Bottomonium at Finite Temperature (from Lattice NRQCD)

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Outline

FASTSUM+ :

M.P. Lombardo (Frascati), D.K. Sinclair (Argonne), A. Aarts/C.R. Allton (Swansea), T. Harris/S.M. Ryan (Trinity), S. Kim (Sejong), J.-I. Skullerud (Maynooth)

+ A. Rothkopf (Heidelberg), Y. Burnier (Lausanne), S.J. Hands (Swansea)

• overall : PRL106 (2011) 061602
• S-wave : JHEP1111 (2011) 103
• SU(2) : PLB 711 (2012) 199
• moving S-wave : JHEP1303 (2013) 084
• P-wave : JHEP1312 (2013) 064
• $N_f = 2 + 1$: JHEP1407 (2014) 097
• finer scan T + $N_f = 2 + 1$: arXiv:1310.6461 + work in preparation (with P. Petreczky (BNL))
• + $\cdots$

Bottomonium in non-zero T?

T. Matsui and H. Satz, PLB178 (1986) 416

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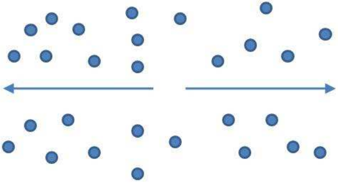


Bottomonium in non-zero T?

- Schroedinger eq.

$$i\frac{\partial\psi}{\partial t} = \mathcal{H}\psi \quad (1)$$

with $\mathcal{H} = 2M - \frac{\nabla^2}{2M} + V(r)$

- $T = 0$, e.g., Cornel potential;

$$V(r) = -\frac{\alpha}{r} + \sigma r \quad (2)$$

- $T \neq 0$, Debye screening;

$$V(r,T) = \frac{\sigma}{\mu(T)}(1 - e^{-\mu(T)r}) - \frac{\alpha}{r}e^{-\mu(T)r} \quad (3)$$

where $\mu(T) = 1/r_D(T)$

Bottomonium in non-zero T?

- F. Karsch, M.T. Mehr, and H. Satz, Z.Phys.C37 (1988) 617.

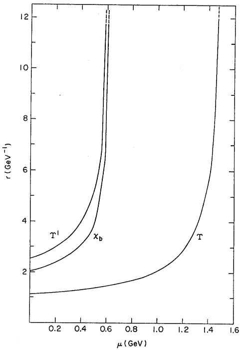


FIGURE 6

Bottomonium in non-zero T?

Let’s make it quantitative !

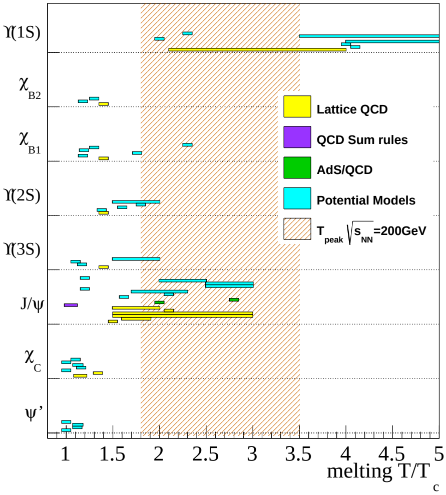
Bottomonium in non-zero T?

- which potential and what parameters should we use? (cf. e.g., P. Petreczky and A. Mocsy, PRD77 (2008) 014501)

- and potential in non-zero T has imaginary part (cf. M. Laine et al, JHEP 0703 (2007) 054)

- Debye screening and Landau damping

Bottomonium in non-zero T?



PHENIX, arXiv:1404.2246

heavy quark propagator in lattice NRQCD

Let's compute non-zero T quarkonium property using lattice QCD !

lattice QCD is a systematically improvable and first principles calculation method

heavy quark propagator in lattice NRQCD

- obtain finite temperature heavy quark potential by lattice calculation $\rightarrow$ solve Schroedinger equation

- obtain spectral function of heavy meson correlator by lattice calculation $\rightarrow$ observe temperature modification of spectrum

- study heavy quarkonium correlators in finite temperature by lattice NRQCD and obtain their NRQCD spectral functions

heavy quark propagator in lattice NRQCD

- various scale on a lattice: lattice cutt-off (a), spacetime volume ($L^3 \times T$), quark mass (M)

- for bottom quark, difficult to satisfy $1/a >> M >> 1/L$ at the same time $\rightarrow$ Effective Field Theory (EFT)

- wide scale separation problem: $M_b \sim 4.5$ GeV and 'binding' energy ~ 450 MeV $\rightarrow$ Effective Field Theory (EFT)

heavy quark propagator in lattice NRQCD

- in Coulomb gauge, Foldy-Wouthuysen-Tani transform heavy quark field of QCD

- in field theory language, NRQCD is an effective theory in which the momentum mode higher than the heavy quark mass, M, is integrated away (but the small parameter is $\mathbf{v} = \frac{\mathbf{p}}{M}$)

- need to show power-counting and factorization etc (cf. Braaten, Bodwin, Lepage, Phys. Rev. D51 (1995) 1125)

- inclusive decay rate = partonic decay rate $\times$ the probability for heavy quark to meet anti-heavy quark

heavy quark propagator in lattice NRQCD

- lattice NRQCD at $T = 0$ is well established: 2012 PDG summary on QCD

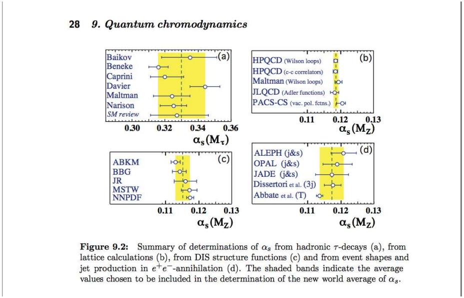

28 *9. Quantum chromodynamics*

Figure 9.2: Summary of determinations of α_s from hadronic τ-decays (a), from lattice calculations (b), from DIS structure functions (c) and from event shapes and jet production in e^+e^--annihilation (d). The shaded bands indicate the average values chosen to be included in the determination of the new world average of α_s.

heavy quark propagator in lattice NRQCD

$$\mathcal{L} = \mathcal{L}_0 + \delta\mathcal{L}, \tag{1}$$

with

$$\mathcal{L}_0 = \psi^\dagger \left(D_\tau - \frac{\mathbf{D}^2}{2M} \right) \psi + \chi^\dagger \left(D_\tau + \frac{\mathbf{D}^2}{2M} \right) \chi, \tag{2}$$

and

$$\begin{aligned}
\delta\mathcal{L} = \quad & -\frac{c_1}{8M^3}\left[\psi^\dagger(\mathbf{D}^2)^2\psi - \chi^\dagger(\mathbf{D}^2)^2\chi\right] \\
& + c_2\frac{ig}{8M^2}\left[\psi^\dagger\left(\mathbf{D}\cdot\mathbf{E} - \mathbf{E}\cdot\mathbf{D}\right)\psi + \chi^\dagger\left(\mathbf{D}\cdot\mathbf{E} - \mathbf{E}\cdot\mathbf{D}\right)\chi\right] \\
& - c_3\frac{g}{8M^2}\left[\psi^\dagger\sigma\cdot\left(\mathbf{D}\times\mathbf{E} - \mathbf{E}\times\mathbf{D}\right)\psi + \chi^\dagger\sigma\cdot\left(\mathbf{D}\times\mathbf{E} - \mathbf{E}\times\mathbf{D}\right)\chi\right] \\
& c_4\frac{g}{2M}\left[\psi^\dagger\sigma\cdot\mathbf{B}\psi - \chi^\dagger\sigma\cdot\mathbf{B}\chi\right].
\end{aligned} \tag{3}$$

heavy quark propagator in lattice NRQCD

- heavy quark propagator in QCD

$$(\gamma_\mu D_\mu + m)G(\mathbf{x},\tau) = S(\mathbf{x})\delta_{t,0} \tag{1}$$

boundary value problem

- heavy quark propagator in NRQCD

$$(D_t - H_0 - \delta H)G(\mathbf{x},\tau) = S(\mathbf{x})\delta_{t,0} \tag{2}$$

initial value problem

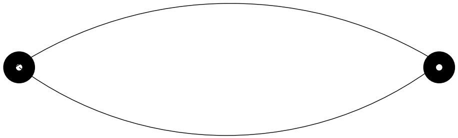
- quarkonium propagator

$$G_{\mathrm{meson}}(\tau) = \sum_{\mathbf{x}} G^\dagger_{ab}(\mathbf{x},\tau)G_{cd}(\mathbf{x},\tau) \tag{3}$$

- gauge field and light quark have “temperature effect”

Remind you (in NRQCD) $\cdots$

bottomonium correlator, $G(\tau, \mathbf{x})$



- NRQCD dispersion relation has undetermined zero point energy

$$E_q = \sqrt{M_q^2 + \mathbf{p}^2} \sim M_q + \frac{\mathbf{p}^2}{2M_q} - \frac{\mathbf{p}^4}{8M_q^3} + \cdots \tag{4}$$

- simulation at zero temperature is required to determine the zero point energy

Remind you (in NRQCD) $\cdots$

- consistent lattice NRQCD requires $M_q a_\tau \sim 1$
- To keep NRQCD as an effective field theory remain valid, $T << M_q$
- In summary, a consistent lattice NRQCD for bottomonium ($M_b = 4.65$ GeV) requires

$$a_\tau \gtrsim \frac{1}{4.65}(\mathrm{GeV}^{-1}) \tag{4}$$

and

$$T = \frac{1}{N_\tau a_\tau} \leq \frac{4.65\mathrm{GeV}^{-1}}{N_\tau} \tag{5}$$

- If we are interested in a few MeV temperature, $N_\tau \sim \mathcal{O}(10)$

spectral function in NRQCD

In QCD,

$$G_\Gamma(\tau) = \sum_{\vec{x}} \langle \overline{\psi}(\tau,\vec{x})\Gamma\psi(\tau,\vec{x})\overline{\psi}(0,\vec{0})\Gamma\psi(0,\vec{0})\rangle \tag{6}$$

$$= \int \frac{d^3p}{(2\pi)^3} \int_0^\infty \frac{d\omega}{2\pi} K(\tau,\omega)\rho_\Gamma(\omega,\vec{p}) \tag{7}$$

and

$$K(\tau,\omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}. \tag{8}$$

• the spectral function of Euclidean correlator has all the information on the finite temperature behavior of a propagator

• numerically ill-posed problem

• Maximum Entropy Method (MEM) is used (cf. M. Asakawa, T. Hatsuda, Y. Nakahara, PPNP46 (2001) 459)

spectral function in NRQCD

• known to have problems (cf. T. Umeda, PRD75 (2007) 094502 and A. Mocsy and P. Petreczky, PRD77 (2008) 014501)

• both the kernel($K(\tau,\omega)$) and the spectral density($\rho_\Gamma(\omega,\vec{p})$) depend on temperature

• constant contribution

• In NRQCD, with $\omega = 2M + \omega'$ and $T/M << 1$, $K(\tau,\omega) \to e^{-\omega\tau}$

$$G(\tau) = \int_{-2M}^{\infty} \frac{d\omega'}{2\pi} \exp(-\omega'\tau)\rho(\omega') \qquad (6)$$

• numerical inverse Laplace transform problem: MEM and New Bayesian Reconstruction (BR) Method (Y. Burnier and A. Rothkopf, PRL 111 (2013) 182003))

spectral function in NRQCD

$$Q = -\frac{\chi^2}{2} + \alpha S \tag{6}$$

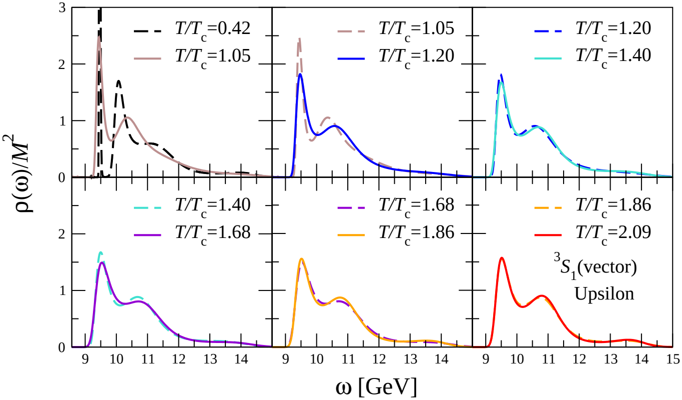
- MEM

$$S = \int d\omega \left[\rho(\omega) - m(\omega) - \rho(\omega) \log \left(\frac{\rho(\omega)}{m(\omega)} \right) \right] \tag{7}$$

- BR

$$S = \int d\omega \left[1 - \frac{\rho(\omega)}{m(\omega)} + \log \left(\frac{\rho(\omega)}{m(\omega)} \right) \right] \tag{8}$$

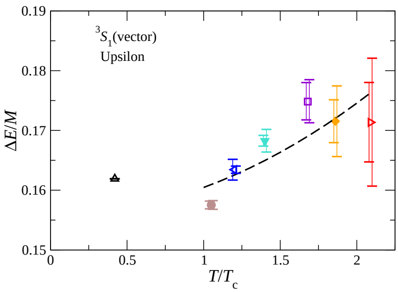
T-dependence of the Υ spectral function

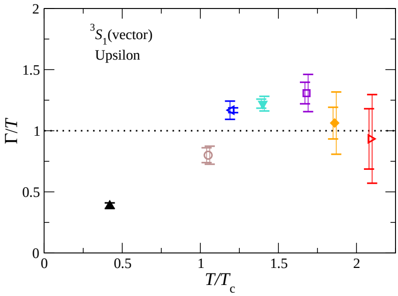


G. Aarts et al, PRL106 (2011) 061602, JHEP1111 (2011) 103, JHEP1407 (2014) 097 (cf. HadSpec. arxiv:0803.3960)

- systematic errors: default model dependency, τ-fitting range dependecy, ω range dependency, Jackknife error

T-dependence of the Υ spectral function



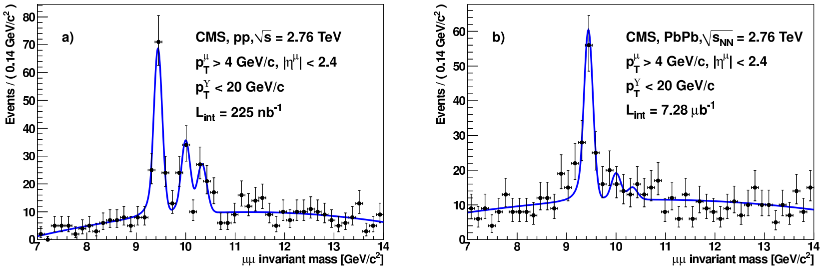


$$\Delta E \sim \alpha_s \frac{T^2}{M}, \qquad \frac{\Gamma}{T} \sim \alpha_s^3 \quad (\alpha_s \sim 0.4) \tag{9}$$

(N.Brambilla, M.A.Escobedo, J. Ghiglieri, J. Soto, A. Vairo, JHEP1009 (2010) 038)

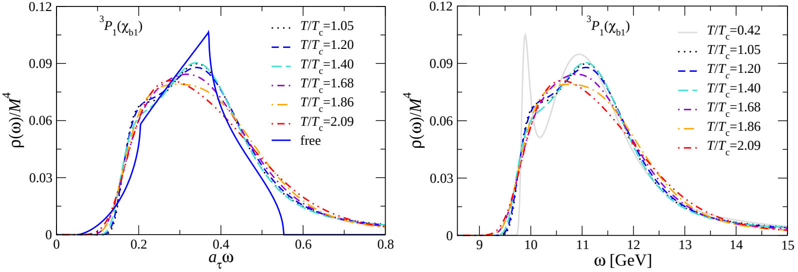
- dissociation may be due to broadening of thermal width not vanishing binding energy (N. Brambilla et al)

CMS collaboration, PRL107 (2011) 052302



- In 2011, CMS collaboration observed disappearance of 2S and 3S upsilon state in Pb-Pb collisions
- sequential suppression

T-dependence of the χ_{b1} spectral function



G. Aarts et al, JHEP1312 (2013) 064, JHEP1407 (2014) 097

• systematic errors: default model dependency, τ-fitting range dependecy, ω range dependency, Jackknife error, reconstruction method dependency

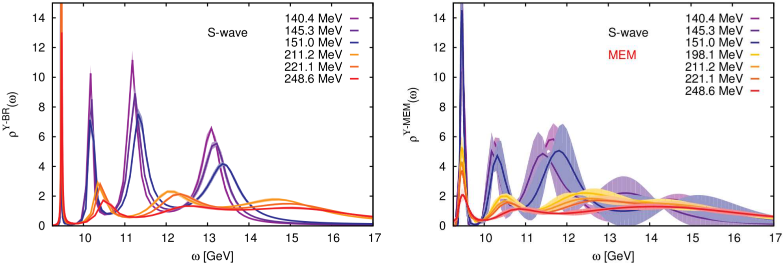
arXiv:1310.6461 and work in preparation

- previous lattice NRQCD approach uses a ‘fixed scale’ lattices (lattice spacing (a_τ) is fixed and N_τ is changed) and uses MEM to reconstruct spectral functions

- new lattice NRQCD approach uses ‘variable scale’ lattices (lattice spacing (a_τ) is changed and N_τ is fixed) and uses MEM and BR to reconstruct spectral functions

arXiv:1310.6461 and work in preparation

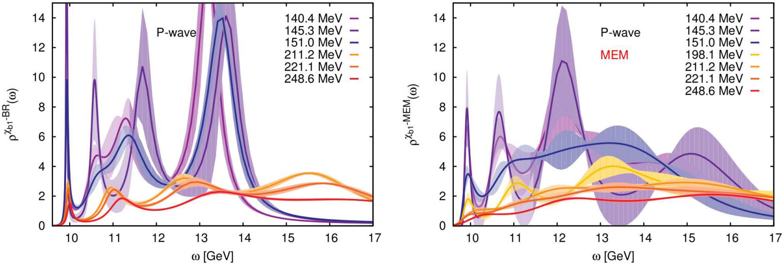
Upsilon spectral function



(cf. HotQCD arxiv:1111.1710)

arXiv:1310.6461 and work in preparation

χ_{b1} spectral function



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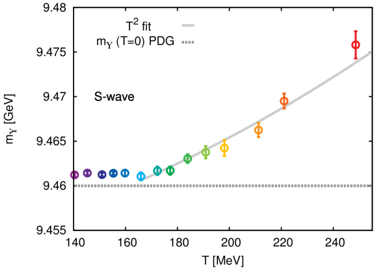
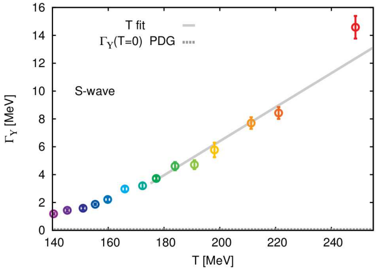
$\Upsilon(^3S_1)$ peak vs. temperature and width vs temperature





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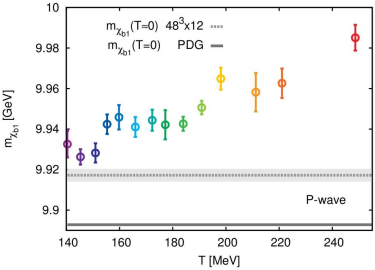
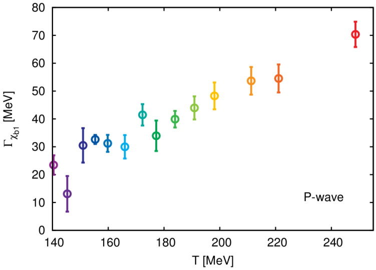
$\chi_{b1}(^3P_1)$ peak vs. temperature and width vs temperature





• systematic errors: default model dependency, τ-fitting range dependecy, ω range dependency, Jackknife error, reconstruction method dependency

Conclusion

• On $T = 0$ and $T \neq 0$, lattice NRQCD + new Baysesian Reconstruction (BR) of spectral function on bottomonium, which is systematically improvable and is based on the first principel of quantum field theory (not a model)

• free from known problem in QCD (constant contribution problem) and improvement from MEM

• from both BR and MEM, the ground state of Υ survives upto $2T_c$ but the excited states are suppressed as the temperature increases above T_c

• 1S peak of Υ channel starts to increase at $T \gtrsim 1.14T_c$ and its width increases monotonically in T

• From BR, the ground state of χ_{b1} retains peak structure even at $1.6T_c$ but from MEM, χ_b (P-wave) melts around $1.3T_c$