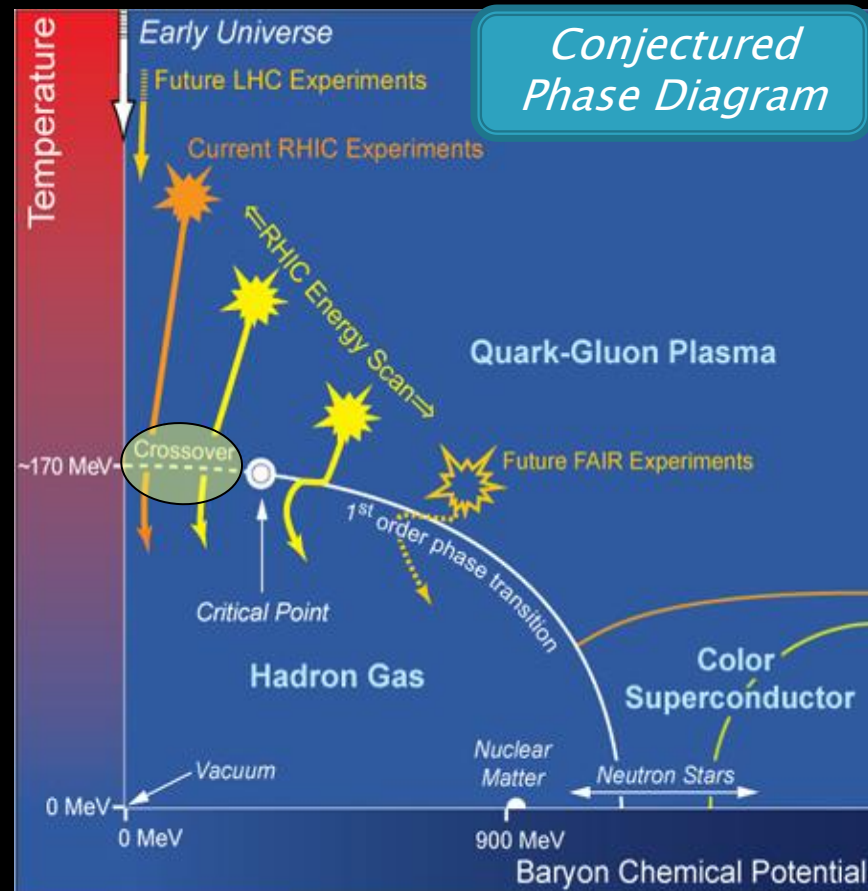


Beam energy dependence of the expansion dynamics in relativistic heavy ion collisions: Indications for the critical end point?

Roy A. Lacey
Stony Brook University

Quantitative study of the QCD phase diagram is a central current focus of our field



A Known known

- *Spectacular achievement: Validation of the crossover transition leading to the QGP*
- ➔ *Necessary requirement for CEP*

Known unknowns

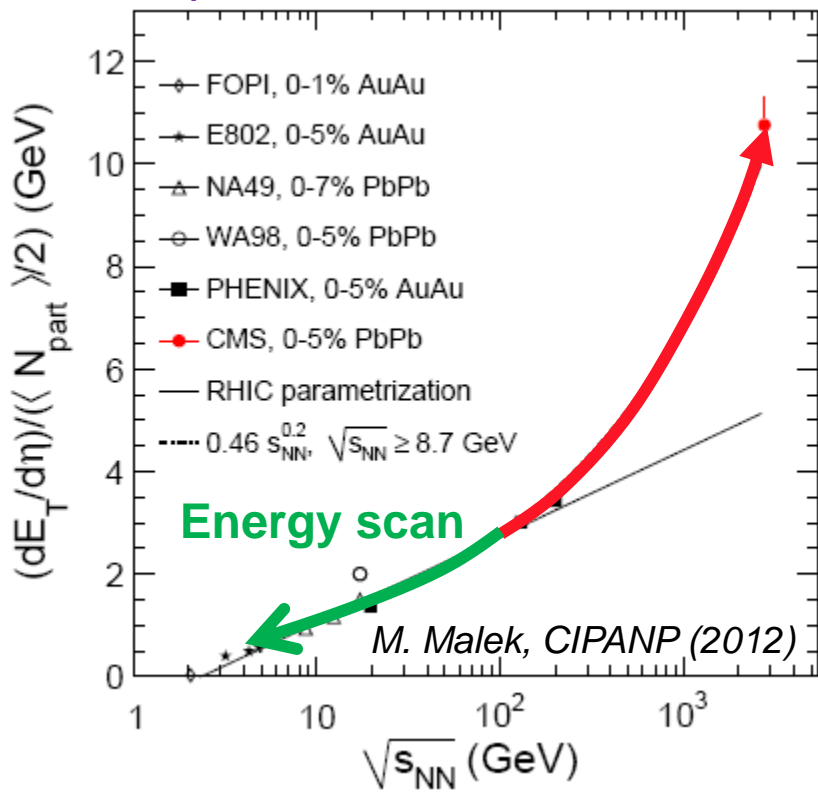
- Location of the critical End point (CEP)?
- Location of phase coexistence regions?
- Detailed properties of each phase?

All are fundamental to the phase diagram of any substance

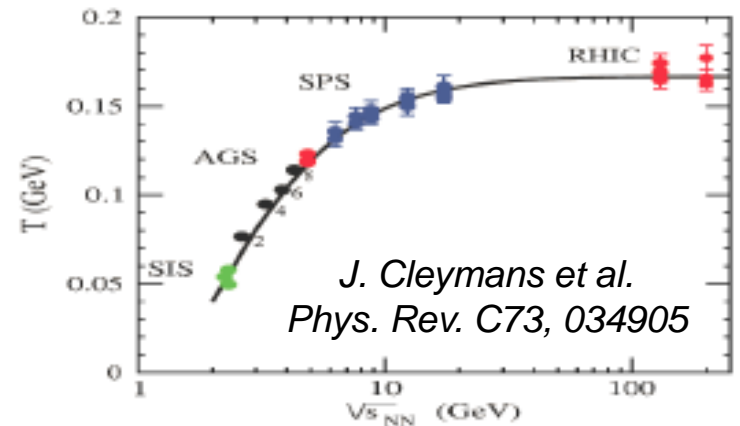
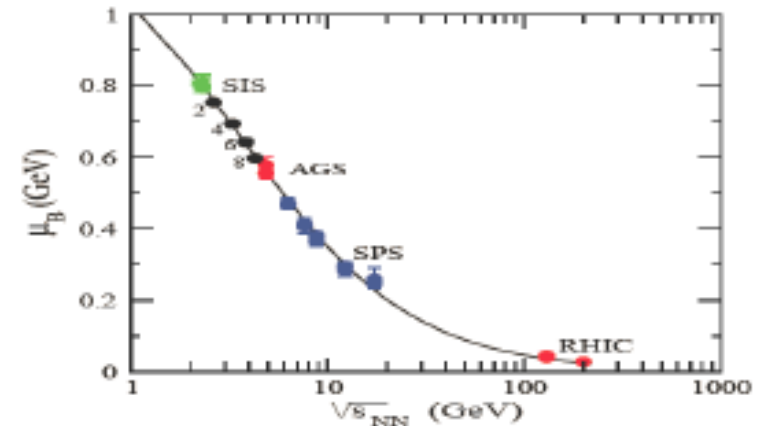
Measurements which span a broad range of the (T, μ_B) -plane are essential for detailed studies of the phase diagram

A Current Strategy

Exploit the RHIC-LHC beam energy lever arm



(μ_B, T) at freeze-out



- LHC → access to high T and small μ_B
 - RHIC → access to different systems and a broad domain of the (μ_B, T) -plane
- RHIC_{BES} to LHC → $\sim 360 \sqrt{s_{NN}}$ increase

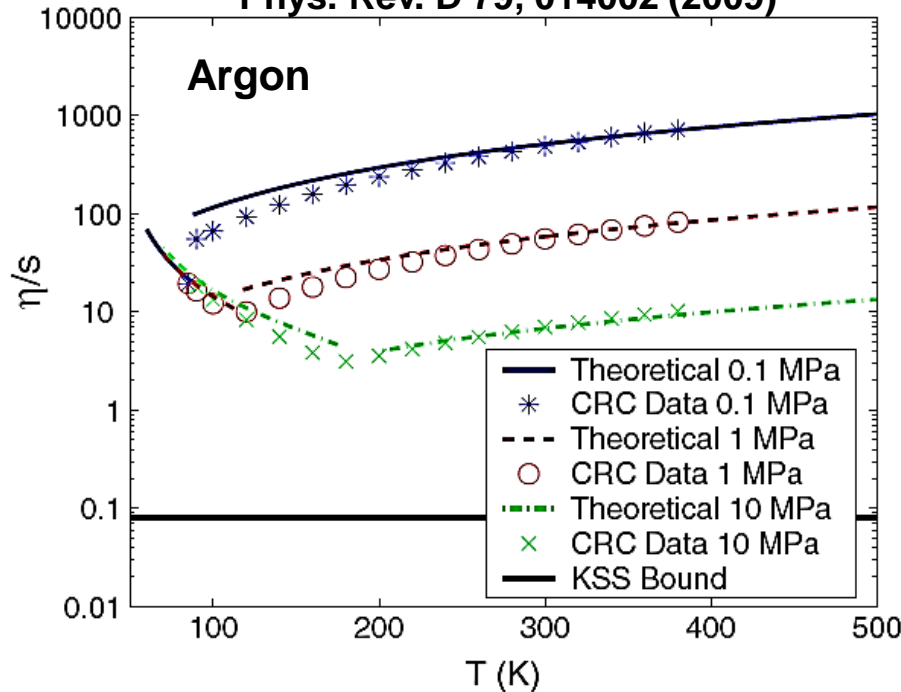
- LHC + BES → access to an even broader domain of the (μ_B, T) -plane

Challenge → leverage signals from full span of energies

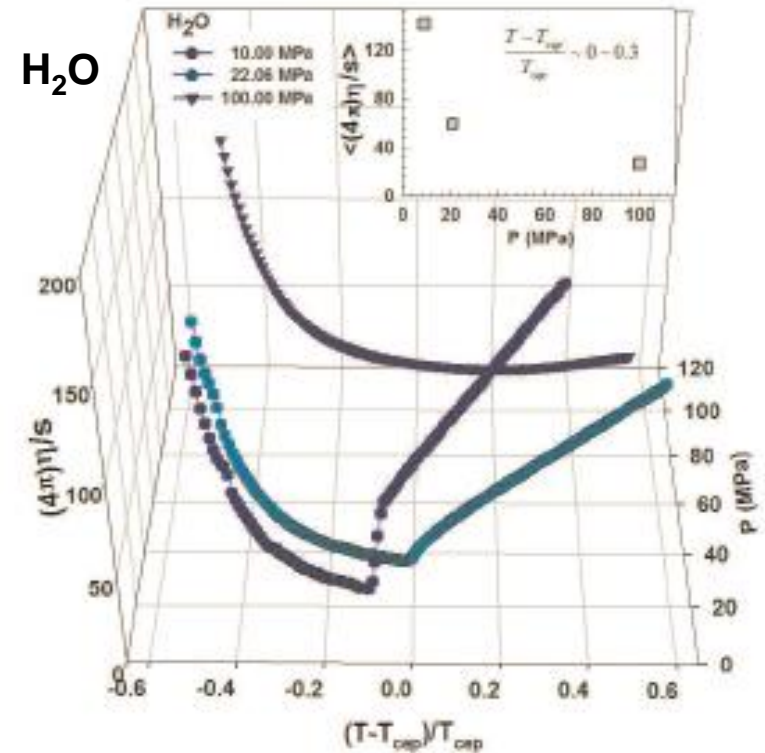
Possible signals

Csernai et. al,
Phys.Rev.Lett. 97 (2006) 152303

A. Dobado et. al,
Phys. Rev. D 79, 014002 (2009)



Lacey et. al,
Phys. Rev.Lett. 98 (2007) 092301
[arXiv:0708.3512](https://arxiv.org/abs/0708.3512) (2008)

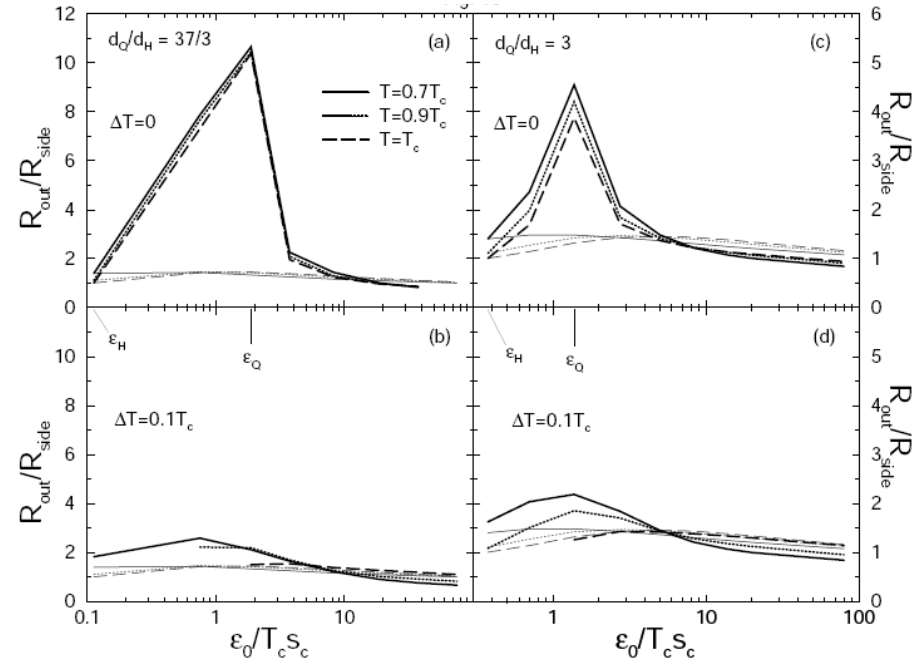
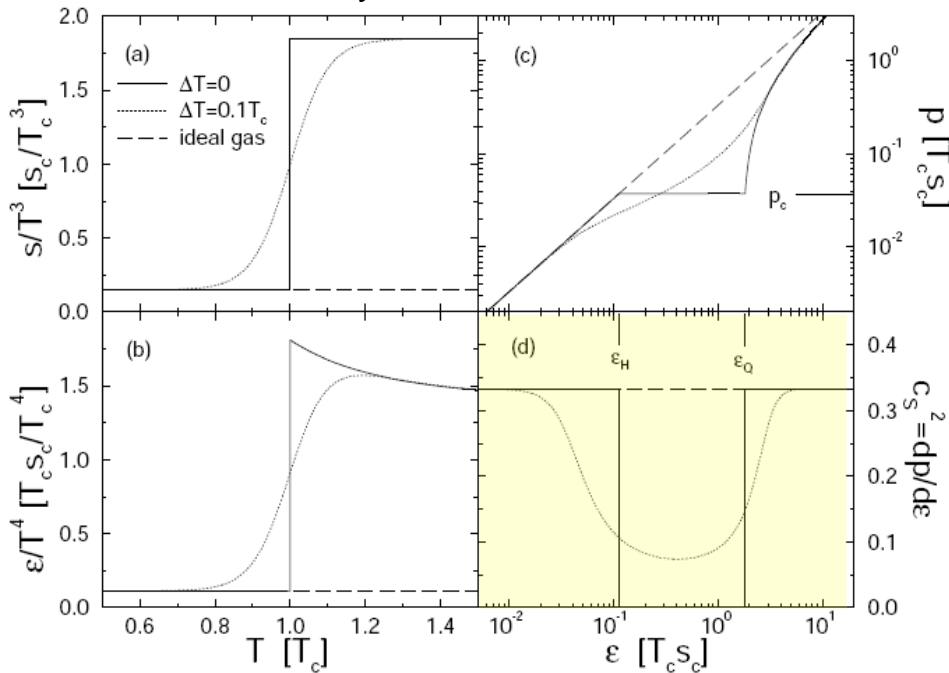


At the CEP or close to it, anomalies in the dynamic properties of the medium can drive abrupt changes in transport coefficients

Anisotropic flow (v_n) measurements are an invaluable probe

Dirk Rischke and Miklos Gyulassy
Nucl.Phys.A608:479-512,1996

Dirk Rischke and Miklos Gyulassy
Nucl.Phys.A608:479-512,1996

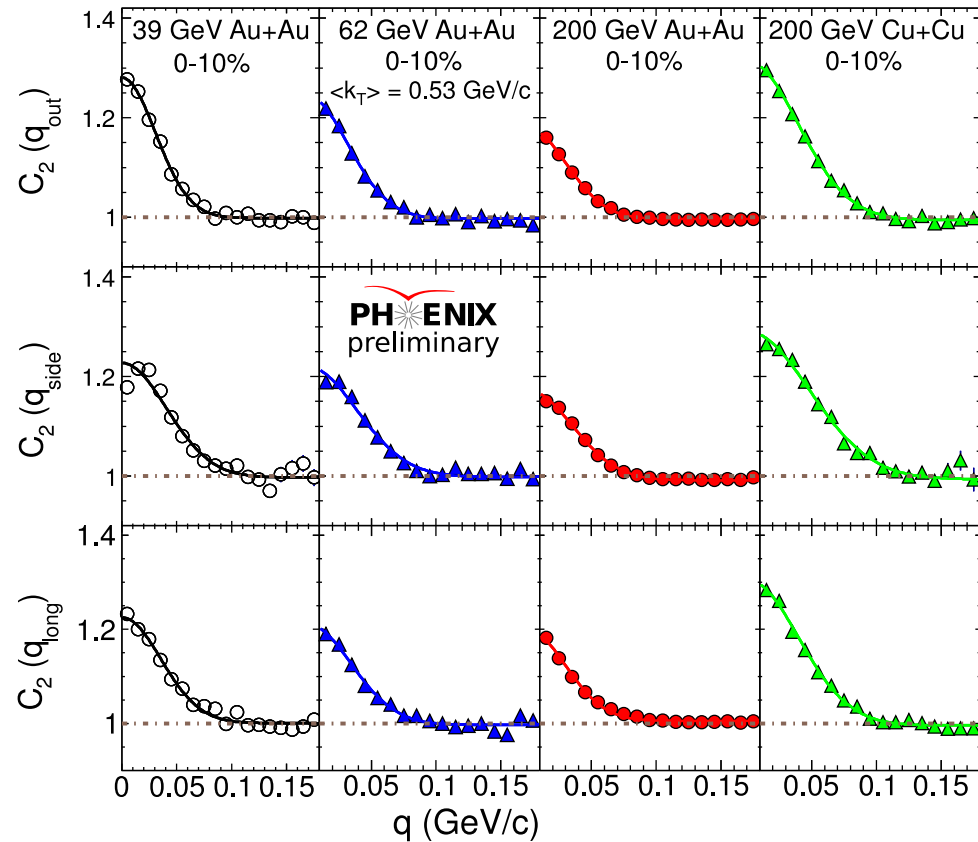
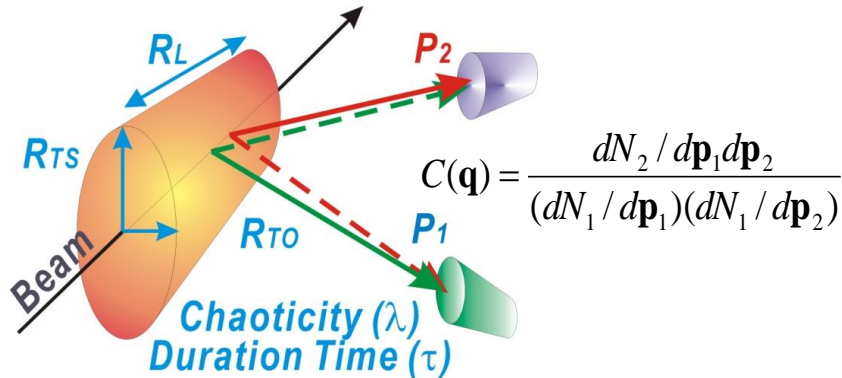


In the vicinity of a phase transition or the CEP, the sound speed is expected to soften considerably.

In the vicinity of a phase transition or the CEP anomalies in the space-time dynamics can enhance the time-like component of emissions.

v_1 and HBT measurements are invaluable probes

Two particle Interferometry Studies



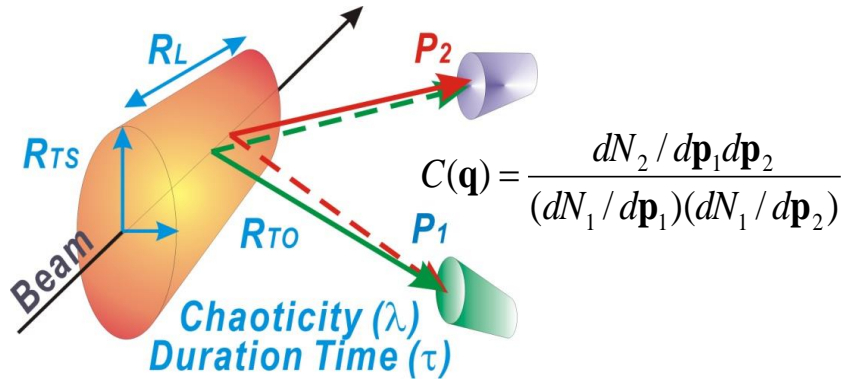
$$C_2(\mathbf{q}) = N[(\lambda(1 + G(\mathbf{q})))F_c + (1 - \lambda)]$$

$$G(\mathbf{q}) \cong \exp(-R_{side}^2 q_{side}^2 - R_{out}^2 q_{out}^2 - R_{long}^2 q_{long}^2)$$

Fits to the correlation functions

→ HBT radii (R_{out} , R_{side} , R_{long}) as a function of centrality, m_T , etc

Two particle Interferometry Studies

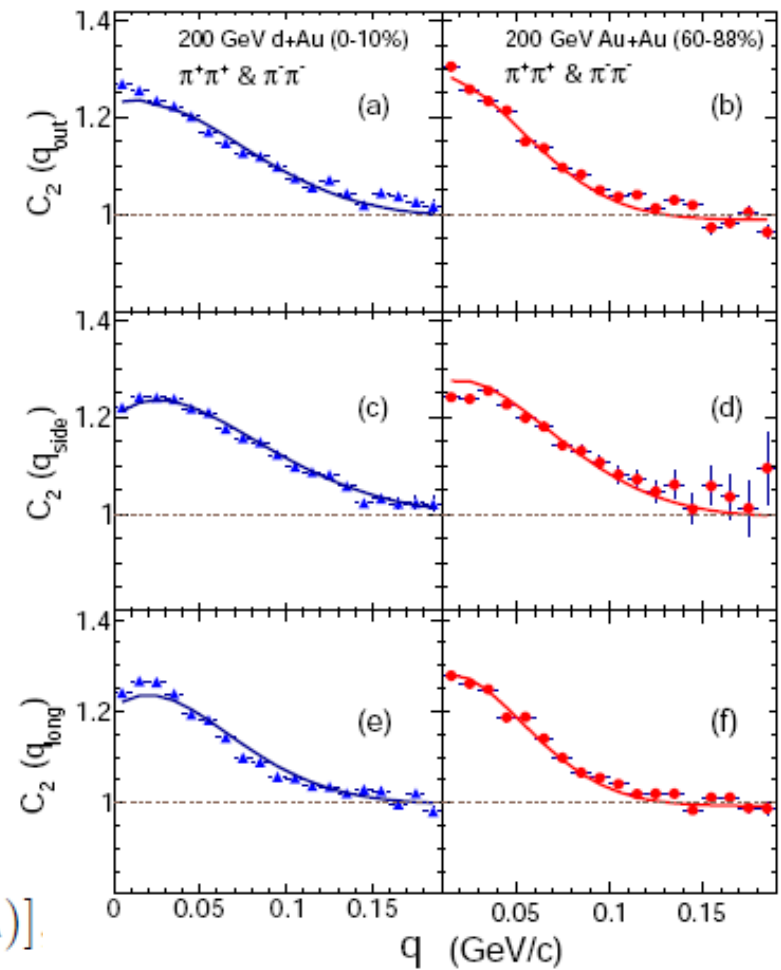


$$C_2(\mathbf{q}) = N[(\lambda(1 + G(\mathbf{q})))F_c + (1 - \lambda)]$$

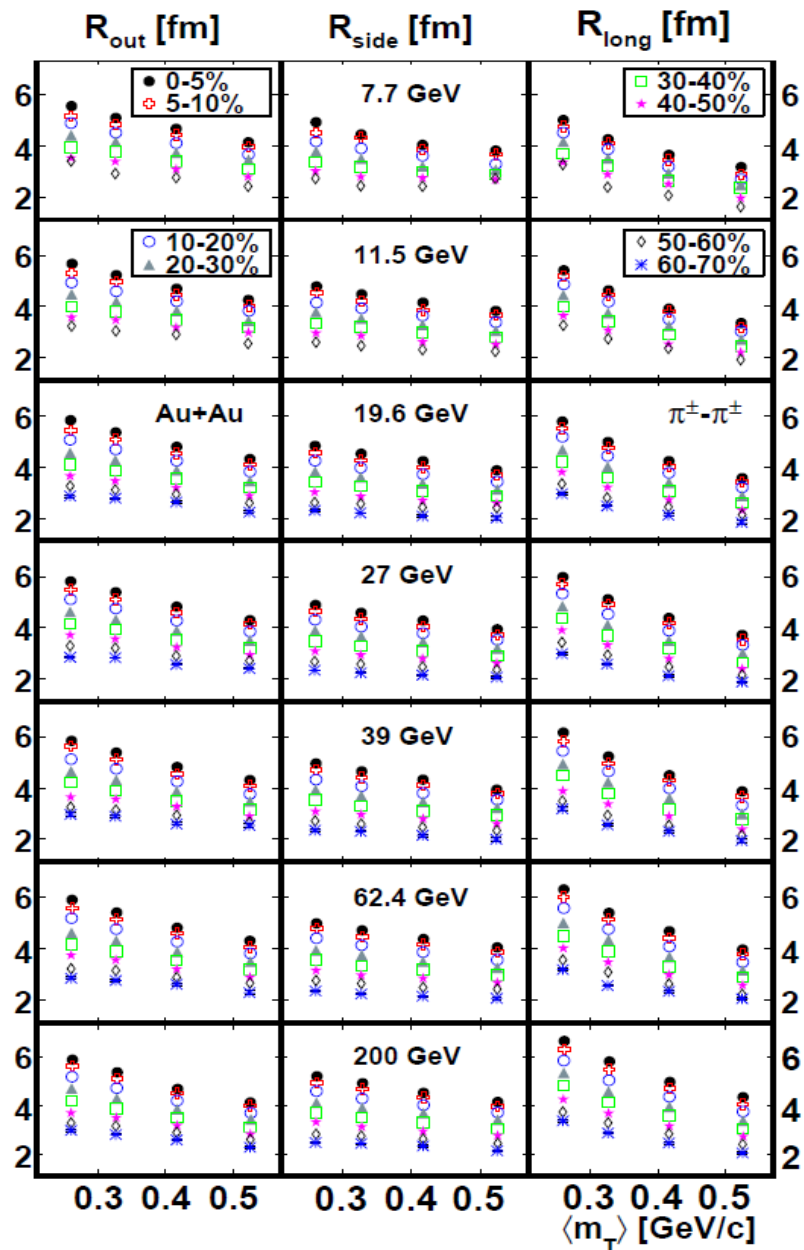
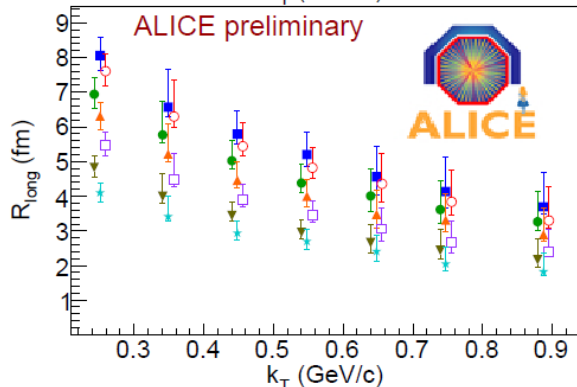
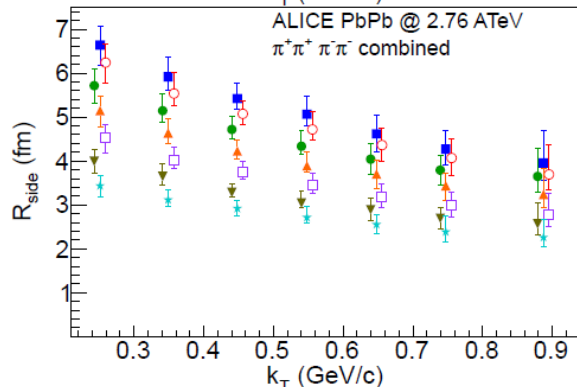
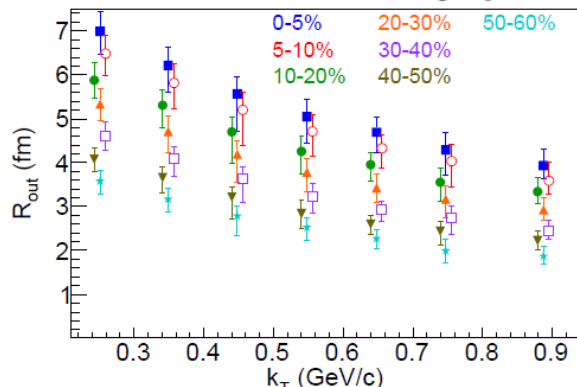
$$G(\mathbf{q}) \cong \exp(-R_{\text{side}}^2 q_{\text{side}}^2 - R_{\text{out}}^2 q_{\text{out}}^2 - R_{\text{long}}^2 q_{\text{long}}^2)$$

Fits to the correlation functions

→ HBT radii (R_{out} , R_{side} , R_{long}) as a function of centrality, m_T , etc

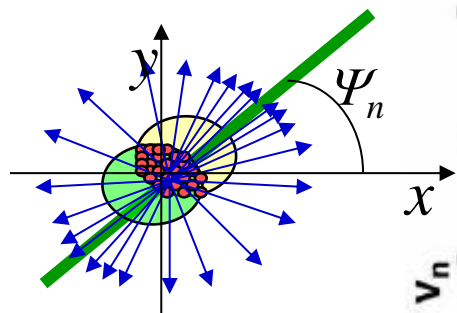


ALICE -PoSWPCF2011

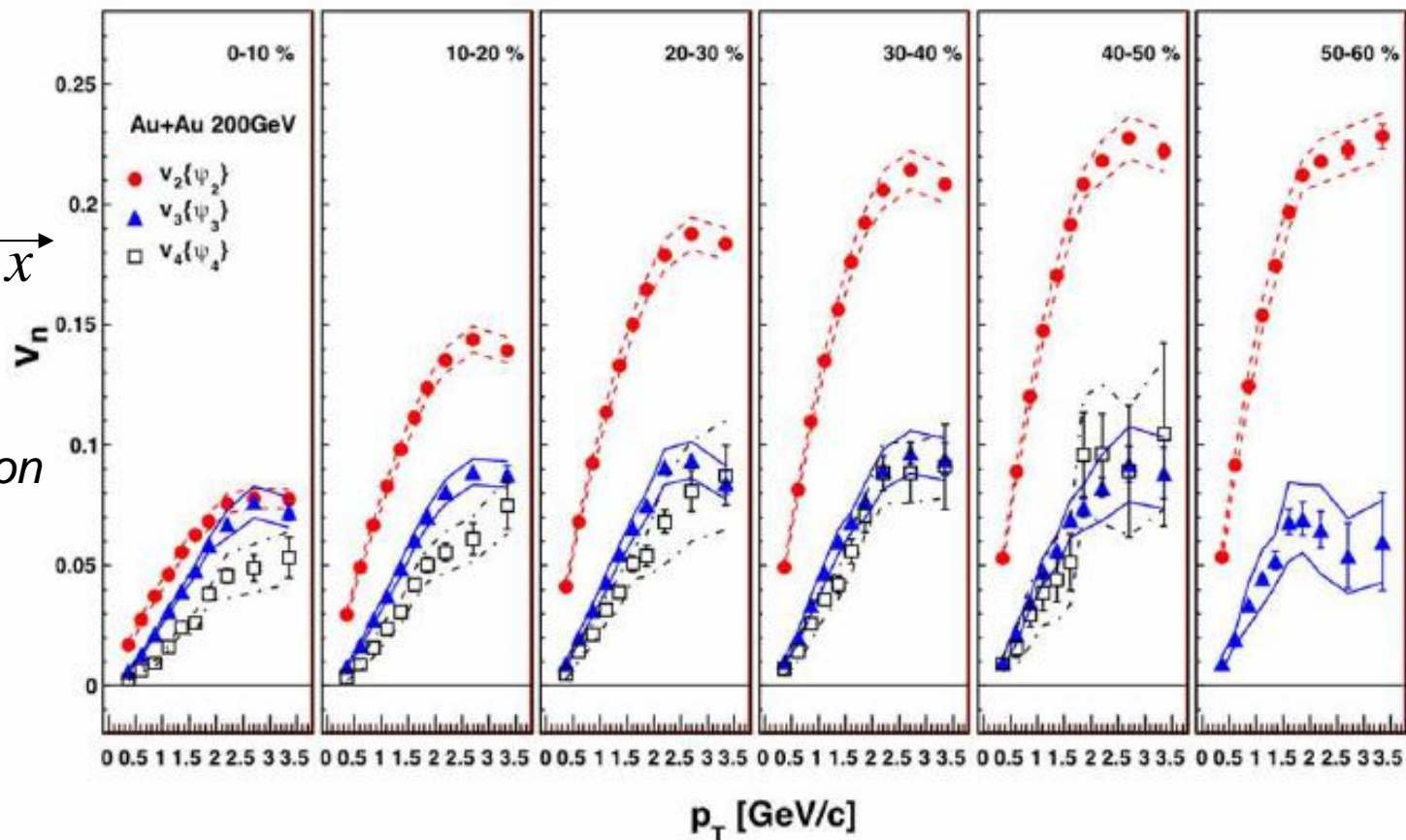


Exquisite data set for combined RHIC-LHC results?

V_n measurements



Measure distribution
relative to Ψ_n

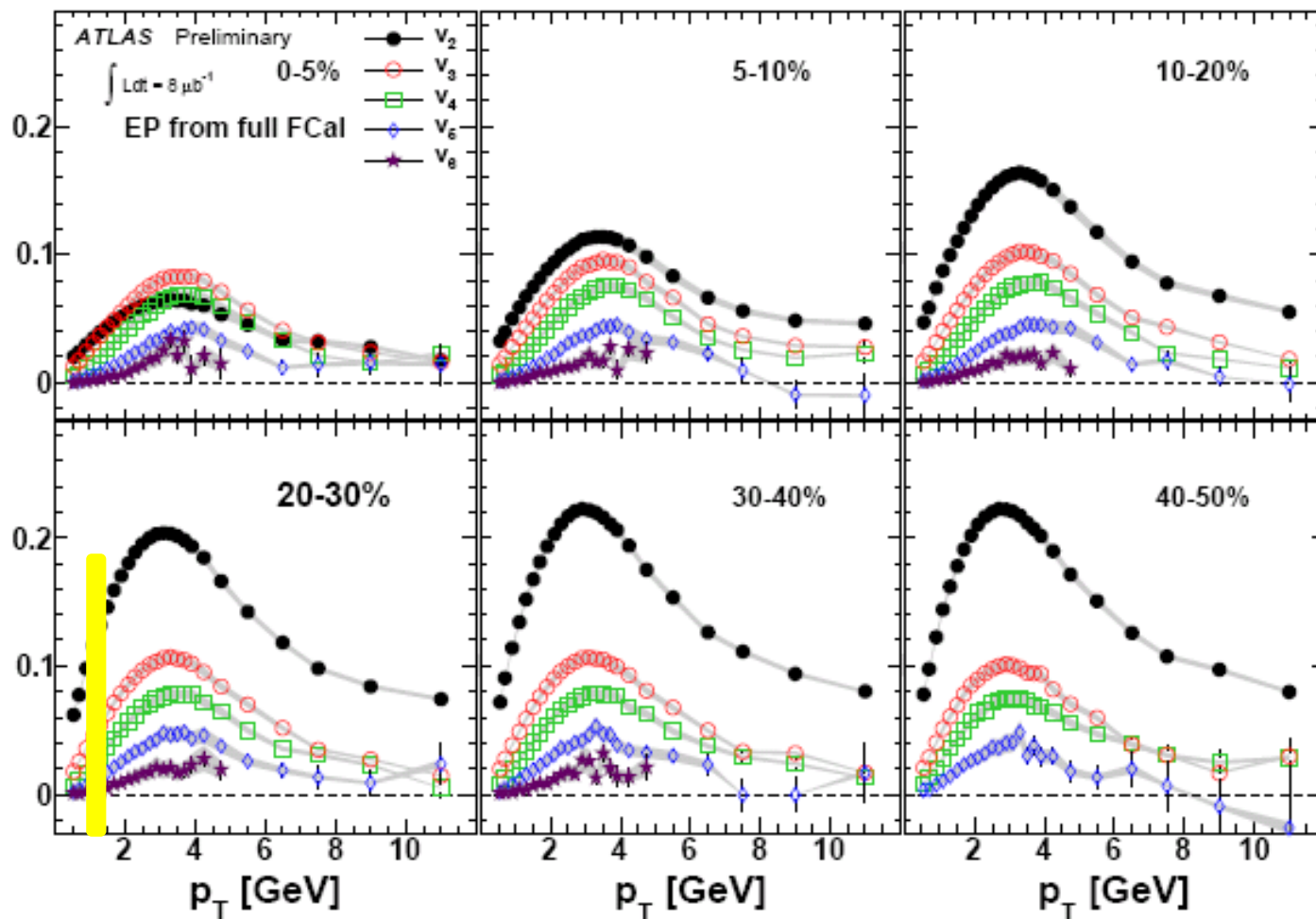


$$v_n = \langle \cos(2n[\varphi - \Psi_n]) \rangle$$

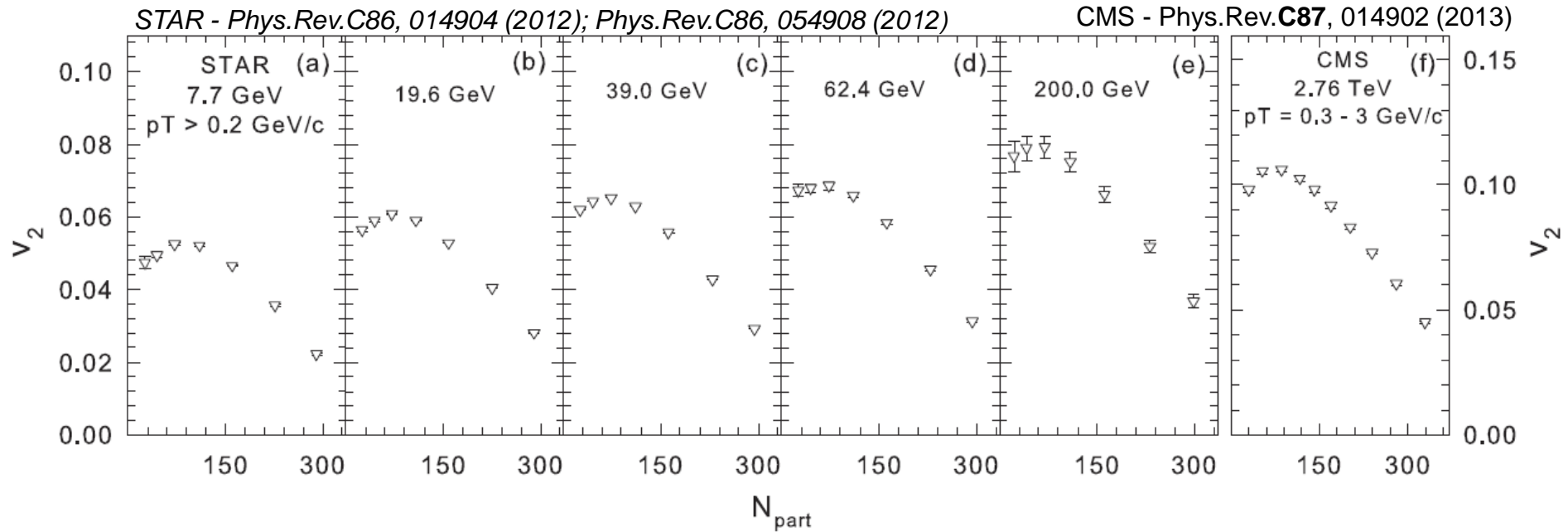
Extensive set of v_n measurements at RHIC and the LHC

$v_n(\psi_n)$ Measurements - ATLAS

ATLAS-CONF-2011-074



High precision double differential Measurements are pervasive



- **Extensive set of measurements now span a broad range of beam energies (T, μ_B).**

Essential Questions

I. *Can the wealth of data be understood in a consistent framework?*

YES!

II. *If it can, what new insight/s are we afforded?*

➤ *Do we see indications for the phase transition / CEP?*

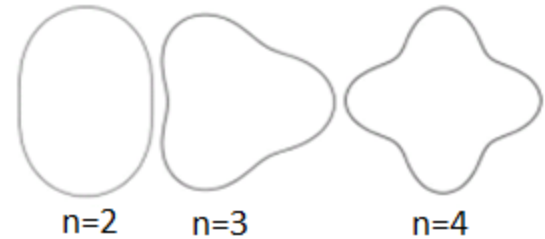
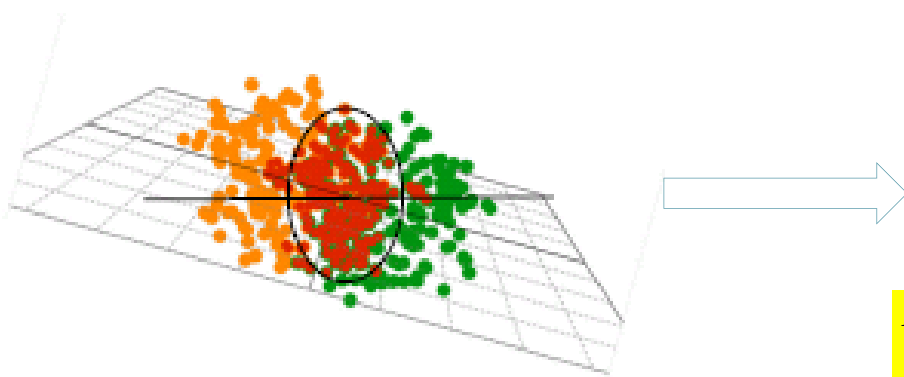
I. *Expansion dynamics is pressure driven and is therefore acoustic!*

➤ *This acoustic property leads to several testable scaling predictions for anisotropic flow and HBT – with implications*

This constitutes an important development

Scaling properties of expansion dynamics

Initial Geometry characterized by many shape harmonics (ϵ_n) \rightarrow drive v_n



$$v_n \propto \epsilon_n$$

HBT scaling expectations:

$$t \propto \bar{R}$$

$$R_{out}, R_{side}, R_{long} \propto \bar{R}$$

Acoustic viscous modulation of v_n

$$\delta T_{\mu\nu}(t, k) = \exp\left(\frac{2\eta}{3s} k^2 \frac{t}{T}\right) \delta T_{\mu\nu}(0)$$

Staig & Shuryak arXiv:1008.3139

$$k = n / \bar{R}$$

$$\delta T_{\mu\nu}(n, t) = \exp(-\beta n^2) \delta T_{\mu\nu}(0), \quad \beta = \frac{2\eta}{3s} \frac{1}{\bar{R}^2} \frac{t}{T}$$

V_n scaling expectations:

n^2 dependence

$$\frac{v_n(p_T)}{\epsilon_n} \propto \exp(-\beta' n^2)$$

v_n is related to v_2

$$\frac{v_n(p_T)}{v_2(p_T)} = \frac{\epsilon_n}{\epsilon_2} \cdot \exp(-\beta'(n^2 - 4))$$

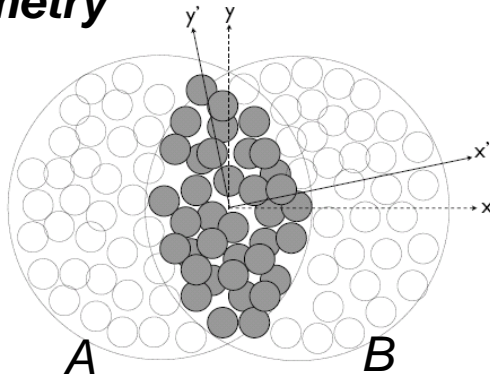
System size dependence

$$\ln\left(\frac{v_n}{\epsilon_n}\right) \propto \frac{-\beta''}{\bar{R}}$$

Each of these scaling expectations can be validated

$$\eta/s \propto \beta', \beta''$$

Geometry



$$\Psi_n^* = \frac{1}{n} \tan^{-1} \left(\frac{S_{ny}}{S_{nx}} \right)$$

$$\varepsilon_n = \langle \cos n(\phi - \psi_n^*) \rangle$$

$$\frac{1}{\bar{R}} = \sqrt{\left(\frac{1}{\sigma_x^2} + \frac{1}{\sigma_y^2} \right)}$$

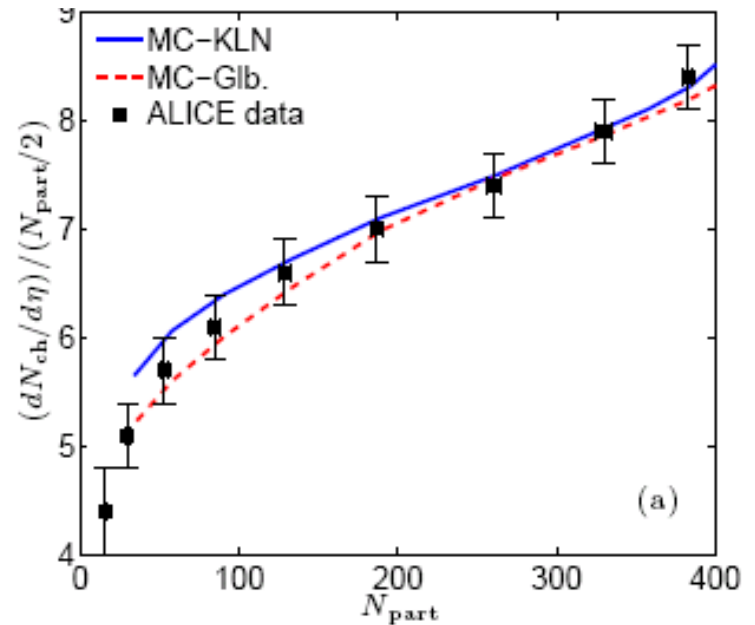
σ_x & $\sigma_y \rightarrow$ RMS widths of density distribution

arXiv:1203.3605

Phys. Rev. C 81, 061901(R) (2010)

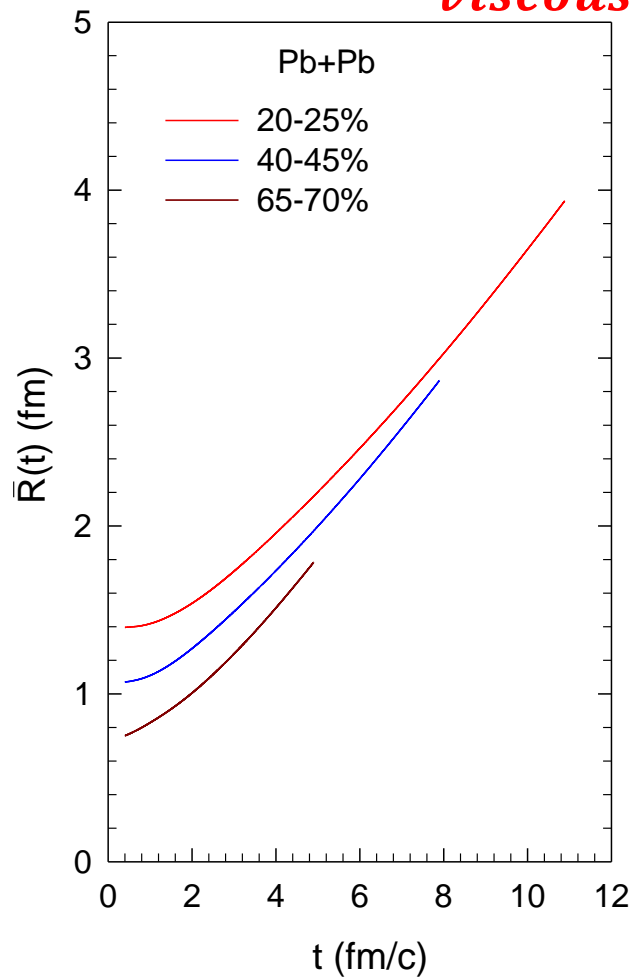
$$S_{nx} \equiv S_n \cos(n\Psi_n^*) = \int d\mathbf{r}_\perp \rho_s(\mathbf{r}_\perp) \omega(\mathbf{r}_\perp) \cos(n\phi)$$

$$S_{ny} \equiv S_n \sin(n\Psi_n^*) = \int d\mathbf{r}_\perp \rho_s(\mathbf{r}_\perp) \omega(\mathbf{r}_\perp) \sin(n\phi),$$



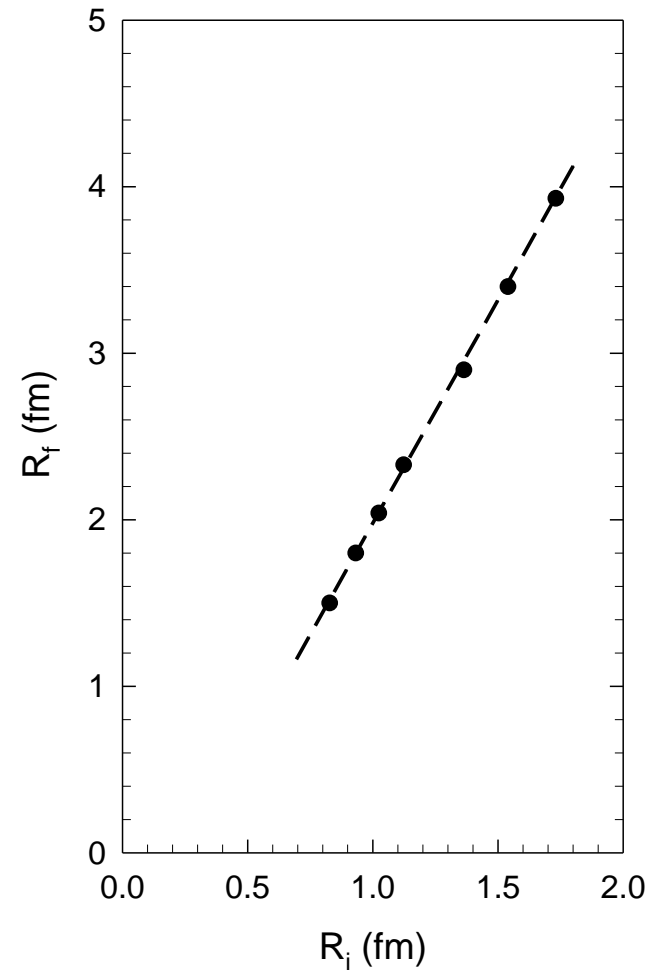
- **Geometric fluctuations included**
- **Geometric quantities constrained by multiplicity density.**

✓ **Characteristic acoustic scaling validated for viscous hydrodynamics**



Freeze-out time varies with initial size

$$t \propto \bar{R}$$

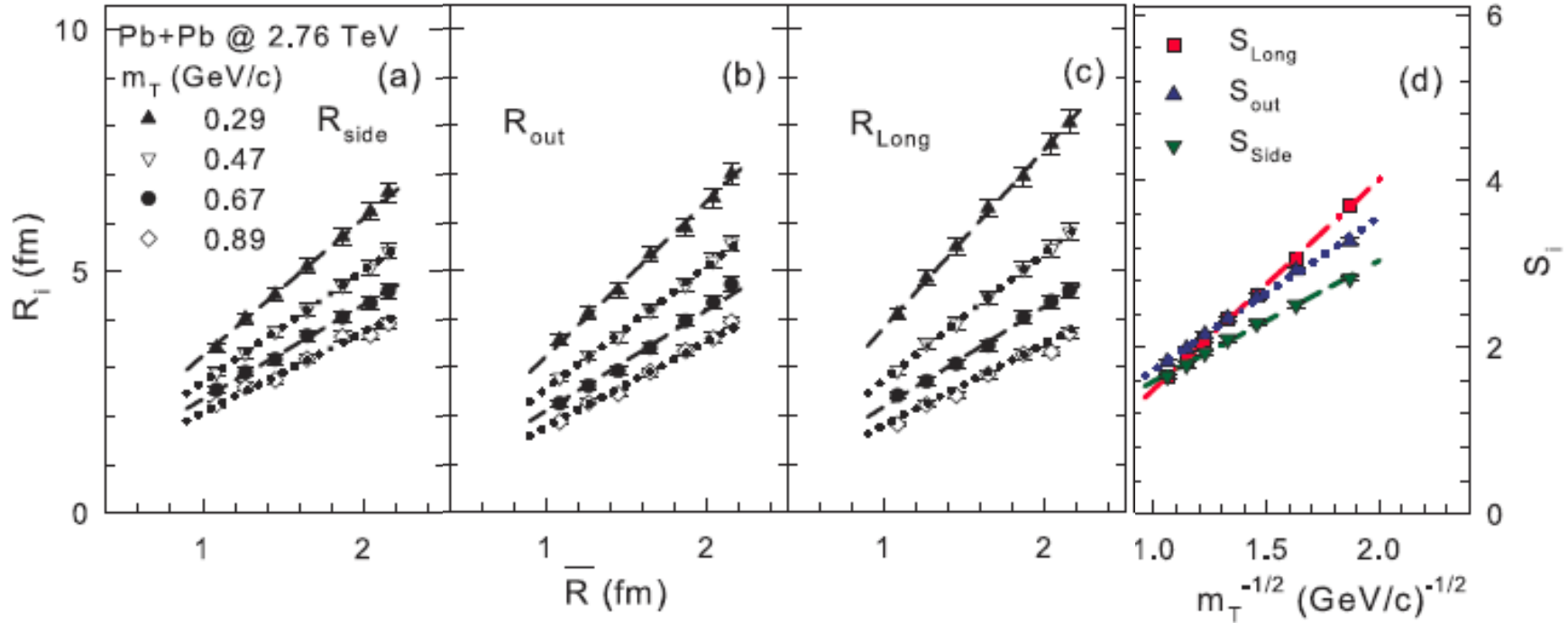
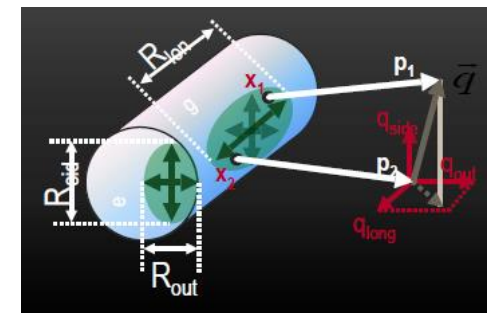


Freeze-out transverse size proportional to initial transverse size

Acoustic Scaling of HBT Radii

$$t \propto \bar{R}$$

$$R_{out}, R_{side}, R_{long} \propto \bar{R}$$



- \bar{R} and m_T scaling of the full LHC data set
- The centrality and m_T dependent data scale to a single curve for each radii.

m_T Scaling of HBT Radii

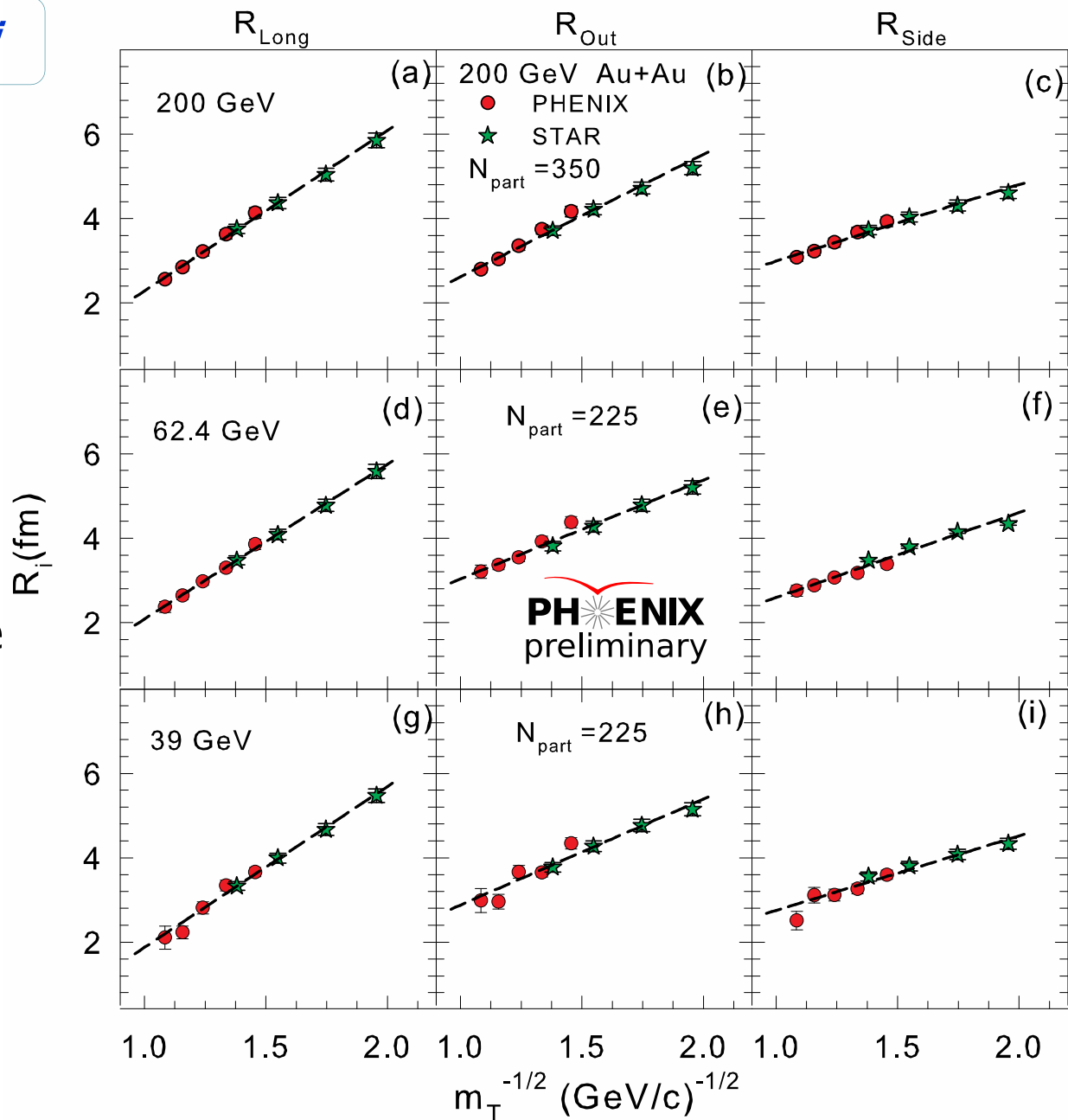
▶ PHENIX and STAR consistent

[arxiv:1403.4972](https://arxiv.org/abs/1403.4972)

▶ all radii linear

◦ $R_i = a + b/\sqrt{m_T}$

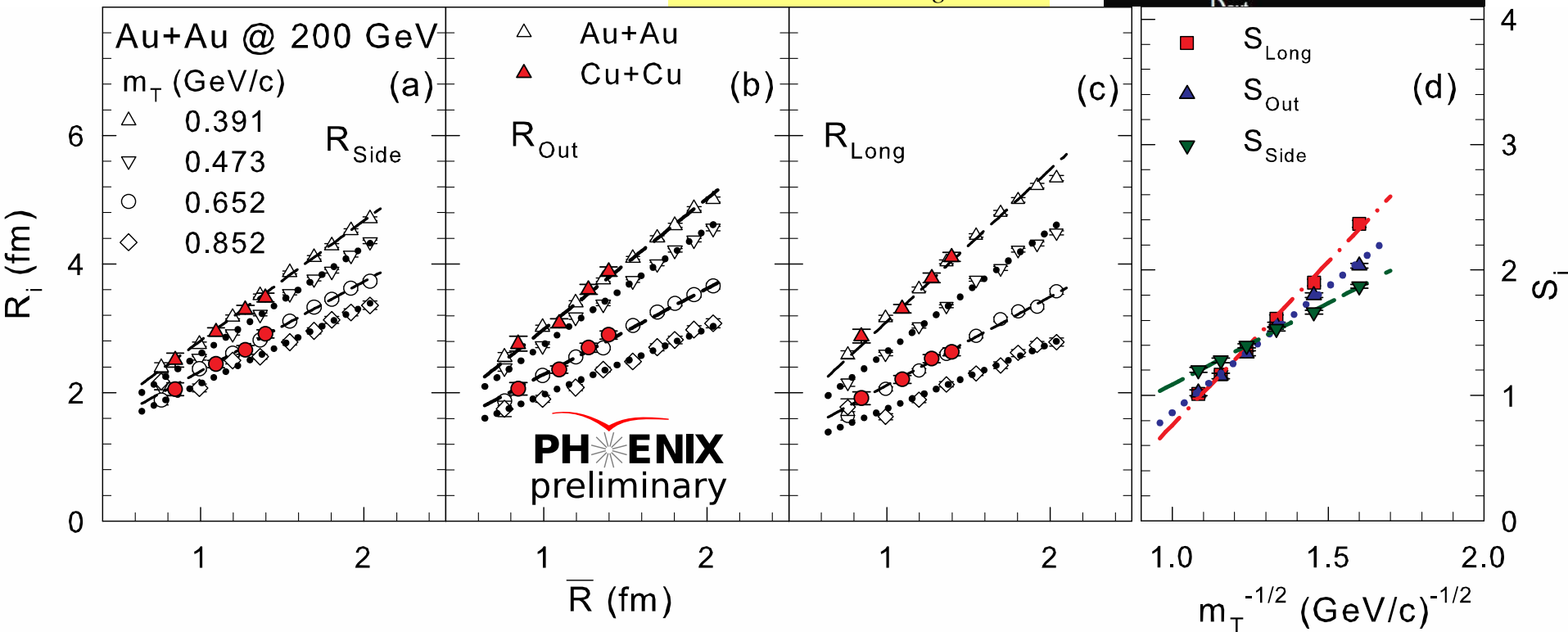
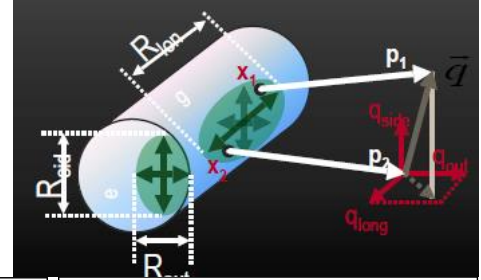
▶ Useful to interpolate to common m_T



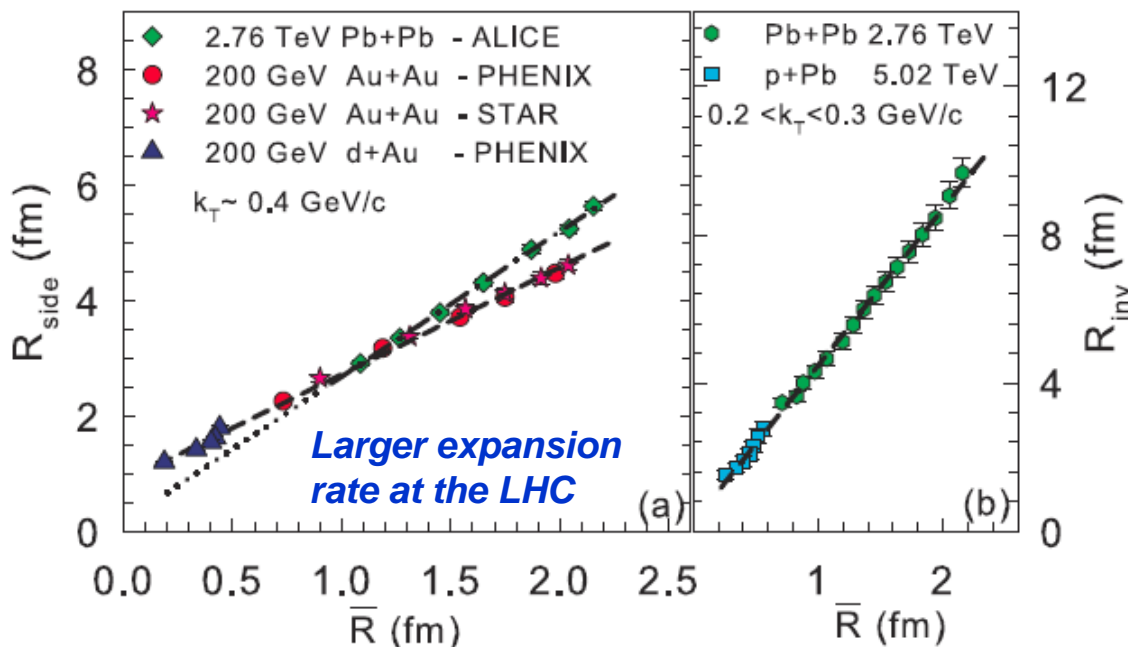
Acoustic Scaling of HBT Radii

$$t \propto \bar{R}$$

$$R_{out}, R_{side}, R_{long} \propto \bar{R}$$



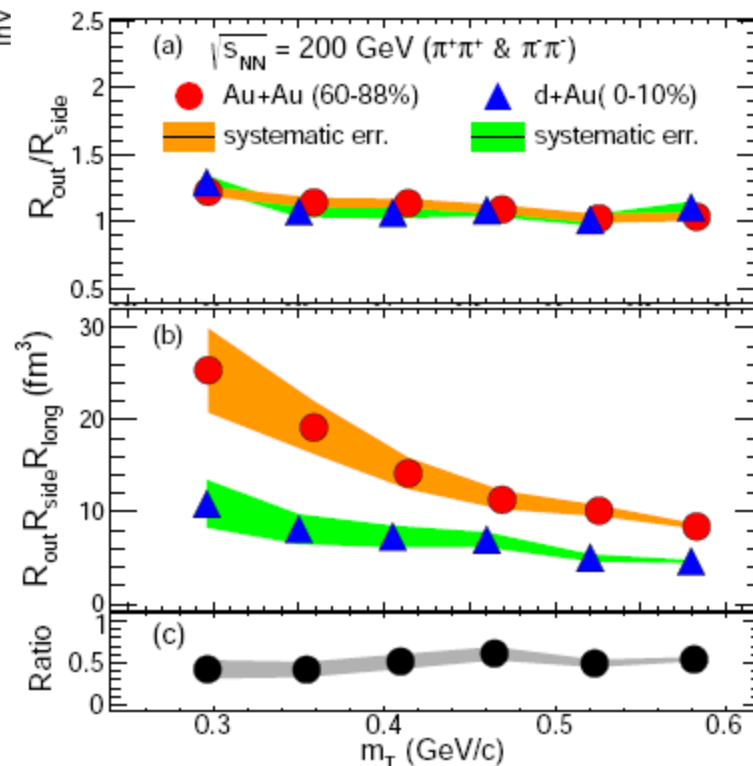
- \bar{R} and m_T scaling of the full RHIC and LHC data sets
- The centrality and m_T dependent data scale to a single curve for each radii.
- Qualitatively similar expansion dynamics at RHIC & LHC



$$t \propto R$$

Exquisitely demonstrated for asymmetric systems \rightarrow similar reaction dynamics

Similar m_T dependence For Au+Au and d+Au



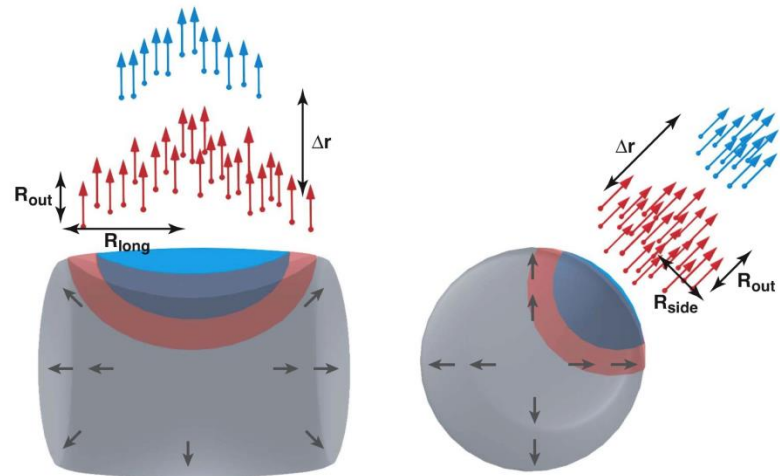
Expansion dynamics similar for Pb+Pb, Au+Au, p+Pb and d+Au
Final-State interactions dominate

Chapman, Scotto, Heinz, PRL.74.4400 (95)

$$R_{side}^2 = \frac{R_{geo}^2}{1 + \frac{m_T}{T} b_T^2}$$

$$R_{out}^2 = \frac{R_{geo}^2}{1 + \frac{m_T}{T} b_T^2} + \underbrace{b_T^2 (Dt)^2}_{\text{red arrow}}$$

$$R_{long}^2 \approx \frac{T}{m_T} t^2$$



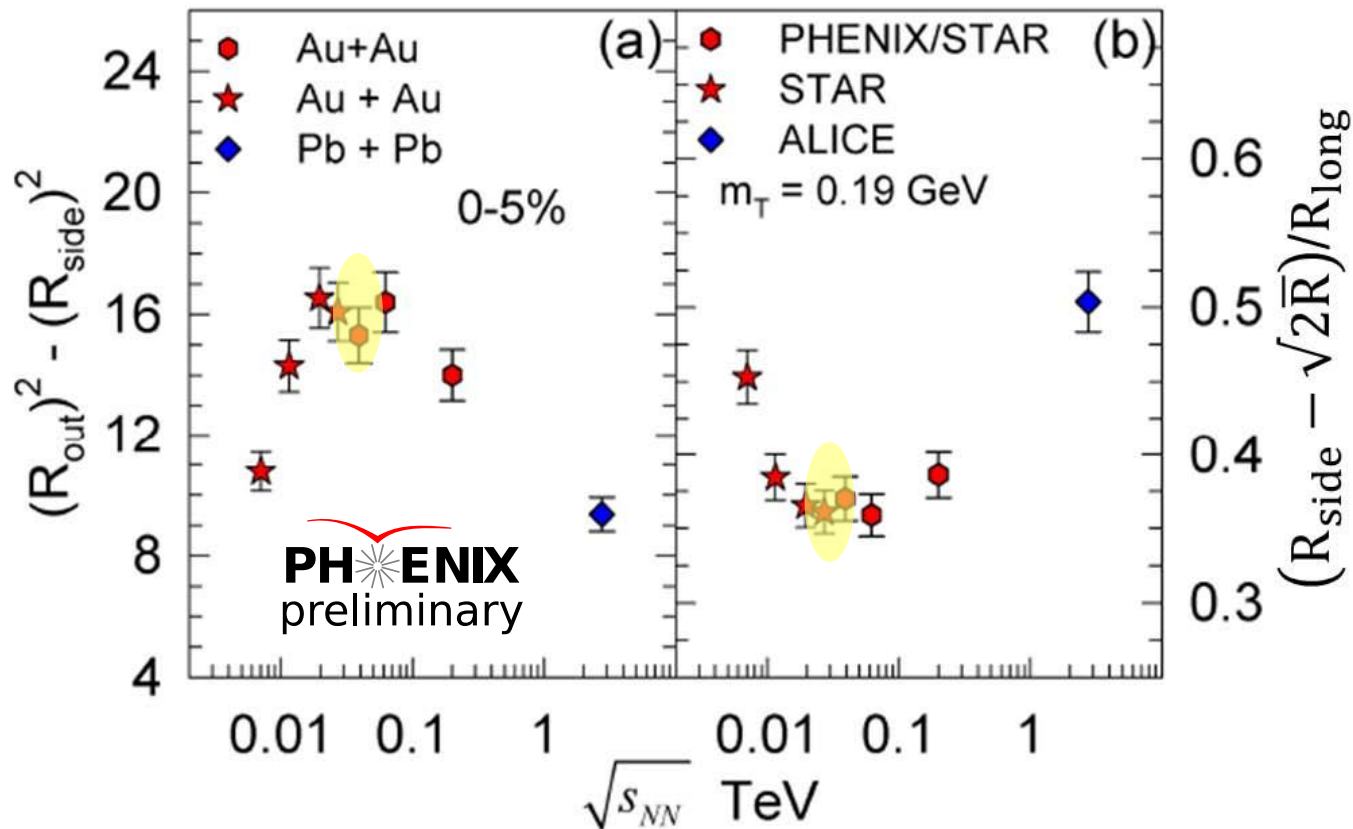
$$R_i = a + \frac{b}{\sqrt{m_T}} \quad \text{Empirical Observation}$$

Makhlin, Sinyukov, ZPC.39.69 (88)

$(R_{out}^2 - R_{side}^2)$ sensitive to emission duration

Anticipate extended emission duration with phase transition/CEP

$\sqrt{s_{NN}}$ dependence of HBT signals



$$R_{long} \propto \tau$$

$$(R_{out}^2 - R_{side}^2) \propto \Delta\tau$$

$$(R - R_i) / R_{long} \propto u$$

$$R_i = \sqrt{2\bar{R}}$$

These non-monotonic patterns signal an important change in the reaction dynamics; CEP? Phase transition?

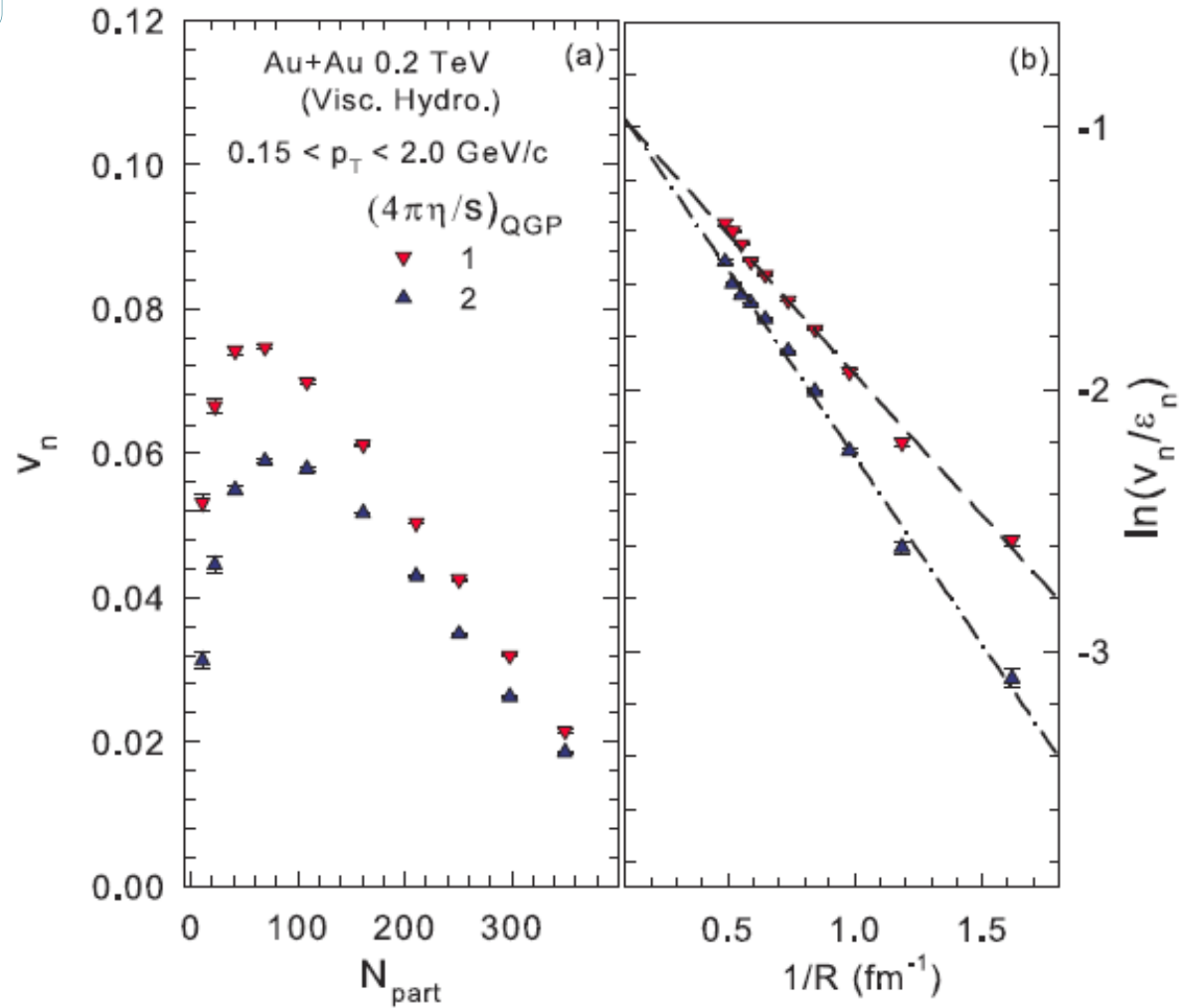
A similar non-monotonic pattern for η/s signals the CEP

Scaling properties of flow

- Viscous Hydrodynamics

$$\ln\left(\frac{v_n}{\varepsilon_n}\right) \propto \frac{-\beta''}{\bar{R}}$$

✓ Characteristic acoustic scaling validated for viscous hydrodynamics

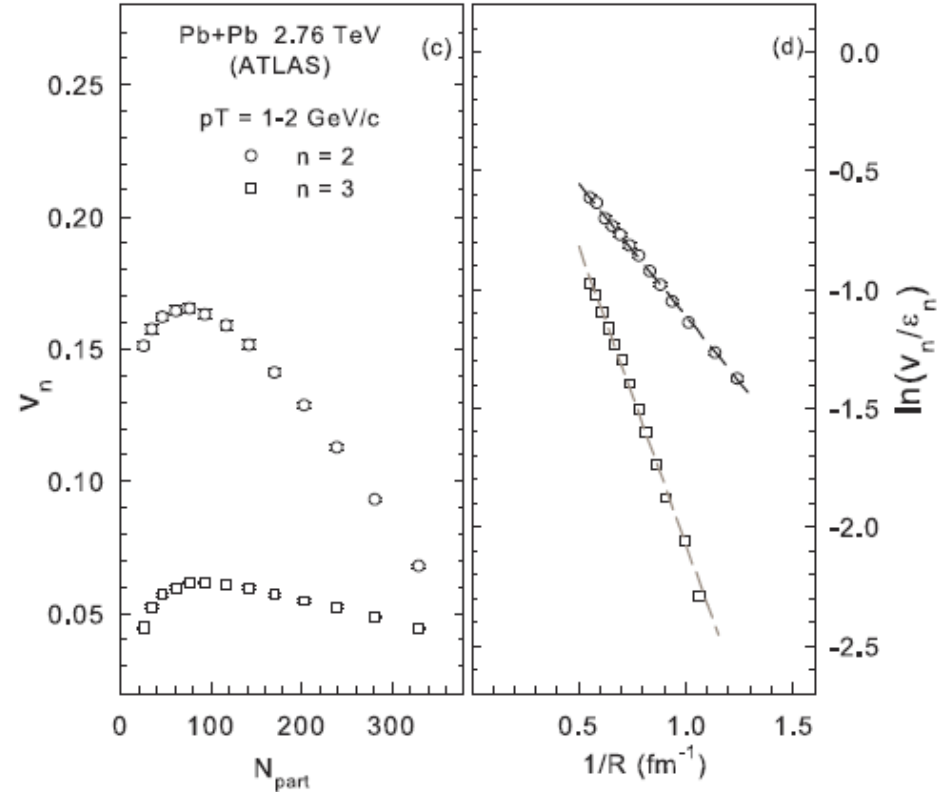
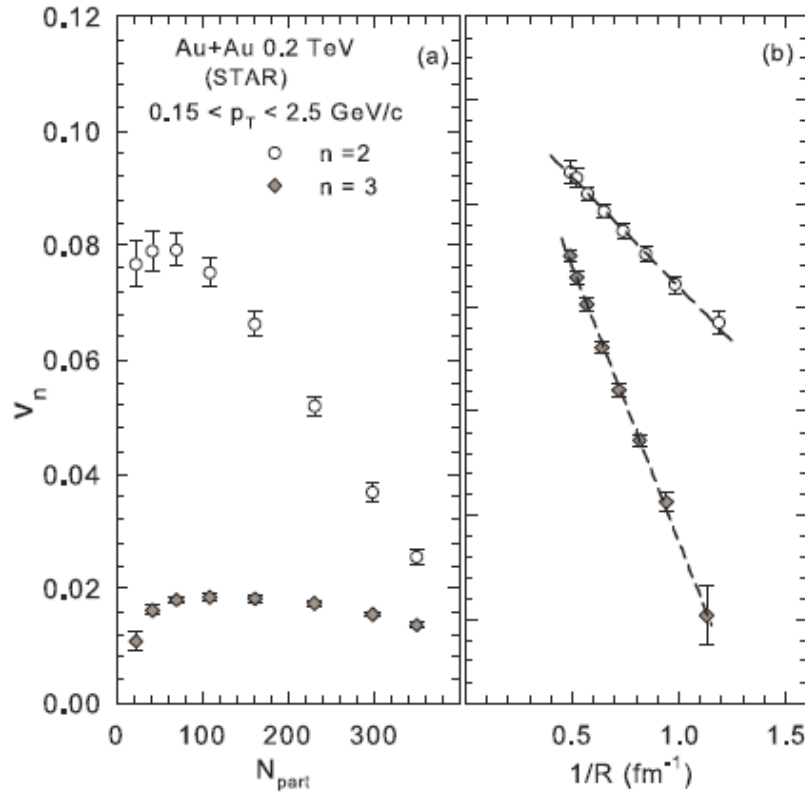


✓ β'' shows clear sensitivity to η/s

✓ Viscous hydrodynamics can be used for calibration

$$\ln\left(\frac{v_n}{\varepsilon_n}\right) \propto \frac{-\beta''}{\bar{R}}$$

Acoustic Scaling – $\frac{1}{\bar{R}}$



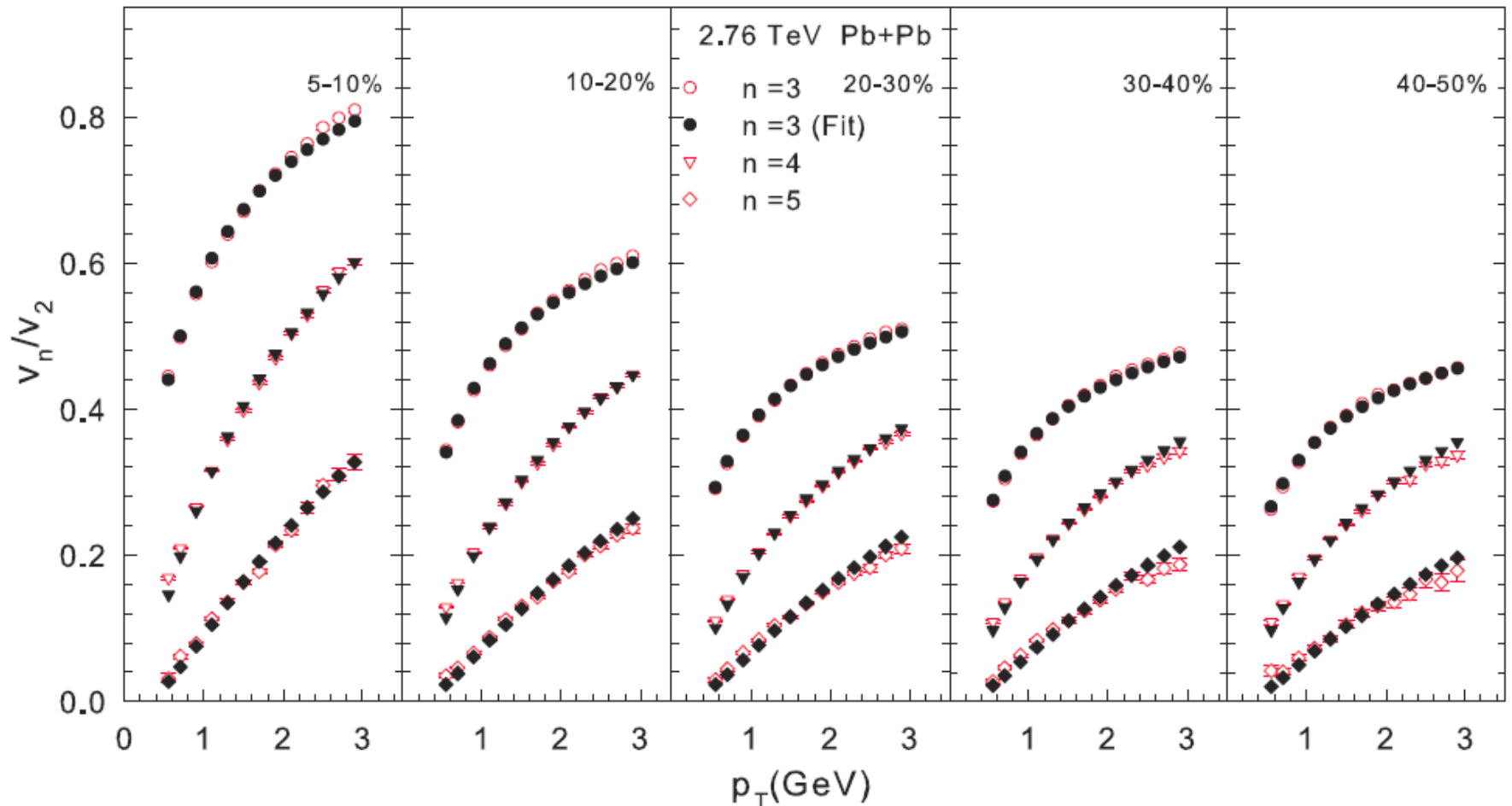
- ✓ **Characteristic $1/\bar{R}$ viscous damping validated with n^2 dependence at RHIC & the LHC**
- ✓ **A further constraint for η/s**

Acoustic Scaling - Ratios

arXiv:1301.0165

$$v_n(p_T) \propto (v_2)^{n/2}$$

$$\frac{v_n(p_T)}{v_2(p_T)} = \frac{\varepsilon_n}{\varepsilon_2} \exp\left(-\beta'(n^2 - 4)\right)$$



- ✓ Characteristic $(n^2 - 4)$ viscous damping validated
- ✓ Constraint for η/s

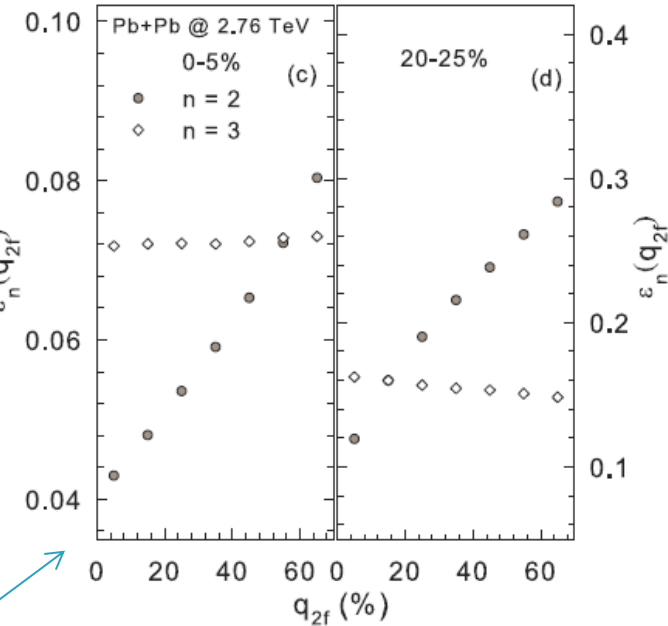
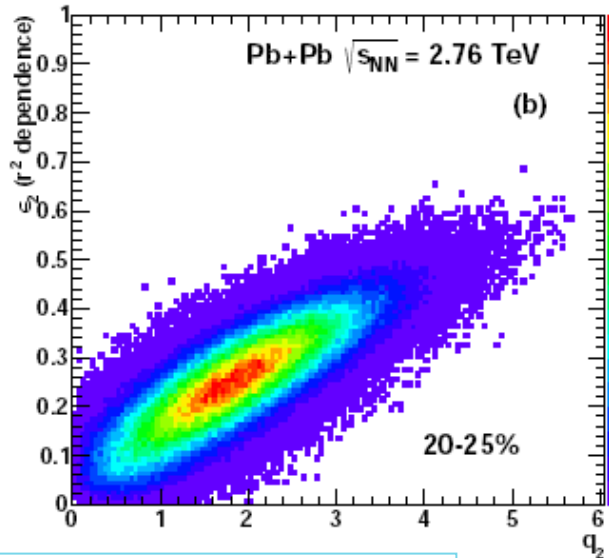
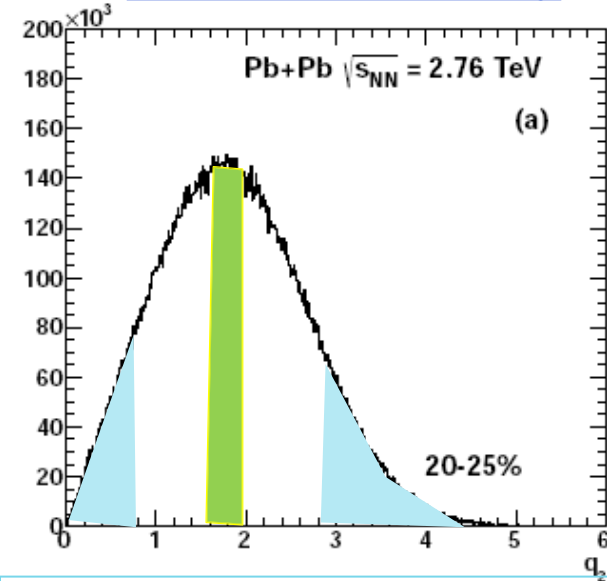
Shape-engineered events

Shape fluctuations lead to a distribution of the Q vector at a fixed centrality

$$Q_{n,x} = \sum_i^M \cos(n\phi_i); \quad Q_{n,y} = \sum_i^M \sin(n\phi_i)$$

$$q_n = Q_n / \sqrt{M}$$

Lacey et. al, arxiv:1311.1728



- Cuts on q_n should change the magnitudes $\langle \epsilon_n \rangle$, $\langle v_n \rangle$, $\langle R_n \rangle$ at a given centrality due to fluctuations
- These magnitudes can influence scaling

- Note characteristic anti-correlation predicted for $v_3(q_2)$ in mid-central events

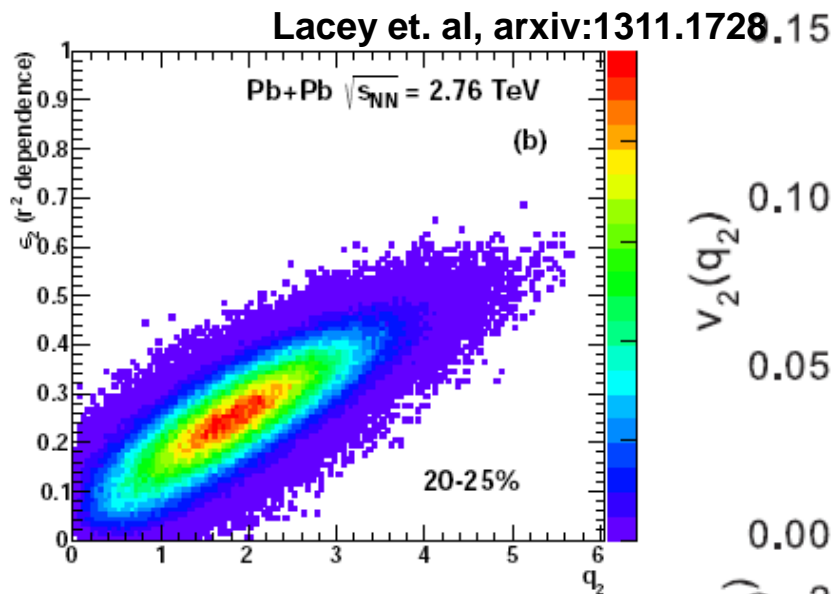
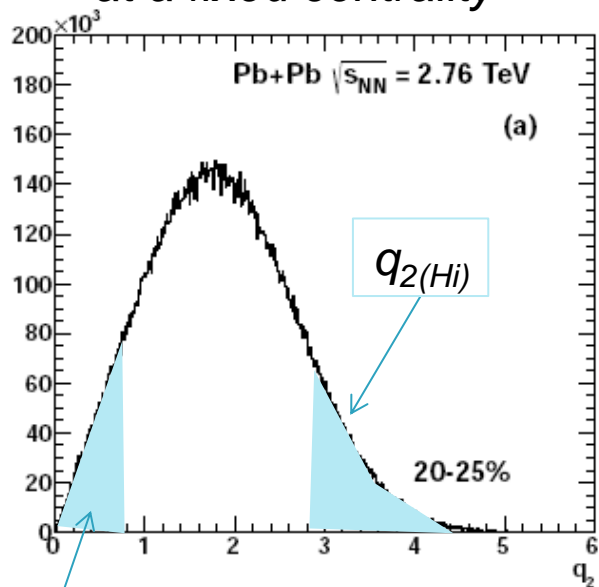
➤ **Crucial constraint for initial-geometry models**

Shape-engineered events

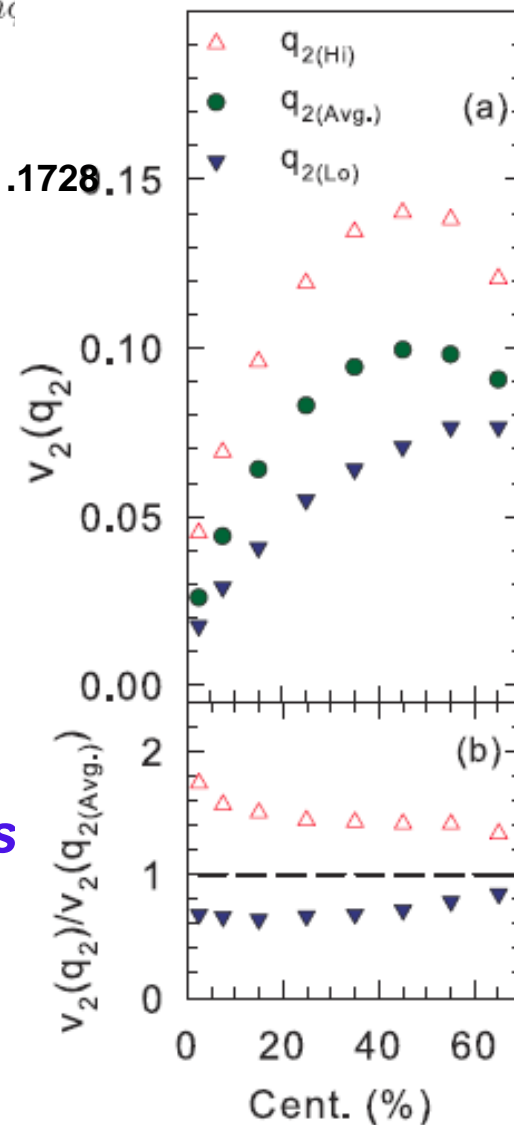
Shape fluctuations lead to a distribution of the Q vector at a fixed centrality

$$Q_{n,x} = \sum_i^M \cos(n\phi_i); \quad Q_{n,y} = \sum_i^M \sin(n\phi_i)$$

$$q_n = Q_n / \sqrt{M}$$



ALICE data

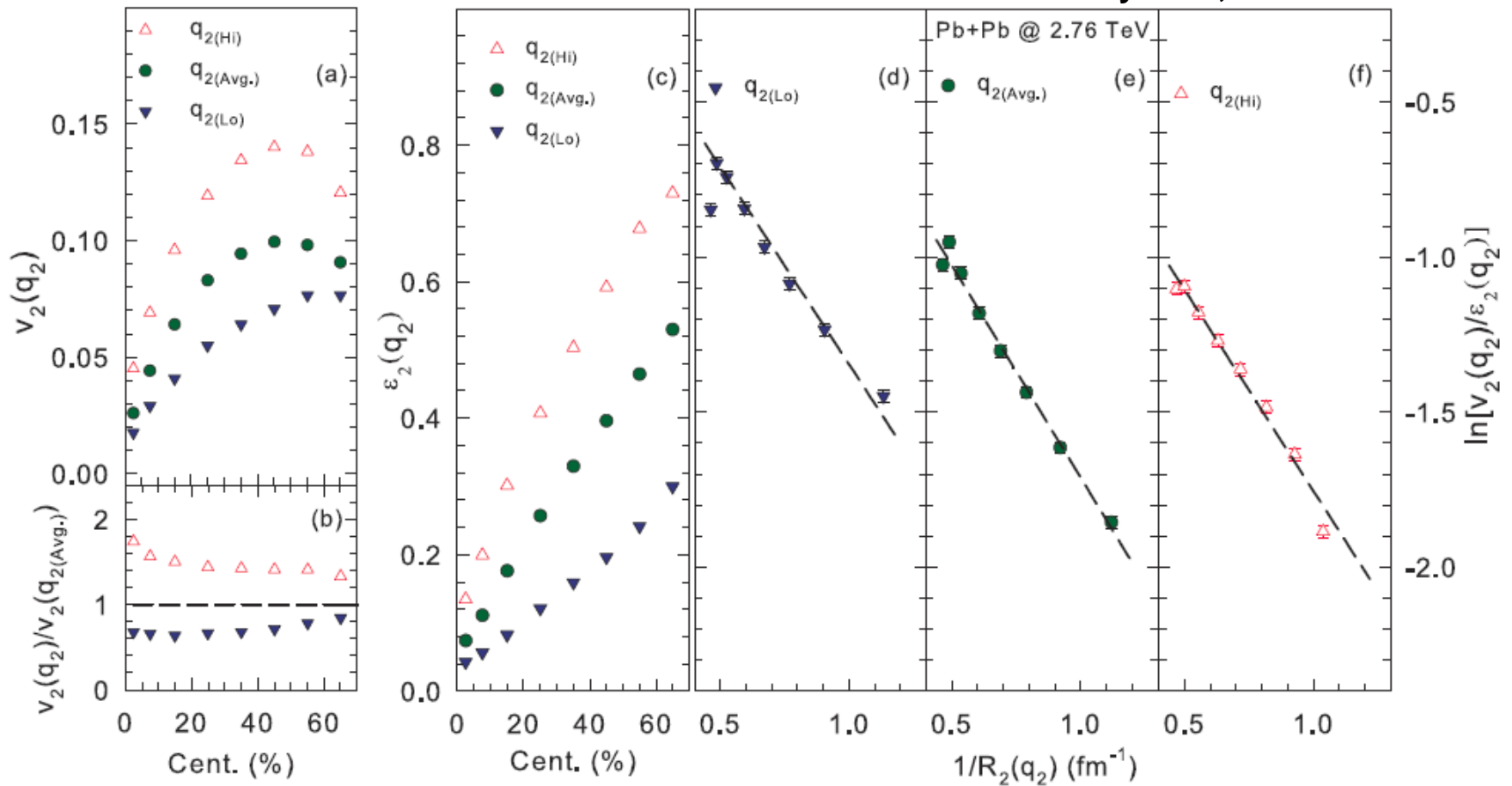


➤ **Cuts on q_n should change the magnitudes $\langle \epsilon_n \rangle, \langle v_n \rangle, \langle R_n \rangle$ at a given centrality due to fluctuations**

➤ **Viable models for initial-state fluctuations should still scale**

Acoustic Scaling of shape-engineered events

Lacey et. al, arxiv:1311.1728

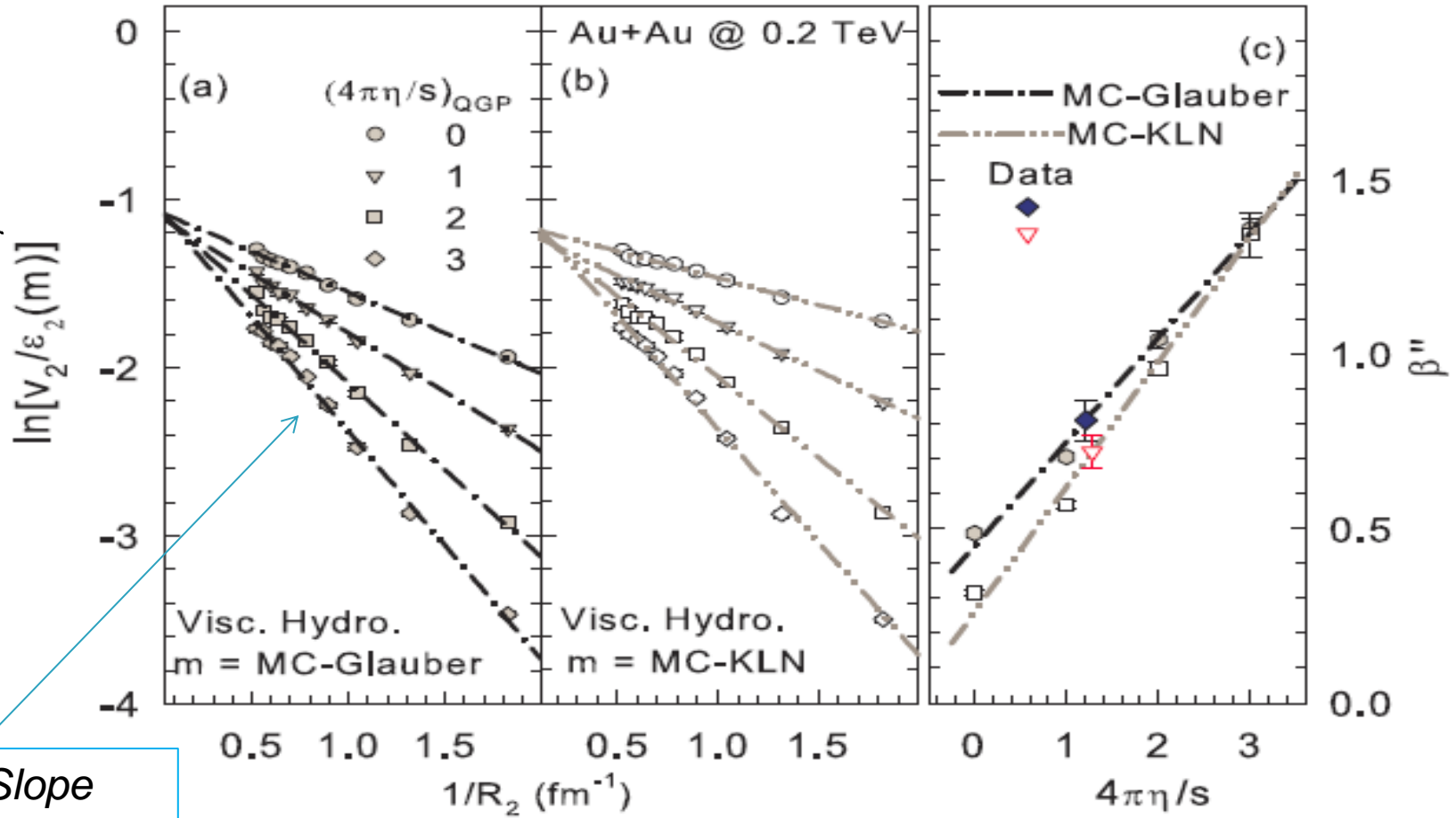


- ✓ **Characteristic $1/\bar{R}$ viscous damping validated for different event shapes at the same centrality**
- ✓ **A further constraint for initial fluctuations model and η/s**

Extraction of η/s

$$\ln\left(\frac{v_n}{\varepsilon_n}\right) \propto \frac{-\beta''}{\bar{R}}$$

Lacey et. al, arxiv:1311.1728



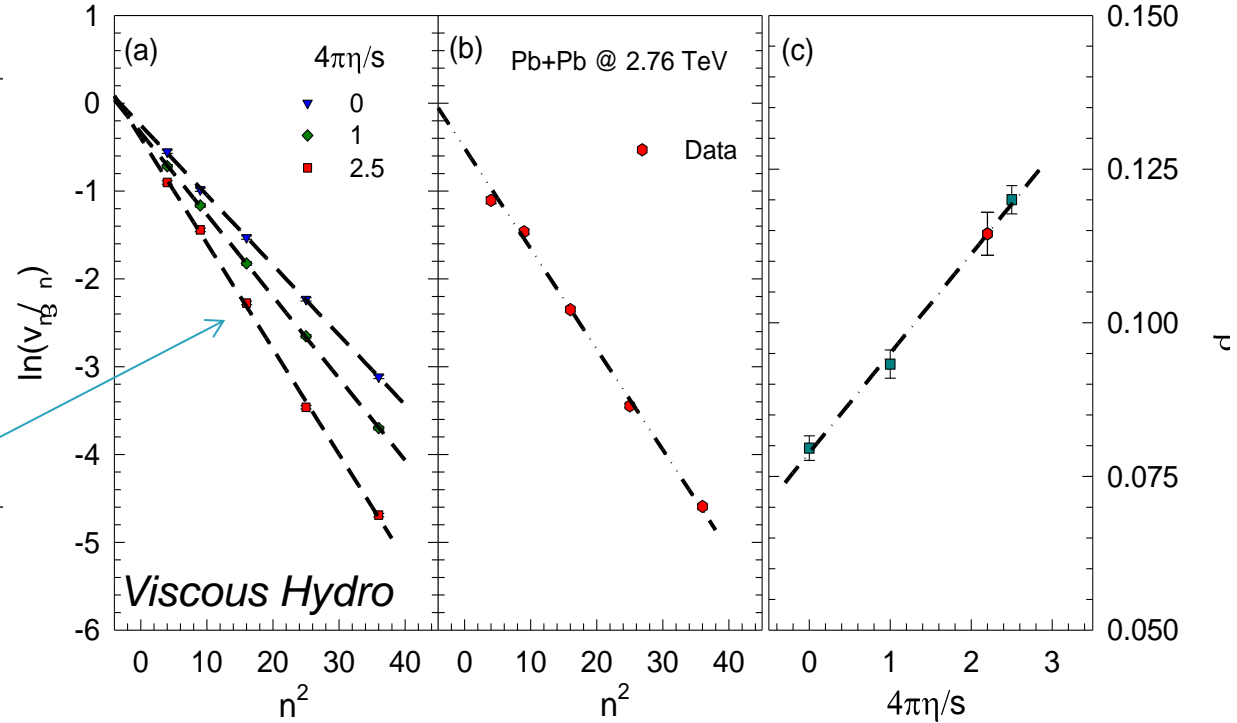
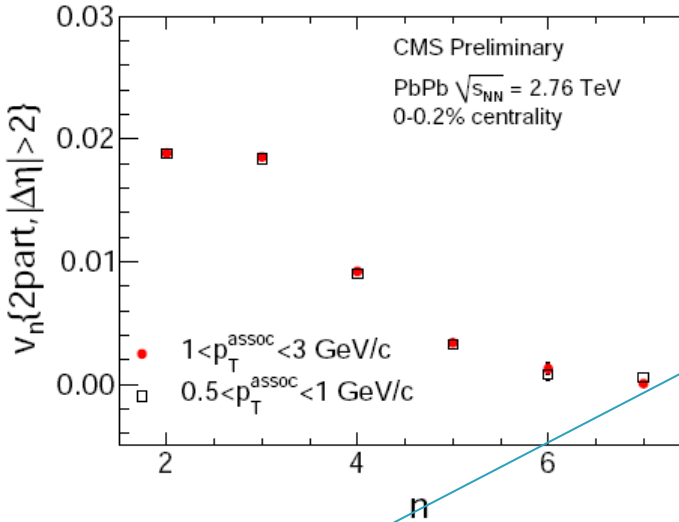
Slope sensitive to $4\pi\eta/s$

Characteristic $1/\bar{R}$ viscous damping validated in viscous hydrodynamics; calibration $\rightarrow 4\pi\eta/s \sim 1.3 \pm 0.2$
Extracted η/s value insensitive to initial conditions

Extraction of η/s

$$\frac{v_n(p_T)}{\mathcal{E}_n} \propto \exp(-\beta' n^2)$$

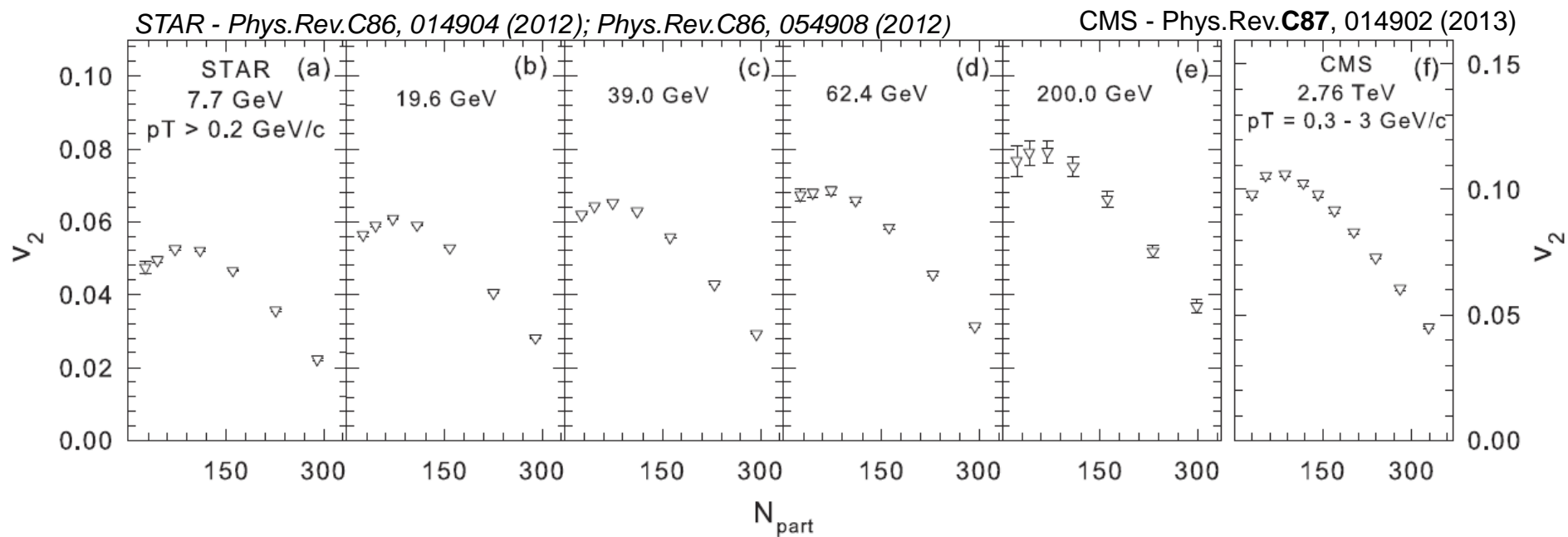
arXiv:1301.0165 & CMS PAS HIN-12-011



Slope sensitive to η/s

n^2 scaling validated in experiment and viscous hydrodynamics;
calibration $\rightarrow 4\pi\eta/s \sim 2.2 \pm 0.2$

arXiv:1305.3341



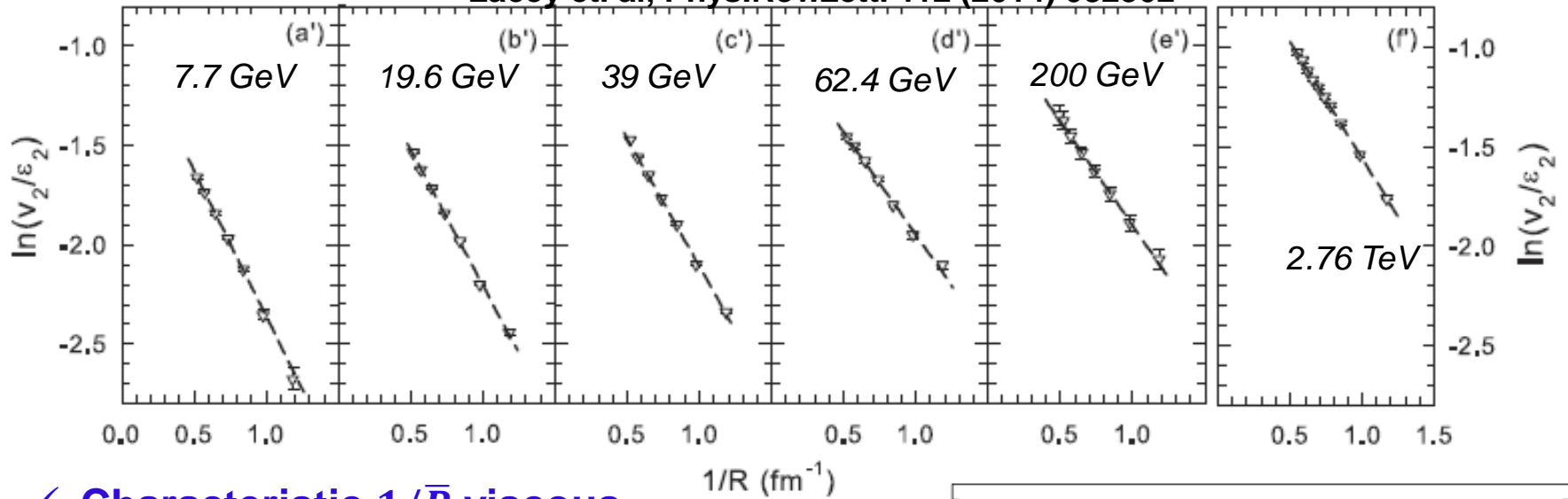
- **An extensive set of measurements now span a broad range of beam energies (T, μ_B).**

$\sqrt{s_{NN}}$ dependence of viscous coefficient

$$\ln\left(\frac{v_n}{\epsilon_n}\right) \propto \frac{-\beta''}{\bar{R}}$$

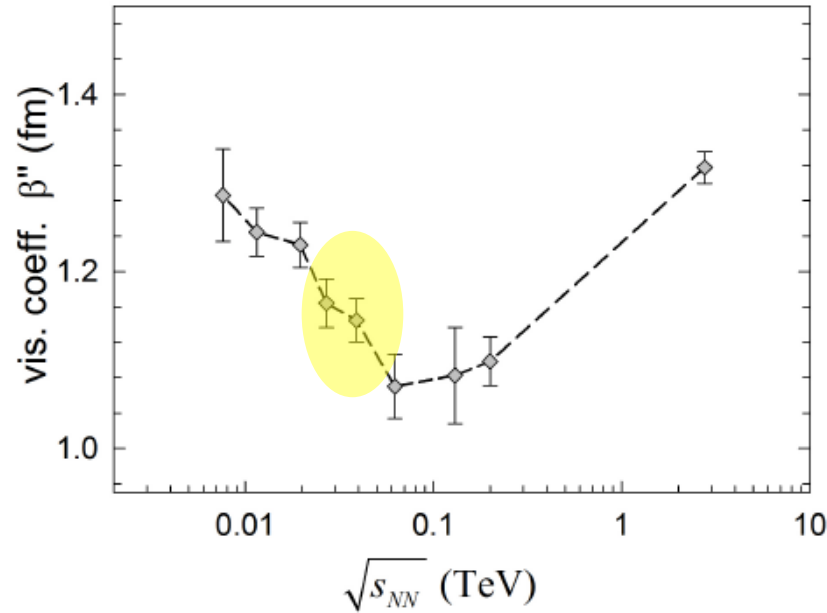
Acoustic Scaling – $\frac{1}{\bar{R}}$ Scaling for the Beam Energy Scan

Lacey et. al, Phys.Rev.Lett. 112 (2014) 082302

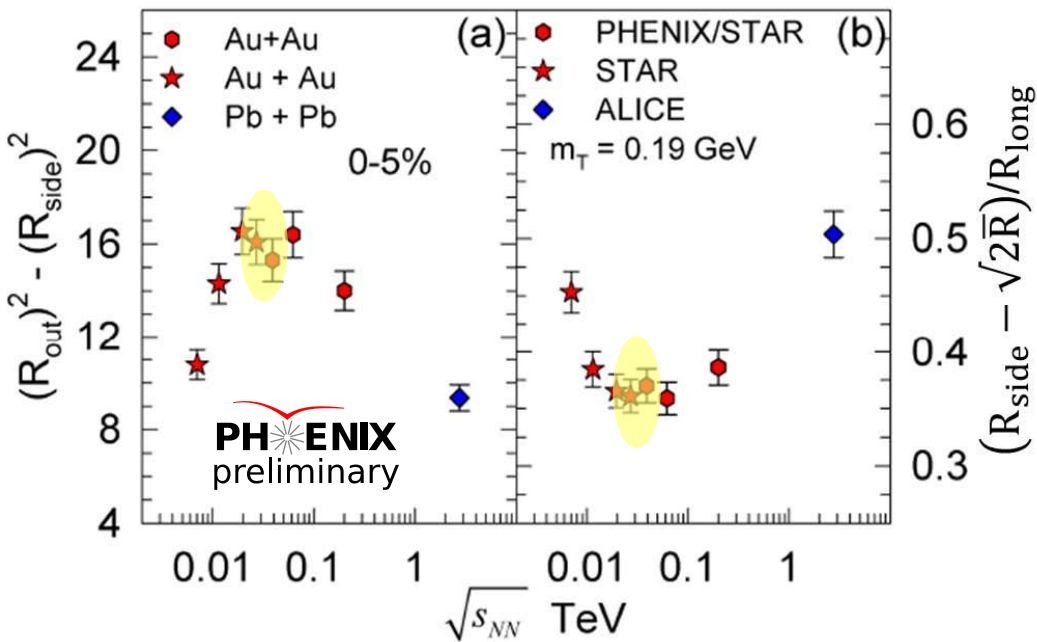


- ✓ Characteristic $1/\bar{R}$ viscous damping validated across beam energies
- ✓ First experimental indication for η/s variation in the (T, μ_B) -plane
- ✓ CEP?

Complimentary signals for similar $\sqrt{s_{NN}}$

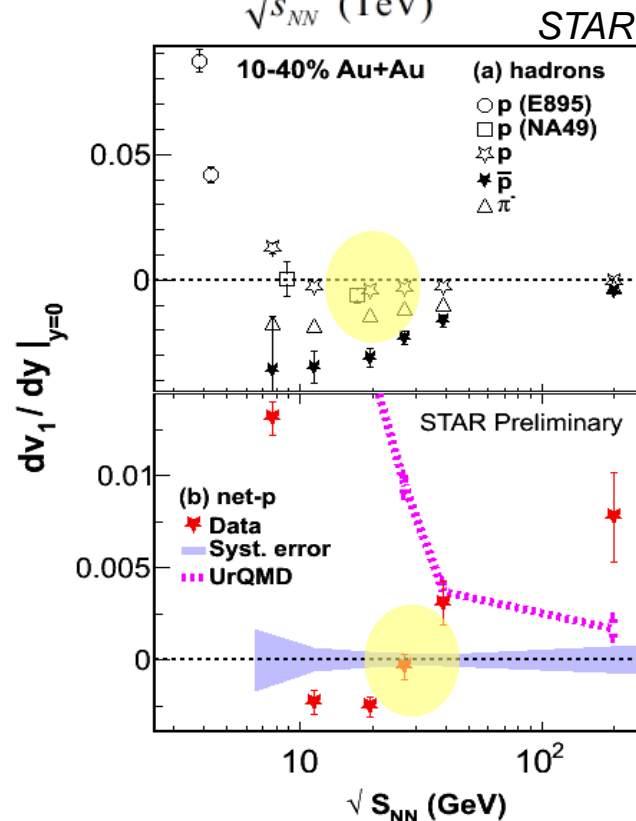
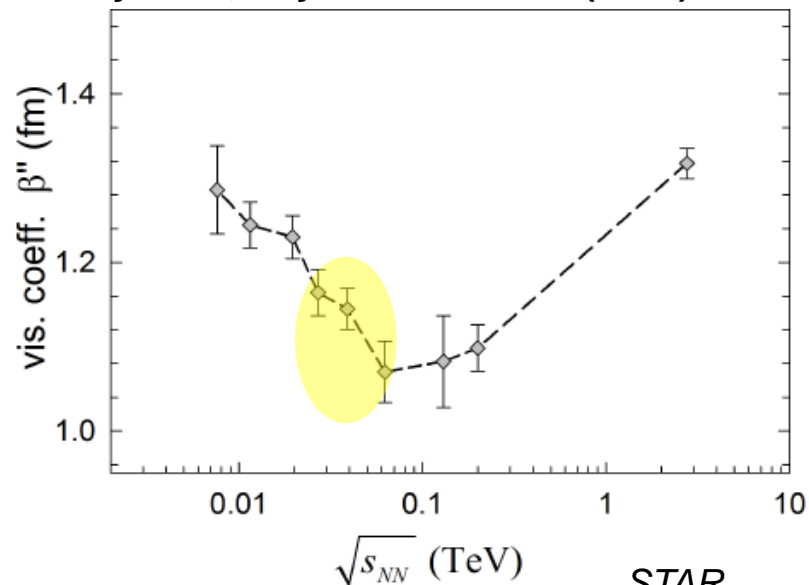


$\sqrt{s_{NN}}$ dependence of HBT signals



Combined results
→ Strongest indications for the CEP to date!

Lacey et. al, Phys.Rev.Lett. 112 (2014) 082302



Acoustic scaling of anisotropic flow and HBT radii lend profound mechanistic insights, as well as new constraints for key observables

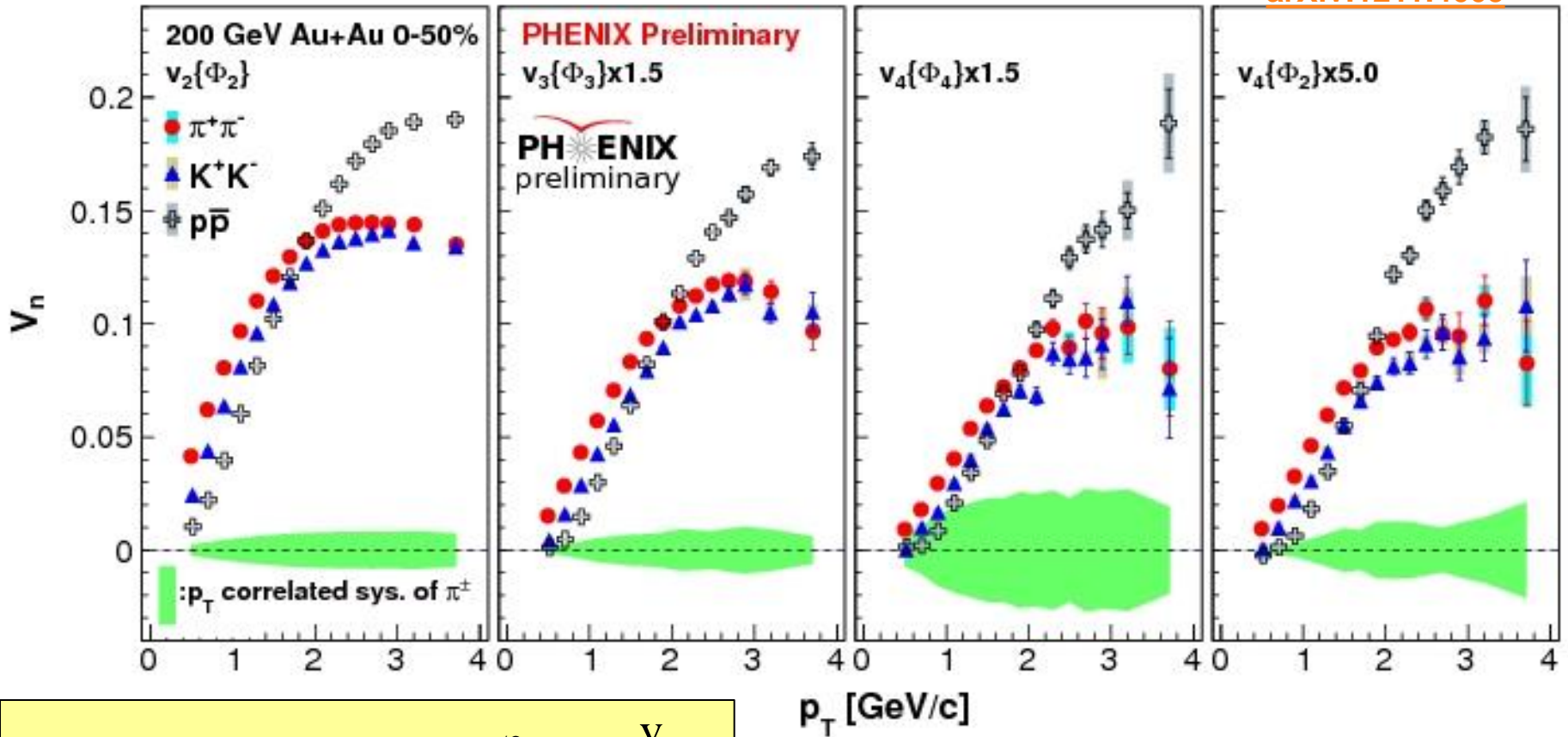
What do we learn?

- **The expansion dynamics is acoustic – “as it should be”**
 - **Validates expected acoustic scaling of flow and HBT radii**
 - ✓ **constraints for $4\pi\eta/s$ & viable initial-state models**
 - ✓ **$4\pi\eta/s$ for RHIC plasma $\sim 1.3 \pm 0.2$ ~ my 2006 estimate**
 - ✓ **$4\pi\eta/s$ for LHC plasma $\sim 2.2 \pm 0.2$**
 - ✓ **Extraction insensitive to initial geometry model**
 - **Characteristic dependence of viscous coefficient β and v_1 , as well as “ c_s ” and $\Delta\tau$ on $\sqrt{s_{NN}}$ give new constraints which could be an indication for reaction trajectories in close proximity to the CEP?**

End

Flow is partonic & Acoustic?

arXiv:1211.4009



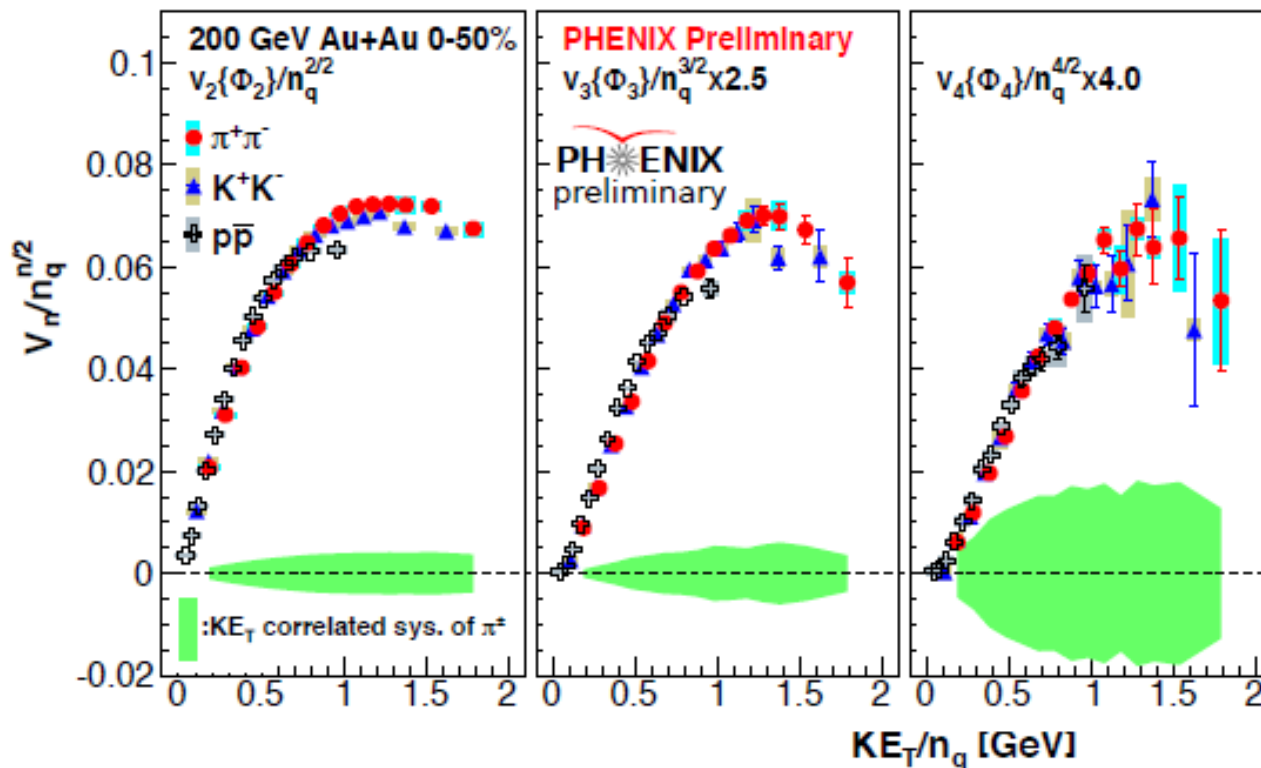
Expectation: $v_n(K E_T) \sim v_2^{n/2}$ or $\frac{V_n}{(n_q)^{n/2}}$

Note species dependence for all v_n

**For partonic flow, quark number scaling expected
 → single curve for identified particle species v_n**

Acoustic Scaling – Ratios

v_n PID scaling



Expectation validated: $v_n(KE_T) \sim v_2^{n/2}$ or $\frac{V_n}{(n_q)^{n/2}}$