednicki

Analytic discussions of non-linear structure formation

Erken,Sikivie,Tam,Yang
Bannik+Sikivie

Sikivie:

1. at $T_\gamma \sim$ keV, "gravitational thermalisation" of axions drives them to a "Bose-Einstein Condensate"
2. axion field can support vortices, which allow caustics in the galactic DM distribution

Rindler-Daller + Shapiro:

1. find analytic solutions representing stable rotating galactic halos formed of scalar field
2. vortices are energetically favoured, for self-interactions of opposite sign from axions (?or smaller masses?)

(recall $V(a) = f_{\rm PQ}^2 m_a^2 [1 - \cos(a/f_{\rm PQ})] \simeq \frac{1}{2} m_a^2 a^2 - \frac{1}{4!} \frac{m_a^2}{f_{\rm PQ}^2} a^4$)

What is a Bose Einstein condensate? (I don't know. Please tell me if you do!)

Important characteristics of a BE condensate seem to be

1. a classical field,

2. carrying a conserved charge,

3. ? whose fourier modes are concentrated at a particular value — most of the "particles" who condense, should coherently do the same thing (but not necc the zero-momentum mode)

consistent with

- BE condensation in equilibrium stat mech, finite T FT, alkali gases.

- LO theory of BE condensates (Boguliubov $\rightarrow$ Pitaevskii) as a classical field

Are the misalignment axions a BE condensate?

1. a classical field yes
2. carrying a conserved charge, in the NR limit, $\approx$ yes
3. ? whose fourier modes are concentrated at a particular value — most of the "particles" who condense, should coherently do the same thing (but not necc the zero-momentum mode) ....umm?

Two approaches:

A: irrelevant question: misalignment axions are a classical field, gives extra pressure which allows axion CDM to differ from WIMPs.

B: Follow Sikivie = misalignment field is not a BE condensate, needs to be to differ from WIMPs, $\Rightarrow$ does gravity put it there?

Saikawa+Yamaguchi+etal
Davidson+Elmer,...

Summary of the paper with Elmer

We can obtain the gravitational interaction rate of Saikawa et al. for the misalignment axions, using classical field theory.

It is a leading order solution of deterministic classical equations, so there is no associated entropy generation, so I think it is incorrect to identify this rate as a "thermalisation rate".

It remains to be shown what those interactions are doing with axions. If the proponents of axion BEC think that the misalignment axions need to forma BEC to differ from WIMPs, then

1. what is the definition of BEC?

2. need to show that the gravitational interaction rate is driving the misalignment axions to that configuration