

A Monte Carlo Study of Multiplicity Fluctuations in Pb–Pb Collisions at LHC Energies

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Association

with **Rohni Sharma**

(University of Jammu)

And helpful advice from **R.C. Hwa**

(University of Oregon)

Plan

- Introduction & Motivation
 - Intermittency
 - Erraticity
- MC Data Analysed
- Methodology & Analysis
- Observations
- Summary

Introduction Motivation

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- Dynamics of the initial processes in the HICs affects the final distribution of the particles produced

S. Gavin & George Moschelli, PRC 85,014905 (2012)

- **Charged particle multiplicity fluctuations**
is one of the sensitive probes of the properties of the system.
- Multiplicity distributions can be analysed in terms of its **Moments**.
- **Factorial Moments**
are one of the convenient tools to study multiplicity fluctuations.
 - with the advantage that it filter out the statistical fluctuations.

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● Factorial Moments

$$F_q(\delta^d) = \frac{\langle n!/(n-q)! \rangle}{\langle n \rangle^q}$$

- first used by A. Bialas and R. Peschanski¹ to explain unexpectedly large fluctuations in high multiplicity events recorded by the JACEE Collaboration.
- where n is bin multiplicity, δ^d bin-size in a d -dimensional phase-space
- and $\langle \dots \rangle$ is over all events (Vertical Moments -- \rightarrow Horizontally averaged Vertical Moments) or bins in an event (Horizontal Moments -- \rightarrow Vertically averaged horizontal moments).
- Suggested as measure to study the fluctuations that are independent of the scale of the bin sizes (δ).
- A power-law

$$F_q(\delta) \propto \delta^{-\varphi_q} \quad \text{over a range of small } \delta$$

or

$$F_q(M) \propto M^{\varphi_q} \quad \text{over number of bins } M \ (M \propto 1/\delta)$$

is termed as **intermittency**

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Introduction & Motivation..

However

- The method of the factorial moments F_q does not fully account for all the fluctuations that the system exhibits.
- Vertically averaged horizontal moments, can gauge the spatial fluctuations, neglecting the event space fluctuations
- On the other hand, horizontally averaged vertical moments lose information about spatial fluctuation and only measure the fluctuations from event-to-event

Improvement

Hwa and Cao* proposed new moments:

Moments of Factorial Moment distributions

- Take into account the spatial fluctuations
- as well as the event space fluctuations.

Referred to as the **Erraticity Analysis of Fluctuations**.

* Z. Cao, R. Hwa, *Phys. Rev. Lett.* **75**, 1268 (1995), *PRD* 53, 6608(1996); *PRD*

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Erraticity..

- In contrast to Sample factorial moments, Cao and Hwa defined event factorial moments as

$$F_q^e(M) = \frac{f_q^e(M)}{[f_1^e(M)]^q}$$

so as to have $P(F_q^e(M))$ distribution for a set of events where

$$f_q^e(M) = \langle n_m(n_m - 1) \dots (n_m - q + 1) \rangle_h$$

with n_m being the multiplicity in the m^{th} bin.

Only bins with $n_m \geq q$ are considered

and q is a positive integer ≥ 2

- $F_q^e(M)$: numerical value, that describes the pattern of distribution of produced particles of the e^{th} event.
- Deviation of $F_q^e(M)$ from $\langle F_q^e(M) \rangle_v$ for each event can be defined as

$$\Phi_q(M) = \frac{F_q^e(M)}{\langle F_q^e(M) \rangle_v}$$

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Erraticity

- Whereas double moments ;
For the vertical p^{th} order moments of the normalised q^{th} order factorial(horizontal) moments can be calculated as

$$C_{p,q}(M) = \langle \Phi_q^p(M) \rangle_v$$

p is a positive real number and $\Phi_q^p(M)$ is

$$\phi_q^p(M) = \frac{[F_q^e(M)]^p}{\langle F_q^e(M) \rangle_v^p}$$

- And the power law behaviour

$$C_{p,q}(M) \propto M^{\psi_q(p)}$$

is referred to as *erraticity* and from this one can define a measure **erraticity index** defined as

$$\mu_q = \frac{d\psi_q(p)}{dp}$$

- independent of M
characterizes the fluctuations of spatial patterns.
quantifies the degree of fluctuations in the generated data.

Advantages at LHC energies

- To understand the subject of dynamical fluctuations both these measure have been used to analyse the experimental data from different collision systems
- However at RHIC and lower energies, the total multiplicities are not enough to avoid substantial averaging that erases some aspects of the fluctuations
- Measures rely on the large bin multiplicities.
- At LHC energies, where multiplicities are higher, it is possible to have detailed study of the local properties in (η, ϕ) space for narrow p_T bins.
- To explore the dynamical properties of the system created in the HIC at these energies with these least studied tools (in the recent past)

Motivation

Intermittency

*The footprints of a **Phase Transition** that has fluctuations of all scales may then be possibly observed in the measurement of intermittency.**

* A. Bialas and R.C. Hwa, Phys. Lett B 253, 436 (1991).

- A study of the second order PT, in Ginzburg-Landau theory by Hwa and Nazirov** found that F_q satisfies power-law behavior

$$F_q \propto F_2^{\beta_q}, \quad \text{referred to as F-scaling}$$

$$\beta_q = (q - 1)^\nu, \quad \nu = 1.304 .$$

- Quark-hadron PT simulated in 2D Ising Model, is found to be in agreement***

** R.C. Hwa and M.T. Nazirov, Phys. Rev. Lett. 69, 741 (1992).

*** Z. Cao, Y. Gao and R.C. Hwa, Z Phys. C 72, 661 (1996).

Erraticity

- Hwa and Yang in their recent work* proposed erraticity index as a measure to distinguish different criticality classes.
- A definitive value of μ from the LHC is called for deeper understanding of the hadronization process.
- For order parameter 4 μ is found to distinguish the four different cases viz., critical, pseudocritical, quasicritical and non-critical.
- $\mu_4 = 1.87 \pm 0.84^*$ for the critical case and $\mu_4 = 4.65 \pm 0.06$ for non-critical case.

* PRC 85, 044914 (2012)

A MultiPhase Transport Model

- In an attempt to develop and understand the methodology we have studied the charged particles generated at midrapidity in Pb–Pb collisions at $\sqrt{s} = 2.76$ TeV using AMPT model (A Multi-Phase Transport Model)*
- Also since this has been quite useful in understanding recent experimental results**

* PRC 61, 067901 (2000), PRC 64, 011902 (2001)

** PRC 84, 044907, (2011)

AMPT

- Hybrid model
- and consists of four main parts
initial conditions, partonic interactions, hadronization and hadron rescattering.
- The initial conditions; are obtained from Heavy Ion Jet Interaction Generator (HIJING) model. There are two versions of the AMPT model
 - default (DF) and
 - string melting(SM)depending on how the partons are hadronized.

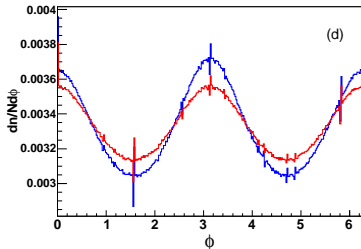
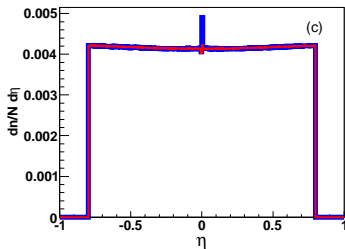
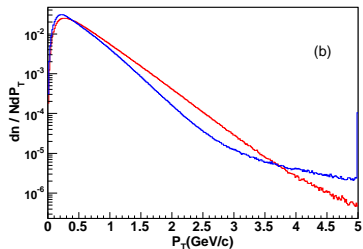
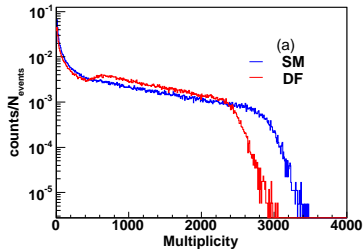
A MultiPhase Transport Model

Data

- Analysis is done for Data generated using both the versions
- with parameters $a = 2.2$, $b = 0.5$, $\mu = 1.8$ and $\alpha = 0.47^*$
- $|\eta| \leq 0.8$ with full azimuthal coverage.
- 5 p_T bins (≤ 1 GeV/c and $\Delta p_T = 0.1$ GeV/c).
- $0.2 \leq p_T \leq 0.3$
- $0.3 \leq p_T \leq 0.4$
- $0.4 \leq p_T \leq 0.5$
- $0.6 \leq p_T \leq 0.7$
- $0.9 \leq p_T \leq 1.0$

One should not integrate over all p_T because the superposition of different patterns at different Δp_T intervals can smear out all recognizable features.

* arXiv:1210.0512v3(2013)



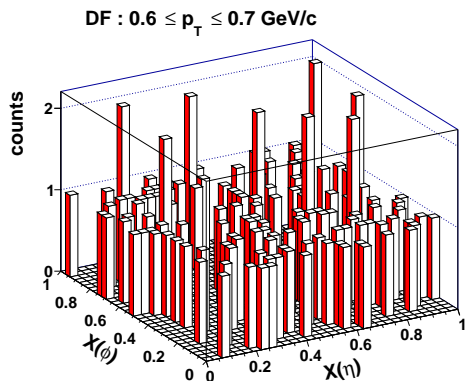
The charged particle data generated by MC:

a) Multiplicity distribution (b) p_T (c) η and (d) ϕ distributions, for DF(red) and SM (blue) version.

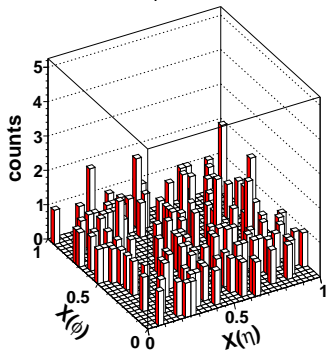
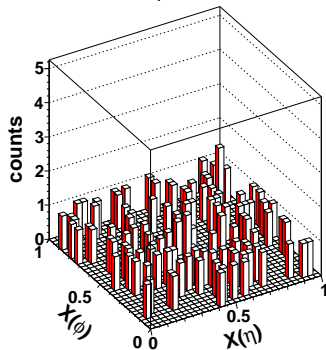
Methodology & Analysis

- The single-particle density distribution in η and ϕ is non flat and thus is converted to the cumulative variable* $X(\eta)$ and $X(\phi)$.
- $(X(\eta), X(\phi))$ phase space of an event in the selected p_T window is binned in a square matrix with M^2 cells.
- M depends on the multiplicity in the Δp_T interval, so that the important part of the M dependence is captured (M : 2 to 32)

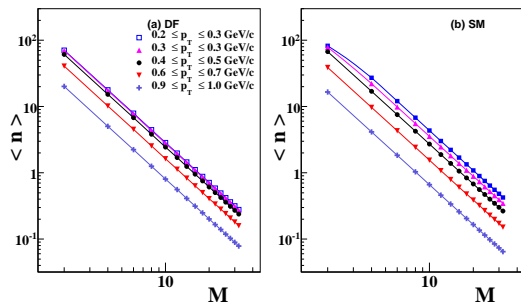
* W. Ochs, Z Phys. C 50, 339 (1991).



$(X(\eta), X(\phi))$ phase space of an event with $M = 32$

(a) DF : $0.6 \leq p_T \leq 0.7$ GeV/c(b) SM : $0.6 \leq p_T \leq 0.7$ GeV/c

$(X(\eta), X(\phi))$ phase space of an event with $M = 32$ in DF and SM case



This figure shows the bin multiplicity dependence on M for the five p_T bins, in case of DF and the SM AMPT model

p_T window	Default $\langle N \rangle$	String Melting $\langle N \rangle$
$0.2 \leq p_T \leq 0.3$	285.2	434.8
$0.3 \leq p_T \leq 0.4$	279.2	355.5
$0.4 \leq p_T \leq 0.5$	243.7	271.6
$0.6 \leq p_T \leq 0.7$	163.3	155.5
$0.9 \leq p_T \leq 1.0$	80.5	66.1

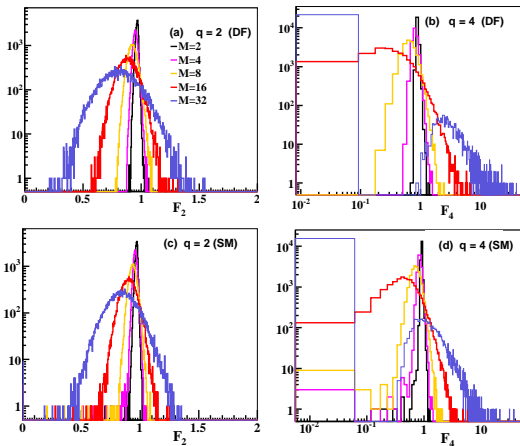
Table : Average Multiplicity of the Simulated Data sets analyzed in different p_T windows

DF: 23424 events with $b \leq 5$

SM: 19669 events with $b \leq 5$

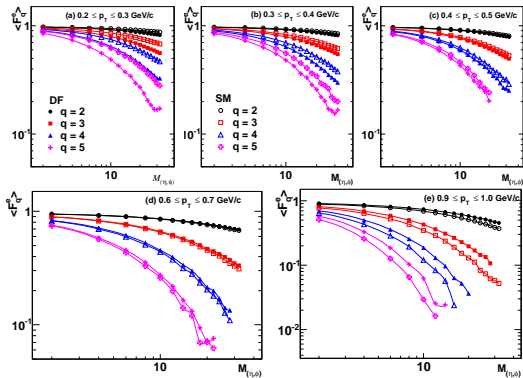
Methodology & Analysis

- Determine event factorial moments
- study M dependence and hence the binning resolution
- $F_q^e(M) = 1$: For Poissonian



- This figure shows $P(F_q^e)$ distributions for order moment $q = 2$ and $q = 4$ for DF and SM ($0.3 \leq p_T \leq 0.4$).
- (a) For $q = 2$ in DF (b) $q = 4$ in DF (c) $q = 2$ in SM and (d) $q = 4$ in SM. M values in multiples of 2 are shown only.
- Distributions become wider as M increases. And for higher q values i.e., 4, at higher M values long tails develop

M dependence of F_q^e



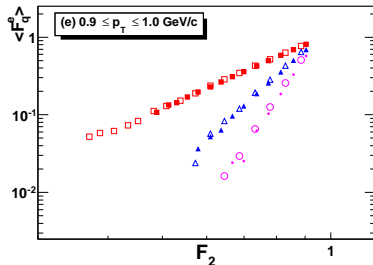
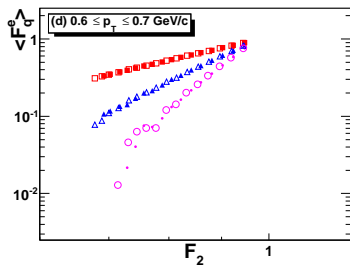
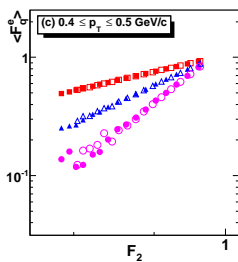
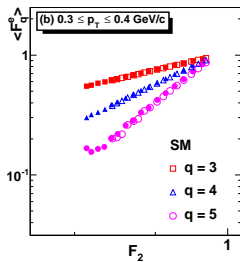
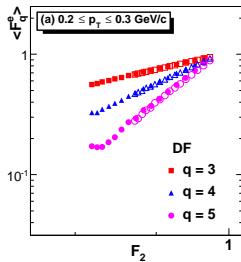
• M dependence of $\langle F_q \rangle$ in various p_T bins. Solid symbols for DF and open symbols for SM

• For $F_q(M)$, we observe $\varphi_q^- < 0$.

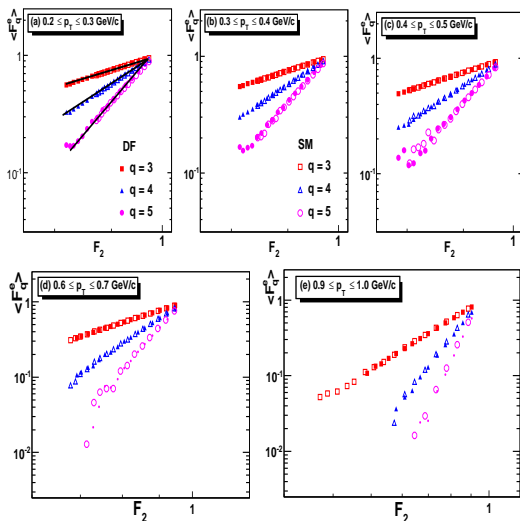
$$F_q^{\text{AMPT}}(M) \propto M^{\varphi_q^-},$$

- Negative intermittency.
- implying that the fluctuations are even less than Poissonian.
- AMPT : there are no rare high-multiplicity spikes anywhere in phase space.

F-Scaling...

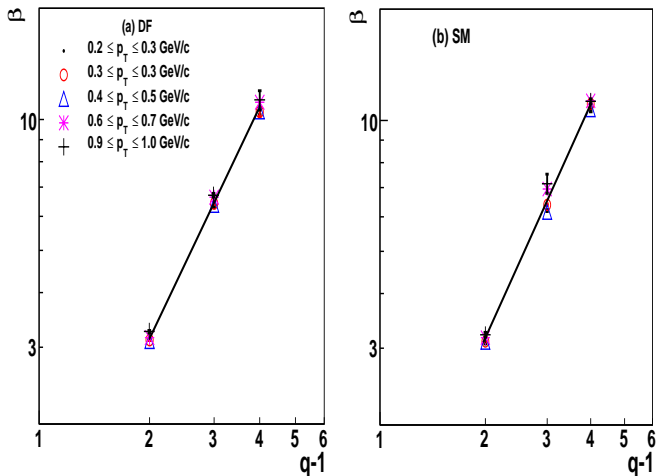


F-Scaling...II



- Linearity in log-log plots
- Linear Fits performed give negative scaling exponent ν_- , defined as $\beta_q^{AMPT} = (q - 1)^{\nu_-}$

β_q vs $(q-1)$



The slopes from these plots give the intermittency index ν

ν_-

p_T	ν_- (Default)	ν_- (String Melting)
$0.2 \leq p_T \leq 0.3$	1.738 ± 0.008	1.753 ± 0.004
$0.3 \leq p_T \leq 0.4$	1.774 ± 0.007	1.793 ± 0.005
$0.4 \leq p_T \leq 0.5$	1.758 ± 0.006	1.755 ± 0.006
$0.6 \leq p_T \leq 0.7$	1.824 ± 0.008	1.869 ± 0.016
$0.9 \leq p_T \leq 1.0$	1.778 ± 0.013	1.781 ± 0.011

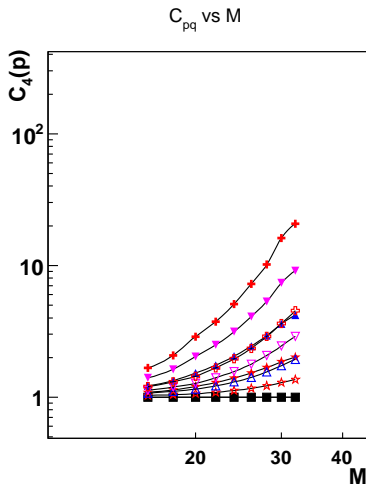
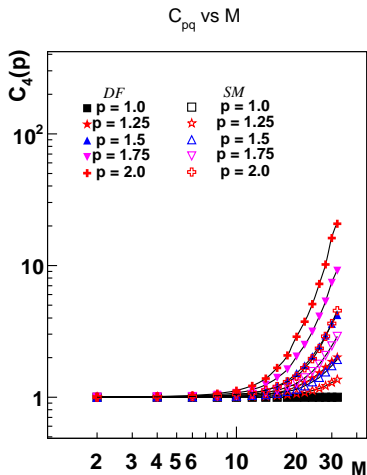
Table : Scaling exponents for negative intermittency in the Default and String Melting versions of the AMPT Model

 ν_-

No comparison with ν as it is fundamentally different from ν_- on account of the difference between the positivity of φ_q and negativity of φ_q^- .

M dependence of the $C_{p,q}$

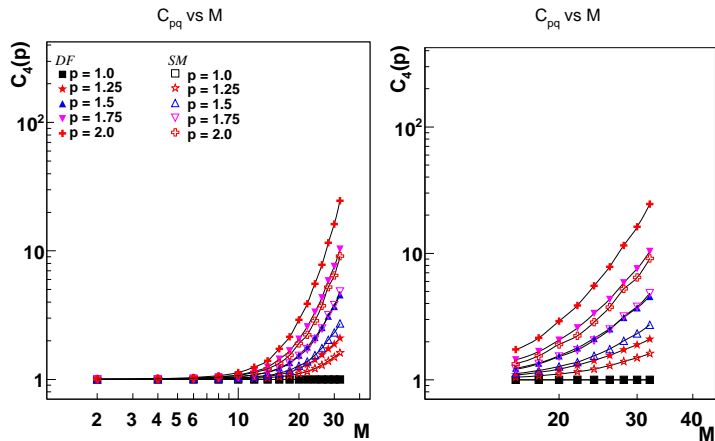
The double moments : Fluctuations in the fluctuations of the spatial patterns



DF and SM : $0.2 \leq p_T \leq 0.3$

M dependence of the $C_{p,q}$

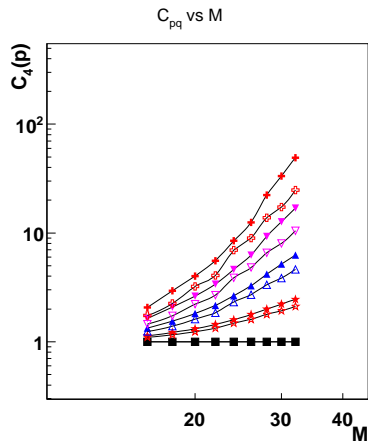
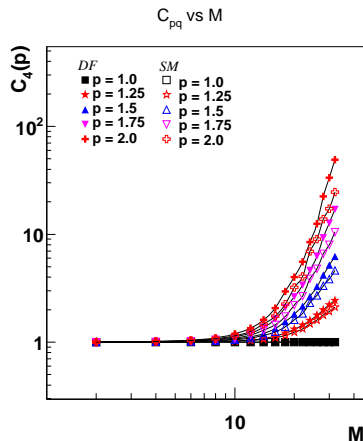
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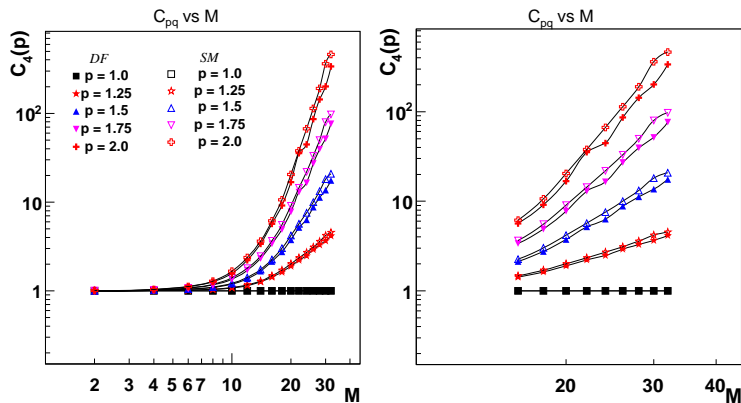
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DF and SM : $0.4 \leq p_T \leq 0.5$

M dependence of the $C_{p,q}$

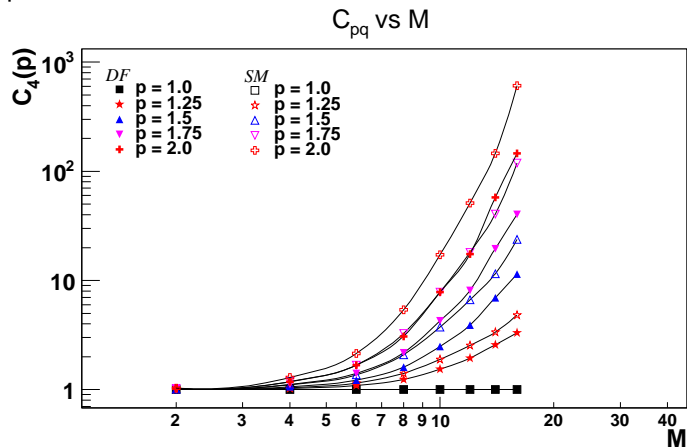
The double moments : Fluctuations in the fluctuations of the spatial patterns



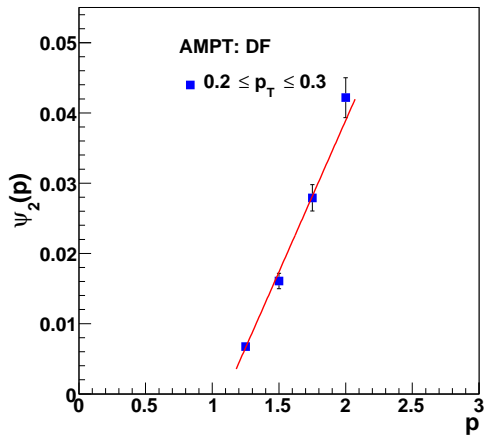
DF and SM : $0.6 \leq p_T \leq 0.7$

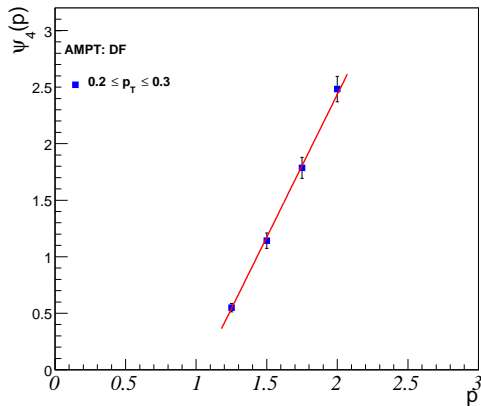
M dependence of the $C_{p,q}$

The double moments : Fluctuations in the fluctuations of the spatial patterns



DF and SM : $0.9 \leq p_T \leq 1.0$

μ_q Erraticity index μ_2 DF : $0.2 \leq p_T \leq 0.3$, $q = 2$

μ_q Erraticity index μ_4 DF : $0.2 \leq p_T \leq 0.3, q = 4$

Erraticity Index... μ

q	p_T	μ	μ
		(Default)	(String Melting)
2	$0.2 \leq p_T \leq 0.3$	0.043 ± 0.002	0.016 ± 0.001
	$0.3 \leq p_T \leq 0.4$	0.045 ± 0.002	0.027 ± 0.001
	$0.4 \leq p_T \leq 0.5$	0.062 ± 0.003	0.048 ± 0.002
	$0.6 \leq p_T \leq 0.7$	0.154 ± 0.008	0.174 ± 0.011
	$0.9 \leq p_T \leq 1.0$	0.739 ± 0.043	1.014 ± 0.064
4	$0.2 \leq p_T \leq 0.3$	4.325 ± 0.243	2.481 ± 0.183
	$0.3 \leq p_T \leq 0.4$	4.532 ± 0.234	3.385 ± 0.235
	$0.4 \leq p_T \leq 0.5$	5.478 ± 0.258	3.935 ± 0.021
	$0.6 \leq p_T \leq 0.7$	5.64 ± 0.203	6.101 ± 0.214
	$0.9 \leq p_T \leq 1.0$	7.484 ± 0.361	7.359 ± 0.305

Table : Erraticity index in DF and SM versions of the AMPT Model, for $q = 2$ and 4 only

Summary

- First attempt to do this study at LHC energy.
- Factorial Moments for the charged particles decrease with increasing bin number contrary to the usual properties of intermittency observed at low energies.
- Events with localization of even moderate multiplicities in the small bins are not observed in the AMPT
- Confirms the absence of PT in AMPT
- Scaling exponents (ν_- and μ_q 's) are determined
- Results can be used to compare with other models and experimental data values.
- A definitive value for μ_4 from experimental data can help uncover interesting physics.

Thank You

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