Observables and inital conditions from self-similar, spheroidal rotating solution of hydrodynamics

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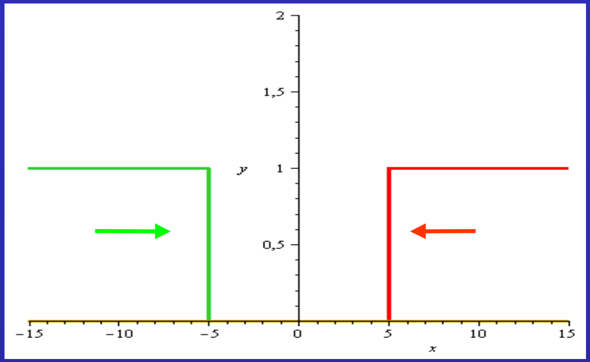
Physically important solutions of PDEs

- ***Travelling waves:***
  arbitrary wave fronts
  u(x,t) ~ g(x-ct), g(x+ct)



- ***Self-similar***

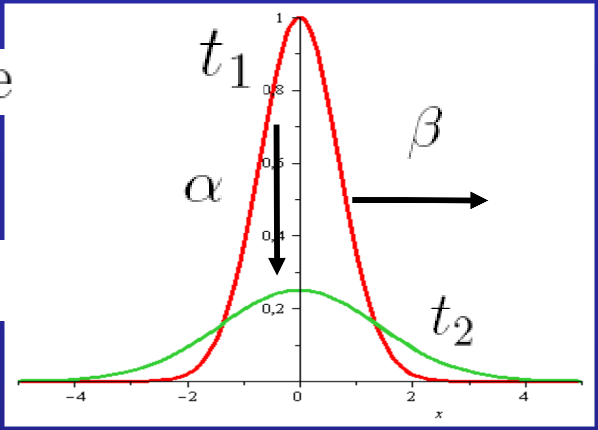
$$u(x,t) = t^{-\alpha} f(x/t^{\beta})$$

Sedov, Barenblatt, Zeldovich

α and β are of primary physical importance

α represents the rate of decay

β is the rate of spread (or contraction if $\beta < 0$)



$t_1 < t_2$ ***in Fourier heat-conduction***

Analytic solutions for non-relativistic fluids

self-similar solutions of the general form of

$u(x,t) = t^{-\alpha} f(x/t^{\beta})$ was used for various 2 and 3 dimensional viscous fluid equations:

Non-compressible and Newtonian

I.F Barna

Commun. In Theo. Phys. 56, (2011) 745

Compressible and Newtonian

I. F. Barna and L. Mátyás

Fluid. Dyn. Res. 46, (2014) 055508

Non-compressible and non- Newtonian

I.F Barna and G. Bognár

almost ready :)

Compressible and non-Newtonian

No idea, toooo complicated :(

not planned to investigate

Our starting equations

Consider the non-relativistic hydrodynamical problem, as given by the continuity, Euler and energy equations:

$$\begin{aligned} \partial_t n + \nabla \cdot (n\mathbf{v}) &= 0\,, \\ \partial_t \mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} &= -(\nabla p)/(mn)\,, \\ \partial_t \epsilon + \nabla \cdot (\epsilon \mathbf{v}) &= -p\nabla\cdot\mathbf{v}\,, \end{aligned}$$

v flow velocity field, n particle number, p pressure, m particle mass, ϵ energy density

The closing Equation of State

$$p = nT\,, \qquad \epsilon = \kappa(T) nT,$$

T temperature, $\kappa(T)$ compressibility for ideal gas 3/2

The Ansatz for the solution

self-similar, ellipsoidally symmetric density and flow profiles:

$$n(t,\mathbf{r}') = n_0 \frac{V_0}{V} \exp\left(-\frac{r_x'^2}{2X^2} - \frac{r_y'^2}{2Y^2} - \frac{r_z'^2}{2Z^2}\right)$$

$$\mathbf{v}'(t,\mathbf{r}') = \left(\frac{\dot X}{X} r_x', \frac{\dot Y}{Y} r_y', \frac{\dot Z}{Z} r_z'\right),$$

where

$(X, Y, Z) = (X(t), Y(t), Z(t))$ $V = XYZ,$

is known from a long time

S.V. Akkelin, T. Csörgő, B. Lukács, Yu. M. Sinyukov and M. Weiner, Phys. Lett. **B505** (2001) 64.

The final ordinary differential equation for the time propagation

$$
\begin{aligned}
\ddot{X}X - X^2\omega^2 = \ddot{Y}Y = \ddot{Z}Z - Z^2\omega^2 &= \frac{T}{m}\,, \\
\dot{T}\frac{d}{dT}(\kappa T) + T\left(\frac{\dot{X}}{X} + \frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\right) &= 0\,,\,, \\
X &= Z \equiv R \\
\dot{X} &= \dot{Z} \equiv \dot{R}
\end{aligned}
$$

where

$T_0 = T(t_0)$ $V_0 = V(t_0)$ n_0

are constants

Additional quantities

tilt angle corresponds now to the angle of rotation

$$\begin{aligned}
\theta(t) &= \theta_0 + \int \mathrm{d}t\, \omega(t), \\
\omega &= \omega_0 \frac{R_0^2}{R^2}, \\
R &= X = Z \neq Y, \\
s &= s_T + s_Z = \frac{r_x^2 + r_z^2}{R^2} + \frac{r_y^2}{Y^2}, \\
V &= XYZ = R^2 Y.
\end{aligned}$$

for the volume

$$\frac{V_0}{V} = \exp\left[\kappa(T) - \kappa(T_0)\right] \exp \int_{T_0}^{T} \frac{dT'}{T'} \kappa(T')$$

or

$$T = T_0 \left(\frac{V_0}{V}\right)^{1/\kappa}$$

The observables I

the emission function at a constant freeze-out temperature

$$S(t,\mathbf{r}',\mathbf{k}') \propto \mathrm{e}^{-\frac{(x\mathbf{k}'-m\mathbf{v}')^2}{2mT_f}-\frac{r_x'^2}{2X_f^2}-\frac{r_y'^2}{2Y_f^2}-\frac{r_z'^2}{2Z_f^2}}\,\delta(t-t_f)$$

single particle spectrum

$$\begin{aligned}
E\frac{d^3n}{d\mathrm{k}'} &\propto E\exp\left(-\frac{k_x'^2}{2mT_x'}-\frac{k_y'^2}{2mT_y'}-\frac{k_z'^2}{2mT_z'}\right),\\
T_x' &= T_f + m(\dot{X}_f^2 + \omega^2 Z^2)\\
T_y' &= T_f + m\dot{Y}_f^2\\
T_z' &= T_f + m(\dot{Z}_f^2 + \omega^2 X^2)\,,
\end{aligned}$$

new terms from the rotation

The observables II

$$
\begin{aligned}
C(\mathbf{K}',\mathbf{q}') &= 1+\lambda \exp\left(-q_x'^2 R_x'^2 - q_y'^2 R_y'^2 - q_z'^2 R_z'^2\right),\\
\mathbf{K}' &= \mathbf{K}_{12}' = 0.5(\mathbf{k}_1' + \mathbf{k}_2'),\\
\mathbf{q}' &= \mathbf{q}_{12}' = \mathbf{k}_1' - \mathbf{k}_2' = (q_x', q_y', q_z'),\\
R_x'^{-2} &= X_f^{-2}\left(1+\frac{m}{T_f}(\dot X_f^2 + Y_f^2\omega^2)\right),\\
R_y'^{-2} &= Y_f^{-2}\left(1+\frac{m}{T_f}\dot Y_f^2\right),\\
R_z'^{-2} &= Z_f^{-2}\left(1+\frac{m}{T_f}(\dot Z_f^2 + X_f^2\omega^2)\right).
\end{aligned}
$$

the two-particle BECF

new terms from the rotation

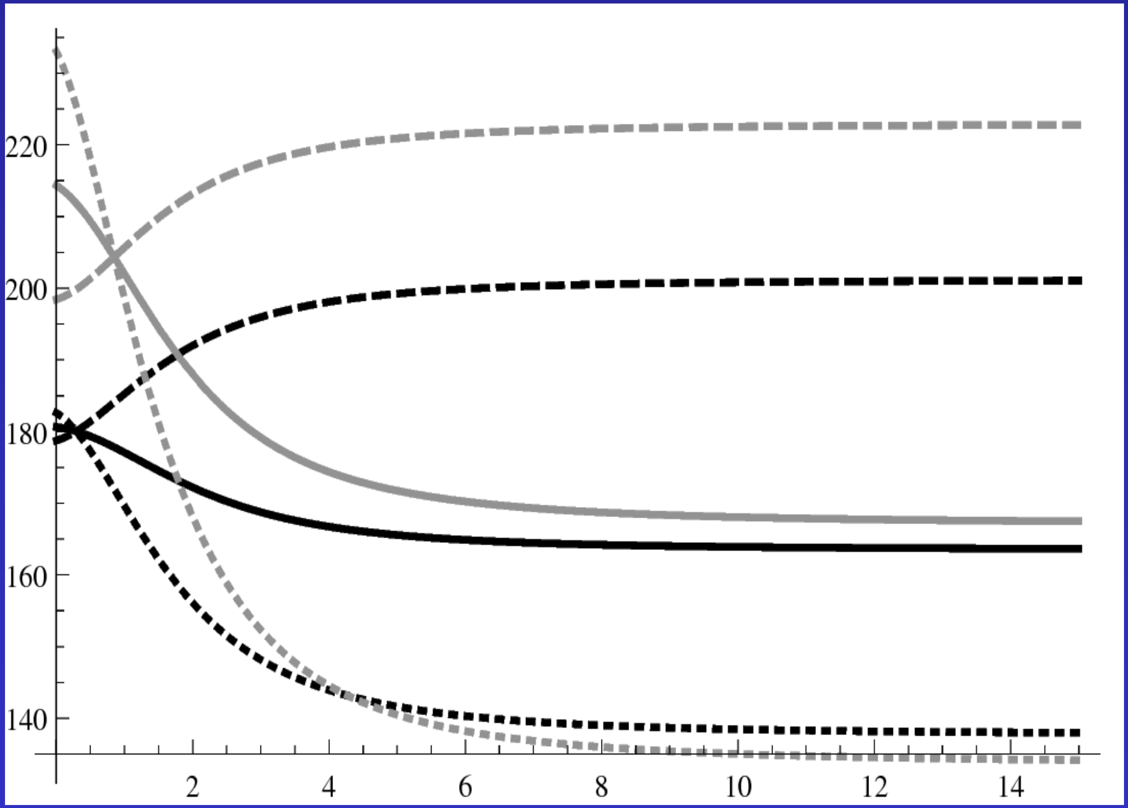
$$
\begin{aligned}
C_2(\mathbf{K},\mathbf{q}) &= 1+\lambda \exp\left(-\sum_{i,j=\mathrm{s,o,l}} q_i q_j R_{ij}^2\right),\\
R_\mathrm{s}^2 &= R_y'^2 \cos^2\phi + R_x^2 \sin^2\phi,\\
R_\mathrm{o}^2 &= R_x^2 \cos^2\phi + R_y'^2 \sin^2\phi + \beta_t^2 \Delta t^2,\\
R_\mathrm{l}^2 &= R_z'^2 \cos^2\theta + R_x'^2 \sin^2\theta + \beta_\mathrm{l}^2 \Delta t^2,\\
R_\mathrm{ol}^2 &= 0 + \beta_t \beta_\mathrm{l} \Delta t^2,\\
R_\mathrm{os}^2 &= (R_x^2 - R_y'^2)\cos\phi \sin\phi,\\
R_\mathrm{sl}^2 &= 0.
\end{aligned}
$$

the transverse momentum of the measured pair

Results I

the ODE solutions for the temperatures

$T_{eff0}(t)$
$T_{x0}(t)$
$T_{y0}(t)$
in MeV

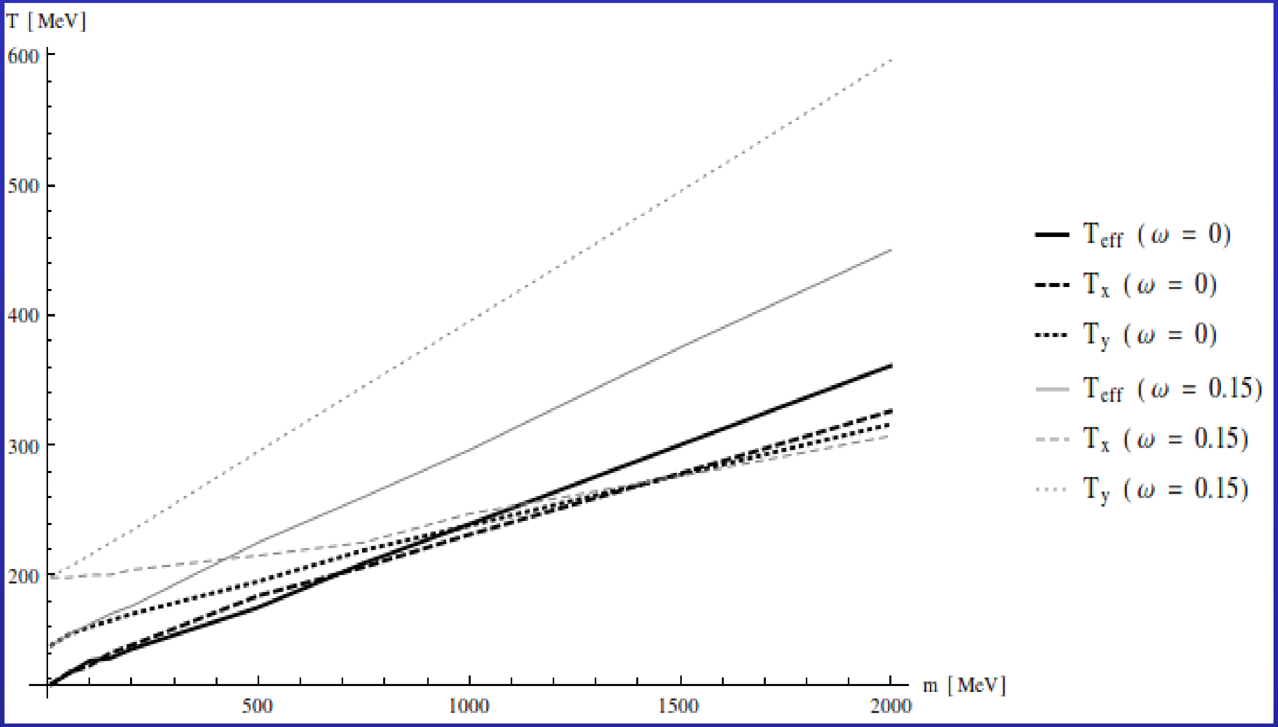


t[fm]

Parameters:
m = 140 MeV
$T_0 = 170$
$R_0 = 2.5$
$R_{0t} = 0.25$
$Y_0 = 4.0$
$\kappa = 1.5$
$\omega_0 = 0.0$ *black*
$\omega_0 = 0.15$ *gray*

Results II

Mass dependence of the temperatures



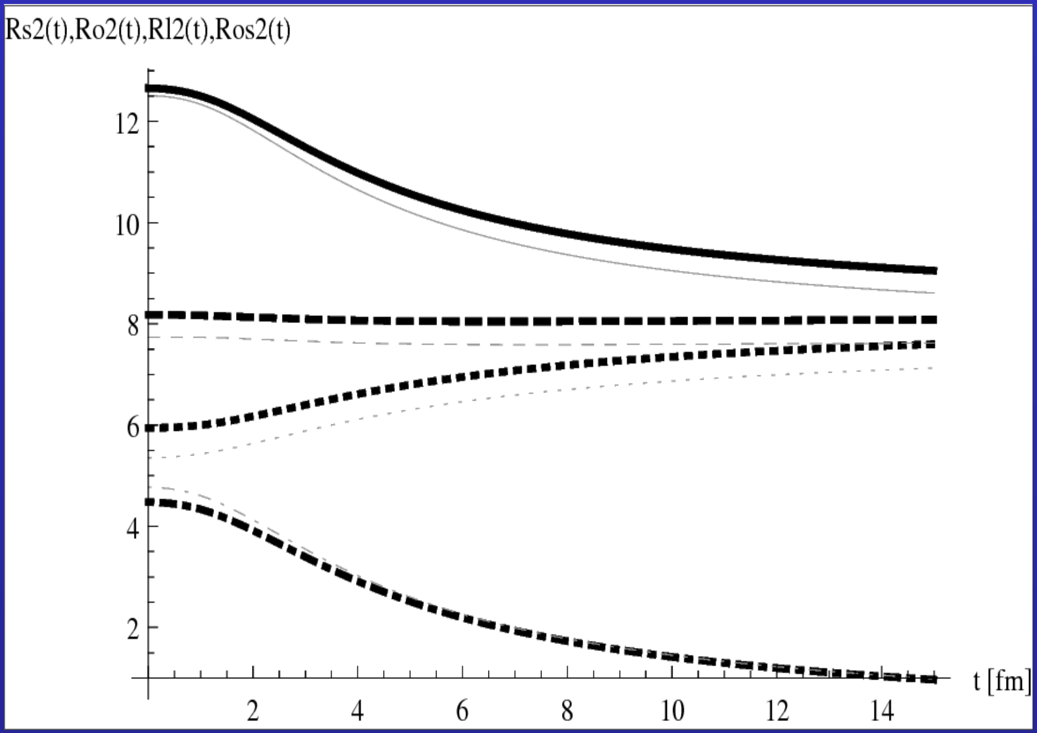
Parameters:
freeze out at
t = 10 fm
T_0 = 170
R_0 = 2.5
R_{0t} =0.25
Y_0 = 4.0
κ = 1.5

Results III

the transverse momentum components

$R_{s2}(t)$
$R_{02}(t)$
$R_{12}(t)$
$R_{os2}(t)$

in MeV/c



Parameters:
m = 140 MeV
T_0 = 170
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R_{0t} =0.25
Y_0 = 4.0
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ω_0 = 0.0 black
ω_0= 0.15 gray

Thank You for Your Attention!

Questions, Remarks, Comments?...