Multipole solutions of hydrodynamics and higher harmonics

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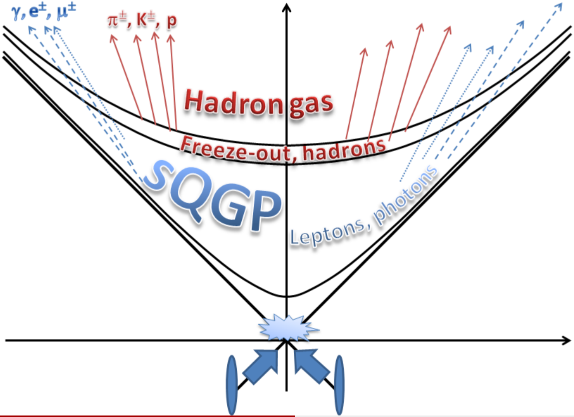
Hydrodynamics in high energy physics

- Strongly interacting QGP discovered at RHIC & created at LHC
- A hot, expanding, strongly interacting, perfect fluid
- Hadrons created at the “chemical” freeze-out
- Hadron distributions decouple at “kinetic” freeze-out



Known solutions of relativistic hydrodynamics

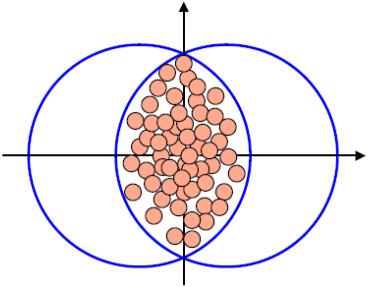
- Many solve the hydro equations numerically
- *Exact, analytic solutions* are important to connect initial and final state
- Famous 1+1D solutions: Landau, Hwa, Bjorken
- Many new 1+1D solutions, few 1+3D, with spherical/axial symmetry
- First truly 3D relativistic solution
  Csörgő, Csernai, Hama, Kodama, Heavy Ion Phys. **A21**, 73 (2004)
- Assumes ellipsoidal symmetry via scaling variable

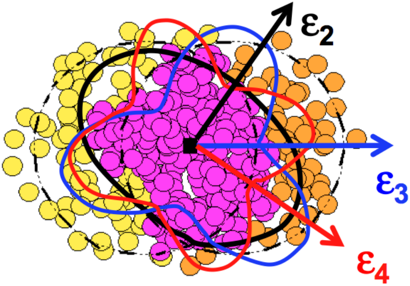
$$s = \frac{x^2}{X^2} + \frac{y^2}{Y^2} + \frac{z^2}{Z^2}$$

- X, Y, Z: time dependent axes of expaning ellipsoid
- Thermodynamical quantities depend only on s
- *Describes hadron data*
  Csanád, Vargyas, Eur. Phys. J. A **44**, 473 (2010)
- *Describes photon & lepton data*
  Csanád, Májer, Central Eur. J. Phys. **10** (2012)
  Csanád, Krizsán, Central Eur.J.Phys. **12** (2014)

Higher order anisotropies?

- Elliptic-like shape $\Rightarrow$ anisotropic particle production
- Anisotropy characterized by $v_2 = \langle \cos 2\phi \rangle$
- Finite number of nucleons $\rightarrow$ higher order anisotropy!





- Successfully utilized in numerical calculations
- *Exact solutions handling this?*

Generalization of elliptic symmetry

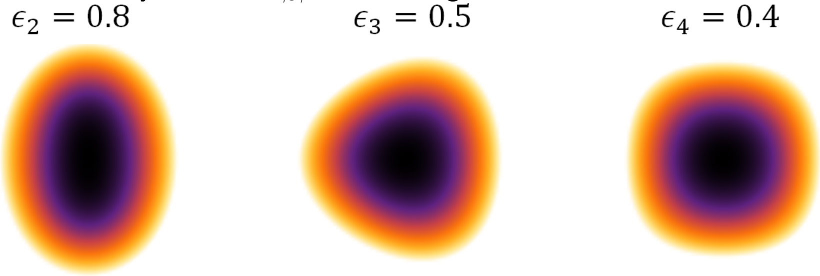
- *How to generalize* the ellipsoidal scaling variable of $s = \frac{x^2}{X^2} + \frac{y^2}{Y^2} + \frac{z^2}{Z^2}$?
- Redefine it via

$$\frac{1}{R^2} = \frac{1}{X^2} + \frac{1}{Y^2} \text{ and } \epsilon = \frac{X^2+Y^2}{X^2-Y^2} \Rightarrow s = \frac{r^2}{R^2}\left(1+\epsilon\cos(2\phi)\right)$$

- Generalize: *N-pole symm.* in transverse plane

$$s = \frac{r^N}{R^N}\left(1+\epsilon_N\cos(N\phi)\right)$$

- ϵ_1 defines only a shift, $\epsilon_{2,3,\ldots}$ interesting



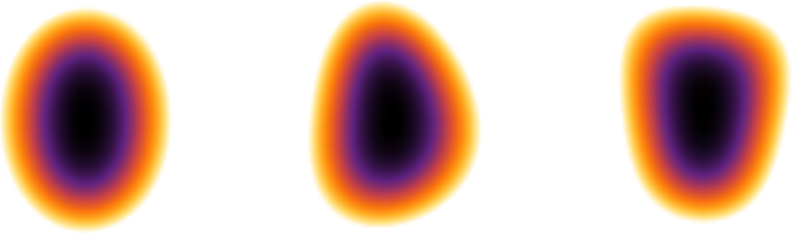
Multipole symmetries combined

- *Multiple symmetries* can be combined:

$$s = \sum_N \frac{r^N}{R^N} \left(1 + \epsilon_N \cos(N(\phi - \psi_N))\right)$$

- Aligned by Nth order reaction planes ψ_N
- Again, $\epsilon_1 = 0$ can be assumed
- R defines time dependent scale: expansion
- Basically any shape can be described, via a “multipole expansion”

$\epsilon_2 = 0.8, \epsilon_3 = 0, \epsilon_4 = 0$ $\quad$ $\epsilon_2 = 0.8, \epsilon_3 = 0.5, \epsilon_4 = 0$ $\quad$ $\epsilon_2 = 0.8, \epsilon_3 = 0.5, \epsilon_4 = 0.4$



New solutions of hydrodynamics

- *New solutions* with multipole symmetries

$$s = \sum_N \frac{r^N}{R^N} \left(1 + \epsilon_N \cos(N(\phi - \psi_N))\right) + \frac{z^N}{Z^N}$$

$$u^\mu = \gamma \left(1, \frac{\dot{R}}{R} r \cos\phi, \frac{\dot{R}}{R} r \sin\phi, \frac{\dot{R}}{R} z\right)$$

$$T = T_f \left(\frac{\tau_f}{\tau}\right)^{3/\kappa} \frac{1}{\nu(s)}$$

- *Observed higher order harmonics*: Maxwell-Jüttner type source function

$$S(x,p) \propto \exp\left[-\frac{p_\mu u^\mu(x)}{T(x)}\right] \delta(\tau - \tau_f) \frac{p_\mu u^\mu}{u^0}$$

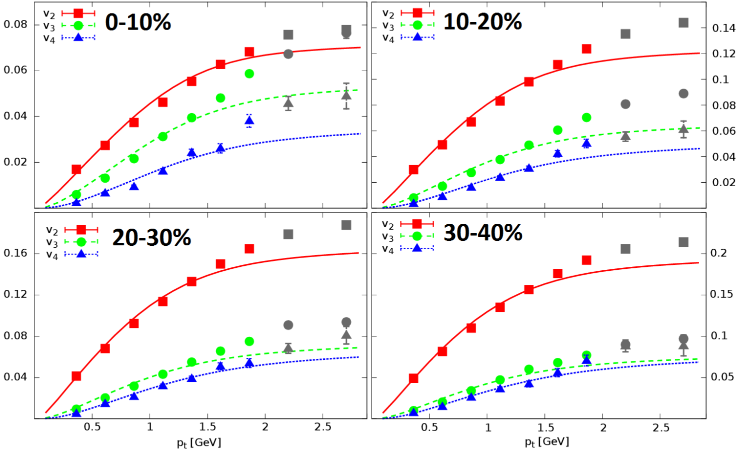
- Momentum distribution $N(p)$ and anisotropies $v_n(p_t)$:

$$N(p) = \int S(x,p) d^4x \text{ and } v_n(p_t) = \langle \cos(n\alpha) \rangle_{N(p)}$$

- Choose *Gaussian profile*, $\nu(s) = e^{bs}$, i.e. b is temperature gradient

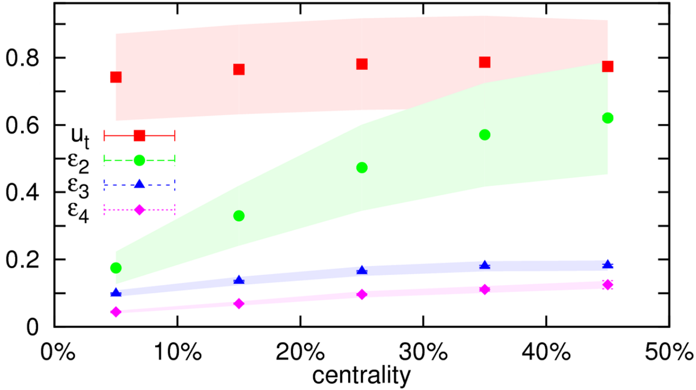
Comparison to PHENIX anisotropy coefficients

- PHENIX measured v_2, v_3 and v_4 in various centrality classes
  Phys. Rev. Lett. **107** (2011) 252301
- Fitted parameters: ϵ_N and transverse flow u_t



Comparison to PHENIX anisotropy coefficients

- *Successful fit*, see details in arXiv:1405.3877
- Transverse flow u_t: minor dependence on centrality
- Strongly influenced by temperature gradient
- ϵ_N increased for peripheral collisions



Multipole velocity field?

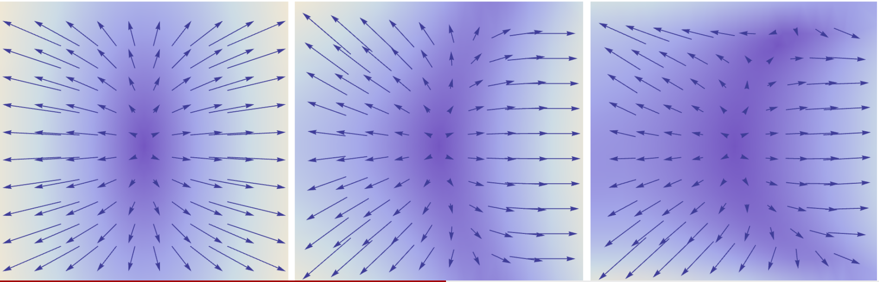
- *Buda-Lund model*: hydro final state parametrization

  Csanád, Csörgő, Lörstad, Nucl.Phys. **A742** (2004) 80
- Add multipole densities (just as above), add *multipole flow*!

$$u^\mu = (\gamma, \partial_x\Phi, \partial_y\Phi, \partial_z\Phi) \text{ with } \Phi = \sum_N \frac{r^N}{NH^{N-1}} \left(1 + \chi_N \cos(N\phi)\right)$$

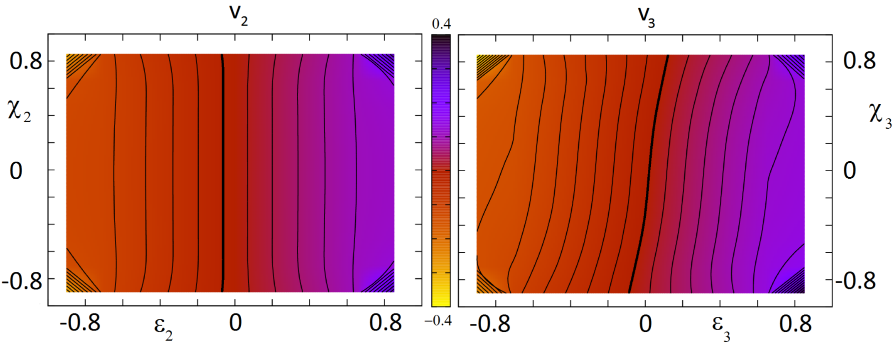
- χ_1 defines only a shift, can be neglected
- H is Hubble-coefficient like (take $N = 1$ with $\chi_1 = 0$)

$\chi_2 = 0.4, \chi_3 = 0.0, \chi_4 = 0.0$ | $\chi_2 = 0.4, \chi_3 = 0.3, \chi_4 = 0.0$ | $\chi_2 = 0.4, \chi_3 = 0.3, \chi_4 = 0.1$



Anisotropy mixing

- *Flow- and density anizotropies mix* in v_N
- Both ϵ_N and χ_N determine v_N

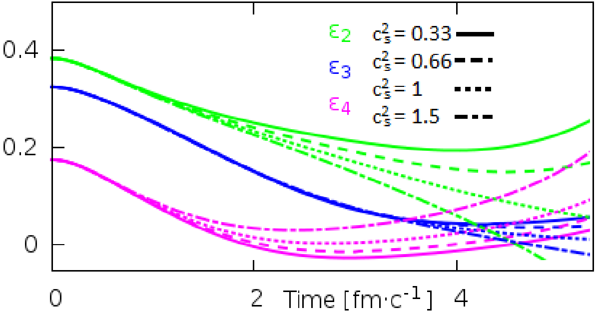


- Especially true for v_3 $(N > 2)$
- Measurement of v_3 does not directly indicate spatial anisotropy
- No such hydrodynamic solutions found yet

Role of other parameters

- Does the value of sound speed play a role?
- How about initial pressure gradient?
- All go down to the EoS
- Note exact solution with temperature dependent EoS

  Csanád, Nagy, Lökös, Eur.Phys.J. **A48** (2012) 173
- Example non-relativistic numerical calculation on eccentricities:



Summary

- Medium of high energy collisions: *hydro expansion*
- Higher order v_n coefficients measure *anisotropy*
- Arise due to *fluctuating initial conditions*
- This work: *first analytic solutions* to describe v_n's
- In agreement with data
- Details in arXiv:1405.3877
- Effect of sound speed, pressure gradient?

Thank you for your attention!

And let me invite you to the 14th Zimanyi School in Budapest

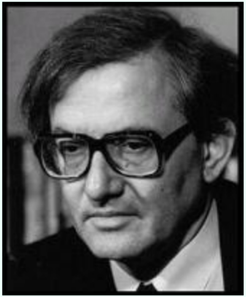
ZIMÁNYI SCHOOL'14

Szinyei M. P.: Meadow with poppies

14. Zimányi

WINTER SCHOOL ON HEAVY ION PHYSICS

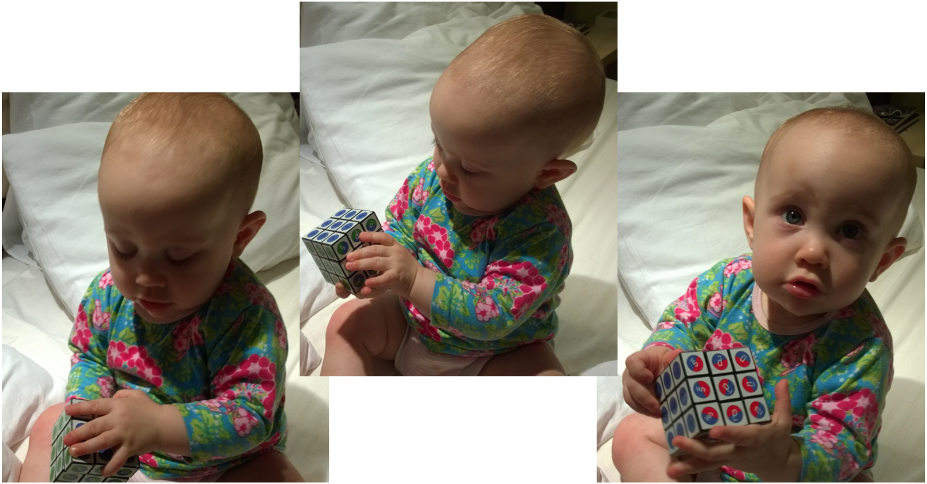
Dec. 1. - Dec. 5., Budapest, Hungary



József Zimányi (1931 - 2006)

http://zimanyischool.kfki.hu/14/

And as a comment on yesterday



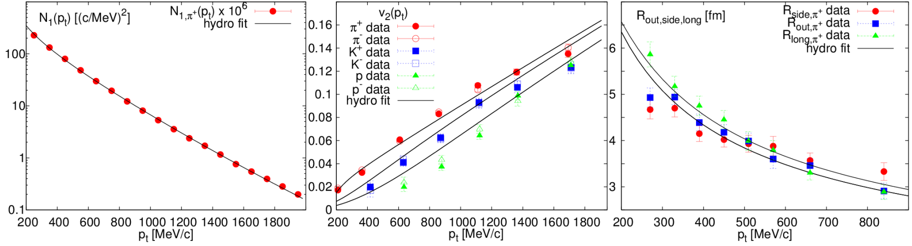
Soft hadron creation in A+A via hydro

- Take first exact, analytic and truly 3D relativistic solution

  Csörgő, Csernai, Hama *et al.*, Heavy Ion Phys. **A21**, 73 (2004), nucl-th/0306004
- Calculate observables for identified hadrons
  - Transverse momentum distribution $N_1(p_t)$
  - Azimuthal asymmetry $v_2(p_t)$
  - Bose-Einstein correlation radii $R_{out,side.long}(p_t)$
- Compared to data successfully (RHIC shown, LHC done as well)

  Csanád, Vargyas, Eur. Phys. J. A **44**, 473 (2010), arXiv:0909.4842

  Data: PHENIX Coll., PRC**69**034909(2004), PRL**91**182301(2003), PRL**93**152302(2004)

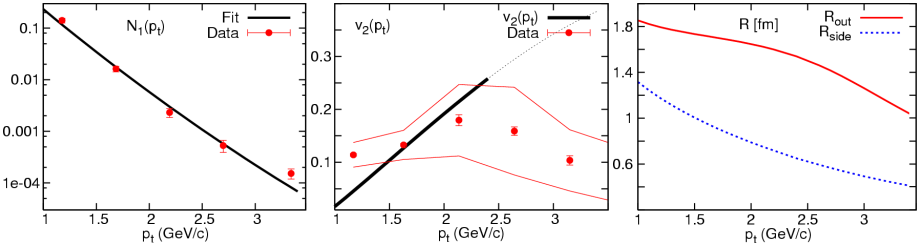


Penetrating probes: photons and leptons

- Photons and leptons are created throughout the evolution
- *Their distribution reveals information about the EoS!*
- Compared to PHENIX data (spectra and flow) successfully
- Predicted photon HBT radii

  Csanád, Májer, Central Eur. J. Phys. **10** (2012), arXiv:1101.1279

  Data: PHENIX Collaboration, arXiv:0804.4168 and arxiv:1105.4126



- Average EoS: $c_s = 0.36 \pm 0.02_{stat} \pm 0.04_{syst}$ (i.e. $\kappa = 7.7$)
- Compatible with soft dilepton data as well