Beam energy scan
using a viscous hydro+cascade model

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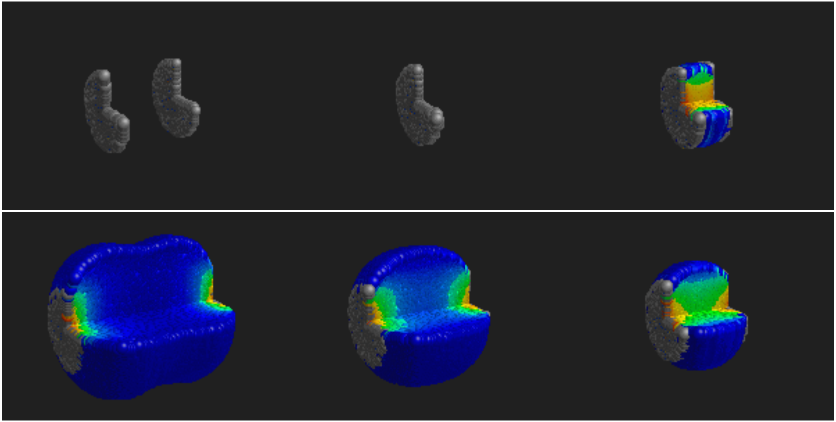
Introduction: heavy ion collision in pictures[1]



Typical size
10 fm $\propto 10^{-14}$m

Typical lifetime
10 fm/c $\propto 10^{-23}$s

10^{-8}sec after the collision: hadrons are detected

[1] https://www.jyu.fi/fysiikka/tutkimus/suurenergia/urhic/anim1.gif/image_view_fullscreen

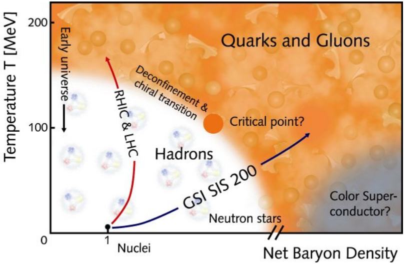
“Stages of Heavy Ion Collision”

1. Initial(pre-thermal) stage
   - Thermalization
2. Hydrodynamic expansion
   - Quark-gluon plasma phase
   - Phase transition
   - Hadron Gas phase
   - Chemical freeze-out
   - End of hydrodynamic regime
3. Kinetic stage
   Kinetic freeze-out
   $\Downarrow$
   Free streaming, then hadrons are detected

$\Leftrightarrow$

1. **Ingredients of hydro+cascade model**:

1. Initial stage model
   Enforced thermalization
2. Hydrodynamic solution
   - Equation of state for hydrodynamics
   - transport coefficients
3. Particlization and switching to a cascade

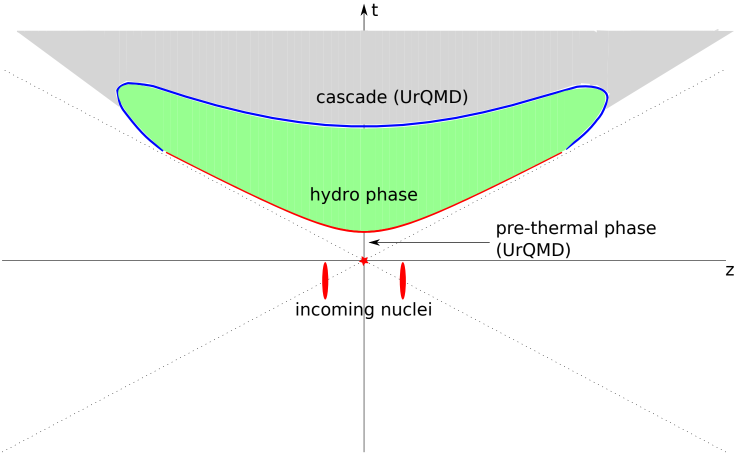


Ingredients essential for beam energy scan studies are marked red.

1. **Ingredients of the model**:

1. Initial stage:
   UrQMD
2. Hydrodynamic solution
   - Equation of state for hydrodynamics:
     Chiral model
     HRG + Bag Model
   - transport coefficients
3. Particlization and switching back to cascade (UrQMD)

Initial conditions for hydro phase



$\tau = \sqrt{t^2 - z^2} = \tau_0$ (red curve):

$\tau_0 = \frac{2R}{\gamma v_z}$

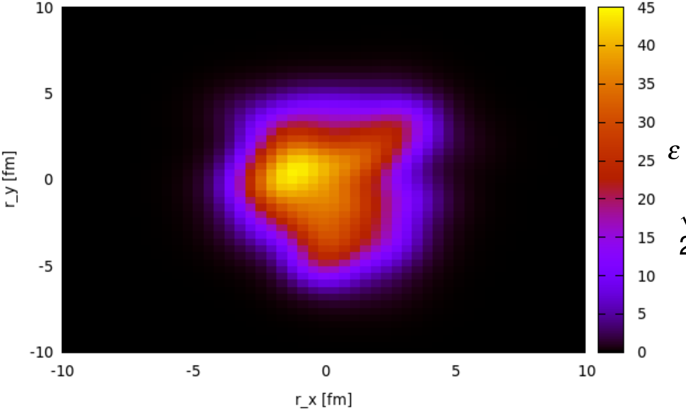
$\{T^{0\mu}, N_b^0, N_q^0\}$ of fluid = averaged $\{T^{0\mu}, N_b^0, N_q^0\}$ of particles

Initial conditions for hydro phase (2)

Fluctuating, but smoothed initial state:

$E \propto \exp\left(-\frac{(x-x_{part})^2+(y-y_{part})^2+\gamma_z^2(z-z_{part})^2}{R^2}\right)$, where $R = 1$ fm

see e.g. H. Petersen et al., Phys.Rev. C78 (2008) 044901



$\sqrt{s_{NN}} = 39$ GeV,
20-30% central

Hydrodynamic phase

The hydrodynamic equations in arbitrary coordinate system:

$$\partial_{;\nu} T^{\mu\nu} = \partial_{\nu} T^{\mu\nu} + \Gamma^{\mu}_{\nu\lambda} T^{\nu\lambda} + \Gamma^{\nu}_{\nu\lambda} T^{\mu\lambda} = 0, \quad \partial_{;\nu} N^{\nu} = 0 \tag{1}$$

where (we choose Landau definition of velocity)

$$T^{\mu\nu} = \varepsilon u^{\mu} u^{\nu} - (p + \Pi)(g^{\mu\nu} - u^{\mu} u^{\nu}) + \pi^{\mu\nu} \tag{2}$$

and $\Delta^{\mu\nu} = g^{\mu\nu} - u^{\mu} u^{\nu}$

Evolutionary equations for shear/bulk, coming from **Israel-Stewart** formalism:

$$< u^{\gamma} \partial_{;\gamma} \pi^{\mu\nu} > = -\frac{\pi^{\mu\nu} - \pi^{\mu\nu}_{\text{NS}}}{\tau_{\pi}} - \frac{4}{3} \pi^{\mu\nu} \partial_{;\gamma} u^{\gamma} \tag{3a}$$

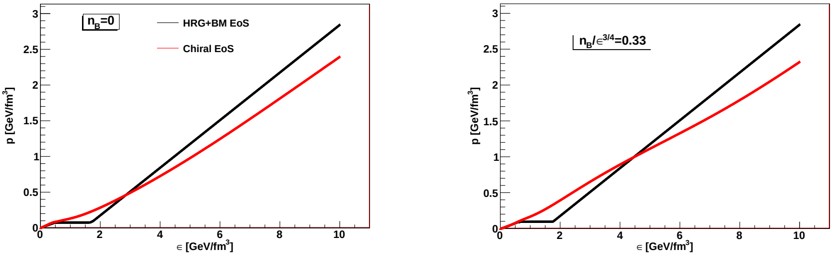
where $< A^{\mu\nu} >= (\frac{1}{2}\Delta^{\mu}_{\alpha}\Delta^{\nu}_{\beta} + \frac{1}{2}\Delta^{\nu}_{\alpha}\Delta^{\mu}_{\beta} - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta})A^{\alpha\beta}$

* Bulk viscosity $\zeta = 0$, charge diffusion=0

See `arXiv:1312.4160` for the details of hydro code and its testing.

Equations of state for hydrodynamic phase

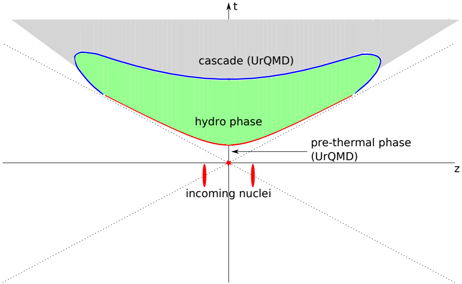
- Chiral model
  - coupled to Polyakov loop to include the deconfinement phase transition
  - good agreement with lattice QCD data at $\mu_B = 0$, also applicable at finite baryon densities
  - (current version) has crossover type PT between hadron and quark-gluon phase at all μ_B
- Hadron resonance gas + Bag Model (a.k.a. `EoS Q`)
  - hadron resonance gas made of u, d quarks including repulsive meanfield
  - the phases matched via Maxwell construction, resulting in 1^{st} order PT



ref: J. Steinheimer, S. Schramm and H. Stocker, J. Phys. G 38, 035001 (2011); P.F. Kolb, J. Sollfrank, and U. Heinz, Phys.Rev. C 62, 054909 (2000).

Fluid→particle transition

$\varepsilon = \varepsilon_{sw} = 0.5$ GeV/fm^3 (end of green zone):
$\{T^{0\mu}, N_b^0, N_q^0\}$ of hadron-resonance gas = $\{T^{0\mu}, N_b^0, N_q^0\}$ of fluid



▷ Momentum distribution from Landau/Cooper-Frye prescription:

$$p^0 \frac{d^3 n_i}{d^3 p} = \int \left(f_{\text{l.eq.}}(x,p) + \delta f(x,p) \right) p^\mu d\sigma_\mu$$

▷ Cornelius subroutine* is used to compute $\Delta\sigma_i$ on transition hypersurface.

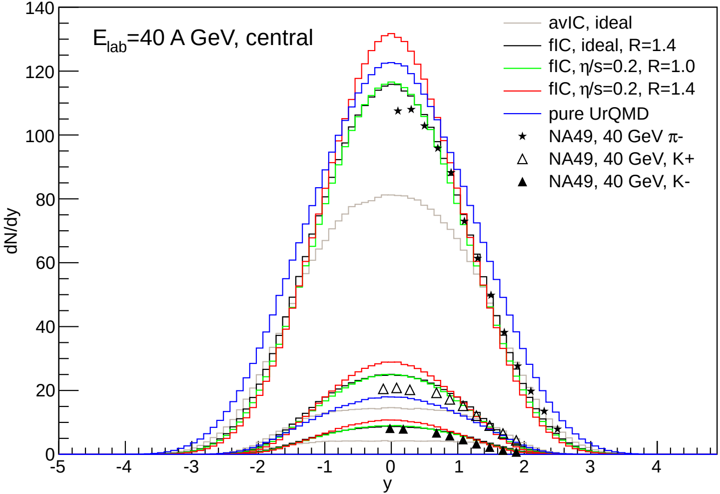
▷ UrQMD cascade is employed after particlization surface.

*Huovinen P and Petersen H 2012, *Eur.Phys.J.* A **48** 171

Results

Results: $E_{\text{lab}} = 40$ A GeV Pb-Pb (SPS)

$\sqrt{s_{NN}} = 8.8$ GeV, 0-5% central collisions ($b = 0 \dots 3.4$ fm) (Chiral EoS only)



→ viscous entropy production

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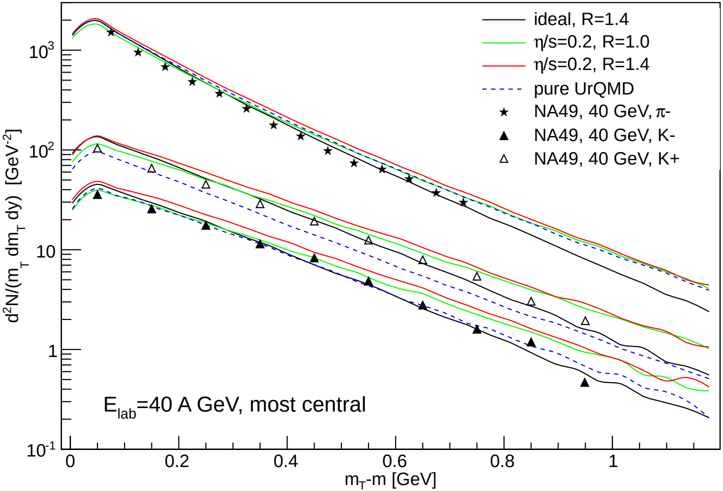
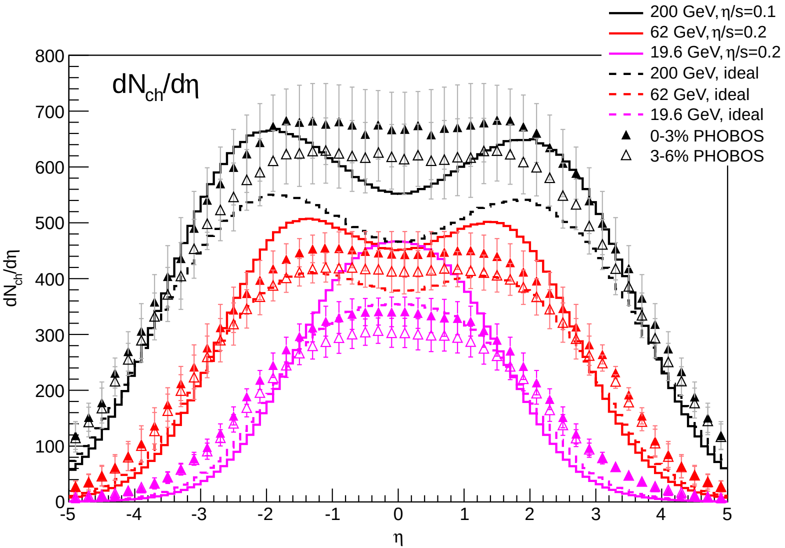


→ viscosity causes stronger transverse expansion

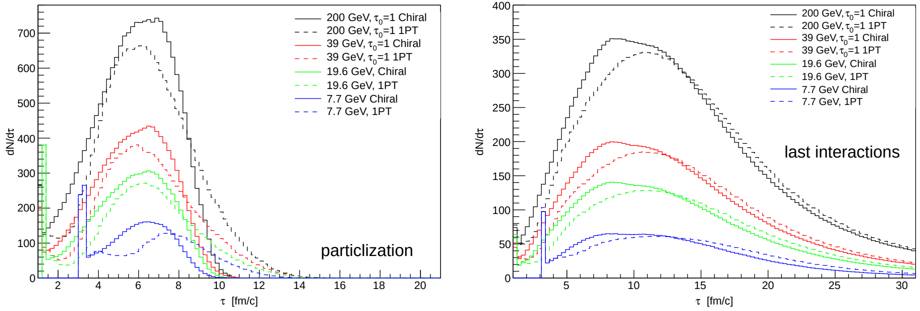
$dN/d\eta$ from RHIC ($\sqrt{s_{NN}} = 19.6, 62.4, 200$ GeV Au-Au)



Fine tuning is required for every energy individually.

Effects of the `EoS Q` compared to Chiral EoS?

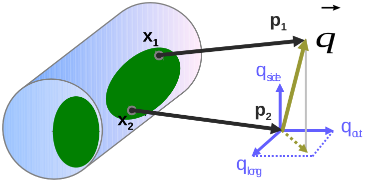
Yes: hydro phase in average lasts longer with `EoS Q`



Can we see it with femtoscopy measurements?

HBT(interferometry) measurements

The only tool for space-time measurements at the scales of 10^{-15}m, 10^{-23}s



$$\vec{q} = \vec{p}_2 - \vec{p}_1$$

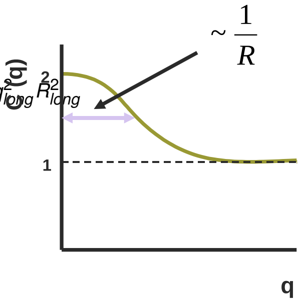
$$\vec{k} = \frac{1}{2}(\vec{p}_1 + \vec{p}_2)$$

$$C(p_1, p_2) = \frac{P(p_1, p_2)}{P(p_1)P(p_2)} = \frac{\text{real event pairs}}{\text{mixed event pairs}}$$

Gaussian approximation of CFs ($q \to 0$):

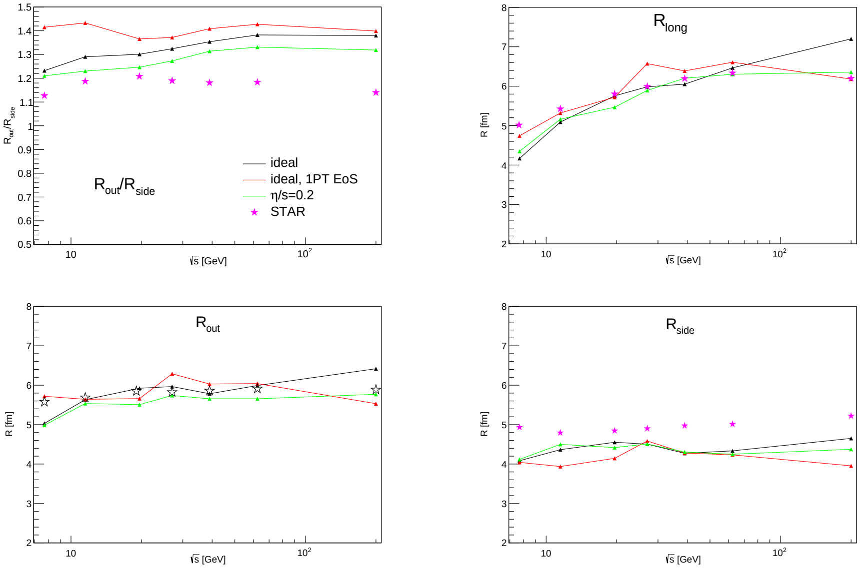
$$C(\vec{k}, \vec{q}) = 1 + \lambda(k) e^{-q_{out}^2 R_{out}^2 - q_{side}^2 R_{side}^2 - q_{long}^2 R_{long}^2}$$

$R_{out}, R_{side}, R_{long}$ (HBT radii) correspond to *homogeneity lengths*, which reflect the space-time scales of emission process



In an event generator, BE/FD two-particle amplitude (anti)symmetrization must be introduced

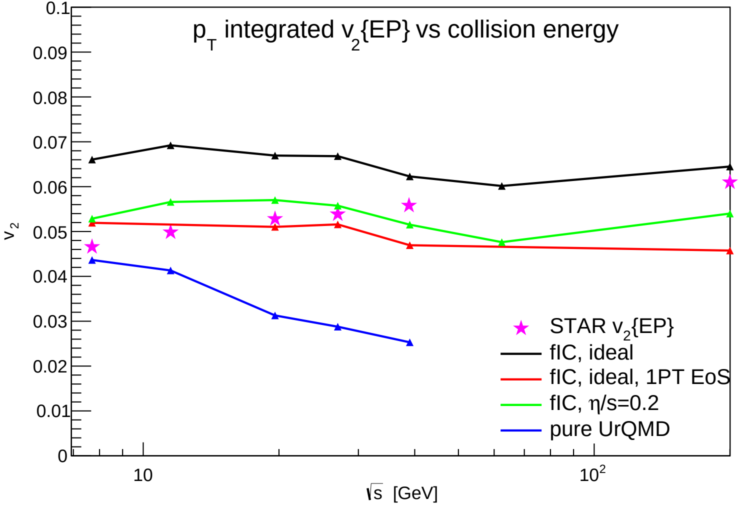
Femtoscopic radii: ideal hydro/Chiral EoS, ideal hydro/EoS Q, visc.hydro/Chiral EoS



Previous results for EoS dependence of HBT in hybrid UrQMD, see Q. Li et al., Phys.Lett.B674:111,2009

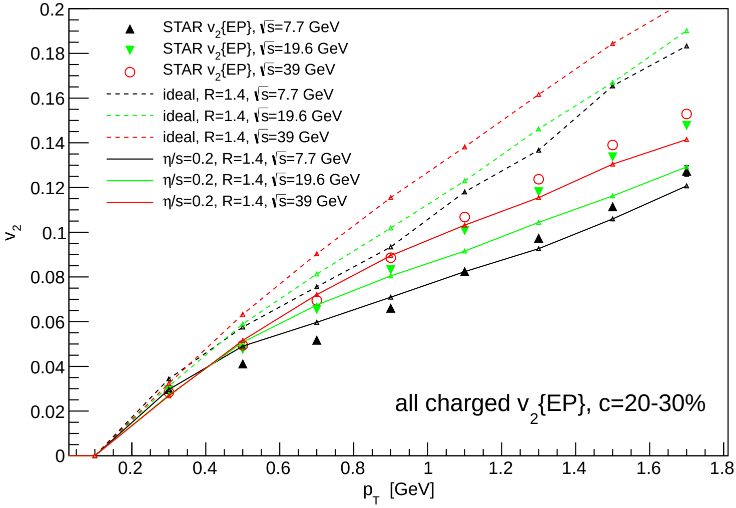
Elliptic flow, $\sqrt{s_{NN}} = 7.7 \ldots 200$ GeV Au-Au

Shear viscosity suppresses the elliptic flow (as expected)
However, with `EoS Q` also suppresses the elliptic flow.



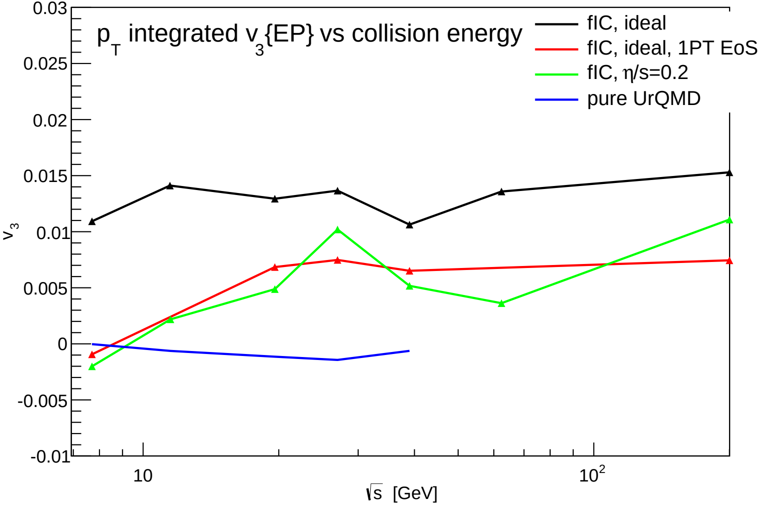
p_T differential v_2

gets systematically better as well with viscous hydro phase



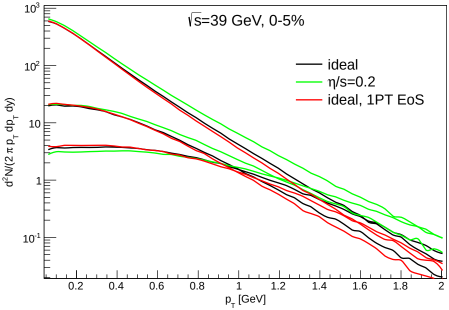
Triangular flow, $\sqrt{s_{NN}} = 7.7 \ldots 200$ GeV Au-Au

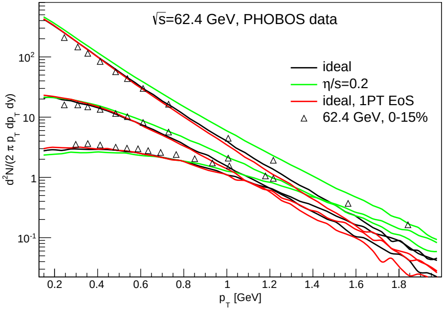
Looks like that v_3 is similarly suppressed, however the statistics (number of events) might not be enough.

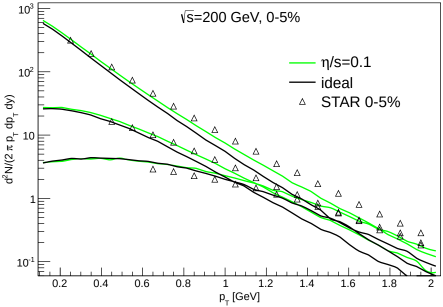


p_T **spectra:**

Radial flow is less affected by the EoS change.

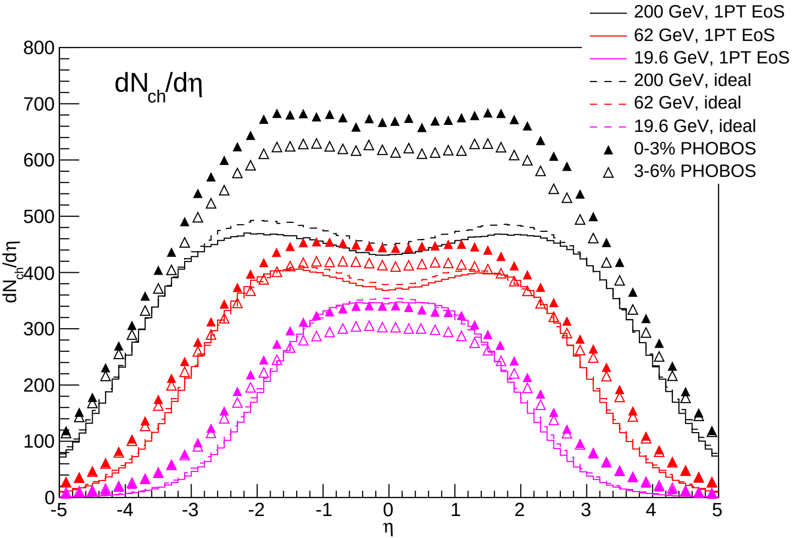






Pseudorapidity distribution of produced hadrons: Chiral EoS vs. EoS Q

No change with `EoS Q`, no longitudinal flow rearrangement.



* $\tau_0 = 1$ fm/c for $\sqrt{s} = 200$ GeV, which gives lower multiplicity

Summary

Viscous hydro + UrQMD model:

- pre-termal stage: UrQMD
- 3+1D viscous hydrodynamics
- EoS at finite μ_B: Chiral model, `EoS Q`

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Conclusions:

- Model applied for $\sqrt{s_{NN}} = 7.7 \ldots 200$ GeV A+A collisions.
- v_2 suggests
  effective $\eta/s > 0.2$ for $\sqrt{s} < 30$ GeV and
  effective $\eta/s < 0.2$ otherwise,
  modulo initial state and EoS used.
  $\Rightarrow \mu_B$ dependent η/s or $\eta/(\varepsilon + p)$?
- `EoS Q` seems to be disfavored by the data (too small v_2 + too small $dN/d\eta$, slightly worse for HBT)
- As usual, more experimental data is needed to e.g. extract η/s less ambiguously.

Summary

Viscous hydro + UrQMD model:

- pre-termal stage: UrQMD
- 3+1D viscous hydrodynamics
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Outlook:

- Different initial state model

Work in progress.

Thank you for your attention!

Extra slides

Hydrodynamic phase

The hydrodynamic equations in arbitrary coordinate system:

$$\partial_{;\nu}T^{\mu\nu}=\partial_{\nu}T^{\mu\nu}+\Gamma^{\mu}_{\nu\lambda}T^{\nu\lambda}+\Gamma^{\nu}_{\nu\lambda}T^{\mu\lambda}=0,\quad \partial_{;\nu}N^{\nu}=0 \qquad (4)$$

where (we choose Landau definition of velocity)

$$T^{\mu\nu}=\varepsilon u^{\mu}u^{\nu}-(p+\Pi)(g^{\mu\nu}-u^{\mu}u^{\nu})+\pi^{\mu\nu} \qquad (5)$$

and $\Delta^{\mu\nu}=g^{\mu\nu}-u^{\mu}u^{\nu}$

Evolutionary equations for shear/bulk, coming from **Israel-Stewart** formalism:

$$<u^{\gamma}\partial_{;\gamma}\pi^{\mu\nu}>=-\frac{\pi^{\mu\nu}-\pi^{\mu\nu}_{\text{NS}}}{\tau_{\pi}}-\frac{4}{3}\pi^{\mu\nu}\partial_{;\gamma}u^{\gamma} \qquad (6a)$$

where

$$<A^{\mu\nu}>=(\frac{1}{2}\Delta^{\mu}_{\alpha}\Delta^{\nu}_{\beta}+\frac{1}{2}\Delta^{\nu}_{\alpha}\Delta^{\mu}_{\beta}-\frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta})A^{\alpha\beta}$$

* Bulk viscosity $\zeta=0$, charge diffusion=0

Coordinate transformations (hydro phase)

Milne coordinates

The coordinate system is defined as follows:

0) $\tau = \sqrt{t^2 - z^2}$

1) $x = x$

2) $y = y$

3) $\eta = \frac{1}{2} ln \frac{t+z}{t-z}$

$$g^{\mu\nu} = diag(1, -1, -1, -1/\tau^2)$$

Nonzero Christoffel symbols are:

$$\Gamma^{\eta}_{\tau\eta} = \Gamma^{\eta}_{\eta\tau} = 1/\tau, \quad \Gamma^{\tau}_{\eta\eta} = \tau$$

$$T^{\mu\nu} = (\varepsilon + p) u^\mu u^\nu - p \cdot g^{\mu\nu}, \text{ where}$$

$$u^\mu = \{\cosh(\eta_f - \eta)\cosh\eta_T, \sinh\eta_T\cos\phi, \sinh\eta_T\sin\phi, \frac{1}{\tau}\sinh(\eta_f - \eta)\cosh\eta_T\}$$

(cf. $u^i_{\text{Cart}} = \{\cosh(\eta_f)\cosh\eta_T, \sinh\eta_T\cos\phi, \sinh\eta_T\sin\phi, \sinh(\eta_f)\cosh\eta_T\}$)

EM conservation equations are

$$\partial_{;\nu} T^{\mu\nu} = 0$$

or

$$\mu = 0: \quad \partial_\nu T^{\tau\nu} + \tau T^{\eta\eta} + \frac{1}{\tau} T^{\tau\tau} = 0$$

$$\mu = 1: \quad \partial_\nu T^{x\nu} + \frac{1}{\tau} T^{x\tau} = 0$$

$$\mu = 2: \quad \partial_\nu T^{y\nu} + \frac{1}{\tau} T^{y\tau} = 0$$

$$\mu = 3: \quad \partial_\nu T^{\eta\nu} + \frac{3}{\tau} T^{\eta\tau} = 0$$

Additional transformations:

$T^{\mu\eta} \to T^{\mu\eta}/\tau,\ \mu \neq \eta,$
$T^{\eta\eta} \to T^{\eta\eta}/\tau^2$

$$\Downarrow\Downarrow$$

$$\partial_\nu(\tau T^{\tau\nu}) + \frac{1}{\tau}(\tau T^{\eta\eta}) = 0$$

$$\partial_\nu(\tau T^{x\nu}) = 0$$

$$\partial_\nu(\tau T^{y\nu}) = 0$$

$$\partial_\nu(\tau T^{\eta\nu}) + \frac{1}{\tau}\tau T^{\eta\tau} = 0$$

Conservative variables are
$Q^\mu = \tau \cdot T^{\tau\mu}$

Closer to numerics:

$$\partial_\mu (T^{\mu\nu}_{\text{id}} + \delta T^{\mu\nu}) = S^\nu, \qquad \text{S=geometrical source terms}$$

$$\partial_\tau \underbrace{(T^{\tau i}_{\text{id}} + \delta T^{\tau i})}_{Q_i} + \partial_j \underbrace{(T^{ji})}_{\text{id.flux}} + \partial_j \underbrace{(\delta T^{ji})}_{\text{visc.flux}} = \underbrace{S^\nu_{\text{id}} + \delta S^\nu}_{\text{source terms}}$$

Finite-volume realization:

$$\frac{1}{\Delta\tau}(Q^{n+1}_{\text{id}} + \delta Q^{n+1} - Q^n_{\text{id}} - \delta Q^n) + \frac{1}{\Delta x}(\Delta F^{n+1/2}_{\text{id}} + \Delta\delta F^{n+1/2}) = S^{n+1/2}_{\text{id}} + \delta S^{n+1/2}$$

now, a small trick:

$$\frac{1}{\Delta\tau}(Q^{n+1}_{\text{id}} + \delta Q^{n+1} \underbrace{- Q^{*n+1}_{\text{id}} + Q^{*n+1}_{\text{id}}}_{=0} - Q^n_{\text{id}} - \delta Q^n) + \frac{1}{\Delta x}(\Delta F_{\text{id}} + \Delta\delta F) = S_{\text{id}} + \delta S$$

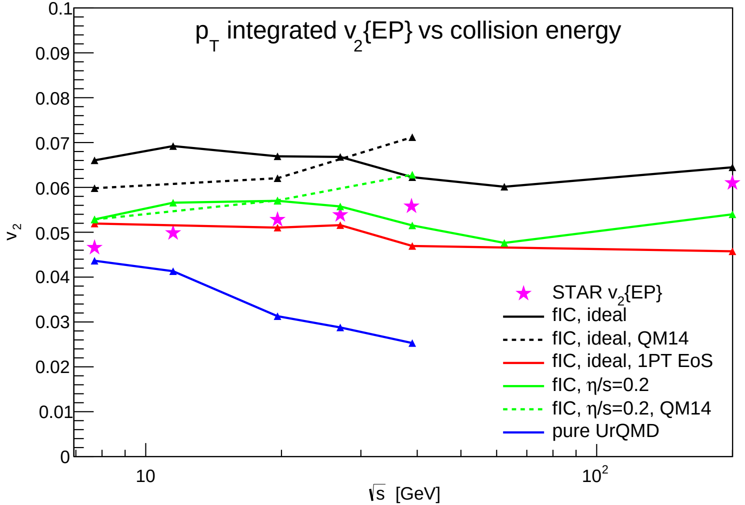
Then,split the equation into two parts[2]:

$$\frac{1}{\Delta t}(Q^{*n+1}_{\text{id}} - Q^n_{\text{id}}) + \frac{1}{\Delta x}\Delta F_{\text{id}} = S_{\text{id}} \quad \text{(using finite volume, HLLE approx)} \qquad (7)$$

$$\frac{1}{\Delta t}(Q^{n+1}_{\text{id}} + \delta Q^{n+1} - Q^{*n+1}_{\text{id}} - \delta Q^n) + \frac{1}{\Delta x}\Delta\delta F = \delta S \quad \text{(Lax-Wendroff)} \qquad (8)$$

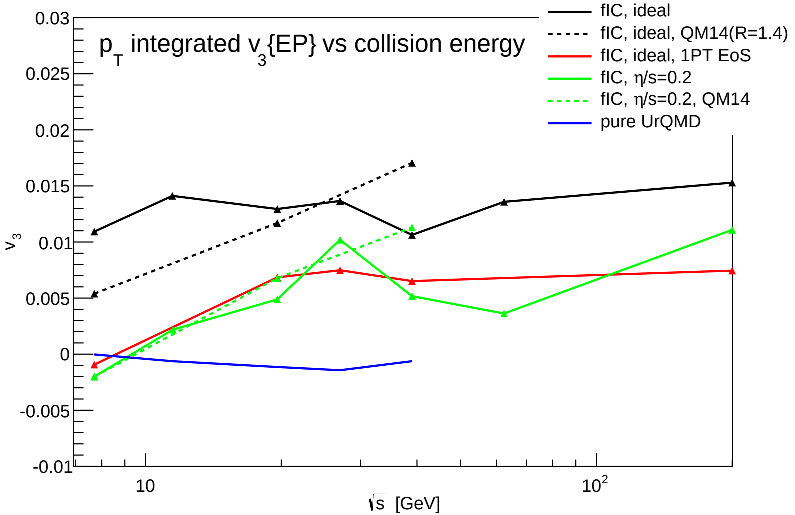
[2] Makoto Takamoto, Shu-ichiro Inutsuka, J.Comput.Phys. 230 (2011) 7002

Comparison to QM14 results: v_2



QM14 results (dashed): hydro starting time is not limited from below

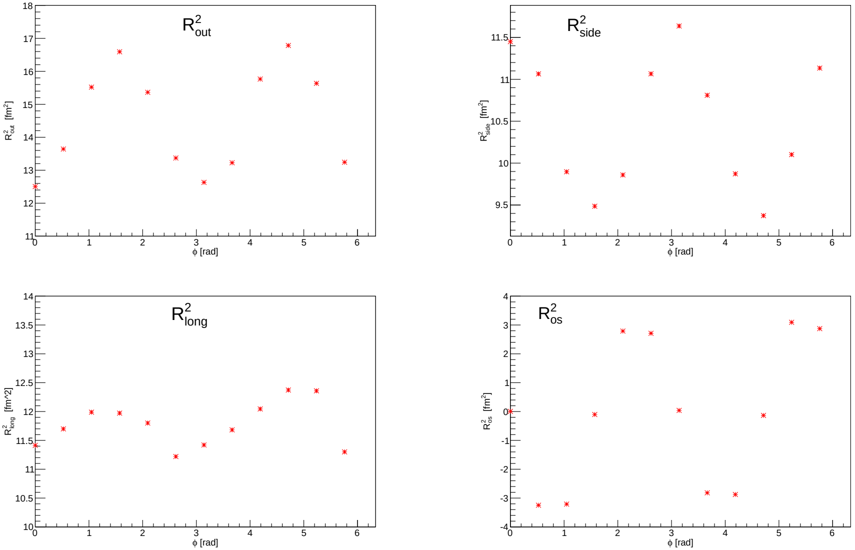
Comparison to QM14 results: v_3



QM14 results (dashed): hydro starting time is not limited from below

Azimuthally-sensitive femtoscopy

$\sqrt{s_{NN}} = 7.7$ GeV, 10-30% central AuAu; $p_T = 0.15 \ldots 0.6$ GeV; $\phi = \psi_{\text{pair}} - \Psi_{\text{RP}}$



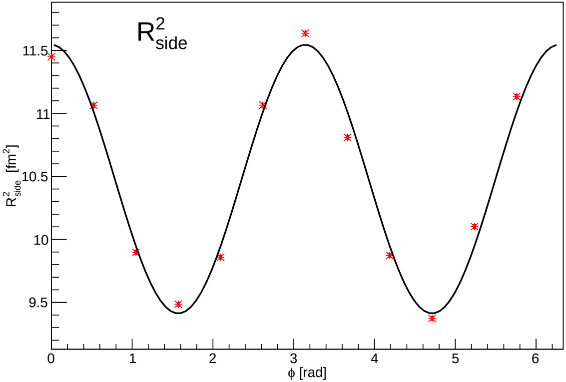
Azimuthally-sensitive femtoscopy

$$R_i^2(\phi) = R_{i,0}^2 + 2\sum_{n=2,4,6...} R_{i,n}^2 \cos(n\phi), \quad i = \text{out,side,long}$$

$$R_i^2(\phi) = 2\sum_{n=2,4,6...} R_{i,n}^2 \sin(n\phi), \quad i = \text{os}$$

solid curve:
$R_{s,0}^2 + 2R_{s,2}^2 \cos(2\phi) \Rightarrow$

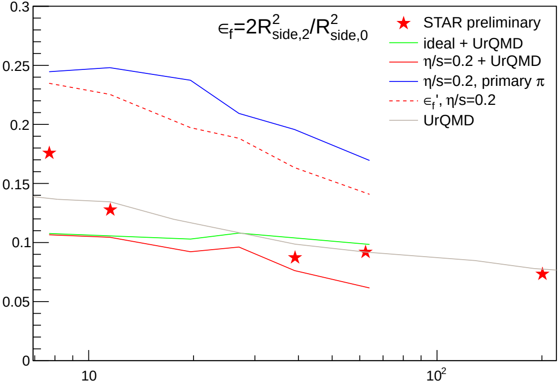
$$\varepsilon_f = 2\frac{R_{\text{side},2}^2}{R_{\text{side},0}^2}$$



F. Retiere and M. Lisa, Phys.Rev. **C**70:044907, 2004

Azimuthally-sensitive femtoscopy

using averaged initial state, single shot hydro



STAR: C. Anson, J.Phys. G**38**:124148,2011

10-30% central AuAu, $p_T = 0.15 \ldots 0.6$ GeV

$$\varepsilon' = \frac{\int (y^2 - x^2) u^\mu d\sigma_\mu}{\int (y^2 + x^2) u^\mu d\sigma_\mu}$$ [1]

Rescatterings and resonance decays decrease the eccentricity

[1] C. Shen, U. Heinz, Phys.Rev. C 85, 054902 (2012)
[2] UrQMD: M.A. Lisa, et al., New J.Phys.13:065006,2011