Generic framework for anisotropic flow analyses with multiparticle azimuthal correlations

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Generic framework

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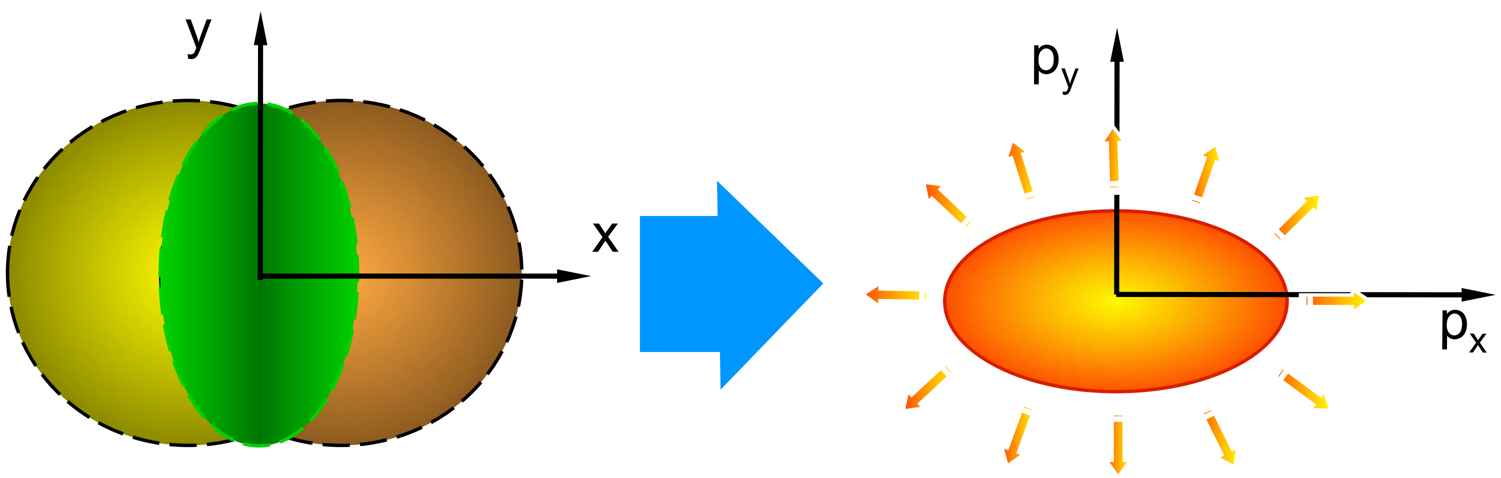
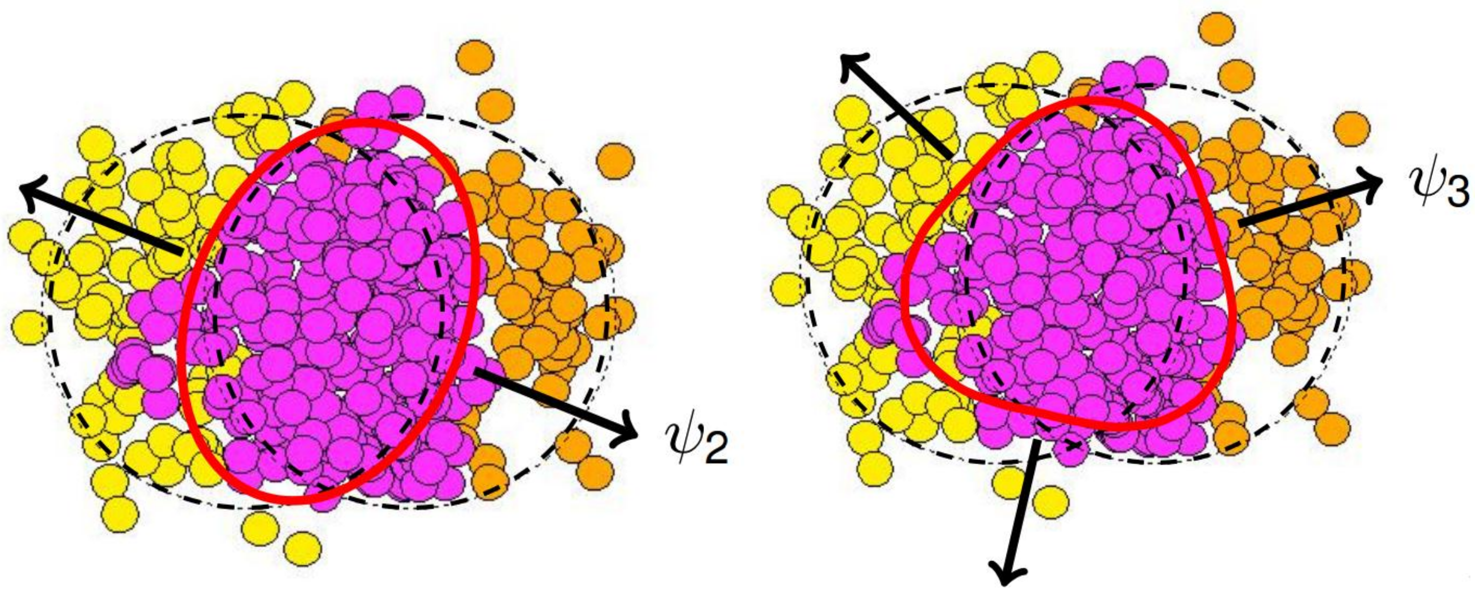
- Builds on the Q-cumulant framework
- Includes acceptance and efficiency corrections
- New observables proposed
- Recursive algorithms extend implementation to *n*-particle correlations
- Analysis of bias from finite detector granularity and particle selection criteria in differential flow analyses

Anisotropic flow

coordinate space

Momentum space





- Anisotropies in momentum space
- Harmonics v_n quantify anisotropic flow
- Ψ_n are symmetry planes
- Instead of estimating Ψ_n => use correlation techniques
- 2-particle correlation: sensitive to non-flow => measure multiparticle correlations

$$\frac{d^3N}{dp_t^2 d\phi dy} = \frac{1}{2\pi}\frac{d^2N}{p_t dp_t dy}\left(1+\sum_{n=1}^{\infty} 2\, v_n \cos\left[n\left(\phi-\Psi_n\right)\right]\right)$$

$$\Psi_n = \frac{1}{n}\tan^{-1}\left(\frac{\sum_i w_i \sin(n\phi_i)}{\sum_i w_i \cos(n\phi_i)}\right)$$

Q-cumulants

- Anisotropic flow with multiparticle cumulants: Borghini *et al.*, PRC 64, 054901 (2001) => approximate: generating functions

- Analytical calculations using Q-vectors: Bilandzic *et al.*, PRC 83, 044913 (2011) => exact measurements - limited to single harmonics

- Q-vector: $Q_n \equiv \sum_{i=1}^{M} e^{in\varphi_i}$

- Using Q-vectors allows for determining: $v_n\{2\}$, $v_n\{4\}$, $v_n\{6\}$, $v_n\{8\}$ in single pass over data

- $v_n\{2\} = \sqrt{\langle v_n^2 \rangle}$,
  $v_n\{4\} = \sqrt[4]{2\langle v_n^2 \rangle^2 - \langle v_n^4 \rangle}$,

Generic equations

- Average *m*-particle correlator of harmonics $n_1, n_2, \ldots, n_m$:

$$\langle m\rangle_{n_1,n_2,\ldots,n_m} \equiv \left\langle e^{i(n_1\varphi_{k_1}+n_2\varphi_{k_2}+\cdots+n_m\varphi_{k_m})}\right\rangle \equiv \frac{\sum\limits_{\substack{k_1,k_2,\ldots,k_m=1\\ k_1\neq k_2\neq\cdots\neq k_m}}^{M} w_{k_1}w_{k_2}\cdots w_{k_m}\, e^{i(n_1\varphi_{k_1}+n_2\varphi_{k_2}+\cdots+n_m\varphi_{k_m})}}{\sum\limits_{\substack{k_1,k_2,\ldots,k_m=1\\ k_1\neq k_2\neq\cdots\neq k_m}}^{M} w_{k_1}w_{k_2}\cdots w_{k_m}}. \quad (1)$$

- Factorisation implies analytical mean value (R. S. Bhalerao *et. al.* PRC 84, 034910 (2011)):

$$\mu_{\langle m\rangle_{n_1,n_2,\ldots,n_m}} \equiv \langle e^{i(n_1\varphi_1+\cdots+n_m\varphi_m)}\rangle = v_{n_1}\cdots v_{n_m} e^{i(n_1\Psi_{n_1}+\cdots+n_m\Psi_{n_m})}. \quad (2)$$

- Numerator of Eq (1) is important:

$$\mathrm{N}\langle m\rangle_{n_1,n_2,\ldots,n_m} \equiv \sum_{\substack{k_1,k_2,\ldots,k_m=1\\ k_1\neq k_2\neq\cdots\neq k_m}}^{M} w_{k_1}w_{k_2}\cdots w_{k_m}\, e^{i(n_1\varphi_{k_1}+n_2\varphi_{k_2}+\cdots+n_m\varphi_{k_m})} \quad (3)$$

- Denominator is the case where $n_1, \ldots, n_m = 0$: $\mathrm{D}\langle m\rangle_{n_1,n_2,\ldots,n_m} = \mathrm{N}\langle m\rangle_{0,0,\ldots,0}$

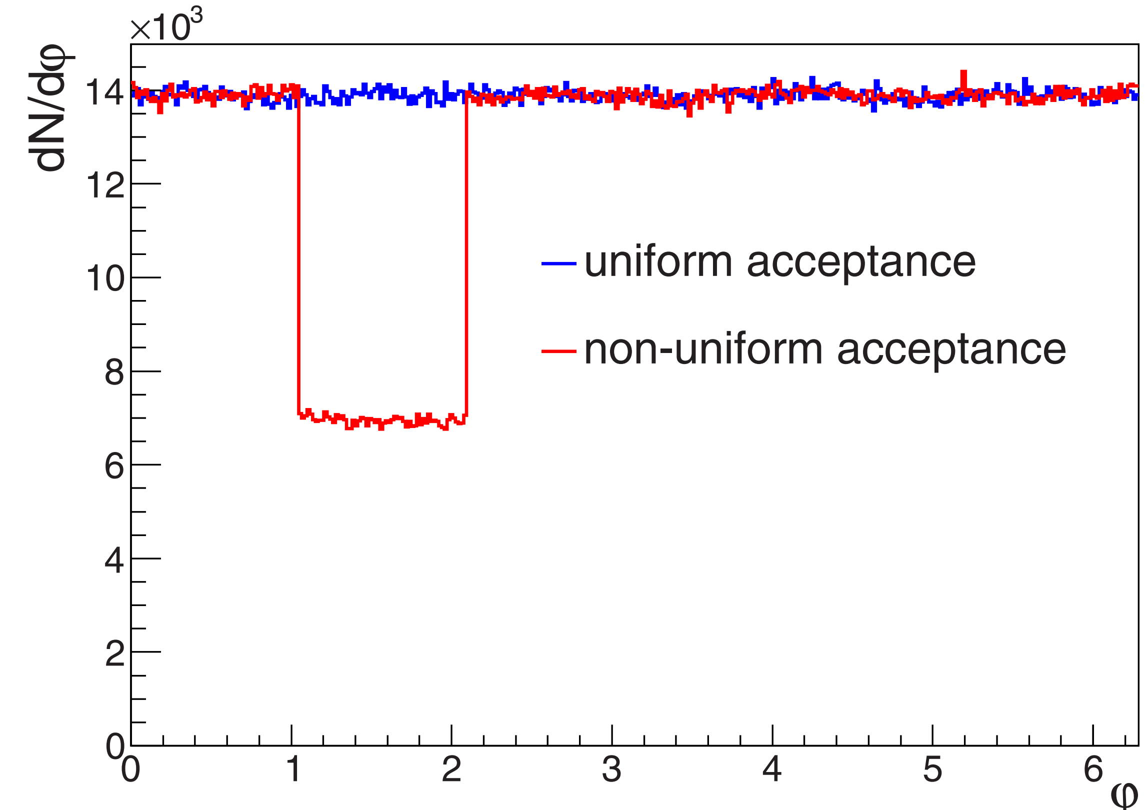
- *2*-particle correlator expressed in terms of (weighted) Q-vectors:

$$\begin{aligned}\mathrm{N}\langle 2\rangle_{n_1,n_2} &= Q_{n_1,1}Q_{n_2,1} - Q_{n_1+n_2,2},\\ \mathrm{D}\langle 2\rangle_{n_1,n_2} &= \mathrm{N}\langle 2\rangle_{0,0} = Q_{0,1}^2 - Q_{0,2}.\end{aligned} \quad (4)$$

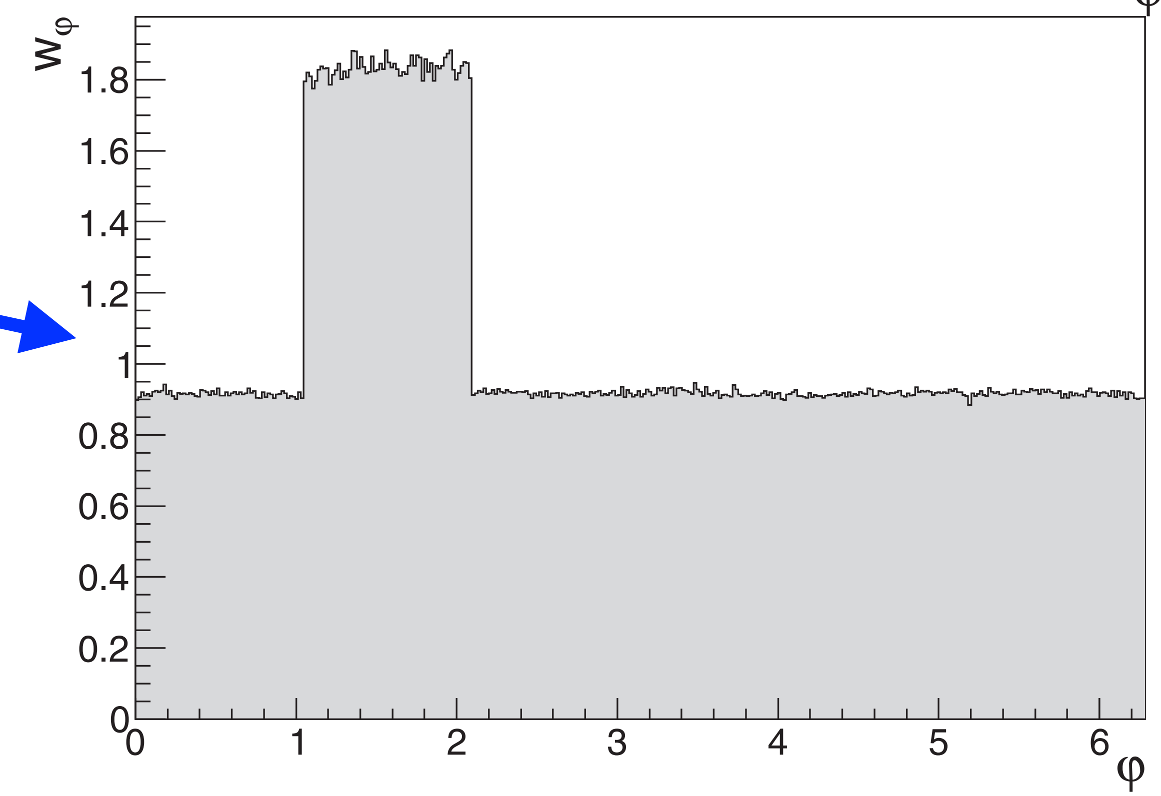
- Straightforwardly generalised to differential flow

Acceptance/efficiency corrections

- New framework allows for using particle weights due to weighted Q-vector: $Q_{n,p} \equiv \sum_{k=1}^{M} w_k^p \, e^{in\varphi_k}$.

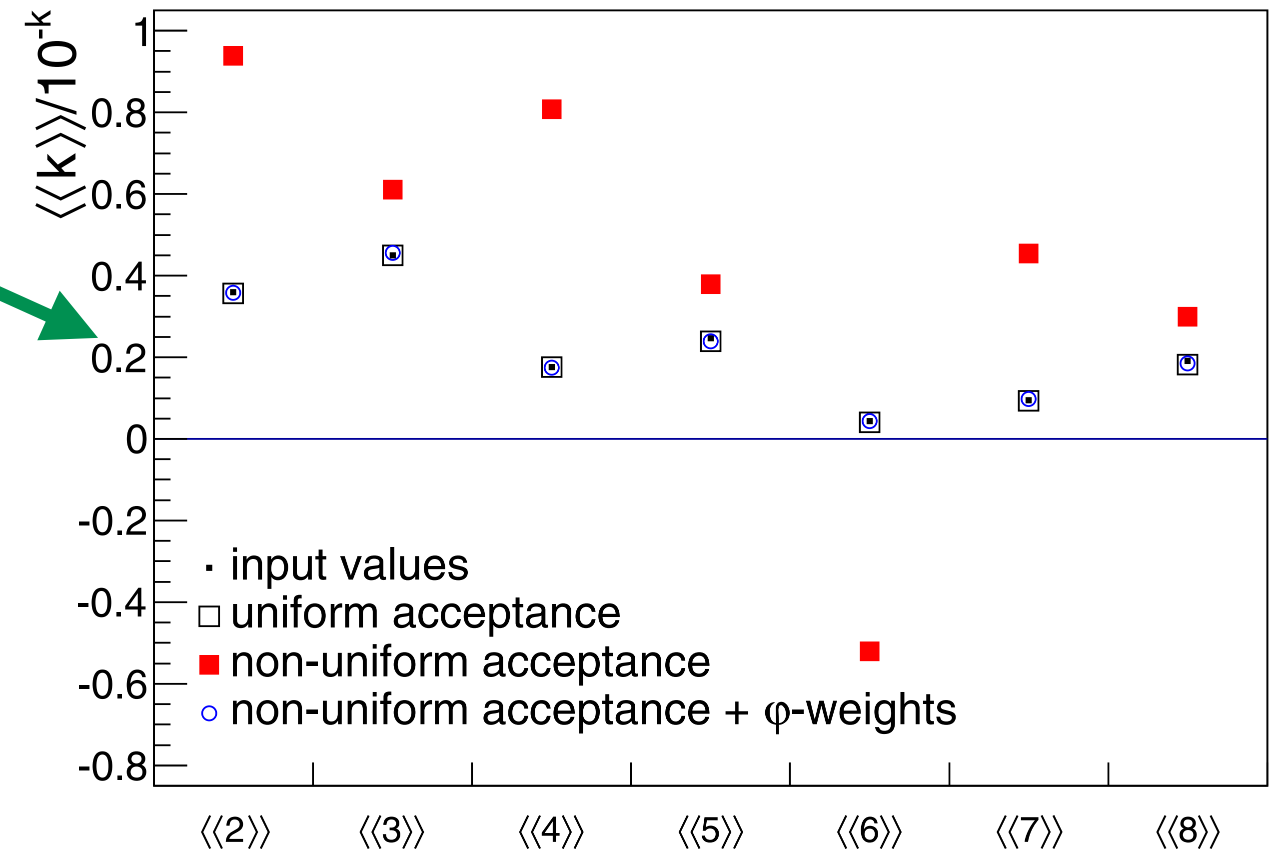


- To correct for non-uniform acceptance:
  - Construct φ-weights from initial pass over data
  - Apply, during analysis…



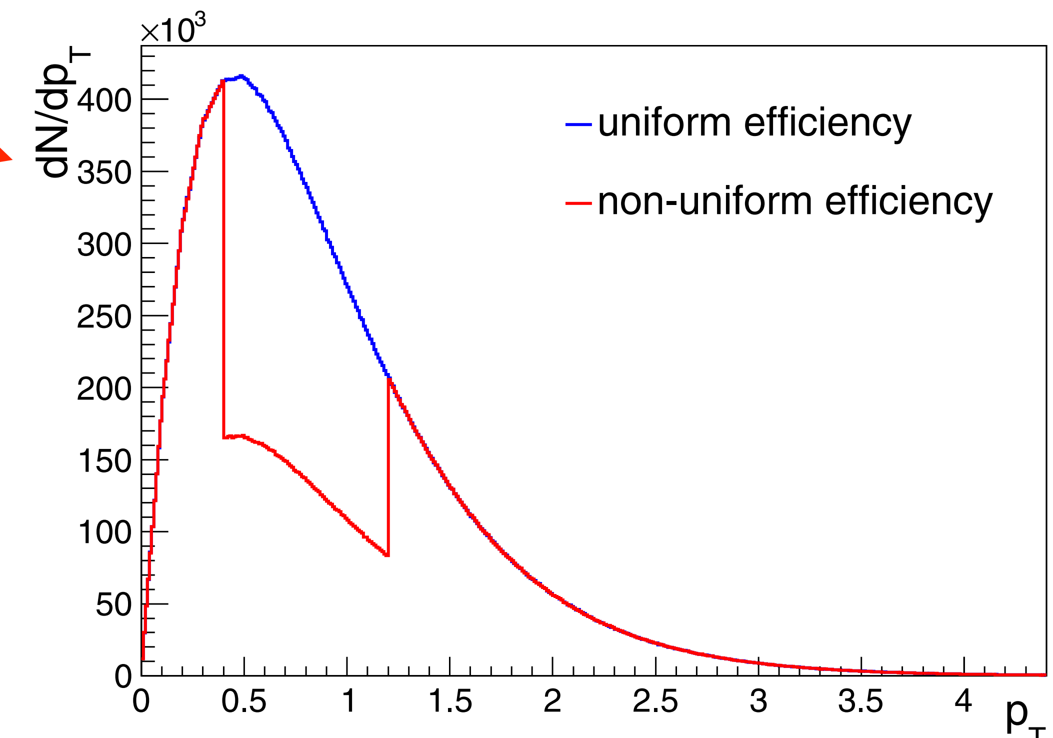
Acceptance/efficiency corrections

- *φ*-weights completely remove the bias



- *Note:* does not work if acceptance is completely missing

- To correct for non-uniform efficiency:
  - Construct p_T-weights from MC
  - Apply weights during analysis, same as for *φ*-weights



New observables 1

- New framework allows access to a many new observables

- Used to probe correlations of fluctuations

- Example of new observables (*standard candles)*:

$$\langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4) \rangle\rangle,$$

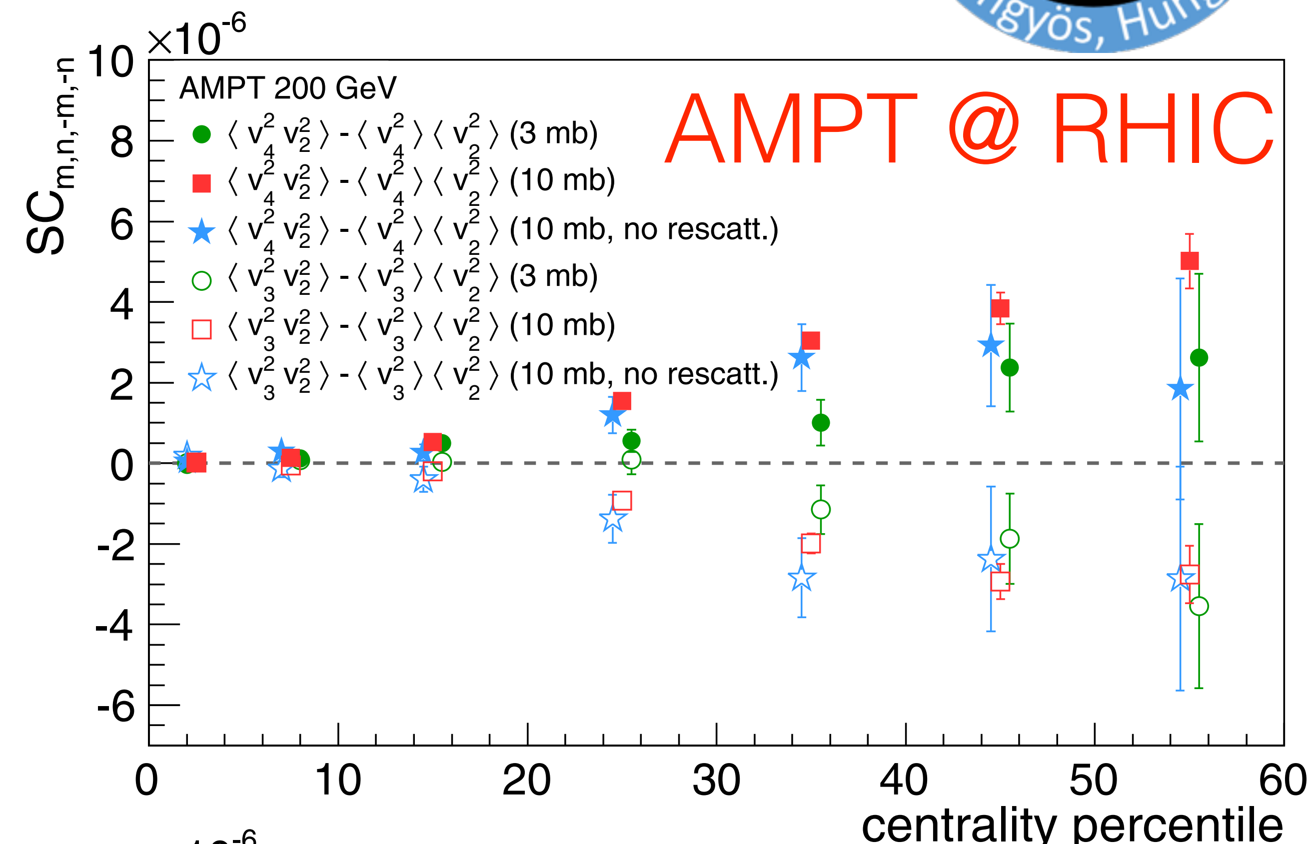
- for $m \neq n$ isotropic parts of four-particle cumulant:

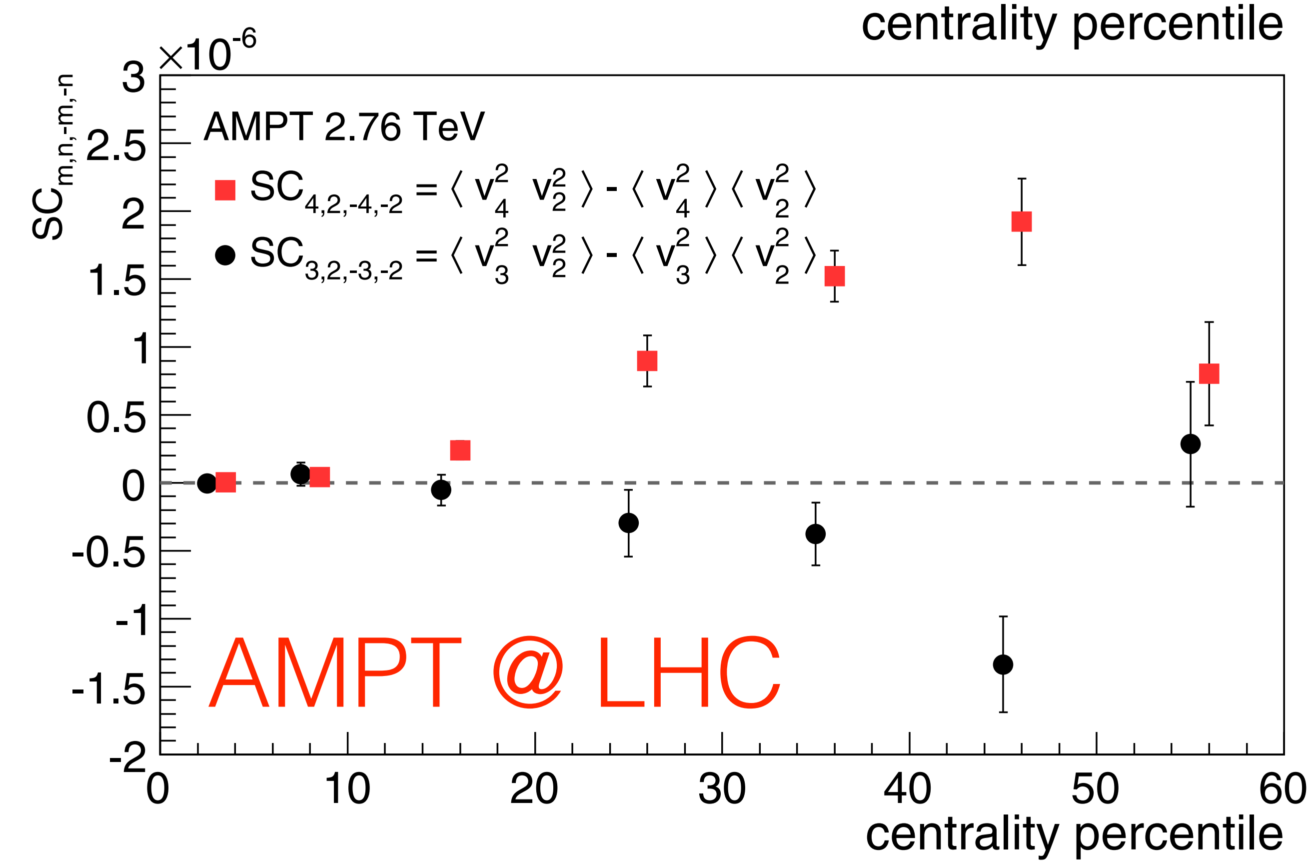
$$\langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4) \rangle\rangle_c = \langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle = SC_{m,n,-m,-n}$$

- Is non-zero only if fluctuations of v_n and v_m are correlated or anti-correlated!

New observables 2

- AMPT with RHIC tunes and LHC tune for new observables: $SC_{3,2,-3,-2}$ and $SC_{4,2,-4,-2}$
- AMPT predicts non-zero values for correlations of event-by-event fluctuations between v_2 and v_4 and anti-correlations between fluctuations of v_2 and v_3
- Provides valuable information on underlying p.d.f.





Higher order correlations

- Using multiparticle correlations: number of terms follows Bell sequence: 1, 2, 5, 15, 52, 203, 877, 4140, 21147…

➡ Direct evaluation impractical

- Higher-order terms can be expressed recursively:

$$
\begin{aligned}
\mathrm{N}\langle 1\rangle_{n_1} &= Q_{n_1,1}, \\
\mathrm{N}\langle 2\rangle_{n_1,n_2} &= \mathrm{N}\langle 1\rangle_{n_1} Q_{n_2,1} - Q_{n_1+n_2,2}, \\
\mathrm{N}\langle 3\rangle_{n_1,n_2,n_3} &= \mathrm{N}\langle 2\rangle_{n_1,n_2} Q_{n_3,1} - \mathrm{N}\langle 1\rangle_{n_1} Q_{n_2+n_3,2} - \mathrm{N}\langle 1\rangle_{n_2} Q_{n_1+n_3,2} + 2Q_{n_1+n_2+n_3,3}, \\
\mathrm{N}\langle 4\rangle_{n_1,n_2,n_3,n_4} &= \mathrm{N}\langle 3\rangle_{n_1,n_2,n_3} Q_{n_4,1} - \mathrm{N}\langle 2\rangle_{n_1,n_2} Q_{n_3+n_4,2} - \mathrm{N}\langle 2\rangle_{n_1,n_3} Q_{n_2+n_4,2} - \mathrm{N}\langle 2\rangle_{n_2,n_3} Q_{n_1+n_4,2} \\
&\quad + 2\mathrm{N}\langle 1\rangle_{n_1} Q_{n_2+n_3+n_4,3} + 2\mathrm{N}\langle 1\rangle_{n_2} Q_{n_1+n_3+n_4,3} + 2\mathrm{N}\langle 1\rangle_{n_3} Q_{n_1+n_2+n_4,3} - 6Q_{n_1+n_2+n_3+n_4,4},
\end{aligned}
$$

➡ C++ and Mathematica code: recursive algorithms and expanded equations for all correlators up to and including $m = 8$ available at: http://www.nbi.dk/~cholm/mcorrelations/

Bias from detector segmentation

- Previous equations derived assuming infinite resolution

- Finite resolution: bias and interference between harmonics

- Detector with *N* equal-sized azimuthal segments, if harmonics *n* > *N/2* are negligible: $E[e^{in\varphi}] \approx v_n e^{in\Psi_n} \frac{\sin \frac{n}{N}\pi}{\frac{n}{N}\pi}.$

- Expectation value:

$$E\left[\langle m\rangle_{n_1,\ldots,n_m}\right] \approx \prod_{k=1}^{m} v_{n_k} e^{in_k\Psi_{n_k}} \frac{\sin \frac{n_k}{N}\pi}{\frac{n_k}{N}\pi}.$$

Bias from detector segmentation

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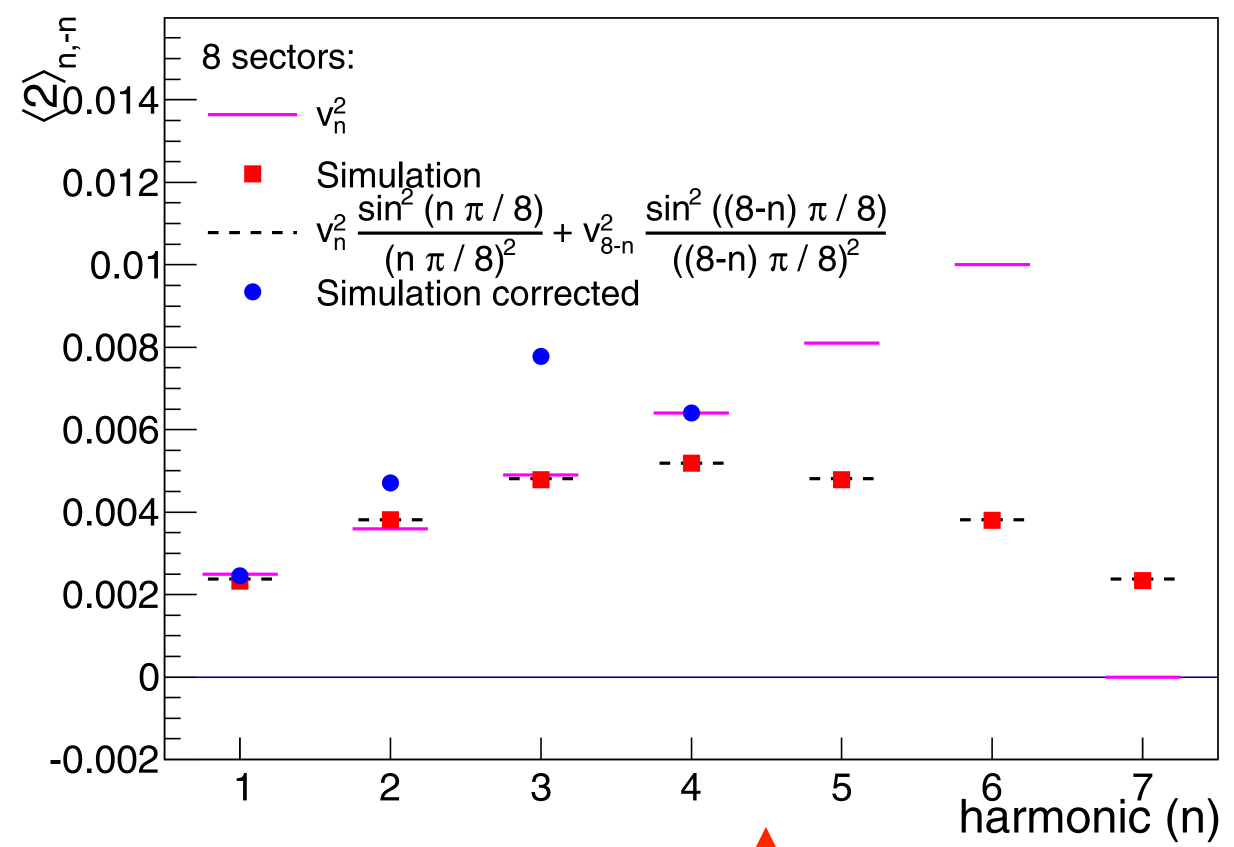


- If $n > N/2$ is non-negligible:

$$E[\langle 2\rangle_{n,-n}] = v_n^2 \frac{\sin^2\left(\frac{n\pi}{N}\right)}{\left(\frac{n\pi}{N}\right)^2} + v_{N-n}^2 \frac{\sin^2\left[\frac{(n-N)\pi}{N}\right]}{\left[\frac{(n-N)\pi}{N}\right]^2}$$

$$= E[\langle 2\rangle_{N-n,n-N}].$$

Bias from particle selection

- In general:
  - 2-particle (reference) cumulant is *enhanced* by flow fluctuations
  - 4-particle (reference) cumulant is *suppressed* by flow fluctuations

$$v\{2\} \approx \langle v \rangle + \frac{1}{2}\frac{\sigma_v^2}{\langle v \rangle} \qquad v\{4\} \approx \langle v \rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v \rangle}$$

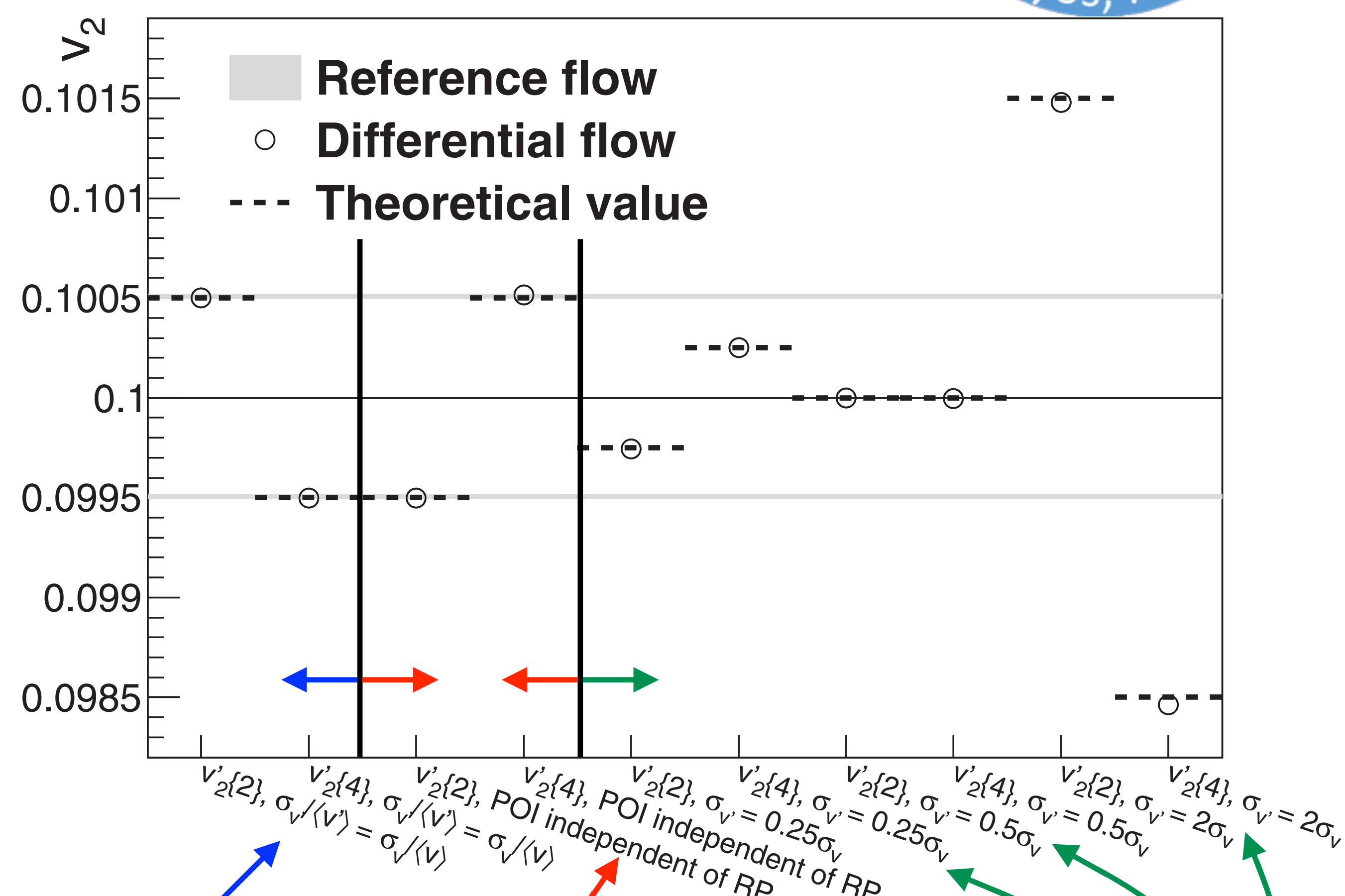
- Differential flow: same behaviour if fluctuations of RPs and POIs are perfectly correlated

- In general: and

$$v'\{2\} \approx \langle v' \rangle \left(1 + \rho \frac{\sigma_{v'}\sigma_v}{\langle v' \rangle \langle v \rangle} - \frac{1}{2}\frac{\sigma_v^2}{\langle v \rangle^2}\right)$$

$$v'\{4\} \approx \langle v' \rangle \left(1 - \rho \frac{\sigma_{v'}\sigma_v}{\langle v' \rangle \langle v \rangle} + \frac{1}{2}\frac{\sigma_v^2}{\langle v \rangle^2}\right)$$

➡ *Enhancement* or *suppression* can happen for both $v_n\{2\}$ and $v_n\{4\}$!



- Perfect correlation and same fluctuations yields same as reference
- No correlation yields opposite result
- Different fluctuations can give any result

Conclusion

- New generic framework to evaluate all multiparticle azimuthal correlations in single pass over particles
- Recursive algorithm found: efficiently computes higher-order correlations
- Particle weights: eliminate bias from efficiency or acceptance issues
- New observables, the standard candles, proposed
- Software package of use to theorists and experimentalists available online
- Bias from finite detector granularity and particle selection criteria are important

Thanks :)

Backup

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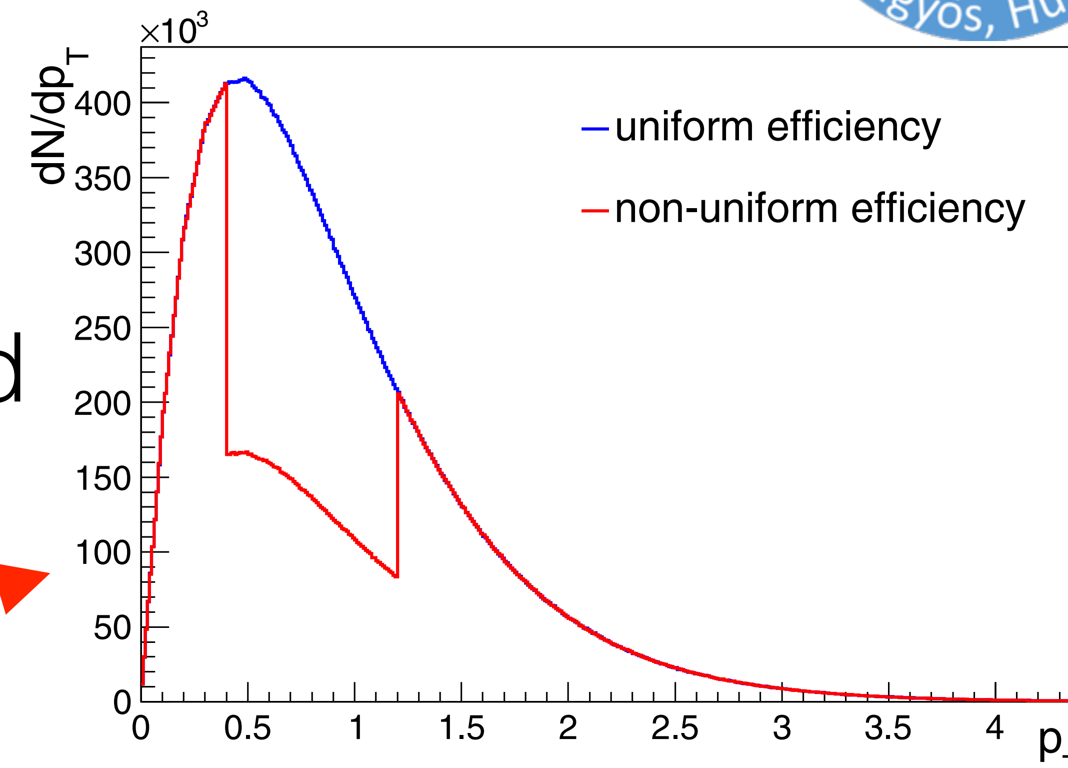

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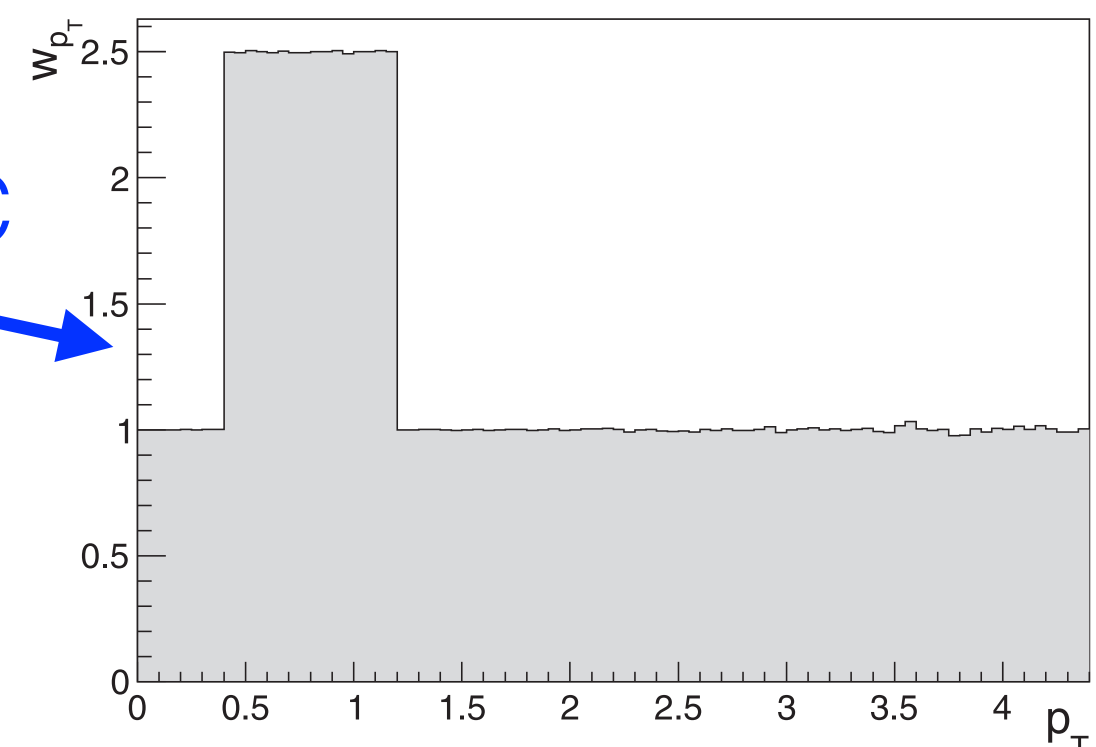

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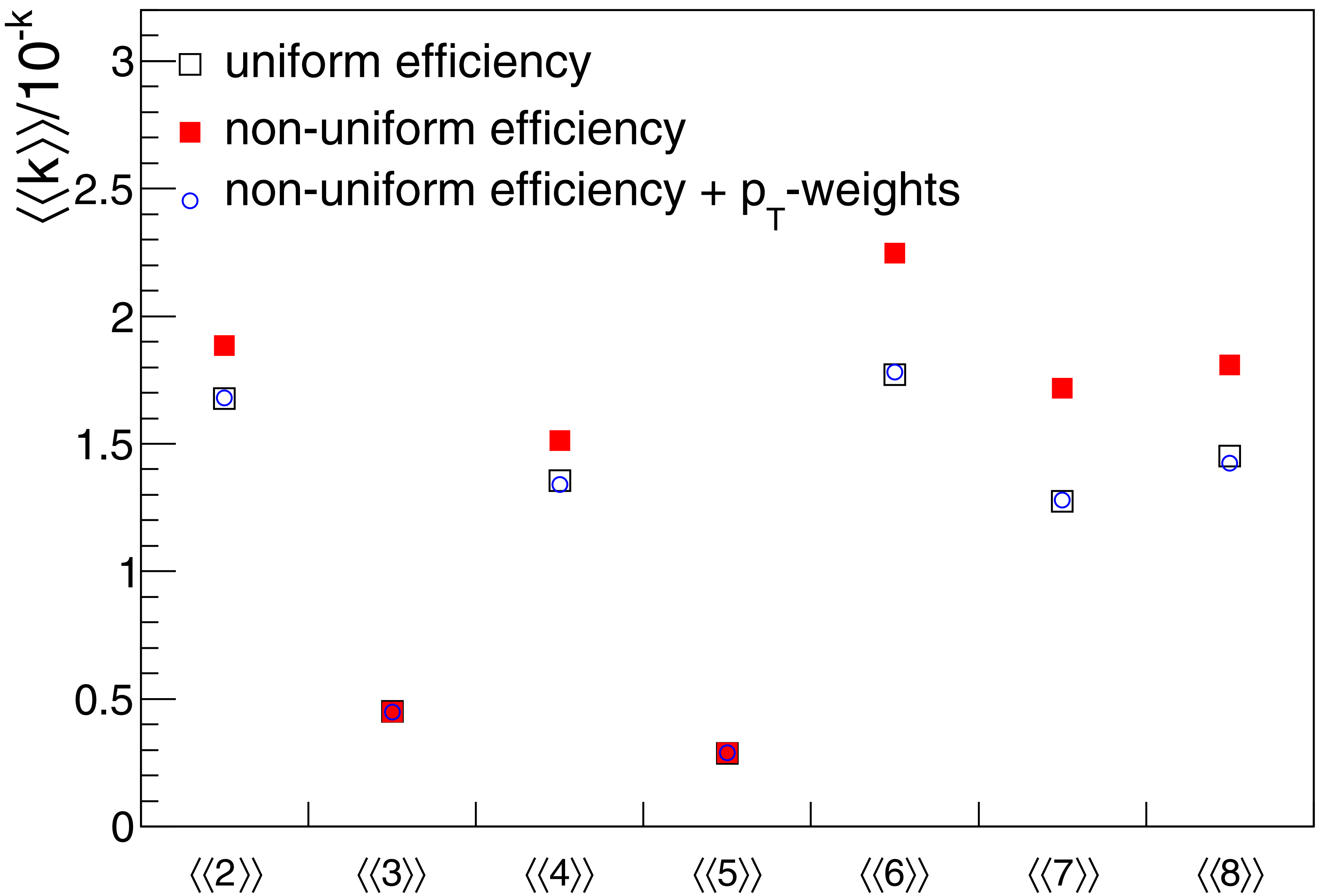

- p_T-weights completely remove the bias