Workshop on Particle Correlations and Femtoscopy
Gyöngyös, August 25-29, 2014

Chaoticity and Coherence in Intensity Interferometry

Cheuk-Yin Wong
Oak Ridge National laboratory

Collaborators:
Weining Zhang, Jie Liu, Peng Ru
Dalian University of Science and Technology

Outline of the talk

- Introduction
- Bose-Einstein condensation of a dense boson system in a potential
- Relation between Bose-Einstein condensation and Bose-Einstein correlation
- Conclusions

A fundamental question on Bose Einstein correlation

- Bose-Einstein correlation occurs only for chaotic sources
- We need a good theory to describe

  -- chaoticity

  -- coherence

  -- transition from coherence to chaoticity

The proper framework to study the above topics is the theory of the Bose-Einstein condensation of bosons in their own mean field potential

Why is Bose-Einstein condensation (BEC) relevant to Bose-Einstein correlations (BEC)?

- Major advances in Bose-Einstein condensation in atomic physics. In particular, the works of Politzer (PRA95), and Naraschewski & Glauber (PRA '99) facilitate the construction of the theoretical framework.

- Glauber in many private communications and in his talk in QM2005 suggested that $\boldsymbol{\lambda} < 1$ may be due to the coherence of the pions in Bose Einstein correlations

How does BEcondensation (and BE correlation coherence) occur for attractively interacting bosons?

1. Bosons with attractive mutual interactions generate a mean field potential, which increases with increasing boson density $\rho(r)$

$$V(r) = -\frac{2\pi}{m} f(0)\rho(r) \approx \frac{1}{2}\hbar\omega \left(\frac{r}{a}\right)^2$$

π - π interaction is attractive in the (S - wave S = 0) and (P - wave S = 1) channels

2. The length scale a is roughly the size of the nuclear radius in a heavy-ion collision. Therefore, $\hbar\omega$ of the underlying mean field increases with increasing density ρ of produced boson

3. $T/\hbar\omega$ dereases with increasing density ρ of produced bosons

4. Higher density of produced bosons $\Rightarrow$ lower values of $T/\hbar\omega$

$\Rightarrow$ greater Bose-Einstein condensation fraction f_0

$\Rightarrow$ greater coherence in Bose-Einstein correlations

We need a theory of Bose-Einstein condensation

Strategy:

1. Place N identical bosons in a harmonic oscillator potential and see how the occupation numbers at different states changes as a function of the temperature T.
2. Bose-Einstein condensation occurs when the occupation number $N_{\mathbf{0}}$ for the lowest state (the condensate state) is a substantial fraction of the total particle number N.
   We study $f_0=N_{\mathbf{0}}/N$ as a function of $T/\hbar\omega$
3. The degree of coherence and chaoticity will change as a function of the $T/\hbar\omega$

C. Y. Wong & W.N.Zhang, Phys. Rev. C76,034905 (‘07)
J. Liu, P. Ru, W.N.Zhang, C.Y. Wong, (J.Phys.G, in press)

BE condensation, canonical, and grand canonical ensemble

Politzer, PRA 54, 5048 (1996)

- In a grand canonical ensemble, we fix μ and T, and we allow the occupation number N_n to vary. We obtain the average occupation number $N_n(\mu,T)=\left\langle \hat{a}_n^+ \hat{a}_n \right\rangle$. The square fluctuation of N_n is given by

$$\left\langle \left(\hat{a}_n^+ \hat{a}_n - \left\langle \hat{a}_n^+ \hat{a}_n \right\rangle \right)^2 \right\rangle \approx N_n \left(N_n + 1 \right)$$

- We cannot treat the lowest $n=0$ state in the grand canonical ensemble.
- We can treat the lowest $n=0$ state in a canonical ensemble of a fixed total N system, and the rest $n \neq 0$ states in a grand canonical ensemble.

Bose-Einstein condensation in a harmonic oscillator potential

Consider N bosons in a potential $\quad V(r) = \frac{1}{2} m\omega^2 r^2$

In a canonical ensemble, the total number of bosons $= N = N_0 + N_T$ is fixed.

The condensate configuration condition is

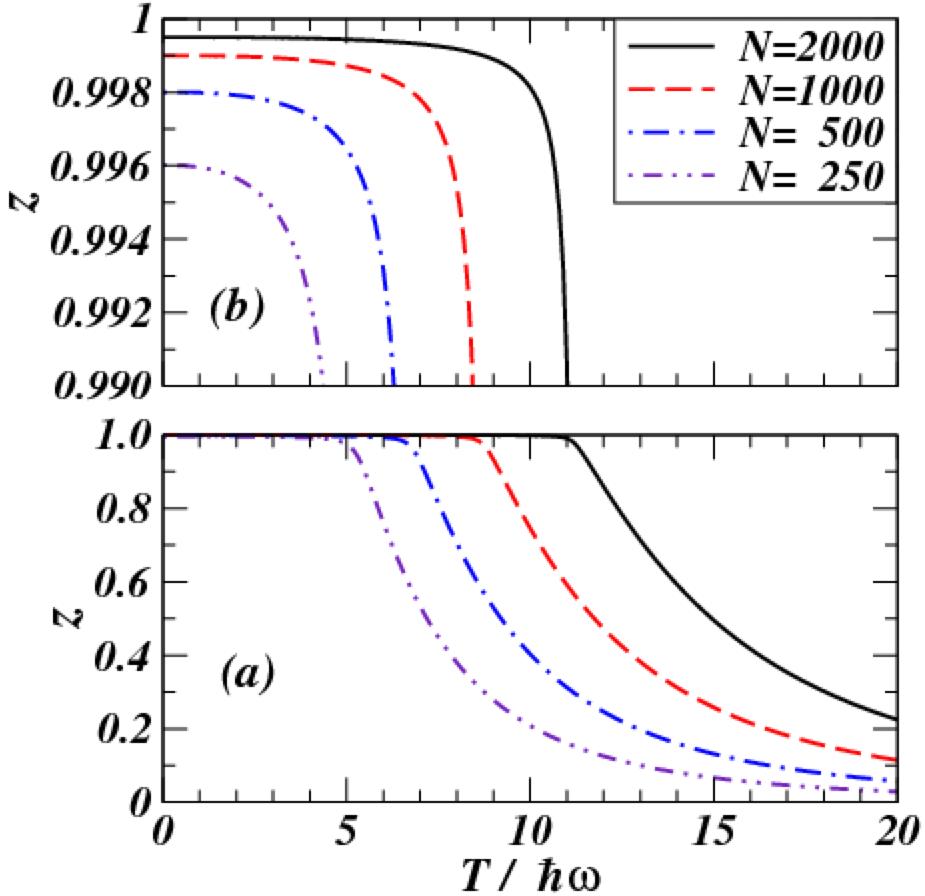
$$N = \frac{z}{1-z} + \sum_{n=1,2,..}^{\infty} \frac{g_n \, z \, e^{-\tilde{\varepsilon}_n/T}}{1 - z \, e^{-\tilde{\varepsilon}_n/T}}, \qquad z = e^{\mu/T} = \text{fugacity}$$

z can be solved as a function of N and $T/\hbar\omega$:

$$\varepsilon_n = (n + 3/2)\hbar\omega, \qquad \tilde{\varepsilon}_n = \varepsilon_n - \varepsilon_0 = n\,\hbar\,\omega,$$

$$g_n = \text{degeneracy} = (n+1)(n+2)/2$$

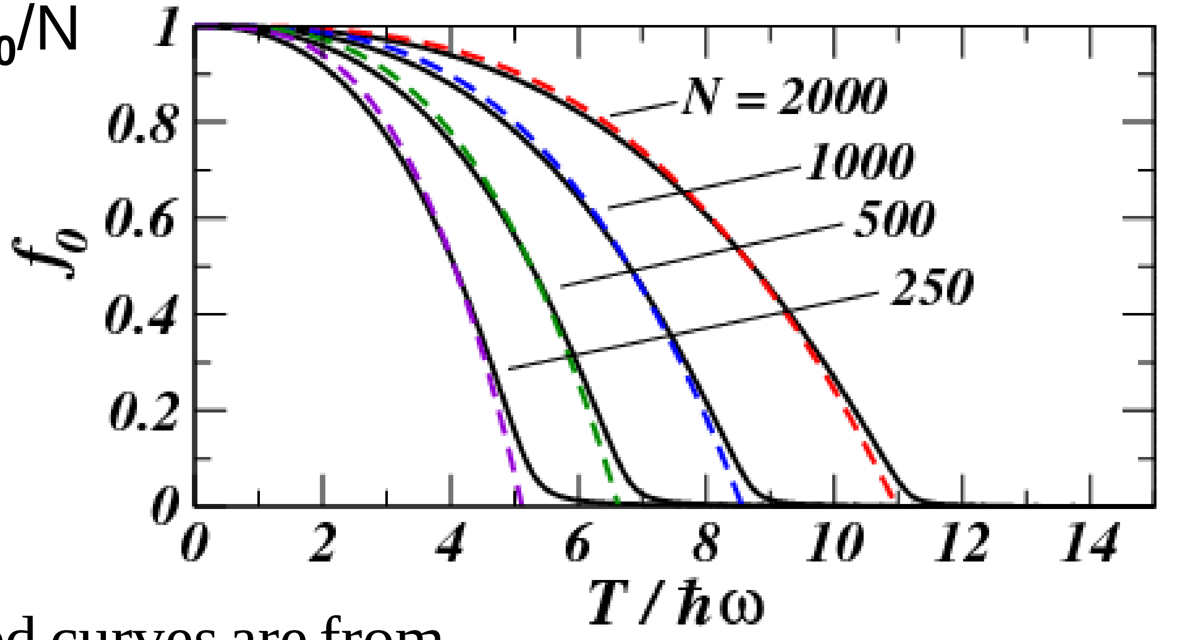
Fugacity z close to 1 corresponds to a large condensate occupation number N_0.



Fugacity z close to 1 corresponds to a large condensate fraction f_0.

$$N_0 = \frac{z}{1-z}\ , \qquad f_0 = \frac{N_0}{N}$$

$f_0 = N_0/N$



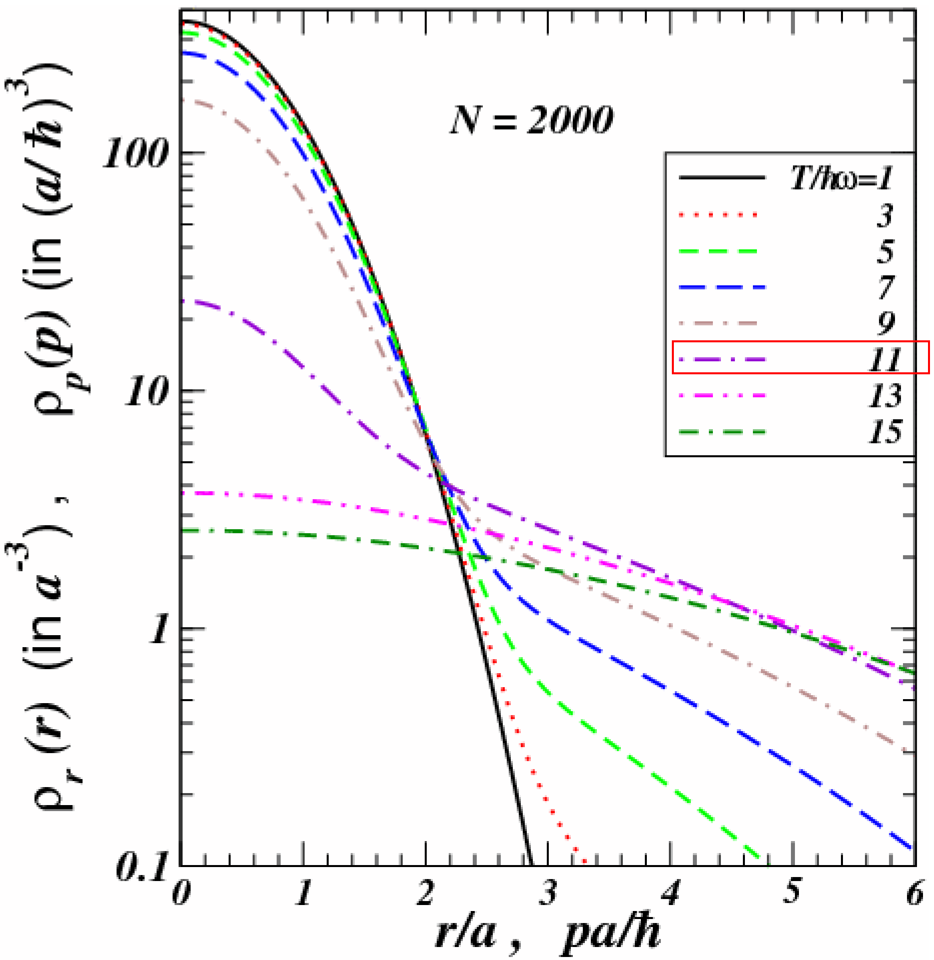
Dashed curves are from the paramtrization

$$f_0(T) = 1 - \left(\frac{T}{T_c}\right)^3$$

The Bose-Einstein condensation transition is a gradual phase transition.

N	$T_c / \hbar\omega$
2000	10.97
1000	8.56
500	6.63
250	5.12

Spatial & momentum densities of the bosons

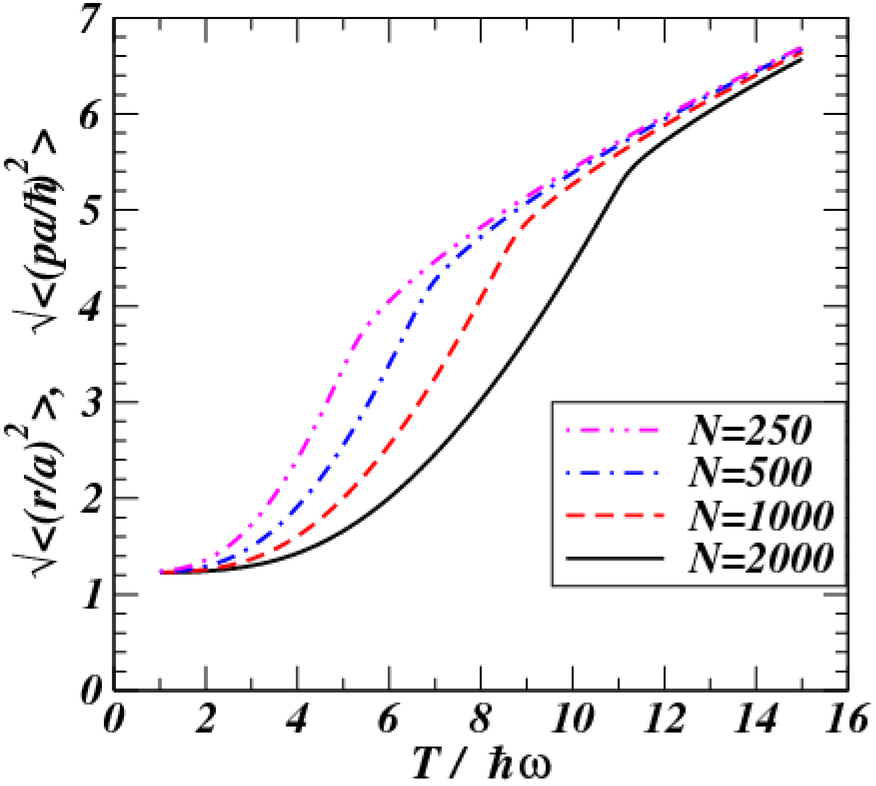


$T_c / \hbar\omega \approx 11$

for $N = 2000$

Density changes abruptly across the condensate temperature T_c

Root-mean-squared radii of the boson system



$T_c / \hbar\omega \approx 11$

for $N = 2000$

Revelance to RHIC & LHC heavy-ion collisions

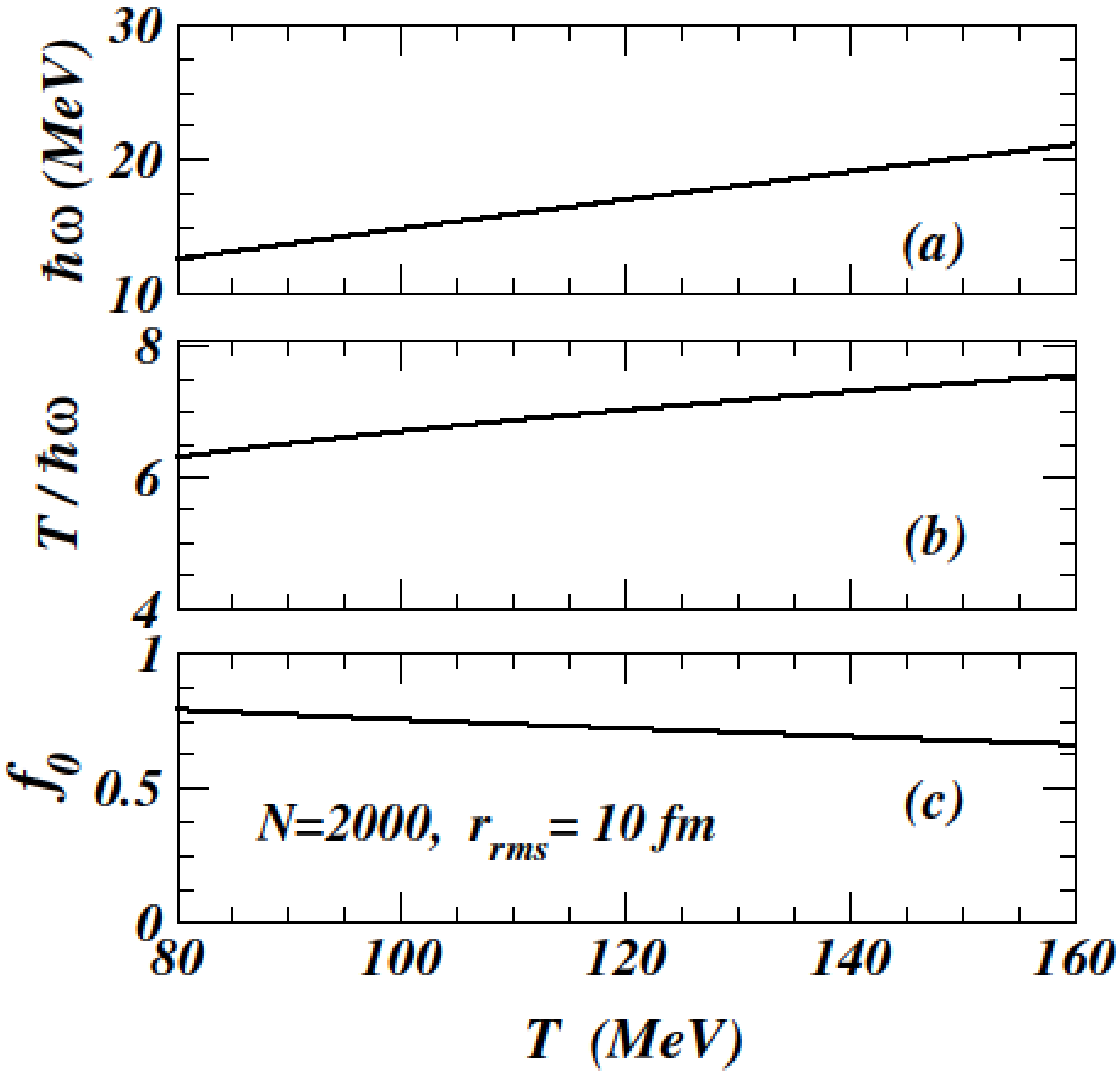
If we have a pion system with

- a root-mean squared radius r_{rms}=10 fm
- a number of 2000 identical pions
- a temperature 80 to 160 MeV

then, what is the condensate fraction f_0 for such a system?

Properties of a non-relativistic pion gas with r_{rms}=10 fm and N=2000

Condensate Fraction f_0



Relation between BE condesation and BE Correlation

We have $N_n = \left\langle \hat{a}_n^+ \hat{a}_n \right\rangle$ and $G^{(1)}(p_1, p_2) = \sum_{n=0}^{\infty} u_n^*(p_1)\, u_n(p_1) \left\langle \hat{a}_n^+ \hat{a}_n \right\rangle$

$$G^{(2)}(p_1, p_2; p_1, p_2) = \sum_{klmn} u_k^*(p_1)\, u_l^*(p_2)\, u_m(p_2) u_n(p_1) \left\langle \hat{a}_k^+ \hat{a}_l^+ \hat{a}_m \hat{a}_n \right\rangle$$

We can re-arrange the above

$$G^{(2)}(p_1, p_2; p_1, p_2) = G^{(1)}(p_1, p_1) G^{(1)}(p_2, p_2) + |G^{(1)}(p_1, p_2)|^2$$

$$+ \sum_{n=0}^{\infty} |u_n(p_1)|^2 |u_n(p_2)|^2 \left\{ \left\langle \hat{a}_n^+ \hat{a}_n^+ \hat{a}_n \hat{a}_n \right\rangle - 2 \left\langle \hat{a}_n^+ \hat{a}_n \right\rangle \left\langle \hat{a}_n^+ \hat{a}_n \right\rangle \right\}$$

We split the last sum into two parts:

$$|u_0(p_1)|^2 |u_0(p_2)|^2 \left\{ \left\langle \hat{a}_0^+ \hat{a}_0^+ \hat{a}_0 \hat{a}_0 \right\rangle - 2 \left\langle \hat{a}_0^+ \hat{a}_0 \right\rangle \left\langle \hat{a}_0^+ \hat{a}_0 \right\rangle \right\} \leftarrow n = 0 \text{ contribution}$$

$$+ \sum_{n=1}^{\infty} |u_n(p_1)|^2 |u_n(p_2)|^2 \left\{ \left\langle \hat{a}_n^+ \hat{a}_n^+ \hat{a}_n \hat{a}_n \right\rangle - 2 \left\langle \hat{a}_n^+ \hat{a}_n \right\rangle \left\langle \hat{a}_n^+ \hat{a}_n \right\rangle \right\} \leftarrow n = 1,2,3 \ldots \text{ contributions}$$

States with $n = 1,2,3,..$ are a grand canonical ensemble,

$$\left\langle \hat{a}_n^+ \hat{a}_n^+ \hat{a}_n \hat{a}_n \right\rangle - \quad \left\langle \hat{a}_n^+ \hat{a}_n \right\rangle \left\langle \hat{a}_n^+ \hat{a}_n \right\rangle \approx \left\langle \hat{a}_n^+ \hat{a}_n \right\rangle \left(\left\langle \hat{a}_n^+ \hat{a}_n \right\rangle + 1 \right)$$

$$\left\langle \hat{a}_n^+ \hat{a}_n^+ \hat{a}_n \hat{a}_n \right\rangle - 2 \left\langle \hat{a}_n^+ \hat{a}_n \right\rangle \left\langle \hat{a}_n^+ \hat{a}_n \right\rangle \approx \left\langle \hat{a}_n^+ \hat{a}_n \right\rangle$$

$$\sum_{n=1}^{\infty} |u_n(p_1)|^2 |u_n(p_2)|^2 \left\{ \left\langle \hat{a}_n^+ \hat{a}_n^+ \hat{a}_n \hat{a}_n \right\rangle - 2 \left\langle \hat{a}_n^+ \hat{a}_n \right\rangle \left\langle \hat{a}_n^+ \hat{a}_n \right\rangle \right\} \text{ can be neglected}$$

The BE correlation function in BE condensation

$$G^{(2)}(p_1,p_2;p_1,p_2)=G^{(1)}(p_1,p_1)G^{(1)}(p_2,p_2)+|G^{(1)}(p_1,p_2)|^2$$
$$+|u_0(p_1)|^2|u_0(p_2)|^2\left\{\left\langle\hat{a}_0^+\hat{a}_0^+\hat{a}_0\hat{a}_0\right\rangle-2\left\langle\hat{a}_0^+\hat{a}_0\right\rangle\left\langle\hat{a}_0^+\hat{a}_0\right\rangle\right\}$$

For states with $n=0$ in a canonical ensemble

$$\left\langle\hat{a}_0^+\hat{a}_0^+\hat{a}_0\hat{a}_0\right\rangle-2\left\langle\hat{a}_0^+\hat{a}_0\right\rangle\left\langle\hat{a}_0^+\hat{a}_0\right\rangle\approx\left\langle\hat{a}_0^+\hat{a}_0\right\rangle\left\langle\hat{a}_0^+\hat{a}_0\right\rangle-2\left\langle\hat{a}_0^+\hat{a}_0\right\rangle\left\langle\hat{a}_0^+\hat{a}_0\right\rangle+O\left(\left\langle\hat{a}_0^+\hat{a}_0\right\rangle\right)$$
$$\approx-\left\langle\hat{a}_0^+\hat{a}_0\right\rangle\left\langle\hat{a}_0^+\hat{a}_0\right\rangle=-N_0^2$$

$$G^{(2)}(p_1,p_2;p_1,p_2)=G^{(1)}(p_1,p_1)G^{(1)}(p_2,p_2)+|G^{(1)}(p_1,p_2)|^2-N_0^2|u_0(p_1)|^2|u_0(p_2)|^2$$

$$C(p_1,p_2)=\frac{G^{(2)}(p_1,p_2;p_1,p_2)}{G^{(1)}(p_1,p_1)G^{(1)}(p_2,p_2)}$$

$$C(p_1,p_2)=1+\frac{|G^{(1)}(p_1,p_2)|^2-N_0^2|u_0(p_1)|^2|u_0(p_2)|^2}{G^{(1)}(p_1,p_1)G^{(1)}(p_2,p_2)}$$

$$G^{(1)}(p_1,p_2)=\sum_{m=0}^{\infty}a_m u_m^*(p_1)u_m(p_2)$$

Bose-Einstein correlation function

$$C(p_1, p_2) = 1 + \frac{|G^{(1)}(p_1, p_2)|^2 - N_0^2 |u_0(p_1)|^2 |u_0(p_2)|^2}{G^{(1)}(p_1, p_2) G^{(1)}(p_1, p_2)}$$

(i) In a coherent source, $|G^{(1)}(p_1, p_2)|^2 \approx N_0^2 |u_0(p_1)|^2 |u_0(p_2)|^2$

$C(p_1, p_2) = 1$, and there is no Bose-Einstein correlation

(ii) In a chaotic source, $|G^{(1)}(p_1, p_2)|^2 >> N_0^2 |u_0(p_1)|^2 |u_0(p_2)|^2$

$$C(p_1, p_2) = 1 + \frac{|G^{(1)}(p_1, p_2)|^2}{G^{(1)}(p_1, p_2) G^{(1)}(p_1, p_2)}$$

and there is Bose-Einstein correlation

(iii) The λ parameter for $p_1 = p_2$ depends on $f_0 = \frac{N_0}{N}$ and $(p_1 + p_2)/2$

This is the BE correlation function for all situations: coherent, chaotic, and transition between coherent and chaotic systems.

Three-particle BE correlation function

$$G^{(3)}(p_1, p_2; p_1, p_2) = G^{(1)}(p_1, p_1)\left[G^{(1)}(p_2, p_2)G^{(1)}(p_3, p_3) + R^{(2)}(p_2, p_3)\right]$$
$$+ G^{(1)}(p_2, p_2)\left[G^{(1)}(p_3, p_3)G^{(1)}(p_1, p_1) + R^{(2)}(p_3, p_1)\right]$$
$$+ G^{(1)}(p_3, p_3)\left[G^{(1)}(p_1, p_1)G^{(1)}(p_2, p_2) + R^{(2)}(p_1, p_2)\right]$$
$$+ R^{(3)}(p_1, p_2, p_3)$$

where

$$G^{(1)}(p_1, p_2) = \sum_{m=0}^{\infty} a_m u_m^*(p_1) u_m(p_2)$$

$$R^{(2)}(p_1, p_2) = |G^{(1)}(p_1, p_2)|^2 - N_0^2 |u_0(p_1)|^2 |u_0(p_2)|^2$$

$$R^{(3)}(p_1, p_2, p_3) = 3\left[\left|G^{(1)}(p_1, p_2)G^{(1)}(p_2, p_3)G^{(1)}(p_3, p_1)\right|^2 - N_0^3 |u_0(p_1)|^2 |u_0(p_2)|^2 |u_0(p_3)|^2\right]$$

(i) In a coherent source, $|G^{(1)}(p_1, p_2)|^2 \approx N_0^2 |u_0(p_1)|^2 |u_0(p_2)|^2$

$$|G^{(1)}(p_1, p_2)G^{(1)}(p_2, p_3)G^{(1)}(p_3, p_1)|^2 \approx N_0^3 |u_0(p_1)|^2 |u_0(p_2)|^2 |u_0(p_3)|^2$$

$R^{(2)}(p_1, p_2) = 0$, $R^{(3)}(p_1, p_2, p_3) = 0$, and there is no Bose - Einstein correlation

(ii) In a chaotic source, $|G^{(1)}(p_1, p_2)|^2 >> N_0^2 |u_0(p_1)|^2 |u_0(p_2)|^2$

$$|G^{(1)}(p_1, p_2)G^{(1)}(p_2, p_3)G^{(1)}(p_3, p_1)|^2 >> N_0^3 |u_0(p_1)|^2 |u_0(p_2)|^2 |u_0(p_3)|^2$$

and there is Bose - Einstein correlation

Numerical evaluation of the BE correlation

$$C(p,q) \equiv C(p_1,p_2) = 1 + \frac{|G^{(1)}(p_1,p_2)|^2 - N_0^2 |u_0(p_1)|^2 |u_0(p_2)|^2}{G^{(1)}(p_1,p_1) G^{(1)}(p_2,p_2)}$$

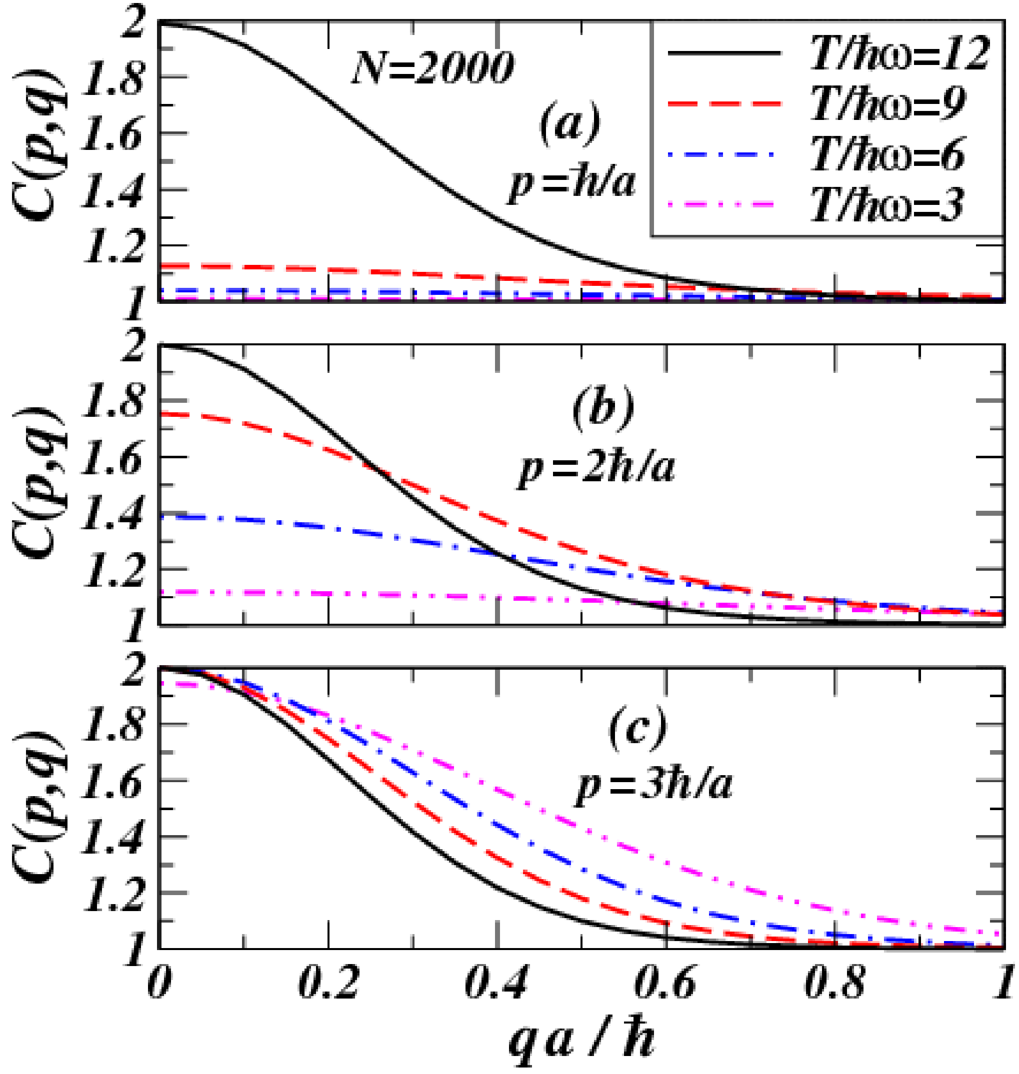
$G^{(1)}(p_1,p_2),\ \ u_0(p_1)$ can be obtained explicitly for a harmonic oscillator:

Naraschewski & Glauber, PRA 59, 4595('99)

$$G^{(1)}(p_1,p_2) = \sum_{k=1}^{\infty} z^k \tilde{G}_0(p_1,p_2;k\beta\hbar\omega)$$

$$\tilde{G}_0(p_1,p_2;\tau) = \left(\frac{a^2}{\pi\hbar^2 (1-e^{-2\tau})} \right)^{3/2} \exp\left(-\frac{a^2}{\hbar^2} \frac{(p_1^2+p_2^2)(\cosh\tau - 1) + (p_1 - p_2)^2}{2\sinh\tau} \right)$$

$$u_0(p) = \left(\frac{a^2}{\pi\hbar^2} \right)^{3/4} \exp\left\{ -\frac{a^2}{\hbar^2} \frac{p^2}{2} \right\}$$



$T_c / \hbar\omega \approx 11$

for $N = 2000$

C(p,q;T) is a senstitive function of p and T.

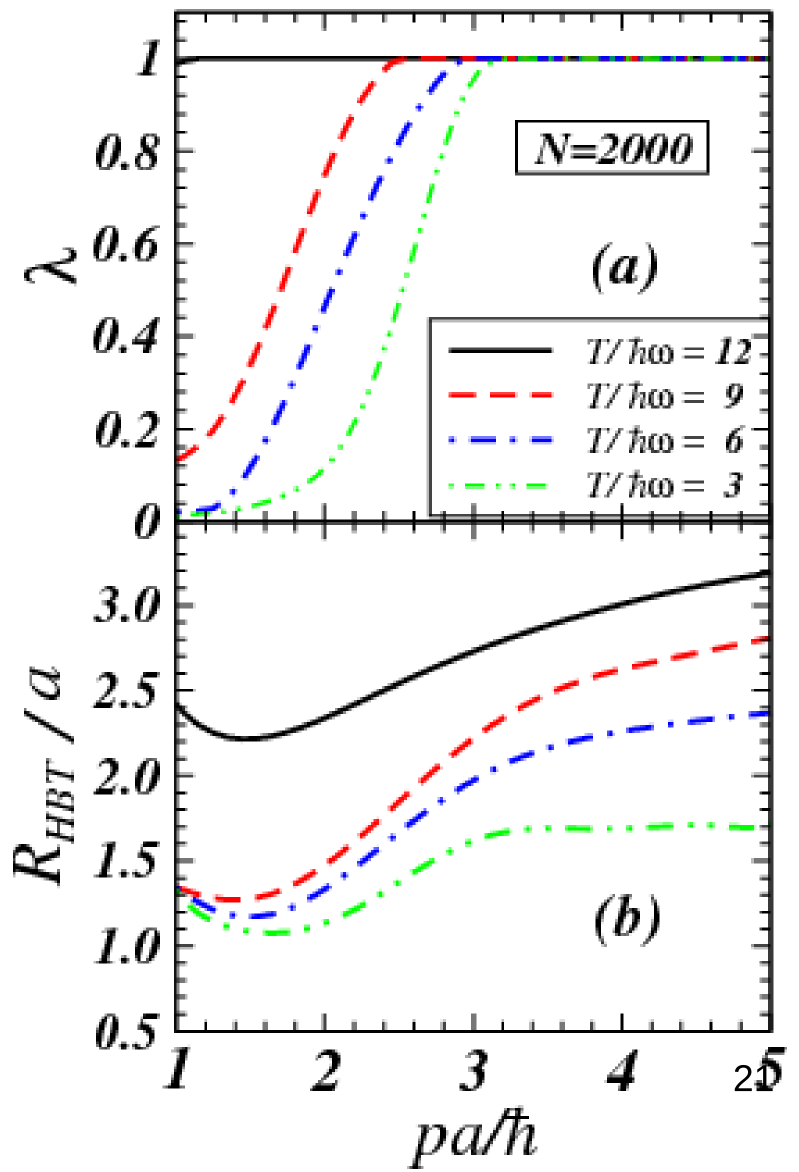
$$T_c / \hbar\omega \approx 11$$

for $N = 2000$

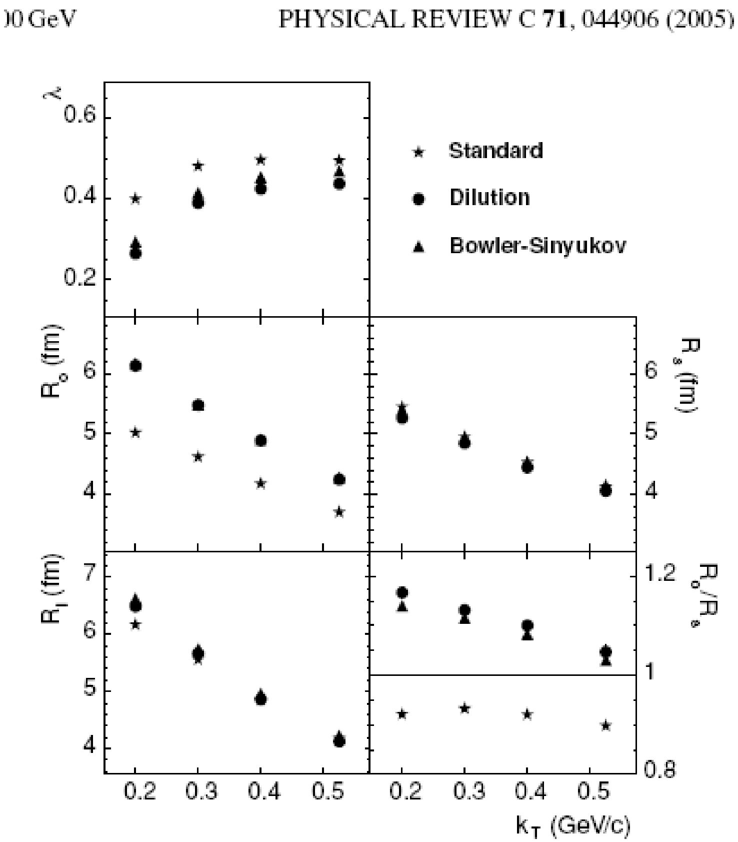
$$C(p,q;T) = 1 + \lambda(p,T)\exp\left\{- R_{\mathrm{HBT}}^2(p,T)\, q^2\right\}$$

$$R_{\mathrm{HBT}}(p,T) = \sqrt{\frac{3}{2}}\,\frac{\hbar}{q_{\mathrm{rms}}(p,T)}$$

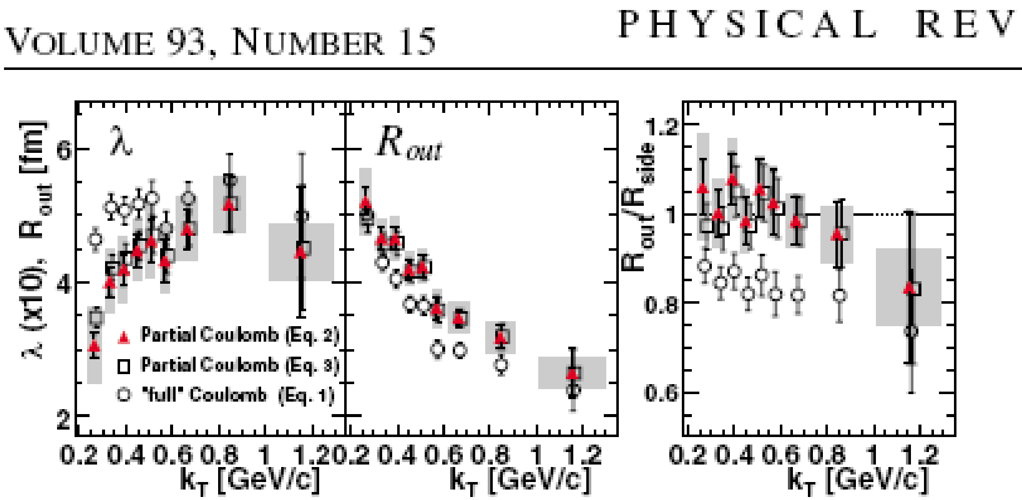
λ increases with increasing p



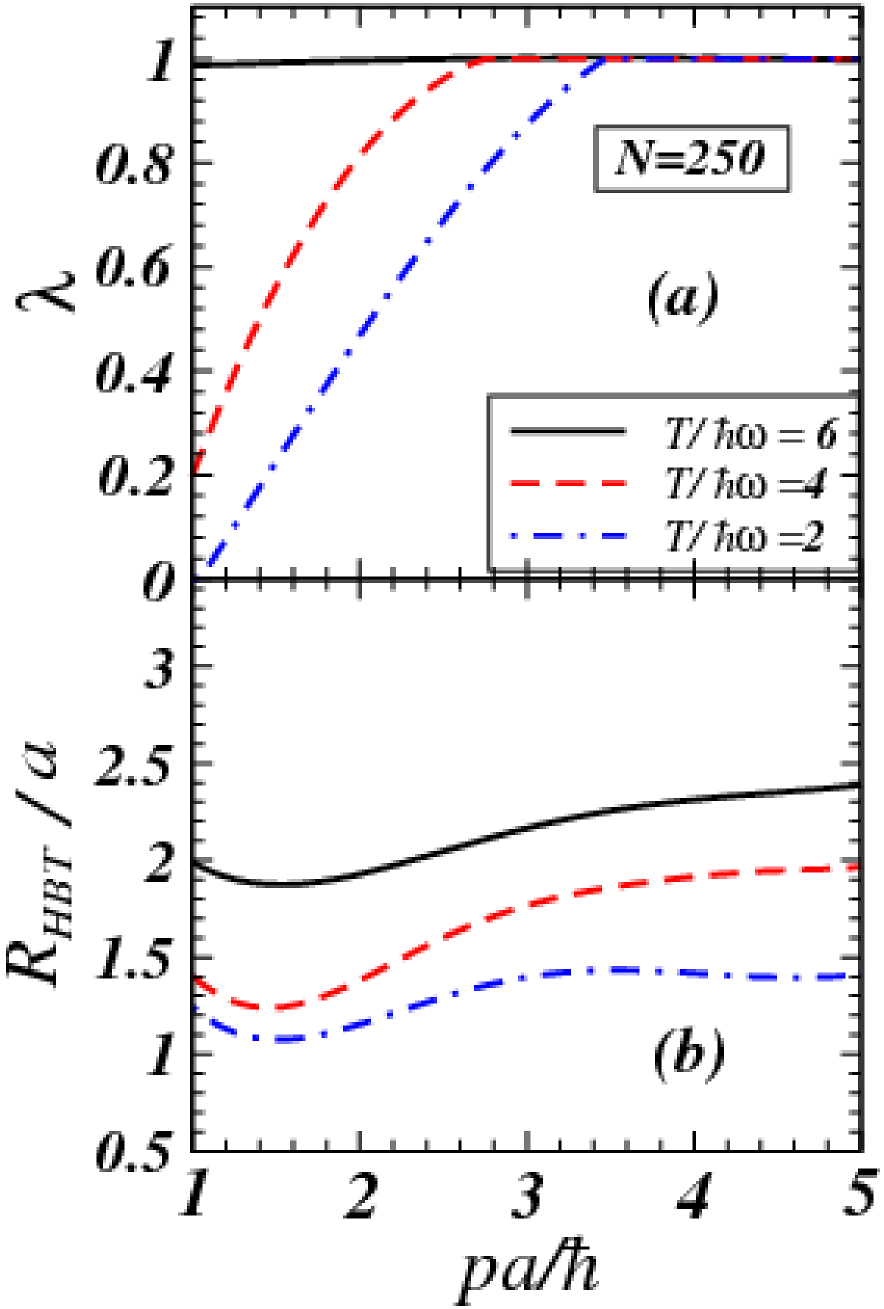
Experimental λ increases with k_T



STAR Au-Au at 200 GeV
PRC 71, 044906 (2005)



PHENIX Au-Au at 200 GeV
PRL 93, 152302 (2004)



$T_c / \hbar\omega \approx 5.1$

for $N = 250$

Conclusions

- A proper framework to study Bose-Einstein correlations is through the theory of Bose-Einstein condensation
- Bose-Einstein condensation leads to coherence and suppression of Bose-Einstein correlations
- The condensate fraction f_0 and coherence depends on
  N, T/ħω and ħω depends on boson density
- A pion gas with the r_{rms},T, and N, typical of those in RHIC and LHC, contains a large condensate fraction f_0 and a high degree of suppression of Bose-Einstein correlation