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Ripples on Relativistically Expanding Fluid

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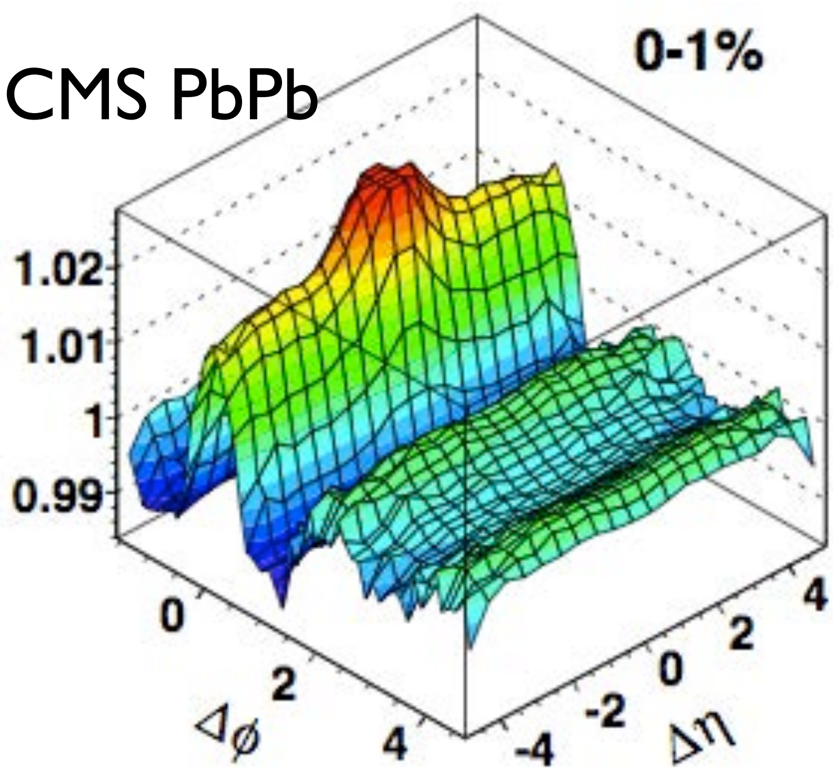
Outline

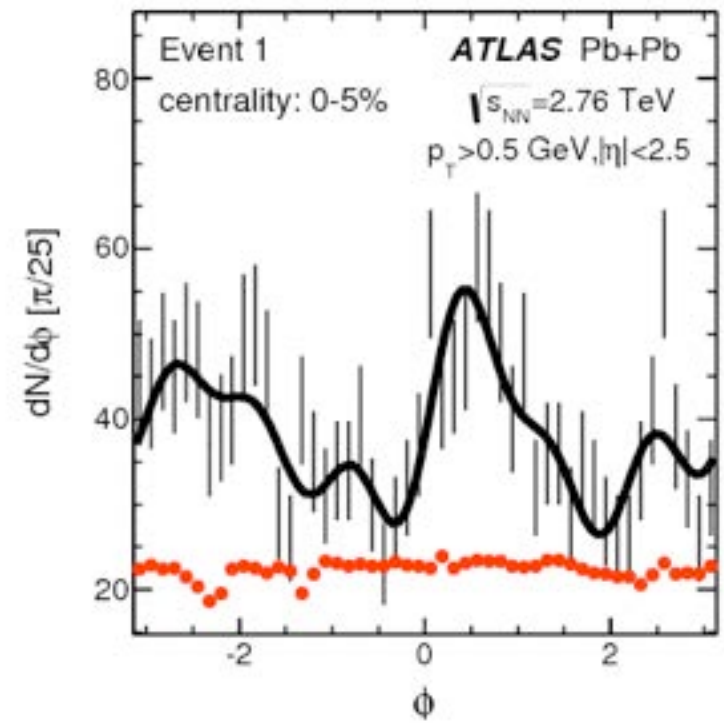
- Introduction: fluctuations in fluid
- Exact sound wave solutions on Bjorken flow
- Exact sound wave solutions on Hubble flow
- Summary & Outlook

Work done in collaboration with:
Shuzhe Shi and Pengfei Zhuang from Tsinghua University, Beijing China

References: Shi, JL, Zhuang, arXiv:1405.4546

Distinctive Correlation Patterns Observed





Specific longitudinal and azimuthal angular correlation patterns are observed experimentally

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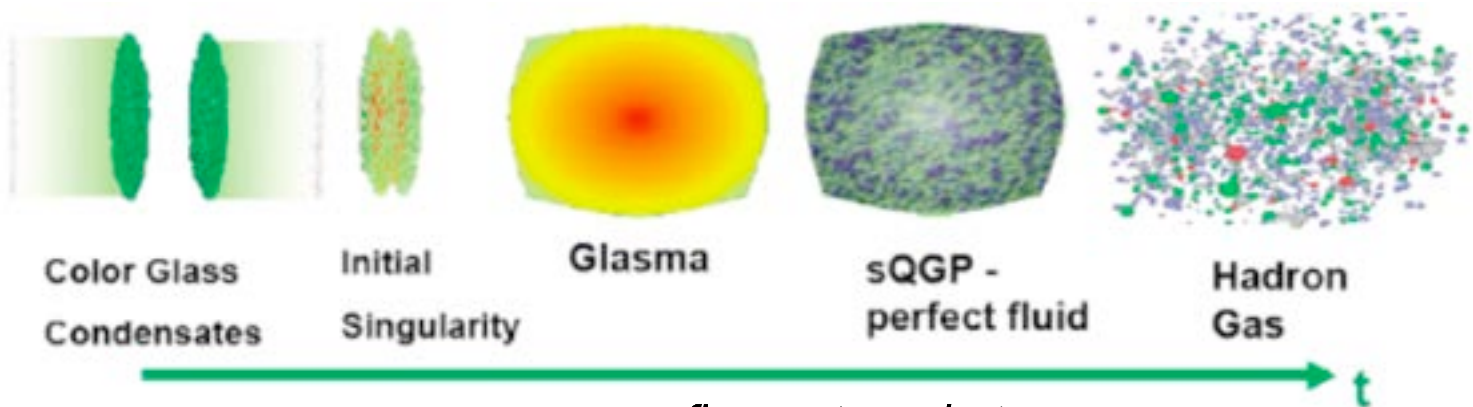
It is widely believed that various fluctuations play curial roles

Fluctuations All Along



initial state fluctuations

fluctuations during the dynamical evolution e.g. jet deposition; hydro fluctuations

freeze-out fluctuations

see e.g. discussions in Kapusta, Muller, Stephanov, PRC2012

To understand the impact of various fluctuations on final state observables, there are in fact TWO QUESTIONS:

* How/when/where/why a certain fluctuation occurs?

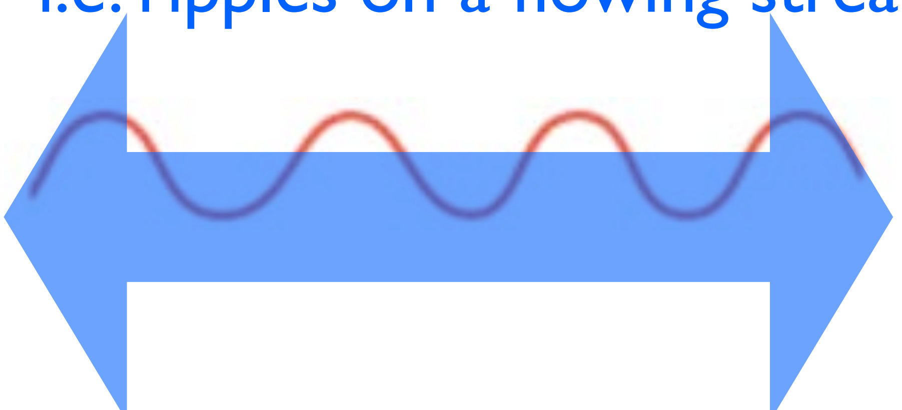
* Once created, how a fluctuation propagates further within the system?

We deal with this question

Sound wave in fluid

The short answer:
a fluctuation (if small) spreads out
in a fluid in the form of (superposed)
sound waves, like ripples on a pond

This however could become highly nontrivial,
if the fluid itself is in motion
(such as the fireball in heavy ion collisions),
i.e. ripples on a flowing stream



Hydrodynamics Framework

The background flow is described by solutions to hydrodynamic equations

$$T^{\mu\nu}{}_{;\mu} = 0$$

In this work we use ideal hydro, and conformal E.o.S

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - g^{\mu\nu} p, \qquad p = c_s^2 \epsilon$$

We use usual convenient coordinates for heavy ion collisions:

$$\begin{aligned} t &= \tau \cosh\eta, & z &= \tau \sinh\eta, \\ x &= \rho \cos\phi, & y &= \rho \sin\phi. \end{aligned}$$

The hydro equations read:

$$\begin{aligned}
0 &= T^{\tau\tau}{}_{,\tau} + T^{\tau\eta}{}_{,\eta} + T^{\tau\rho}{}_{,\rho} + T^{\tau\phi}{}_{,\phi} + \tau T^{\eta\eta} + \frac{1}{\tau} T^{\tau\tau} + \frac{1}{\rho} T^{\rho\tau}, \\
0 &= T^{\eta\tau}{}_{,\tau} + T^{\eta\eta}{}_{,\eta} + T^{\eta\rho}{}_{,\rho} + T^{\eta\phi}{}_{,\phi} + \frac{3}{\tau} T^{\eta\tau} + \frac{1}{\rho} T^{\rho\eta}, \\
0 &= T^{\rho\tau}{}_{,\tau} + T^{\rho\eta}{}_{,\eta} + T^{\rho\rho}{}_{,\rho} + T^{\rho\phi}{}_{,\phi} - \rho T^{\phi\phi} + \frac{1}{\rho} T^{\rho\rho} + \frac{1}{\tau} T^{\tau\rho}, \\
0 &= T^{\phi\tau}{}_{,\tau} + T^{\phi\eta}{}_{,\eta} + T^{\phi\rho}{}_{,\rho} + T^{\phi\phi}{}_{,\phi} + \frac{3}{\rho} T^{\rho\phi} + \frac{1}{\tau} T^{\tau\phi}.
\end{aligned}$$

Hydrodynamic Framework

The background flow is described by solutions to hydrodynamic equations

$$T^{\mu\nu}{}_{;\mu} = 0$$

In this work we use ideal hydro, and conformal E.o.S

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - g^{\mu\nu}p, \qquad p = c_s^2\epsilon$$

Sound wave modes are described in a perturbative way as solutions to <u>linearized hydro equations</u> on top of given background flow solution

$$p = p_0 + p_1 \qquad u^\mu = u_0^\mu + u_1^\mu$$

$$\longrightarrow \quad T^{\mu\nu} \approx \{T_0^{\mu\nu}\} + \{T_1^{\mu\nu}\} \quad \longrightarrow \quad T_1^{\mu\nu}{}_{;\mu} = 0.$$

Linearized Hydrodynamic Equations

Sound wave modes are described in a perturbative way as solutions to linearized hydro equations on top of given background flow solution

$$p = p_0 + p_1 \qquad u^\mu = u_0^\mu + u_1^\mu$$

$$\longrightarrow T^{\mu\nu} \approx \{T_0^{\mu\nu}\} + \{T_1^{\mu\nu}\} \longrightarrow T_1^{\mu\nu}{}_{;\mu} = 0.$$

In the coordinates we use, the linearized eqs. read:

$$\begin{aligned}
0 &= \frac{p_0 u_1^\rho}{\rho} + \frac{p_1}{\tau} + \frac{1}{1+c_s^2} p_{1,\tau} + p_0(u_{1,\eta}^\eta + u_{1,\rho}^\rho + u_{1,\phi}^\phi), \\
0 &= p_0 u_{1,\tau}^\eta + \frac{2-c_s^2}{\tau} p_0 u_1^\eta + \frac{c_s^2}{1+c_s^2} \frac{p_{1,\eta}}{\tau^2}, \\
0 &= p_0 u_{1,\tau}^\rho - \frac{c_s^2}{\tau} p_0 u_1^\rho + \frac{c_s^2}{1+c_s^2} p_{1,\rho}, \\
0 &= p_0 u_{1,\tau}^\phi - \frac{c_s^2}{\tau} p_0 u_1^\phi + \frac{c_s^2}{1+c_s^2} \frac{p_{1,\phi}}{\rho^2}.
\end{aligned}$$

On Top of Bjorken Flow

One well-known solution is the Bjorken flow

We use usual convenient coordinates for heavy ion collisions:

$$t = \tau \cosh\eta, \qquad z = \tau \sinh\eta,$$
$$x = \rho \cos\phi, \qquad y = \rho \sin\phi.$$

The hydro equations read:

$$0 = T^{\tau\tau}{}_{,\tau} + T^{\tau\eta}{}_{,\eta} + T^{\tau\rho}{}_{,\rho} + T^{\tau\phi}{}_{,\phi} + \tau T^{\eta\eta} + \frac{1}{\tau}T^{\tau\tau} + \frac{1}{\rho}T^{\rho\tau},$$
$$0 = T^{\eta\tau}{}_{,\tau} + T^{\eta\eta}{}_{,\eta} + T^{\eta\rho}{}_{,\rho} + T^{\eta\phi}{}_{,\phi} + \frac{3}{\tau}T^{\eta\tau} + \frac{1}{\rho}T^{\rho\eta},$$
$$0 = T^{\rho\tau}{}_{,\tau} + T^{\rho\eta}{}_{,\eta} + T^{\rho\rho}{}_{,\rho} + T^{\rho\phi}{}_{,\phi} - \rho T^{\phi\phi} + \frac{1}{\rho}T^{\rho\rho} + \frac{1}{\tau}T^{\tau\rho},$$
$$0 = T^{\phi\tau}{}_{,\tau} + T^{\phi\eta}{}_{,\eta} + T^{\phi\rho}{}_{,\rho} + T^{\phi\phi}{}_{,\phi} + \frac{3}{\rho}T^{\rho\phi} + \frac{1}{\tau}T^{\tau\phi}.$$

The background flow solution reads:

$$p_0(\tau) = p(\tau_0)\tau_0^{1+c_s^2}/\tau^{1+c_s^2} \qquad u_0^\mu(\tau) = (1,0,0,0)$$

Linearized Eqs. on top of Bjorken Flow

The linearized equations now read:

$$0 = \frac{p_0 u_1^\rho}{\rho} + \frac{p_1}{\tau} + \frac{1}{1+c_s^2} p_{1,\tau} + p_0 (u^\eta_{1,\eta} + u^\rho_{1,\rho} + u^\phi_{1,\phi}),$$

$$0 = p_0 u^\eta_{1,\tau} + \frac{2-c_s^2}{\tau} p_0 u_1^\eta + \frac{c_s^2}{1+c_s^2} \frac{p_{1,\eta}}{\tau^2},$$

$$0 = p_0 u^\rho_{1,\tau} - \frac{c_s^2}{\tau} p_0 u_1^\rho + \frac{c_s^2}{1+c_s^2} p_{1,\rho},$$

$g^{\mu\nu} u_\mu u_\nu = 1$ requires $u_1^\tau = 0$

$$0 = p_0 u^\phi_{1,\tau} - \frac{c_s^2}{\tau} p_0 u_1^\phi + \frac{c_s^2}{1+c_s^2} \frac{p_{1,\phi}}{\rho^2}.$$

The strategy is to go toward 2nd order differentiation with separable variables, by combining these eqns into:

$\delta \equiv p_1/p_0 \qquad \delta(\tau,\eta,\rho,\phi) = \delta_\parallel(\tau,\eta)\delta_\perp(\rho,\phi)$

$$\delta_{\perp,\rho\rho} + \frac{1}{\rho}\delta_{\perp,\rho} + \frac{1}{\rho^2}\delta_{\perp,\phi\phi} = -\omega^2\delta_\perp,$$

$$\tau^{1+c_s^2}(\tau^{-1-c_s^2}\delta_{\parallel,\eta\eta})_{,\tau} = (3-c_s^2)\omega^2\tau\delta_\parallel + (\omega^2\tau^2 - 2 + c_s^2)\delta_{\parallel,\tau} + (3c_s^{-2}-2)\tau\delta_{\parallel,\tau\tau} + c_s^{-2}\tau^2\delta_{\parallel,\tau\tau\tau}$$

N.B. One can also study simpler cases: purely longitudinal and purely transverse sound wave

General Sound Wave Solutions

The equations after separation indicate at the following form for eigen-modes:

$$\delta_\perp(\rho,\phi) \sim J_m(\omega\rho)\,e^{im\phi},$$
$$\delta_\parallel(\tau,\eta) \sim e^{ik\eta}\times W(\tau).$$

These eventually lead to the following general solutions:

$$\delta(\tau,\eta,\rho,\phi) = \sum_m \int_0^\infty d\omega \int_{-\infty}^\infty dk\; e^{ik\eta} J_m(\omega\rho) e^{im\phi} W(\tau),$$

with auxiliary functions:

$$W(\tau) \equiv A_{k,\omega,m}W_1(\tau) + B_{k,\omega,m}W_2(\tau) + C_{k,\omega,m}W_3(\tau),$$
$$W_1(\tau) = \left(\frac{\tau}{\tau'}\right)^{-\frac{1-c_s^2}{2}-\alpha_k}(\omega\tau)^{\alpha_k}\left[\frac{1-c_s^2-2\alpha_k}{2}J_{-\alpha_k}(c_s\omega\tau) - (c_s\omega\tau)J_{1-\alpha_k}(c_s\omega\tau)\right],$$
$$W_2(\tau) = \left(\frac{\tau}{\tau'}\right)^{-\frac{1-c_s^2}{2}+\alpha_k}(\omega\tau)^{-\alpha_k}\left[\frac{1-c_s^2-2\alpha_k}{2}J_{\alpha_k}(c_s\omega\tau) + (c_s\omega\tau)J_{\alpha_k-1}(c_s\omega\tau)\right],$$
$$W_3(\tau) = (\omega\tau)^{1+c_s^2}\;{}_1F_2\left[2;\frac{7+c_s^2+2\alpha_k}{4},\frac{7+c_s^2-2\alpha_k}{4};-c_s^2\omega^2\tau^2/4\right]$$

three coefficients for three independent modes,
entirely fixed from given perturbation at occurring time.

On Top of Hubble Flow

Another well-known solution is the 3D Hubble flow

We use following convenient coordinates:

$$t = \tau \cosh\eta, \qquad z = \tau \sinh\eta \cos\theta,$$
$$x = \tau \sinh\eta \sin\theta \cos\phi, \qquad y = \tau \sinh\eta \sin\theta \sin\phi,$$

The hydro equations read:

$$\begin{aligned}
T^{\tau\lambda}_{\ \ ;\lambda} &= T^{\tau\tau}_{\ \ ,\tau} + T^{\tau\eta}_{\ \ ,\eta} + T^{\tau\theta}_{\ \ ,\theta} + T^{\tau\phi}_{\ \ ,\phi} + \Gamma^{\eta}_{\ \tau\eta} T^{\tau\tau} + \Gamma^{\tau}_{\ \eta\eta} T^{\eta\eta} + \Gamma^{\theta}_{\ \tau\theta} T^{\tau\tau} + \Gamma^{\theta}_{\ \eta\theta} T^{\tau\eta} \\
&+ \Gamma^{\tau}_{\ \theta\theta} T^{\theta\theta} + \Gamma^{\phi}_{\ \tau\phi} T^{\tau\tau} + \Gamma^{\phi}_{\ \eta\phi} T^{\tau\eta} + \Gamma^{\phi}_{\ \theta\phi} T^{\tau\theta} + \Gamma^{\tau}_{\ \phi\phi} T^{\phi\phi}, \\
T^{\eta\lambda}_{\ \ ;\lambda} &= T^{\eta\tau}_{\ \ ,\tau} + T^{\eta\eta}_{\ \ ,\eta} + T^{\eta\theta}_{\ \ ,\theta} + T^{\eta\phi}_{\ \ ,\phi} + 3\Gamma^{\eta}_{\ \eta\tau} T^{\tau\eta} + \Gamma^{\theta}_{\ \tau\theta} T^{\eta\tau} + \Gamma^{\theta}_{\ \eta\theta} T^{\eta\eta} + \Gamma^{\eta}_{\ \theta\theta} T^{\theta\theta} \\
&+ \Gamma^{\phi}_{\ \tau\phi} T^{\eta\tau} + \Gamma^{\phi}_{\ \eta\phi} T^{\eta\eta} + \Gamma^{\phi}_{\ \theta\phi} T^{\eta\theta} + \Gamma^{\eta}_{\ \phi\phi} T^{\phi\phi}, \\
T^{\theta\lambda}_{\ \ ;\lambda} &= T^{\theta\tau}_{\ \ ,\tau} + T^{\theta\eta}_{\ \ ,\eta} + T^{\theta\theta}_{\ \ ,\theta} + T^{\theta\phi}_{\ \ ,\phi} + 3\Gamma^{\theta}_{\ \theta\tau} T^{\tau\theta} + \Gamma^{\eta}_{\ \tau\eta} T^{\theta\tau} + 3\Gamma^{\theta}_{\ \theta\eta} T^{\eta\theta} + \Gamma^{\phi}_{\ \tau\phi} T^{\theta\tau} \\
&+ \Gamma^{\phi}_{\ \eta\phi} T^{\theta\eta} + \Gamma^{\phi}_{\ \theta\phi} T^{\theta\theta} + \Gamma^{\theta}_{\ \phi\phi} T^{\phi\phi}, \\
T^{\phi\lambda}_{\ \ ;\lambda} &= T^{\phi\tau}_{\ \ ,\tau} + T^{\phi\eta}_{\ \ ,\eta} + T^{\phi\theta}_{\ \ ,\theta} + T^{\phi\phi}_{\ \ ,\phi} + 3\Gamma^{\phi}_{\ \phi\tau} T^{\tau\phi} + \Gamma^{\eta}_{\ \tau\eta} T^{\phi\tau} + 3\Gamma^{\phi}_{\ \phi\eta} T^{\eta\phi} + \Gamma^{\theta}_{\ \tau\theta} T^{\phi\tau} \\
&+ \Gamma^{\theta}_{\ \eta\theta} T^{\phi\eta} + 3\Gamma^{\phi}_{\ \phi\theta} T^{\theta\phi}.
\end{aligned}$$

In these coordinates, the background flow solution reads:

$$p_0 = \frac{p_0(\tau_0)\tau_0^{3(1+c_s^2)}}{\tau^{3(1+c_s^2)}}\,, \quad u_0^{\mu} = (1,0,0,0).$$

Linearized Eqs. on top of Hubble Flow

The linearized equations now read:

$$
\begin{aligned}
0 &= \frac{1}{1+c_s^2} p_{1,\tau} + \frac{3}{\tau} p_1 + p_0 u^\eta_{1,\eta} + p_0 u^\theta_{1,\theta} + p_0 u^\phi_{1,\phi} + 2\frac{\cosh\eta}{\sinh\eta} p_0 u_1^\eta + \frac{\cos\theta}{\sin\theta} p_0 u_1^\theta \\
0 &= p_0 u^\eta_{1,\tau} + \frac{2-3c_s^2}{\tau} p_0 u_1^\eta + \frac{c_s^2}{1+c_s^2} \frac{p_{1,\eta}}{\tau^2} \\
0 &= p_0 u^\theta_{1,\tau} + \frac{2-3c_s^2}{\tau} p_0 u_1^\theta + \frac{c_s^2}{1+c_s^2} \frac{1}{\tau^2 \sinh^2\eta} p_{1,\theta} \\
0 &= p_0 u^\phi_{1,\tau} + \frac{2-3c_s^2}{\tau} p_0 u_1^\phi + \frac{c_s^2}{1+c_s^2} \frac{1}{\tau^2 \sinh^2\eta \ \sin^2\theta} p_{1,\phi} \,.
\end{aligned}
$$

The strategy is to go toward 2nd order differentiation with separable variables, by combining these eqns into:

$$
\begin{aligned}
& c_s^{-2}\tau^2 p_{1,\tau\tau} + (3+8c_s^{-2})\tau p_{1,\tau} + 12(1+c_s^{-2}) p_1 \\
& \quad = p_{1,\eta\eta} + 2\frac{\cosh\eta}{\sinh\eta} p_{1,\eta} + \frac{1}{\sinh^2\eta}\left(p_{1,\theta\theta} + \frac{\cos\theta}{\sin\theta} p_{1,\theta} + \frac{1}{\sin^2\theta} p_{1,\phi\phi}\right).
\end{aligned}
$$

General Sound Wave Solutions

The equations after separation indicate at the following form for eigen-modes:

$$p_1(\tau,\eta,\theta,\phi) = \sum_{l,m} p_{l,m}(\tau,\eta) Y_l^m(\theta,\phi)$$

These eventually lead to the following general solutions:

$$\frac{p_1(\tau,\eta,\theta,\phi)}{p_0} = (\frac{\tau}{\tau'})^{\frac{3c_s^2-1}{2}} \sum_{l,m} \int_{-\infty}^{\infty} a_{l,m}(k) \cos\left[\beta_k \ln(\tau/\tau')\right] R_l(k,\eta) Y_l^m(\theta,\phi)\, \mathrm{d}k$$
$$+(\frac{\tau}{\tau'})^{\frac{3c_s^2-1}{2}} \sum_{l,m} \int_{-\infty}^{\infty} b_{l,m}(k) \sin\left[\beta_k \ln(\tau/\tau')\right] R_l(k,\eta) Y_l^m(\theta,\phi)\, \mathrm{d}k,$$

with special functions:

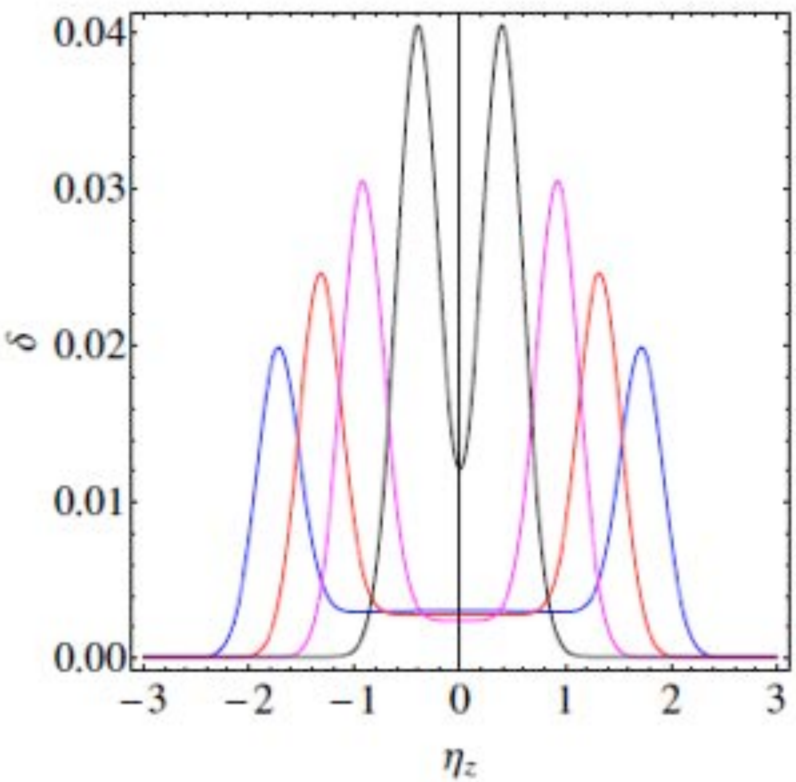
$$R_l(k,\eta) = \sqrt{\frac{\Gamma(l+1+ik)\Gamma(l+1-ik)}{\pi 2^{2l+2}\Gamma(l+3/2)^2}} \sinh^l \eta \; {}_2F_1(\frac{l+1+ik}{2}, \frac{l+1-ik}{2}, l+3/2, -\sinh^2\eta)$$

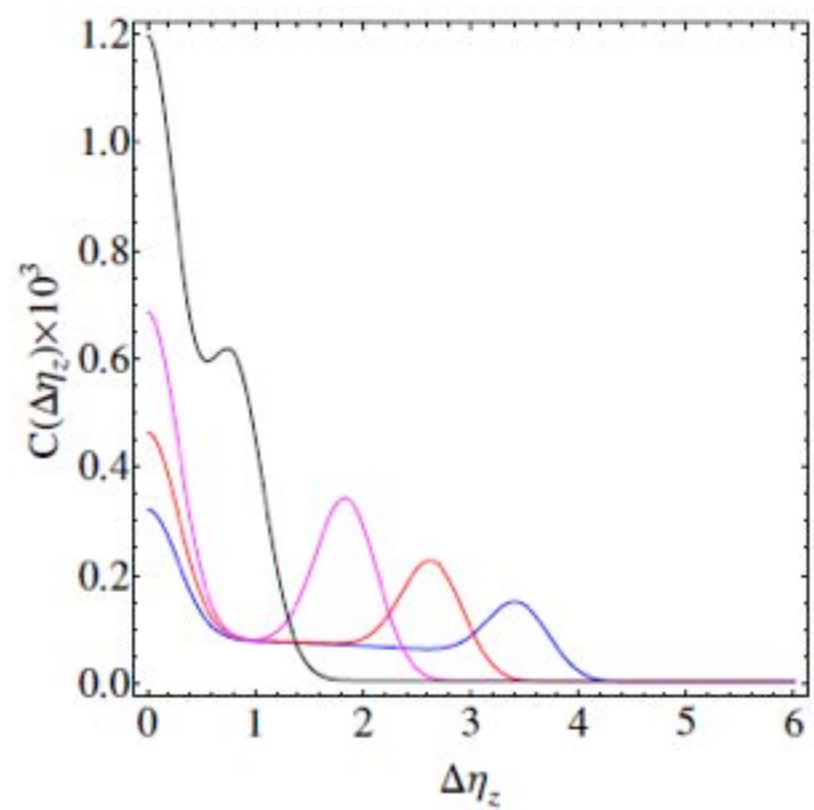
the two coefficients for radially inward/outward modes, entirely fixed from given perturbation at occurring time.

Showcasing Resulting Correlations

As an example, we show pressure-pressure rapidity correlations arising from an earlier Guassian perturbation

$$C(\Delta\eta) = \int d\eta_1 \delta(\tau, \eta_1)\delta(\tau, \eta_1 + \Delta\eta)$$





longitudinal wave on top of Bjorken flow

Showcasing Resulting Correlations

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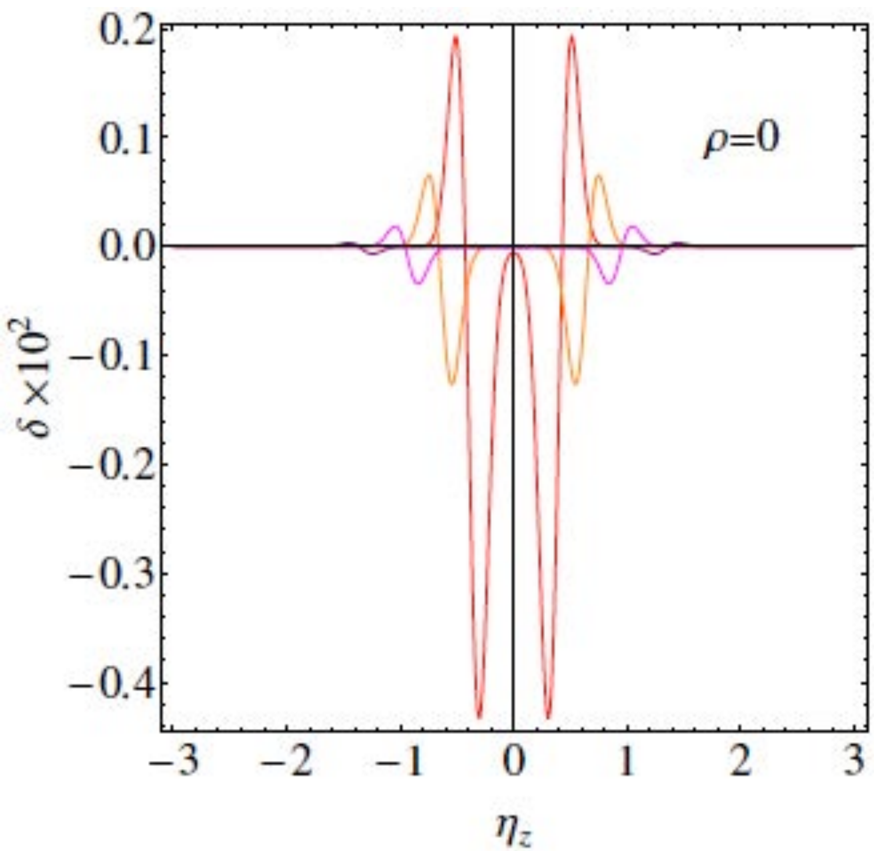
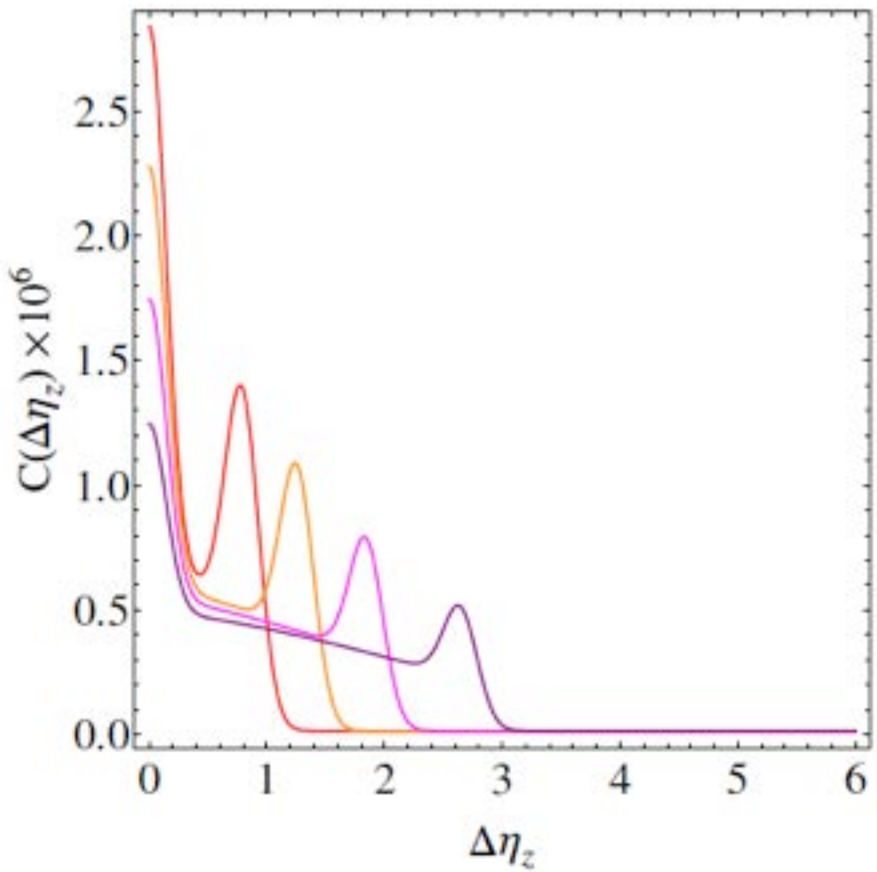
$$C(\Delta\eta) = \int d\eta_1 \delta(\tau, \eta_1)\delta(\tau, \eta_1 + \Delta\eta)$$



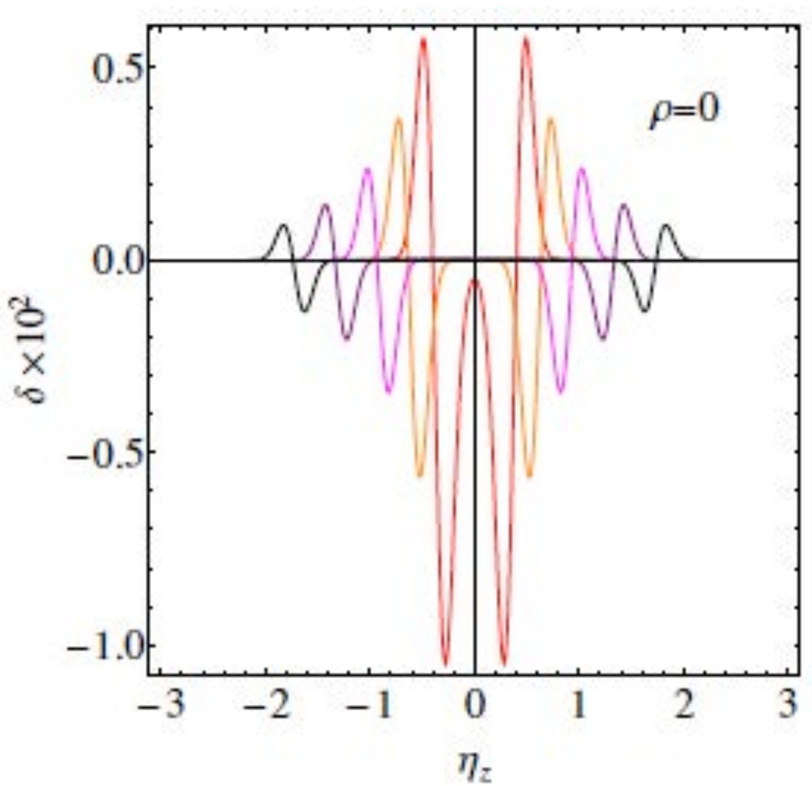


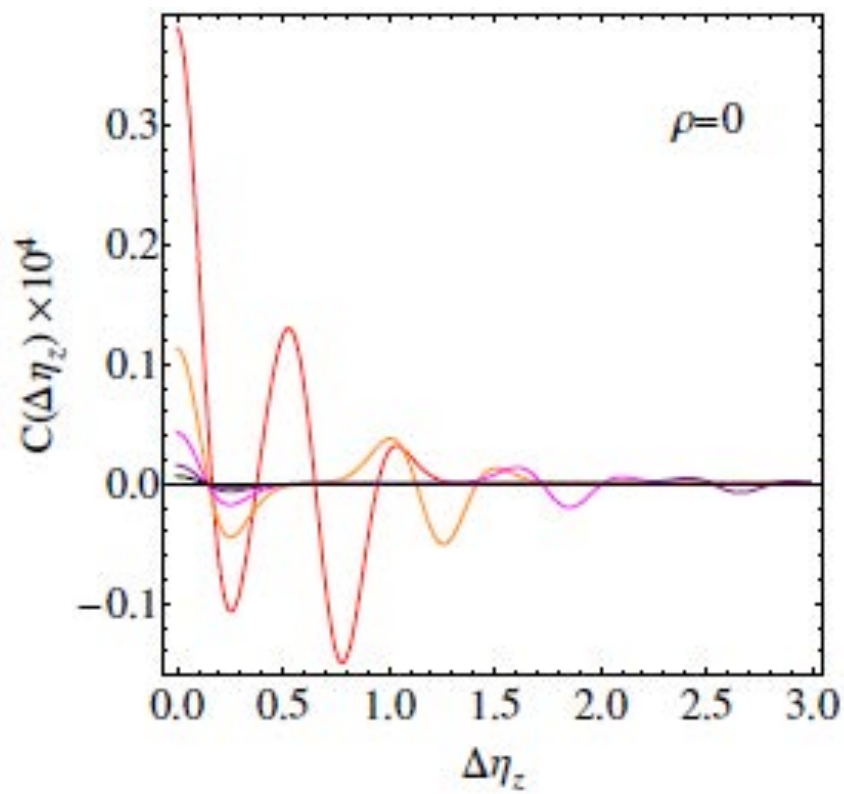
3D wave on top of Bjorken flow

Showcasing Resulting Correlations

As an example, we show pressure-pressure rapidity correlations arising from an earlier Guassian perturbation

$$C(\Delta\eta) = \int d\eta_1 \delta(\tau, \eta_1)\delta(\tau, \eta_1 + \Delta\eta)$$





3D wave on top of Hubble flow

Summary & Outlook

* It is important to understand how fluctuations, after occurrence, propagate in relativistically expanding fluid.

* We have found complete and analytic sound wave solutions on top of Bjorken flow.

* We have found complete and analytic sound wave solutions on top of Hubble flow.

* These sound waves can lead to nontrivial correlation patterns after propagating for certain time.

* Outlook for future work:
  application to phenomenology (hydro fluc., jet, ...);
  extension to other flow backgrounds;
  analysis of viscous effects.

Thank you!

BACKUP: Velocity Field for Solutions on top of Bjorken Flow

$$\delta(\tau,\eta,\rho,\phi) = \sum_m \int_0^\infty d\omega \int_{-\infty}^\infty dk\; e^{ik\eta} J_m(\omega\rho) e^{im\phi} W(\tau),$$

$$u_1^\eta(\tau,\eta,\rho,\phi) = u_1^\eta(\tau',\eta,\rho,\phi) + \frac{c_s^2}{1+c_s^2}\tau^{c_s^2-2} u_{\perp,\eta}(\tau,\eta,\rho,\phi),$$

$$u_1^\rho(\tau,\eta,\rho,\phi) = u_1^\rho(\tau',\eta,\rho,\phi) + \frac{c_s^2}{1+c_s^2}\tau^{c_s^2} u_{\perp,\rho}(\tau,\eta,\rho,\phi),$$

$$u_1^\phi(\tau,\eta,\rho,\phi) = u_1^\phi(\tau',\eta,\rho,\phi) + \frac{c_s^2}{1+c_s^2}\frac{\tau^{c_s^2}}{\rho^2} u_{\perp,\phi}(\tau,\eta,\rho,\phi),$$

$$u_\perp(\tau,\eta,\rho,\phi) = \sum_m \int_0^\infty d\omega \int_{-\infty}^\infty dk\; e^{ik\eta} J_m(\omega\rho) e^{im\phi} \int_{\tau'}^{\tau} d\tilde{\tau}\; \tilde{\tau}^{-c_s^2} W(\tilde{\tau}),$$

BACKUP: Velocity Field for Solutions on top of Hubble Flow

$$
\begin{aligned}
u_1^\eta(\tau,\eta,\theta,\phi) &= (\frac{\tau'}{\tau})^{2-3c_s^2} u_1^\eta(\tau',\eta,\theta,\phi) + \frac{c_s^2}{1+c_s^2} u_{\perp,\eta}(\tau',\eta,\theta,\phi), \\
u_1^\theta(\tau,\eta,\theta,\phi) &= (\frac{\tau'}{\tau})^{2-3c_s^2} u_1^\theta(\tau',\eta,\theta,\phi) + \frac{c_s^2}{1+c_s^2} \frac{1}{\sinh^2\eta} u_{\perp,\theta}(\tau',\eta,\theta,\phi), \\
u_1^\phi(\tau,\eta,\theta,\phi) &= (\frac{\tau'}{\tau})^{2-3c_s^2} u_1^\phi(\tau',\eta,\theta,\phi) + \frac{c_s^2}{1+c_s^2} \frac{1}{\sinh^2\eta\ \sin^2\theta} u_{\perp,\phi}(\tau',\eta,\theta,\phi)
\end{aligned}
$$

$$
\begin{aligned}
u_\perp(\tau,\eta,\theta,\phi) &= \frac{1}{\tau}\sum_{l,m}\int_{-\infty}^{\infty} \frac{\beta_k b_{l,m}(k) + (3c_s^2-1)a_{l,m}(k)}{(3c_s^2-1)^2+\beta_k^2} \left[\cos\left(\beta_k \ln\frac{\tau}{\tau'}\right) - 1\right] R_l(k,\eta) Y_l^m(\theta,\phi)\ \mathrm{d}k \\
&+ \frac{1}{\tau}\sum_{l,m}\int_{-\infty}^{\infty} \frac{-\beta_k a_{l,m}(k) + (3c_s^2-1)b_{l,m}(k)}{(3c_s^2-1)^2+\beta_k^2} \sin\left(\beta_k \ln\frac{\tau}{\tau'}\right) R_l(k,\eta) Y_l^m(\theta,\phi)\ \mathrm{d}k,
\end{aligned}
$$