

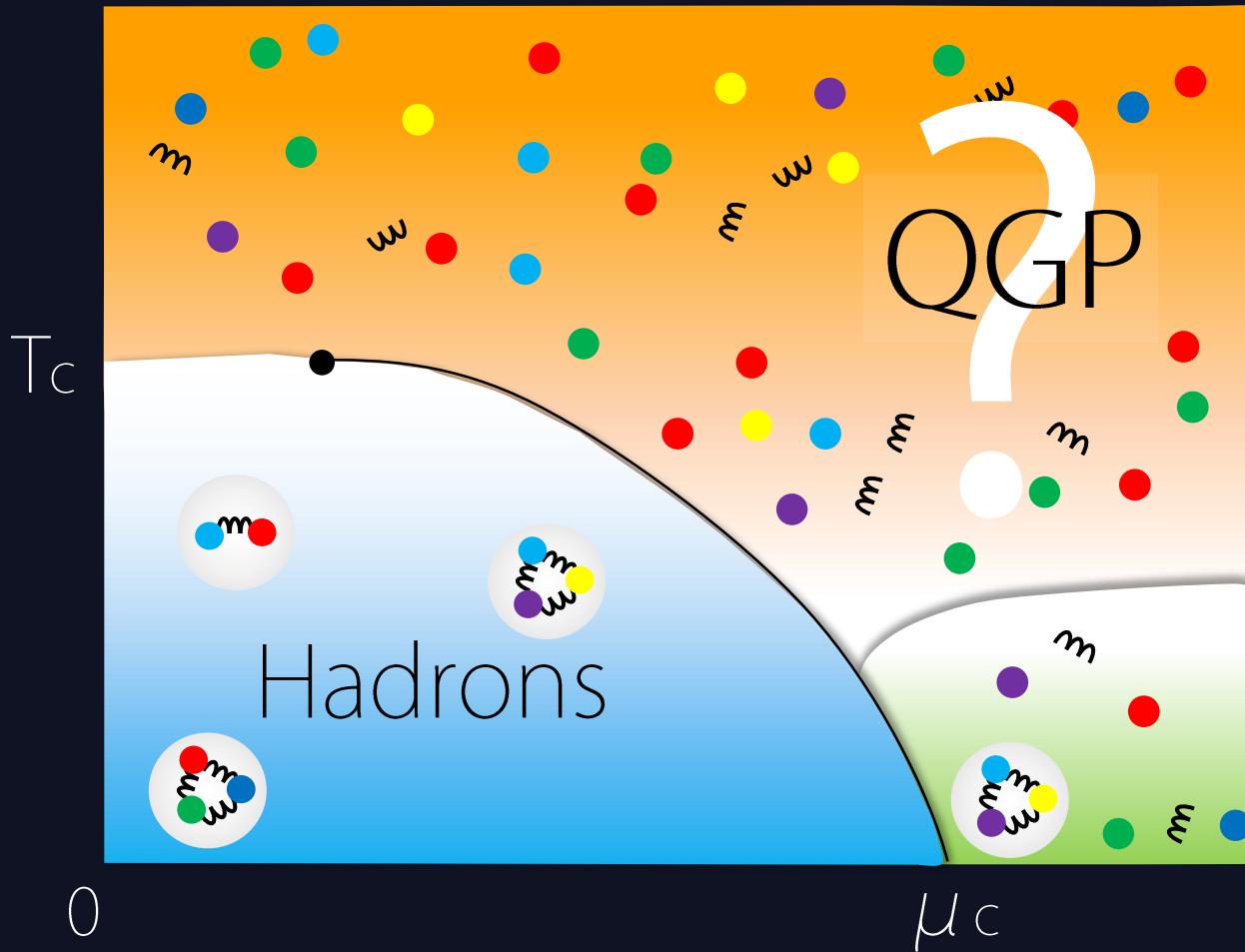
Diffusion Phenomenon of Electric Charge and Fluctuation observed at ALICE

Miki Sakaida (Osaka University)

Masayuki Asakawa, Masakiyo Kitazawa

QCD Phase Diagram

Temperature

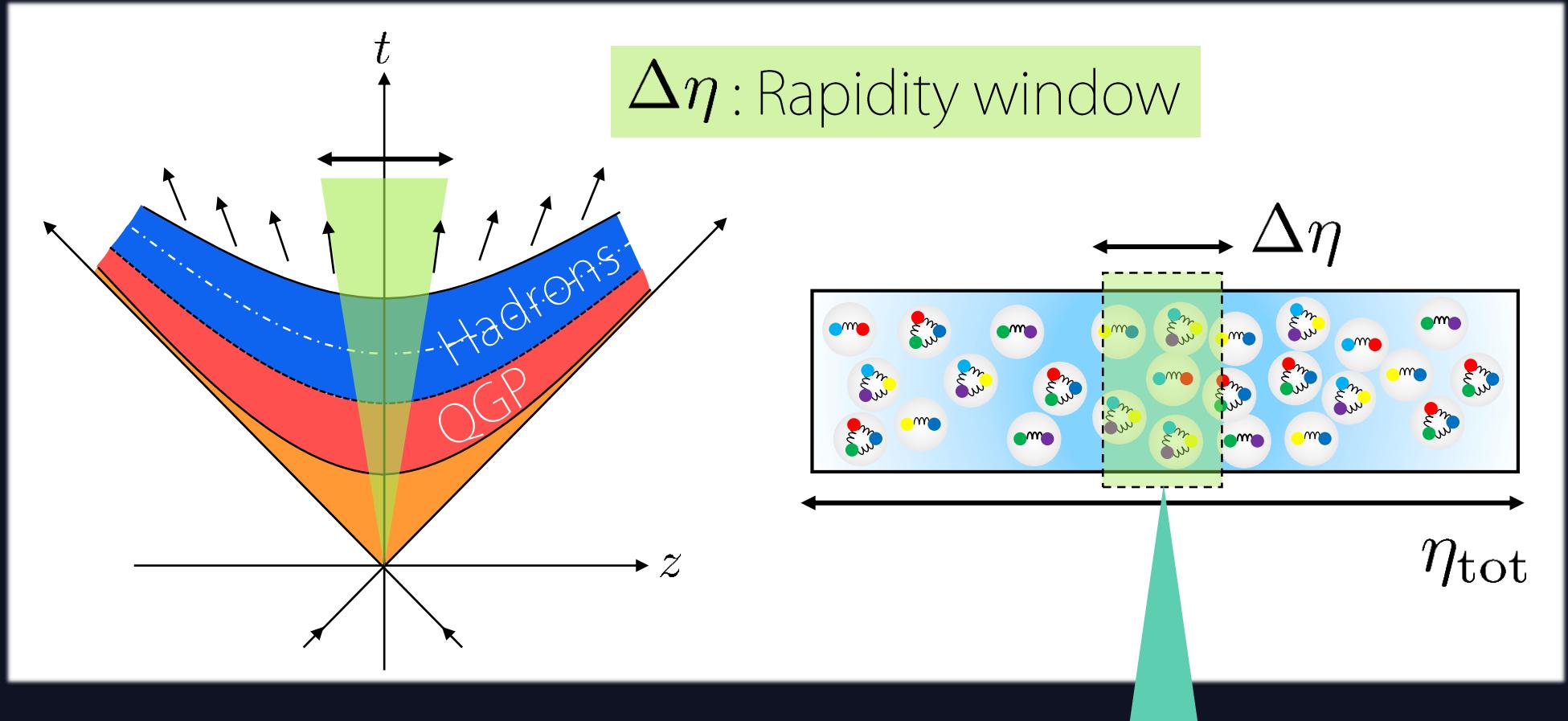


Baryon Chemical
Potential

Fluctuations of Conserved Charges
tell us much Information
about the Hot Medium

Fluctuations of Conserved Charges

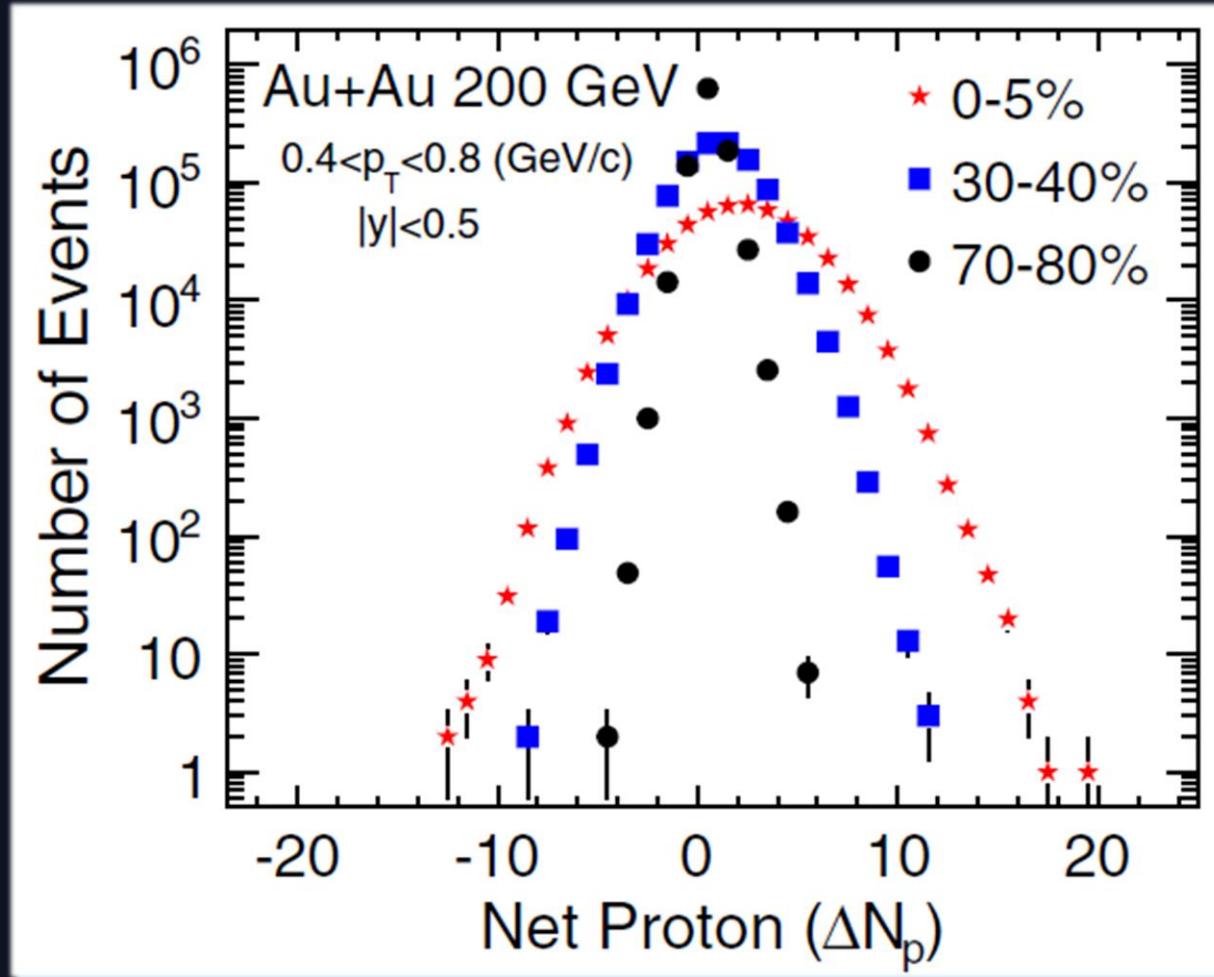
Event-by-Event Charge Fluctuations



The number of conserved charges are counted in a given $\Delta\eta$ in each event

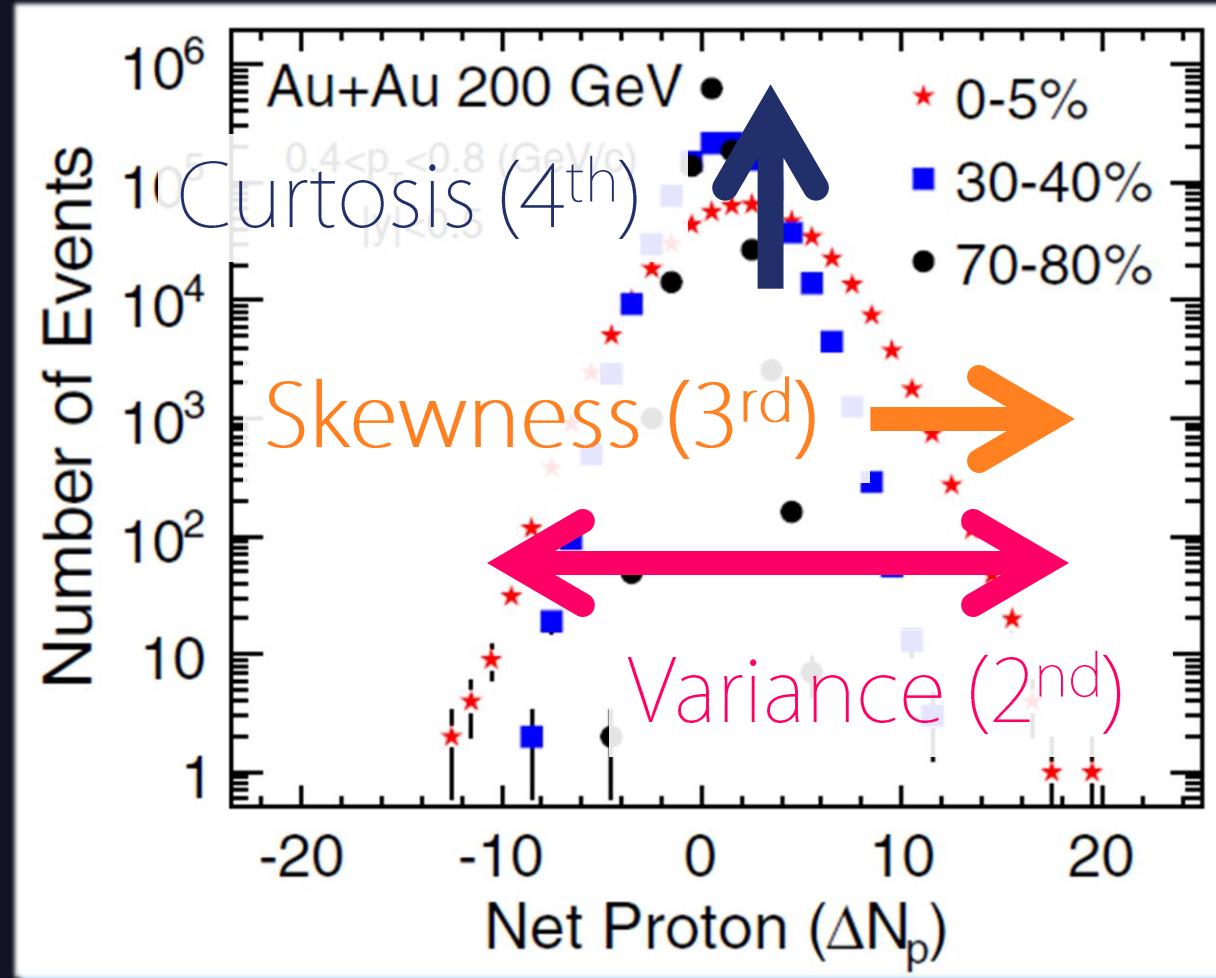
Event-by-Event Charge Fluctuations

STAR, PRL105 (2010)



Event-by-Event Charge Fluctuations

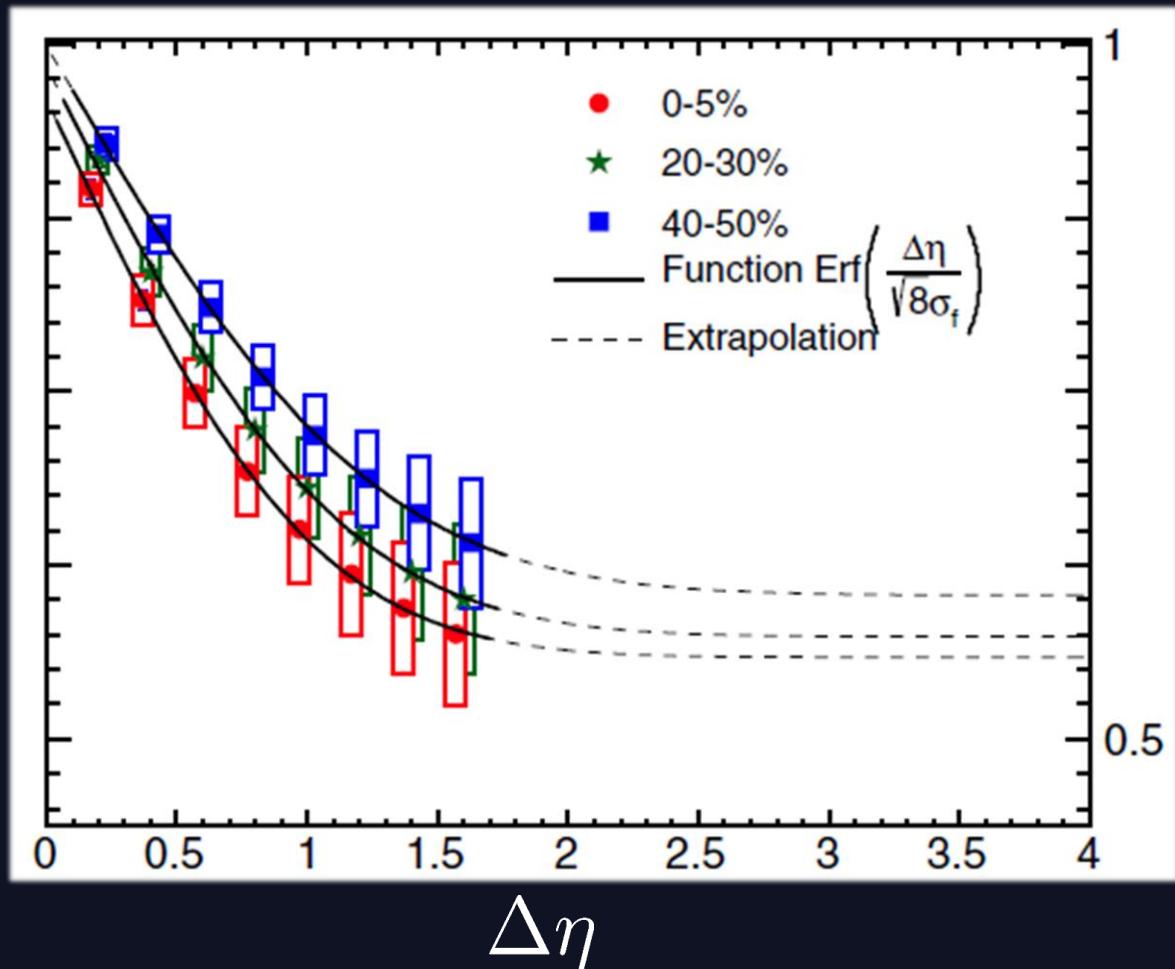
STAR, PRL105 (2010)



In HICs,
Higher order
Fluctuations
can also
be observed !!

$\Delta\eta$ Dependence of Charge Fluctuation @ ALICE

ALICE, PRL110, 152301 (2013)

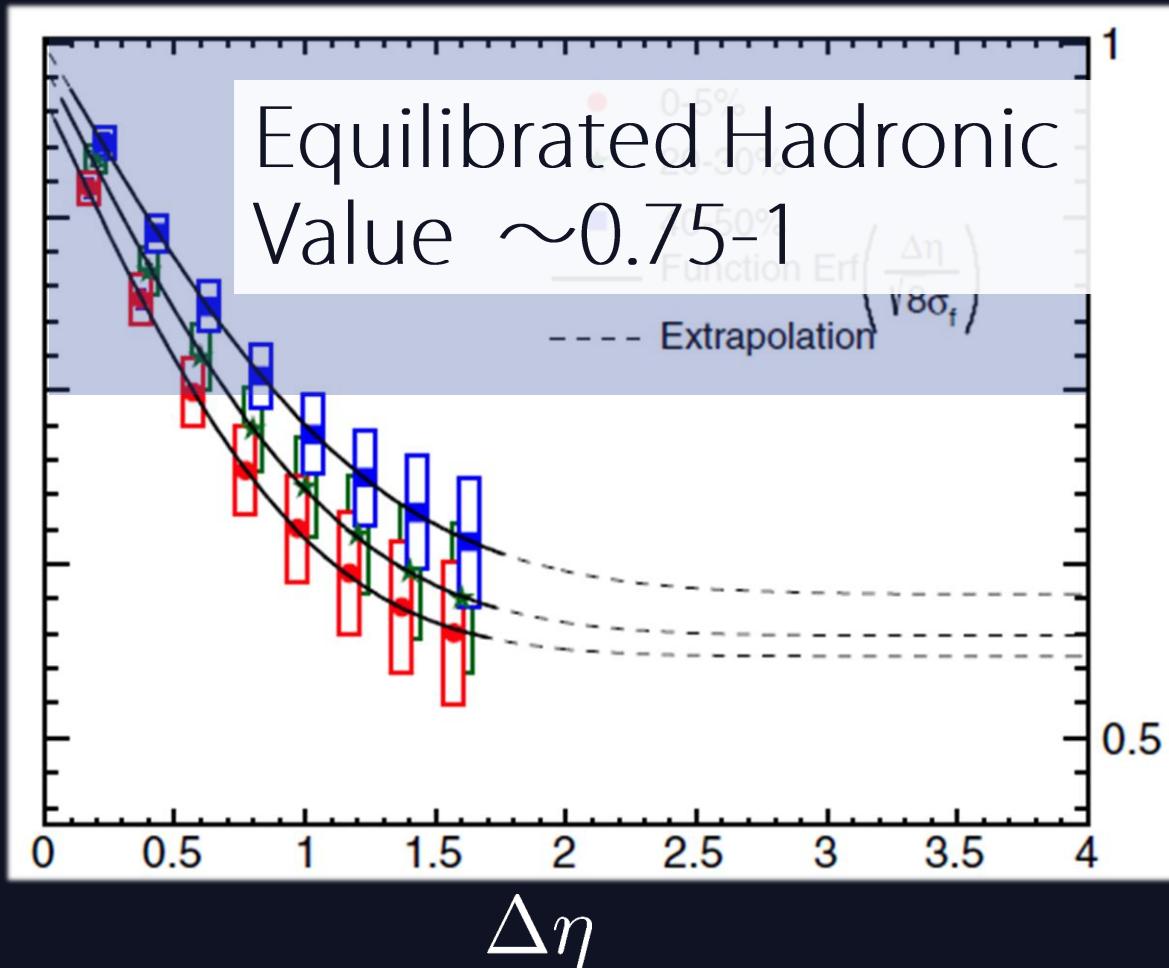


2nd order
Electric Charge
Fluctuation

$$\frac{\langle \delta N_Q^{(\text{net})2} \rangle_c}{\langle N_Q^+ + N_Q^- \rangle_c}$$

$\Delta\eta$ Dependence of Charge Fluctuation @ ALICE

ALICE, PRL110, 152301 (2013)



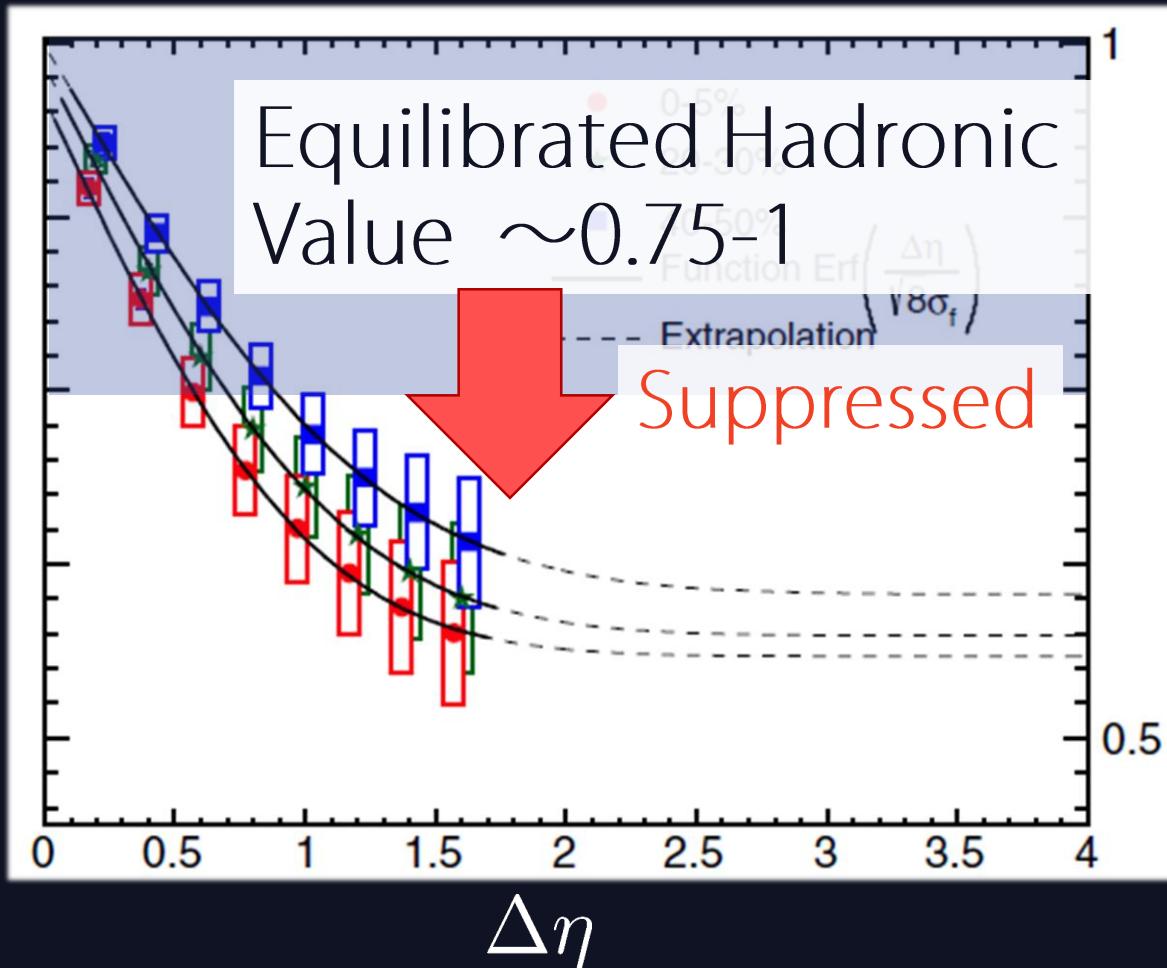
2nd order
Electric Charge
Fluctuation

$$\frac{\langle \delta N_Q^{(\text{net})} \rangle^2}{\langle N_Q^+ + N_Q^- \rangle}$$

Asakawa, Heinz, Muller (2000)
Jeon, Koch (2000)

$\Delta\eta$ Dependence of Charge Fluctuation @ ALICE

ALICE, PRL110, 152301 (2013)



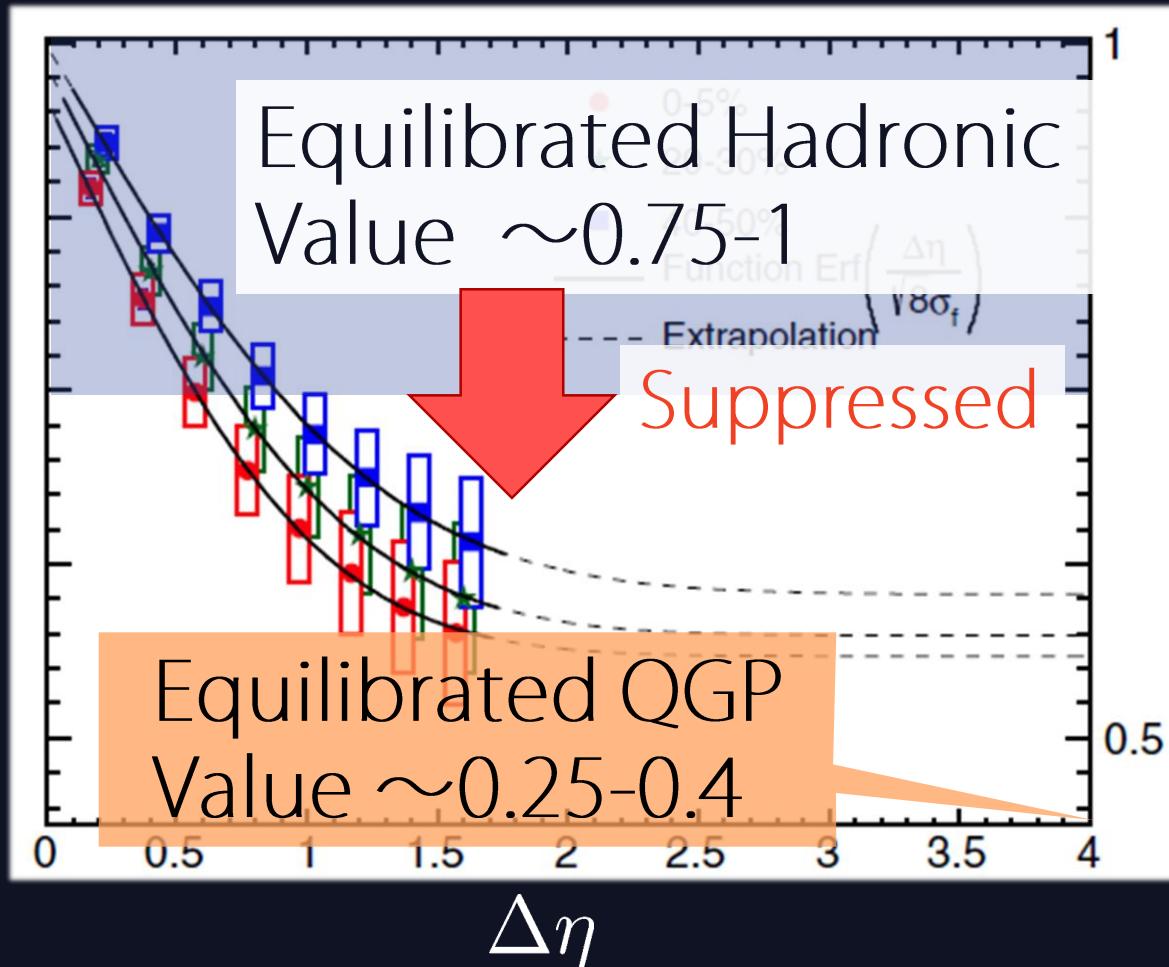
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$\Delta\eta$ Dependence of Charge Fluctuation @ ALICE

ALICE, PRL110, 152301 (2013)



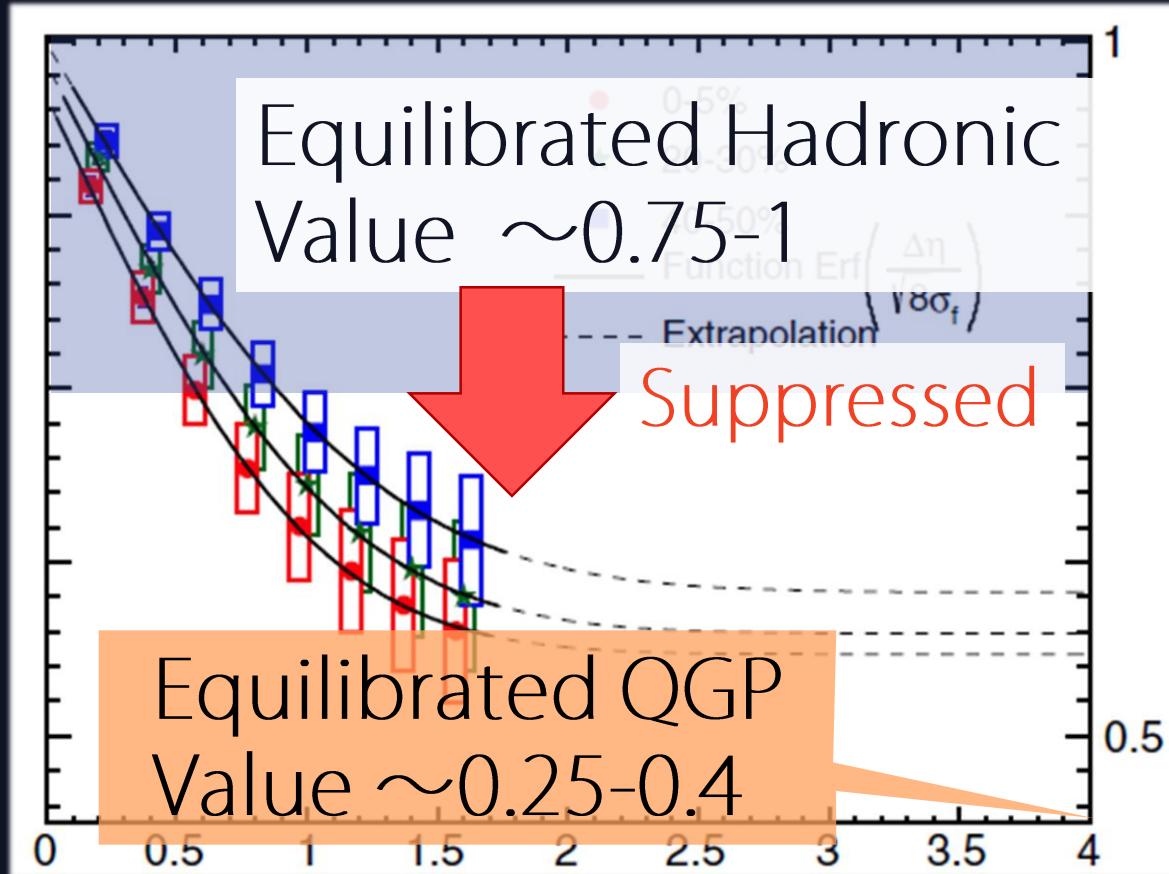
2nd order
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$\Delta\eta$ Dependence of Charge Fluctuation @ ALICE

ALICE, PRL110, 152301 (2013)



2nd order
Electric Charge
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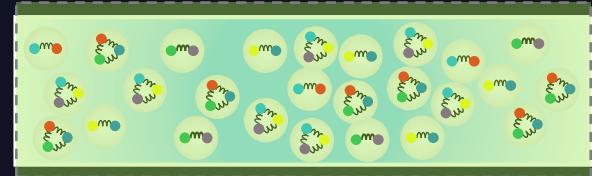
Asakawa, Heinz, Muller (2000)
Jeon, Koch (2000)

Fluctuations are not Equilibrated at freeze-out(?)
We can observe QGP Fluctuations for Larger $\Delta\eta$ (?)

Problem

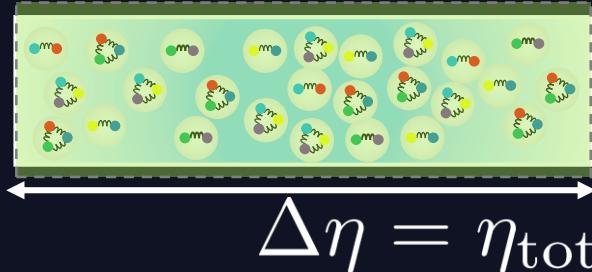
Global Charge Conservation (GCC)

If one looks at the Total System,
Conserved Charge
does NOT Fluctuate !!



Global Charge Conservation (GCC)

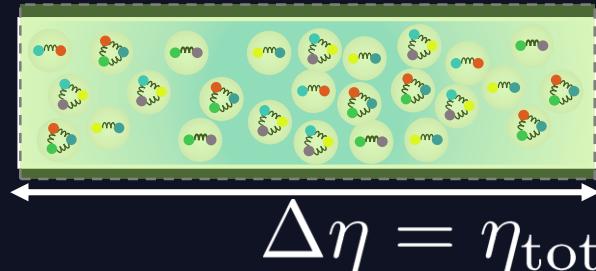
If one looks at the Total System,
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※ GCC Effect also causes Suppression !!

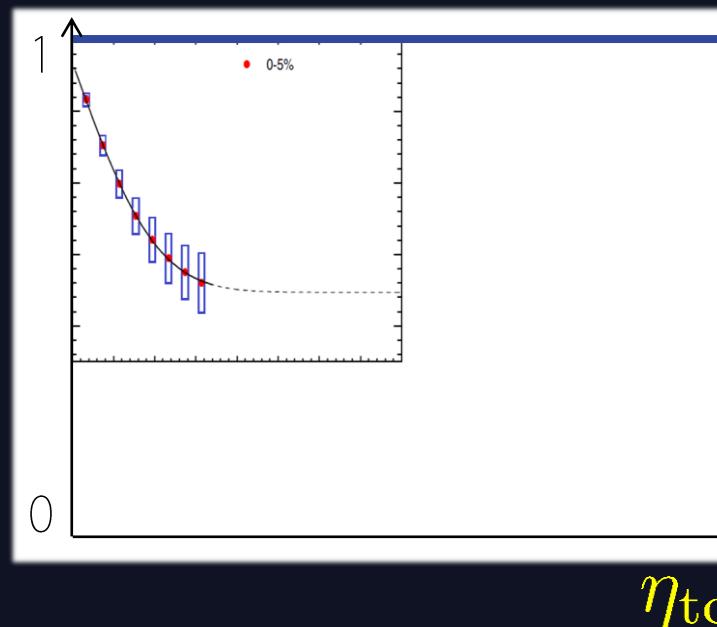
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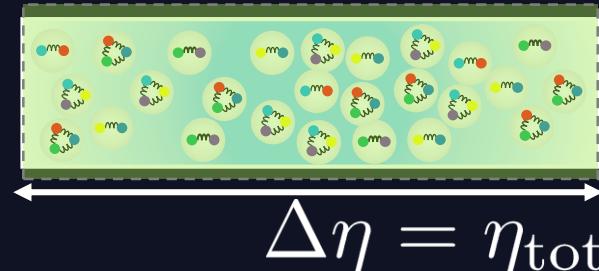
2nd order Fluctuation $\langle Q_{(\text{net})}^2 \rangle_c / \langle Q_{(\text{tot})} \rangle_c$



Equilibrated Hadronic Value
(NOT Consider GCC Effect)

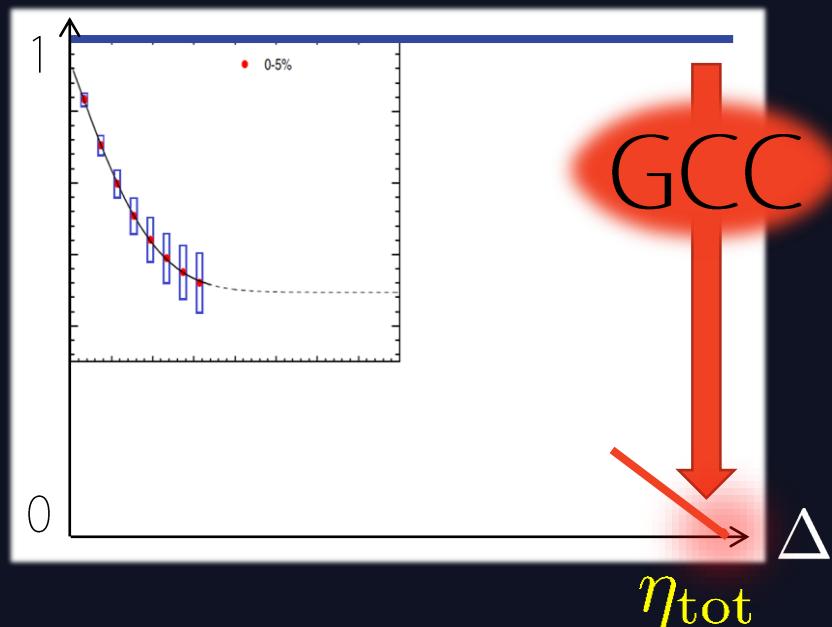
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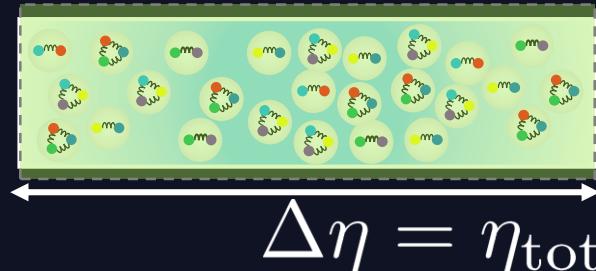
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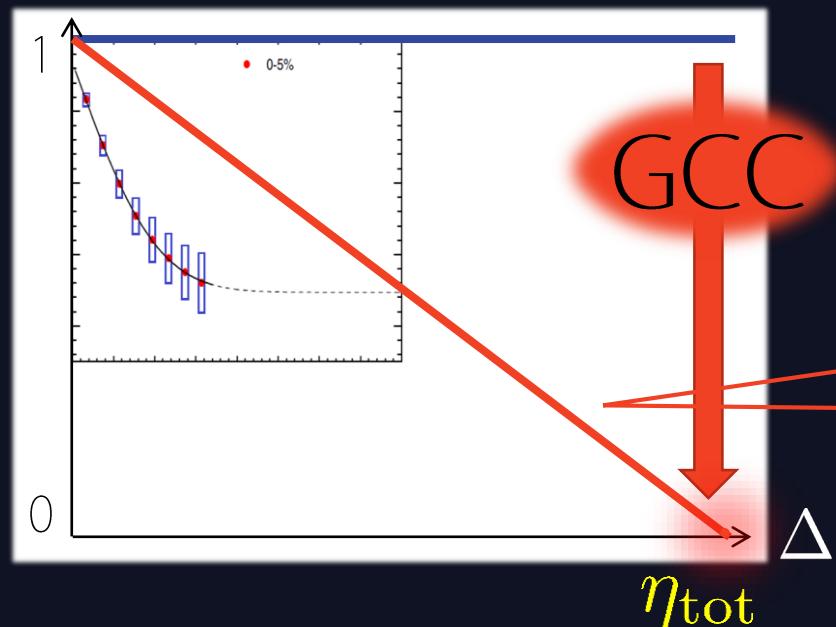
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2nd order Fluctuation $\langle Q_{(\text{net})}^2 \rangle_c / \langle Q_{(\text{tot})} \rangle_c$



Equilibrated Hadronic Value
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Naïve Estimate of GCC Effect

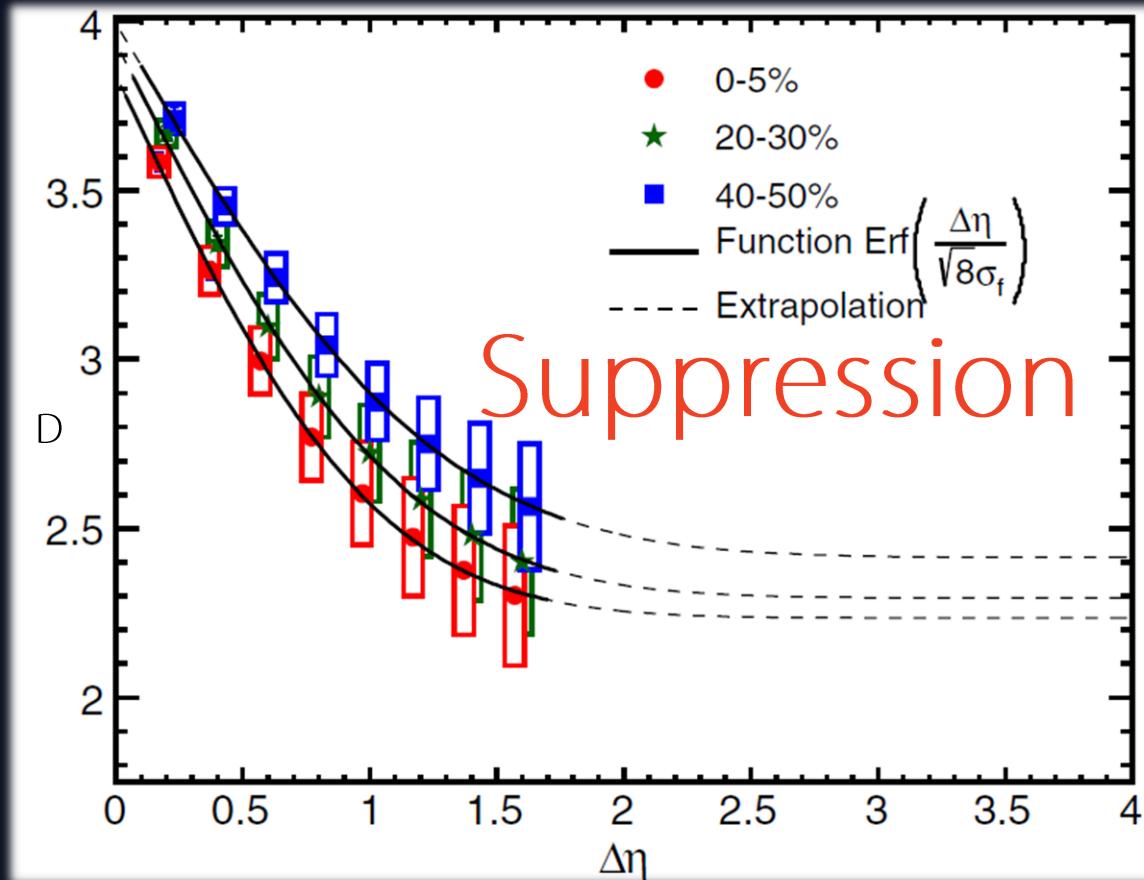
$$\langle \delta N^2 \rangle_{\text{GCC}} = \langle \delta N^2 \rangle_{\text{inf}} \times \left(1 - \frac{\Delta \eta}{\eta_{\text{tot}}} \right)$$

Δη: Rap. Win.
η_{tot}

Bleicher, Jeon, Koch (2000)

What does the Suppression imply ?

Fluctuation is NOT equilibrated??

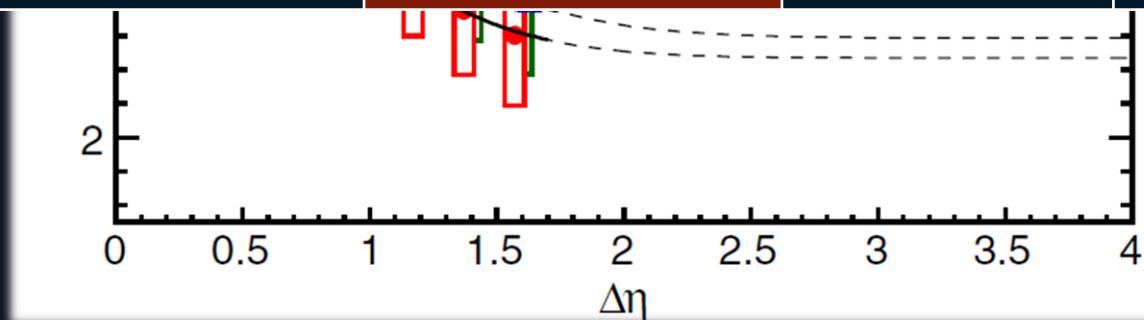


Global Charge Conservation ??

What does the Suppression imply ?

Fluctuation is NOT equilibrated??

| Previous Study | Global Charge Conservation | Time Evolution | Higher Fluctuations |
|----------------------------------|----------------------------|----------------|---------------------|
| Bleicher, Jeon, Koch (2000) | ○ | ✗ | ✗ |
| Shuryak, Stephanov (2001) | ✗ | ○ | ✗ |
| Kitazawa, Asakawa, Ono (2013) | ✗ | ○ | ○ |



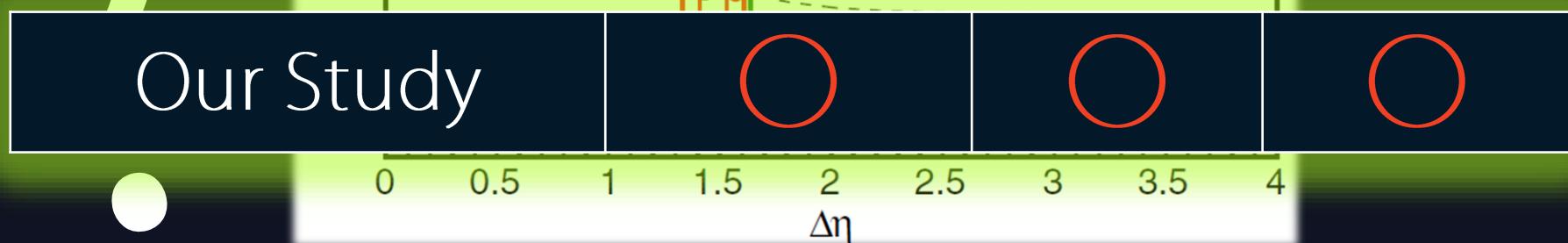
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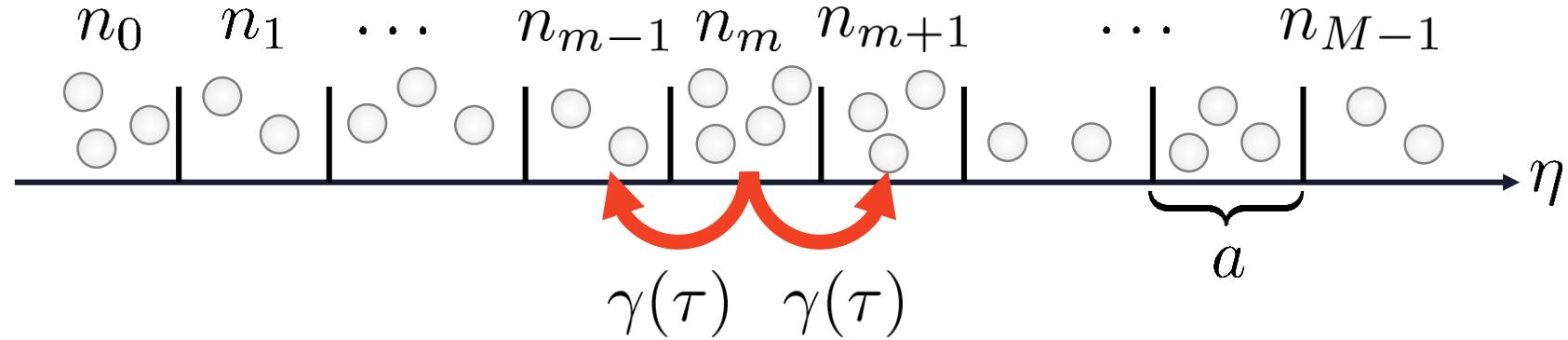


GCC Effects on Time evolution of Fluctuations

Method

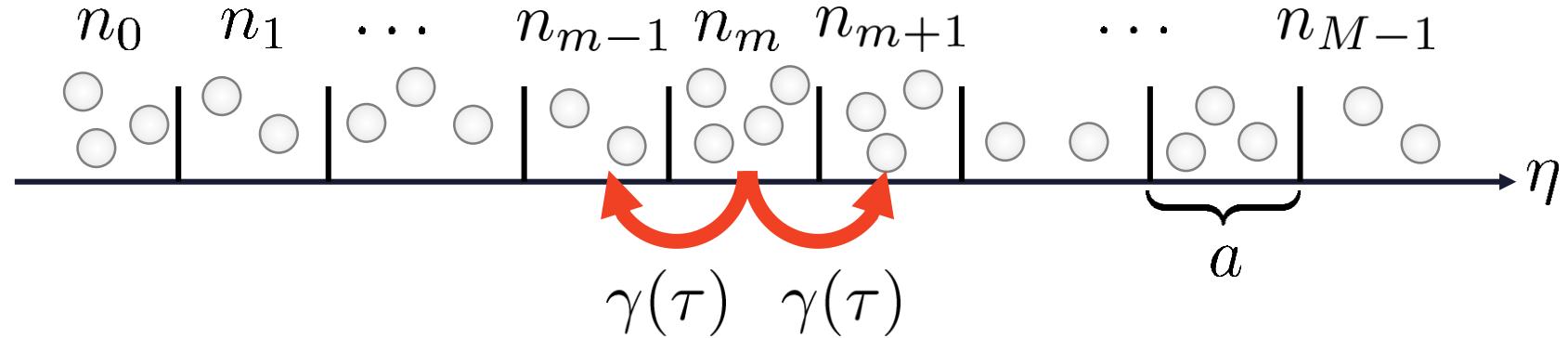
Diffusion Model For Hadrons (1D Brownian Motion)

Kitazawa, Asakawa, Ono, Phys. Rev. B728, 386 (2013)



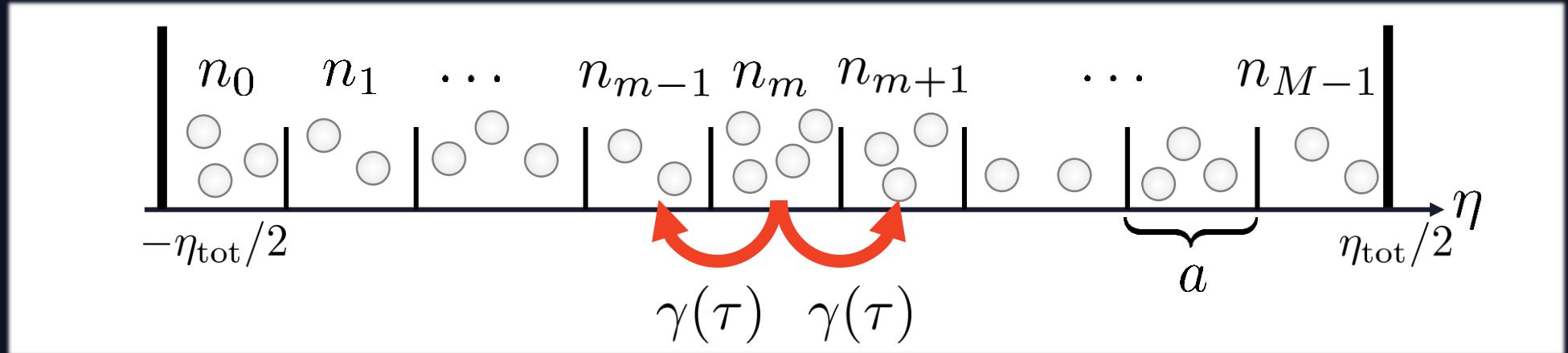
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Diffusion Master
Equation

Diffusion Model For Hadrons (1D Brownian Motion)

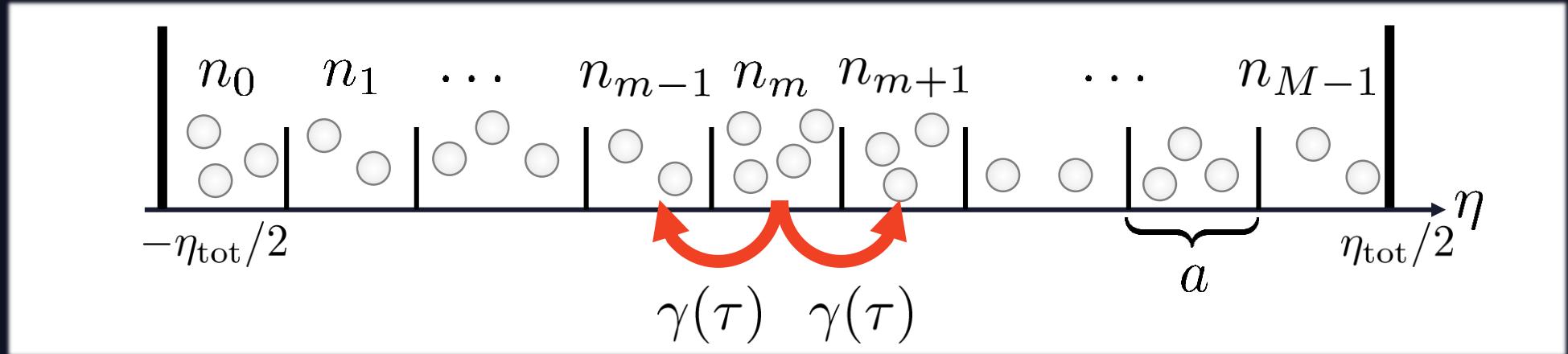


Diffusion Master
Equation

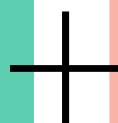


Boundary Condition(GCC Effect)
Particles do not flow in/out.

Diffusion Model For Hadrons (1D Brownian Motion)



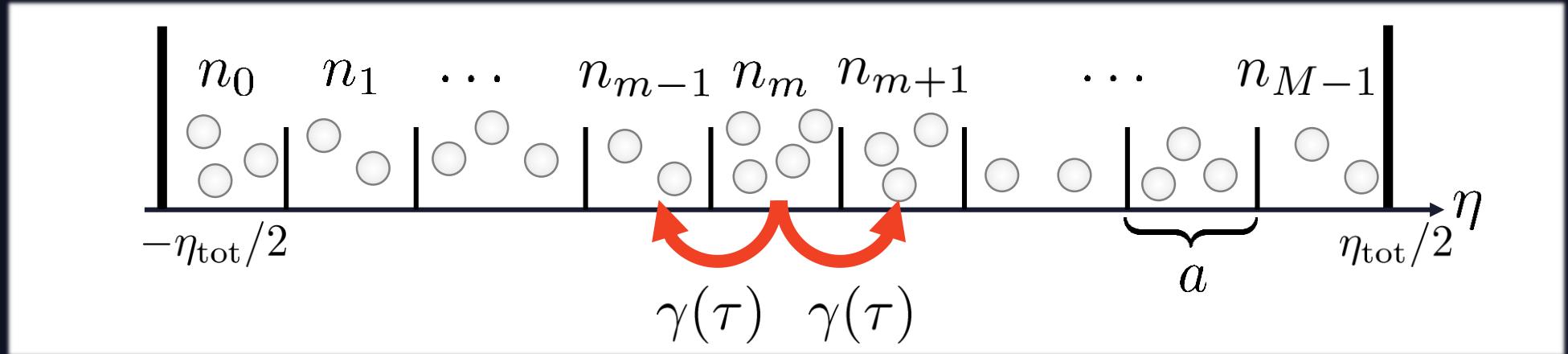
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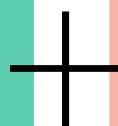
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-
- * Diffusion from Hadronization to Thermal Freeze-out
 - * Initial Condition : Fluctuations in Thermal QGP

Diffusion Model For Hadrons (1D Brownian Motion)



Diffusion Master
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Boundary Condition(GCC Effect)
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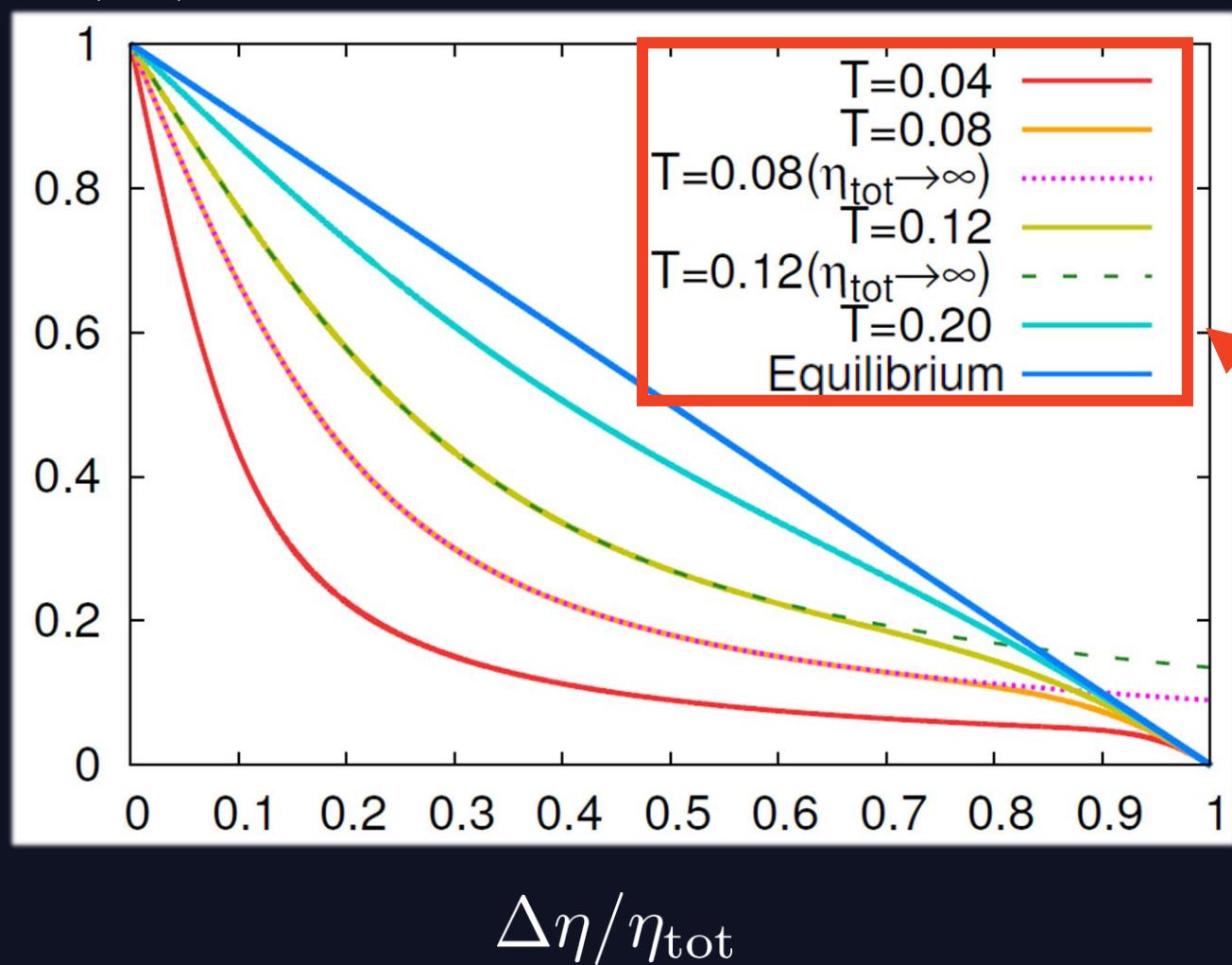
-
- * Diffusion from Hadronization to Thermal Freeze-out
 - * Initial Condition : Fluctuations in Thermal QGP

Rapidity Window Dependence of Charge Fluctuations

Results

$\Delta\eta$ Dep. of Fluctuations (No Initial Fluctuation)

$$\langle Q_{(\text{net})}^2 \rangle_c / \langle Q_{(\text{tot})} \rangle_c$$

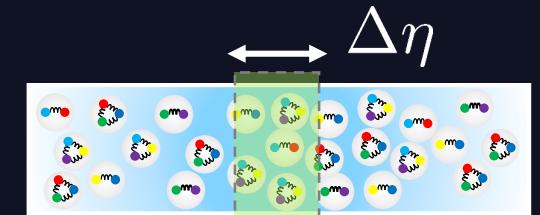


$$d(\tau) = \sqrt{2 \int_{\tau_0}^{\tau} D(\tau') d\tau'}$$

: Average Diffusion Length

$D(\tau)$: Diffusion Coefficient

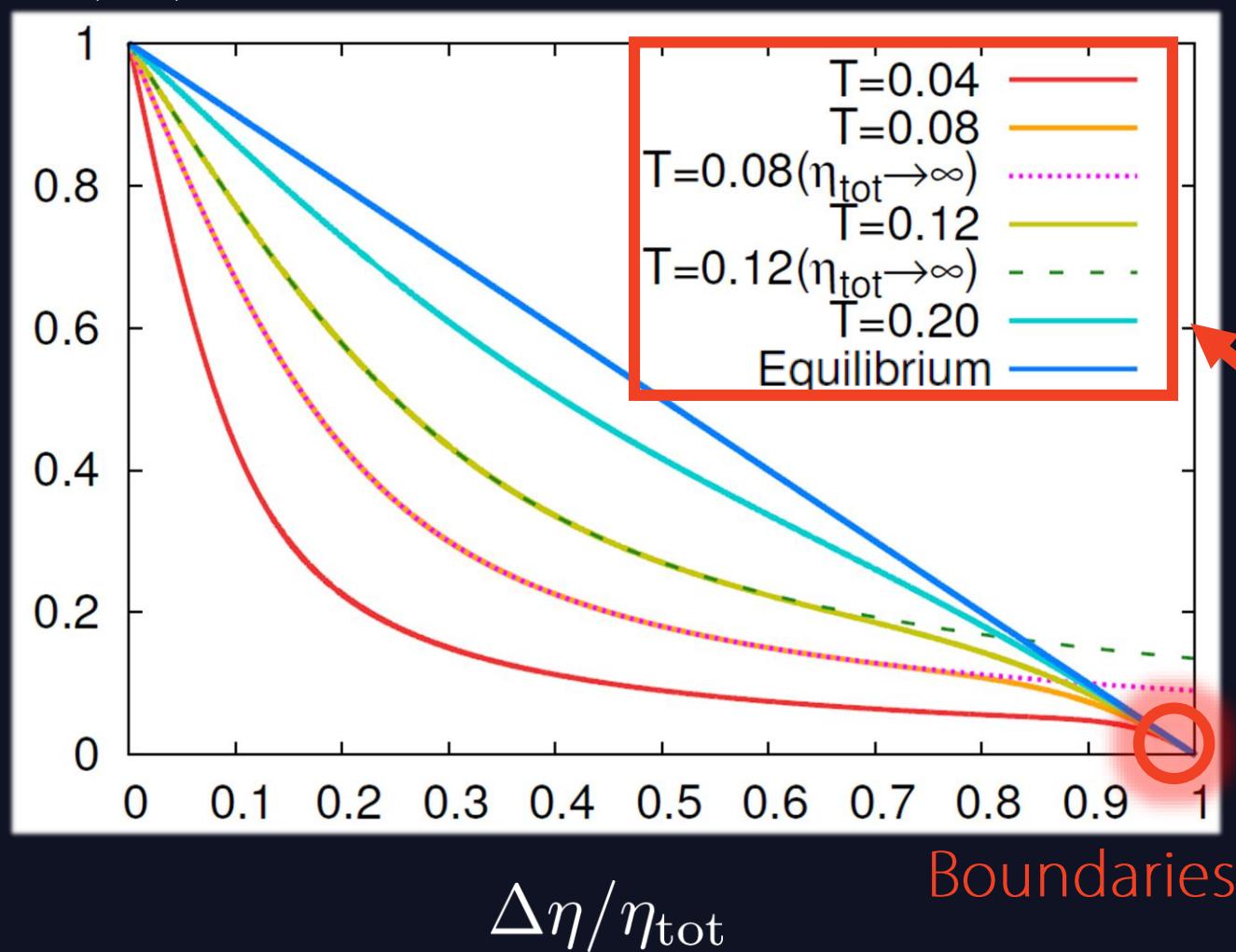
$$T = \frac{d(\tau)}{\eta_{\text{tot}}}$$



η_{tot} : Total Rapidity Length

$\Delta\eta$ Dep. of Fluctuations (No Initial Fluctuation)

$$\langle Q_{(\text{net})}^2 \rangle_c / \langle Q_{(\text{tot})} \rangle_c$$

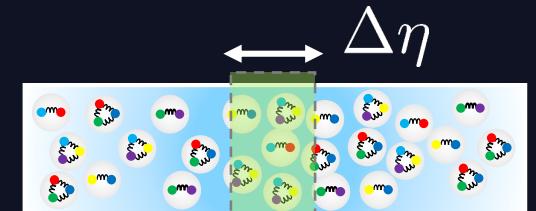


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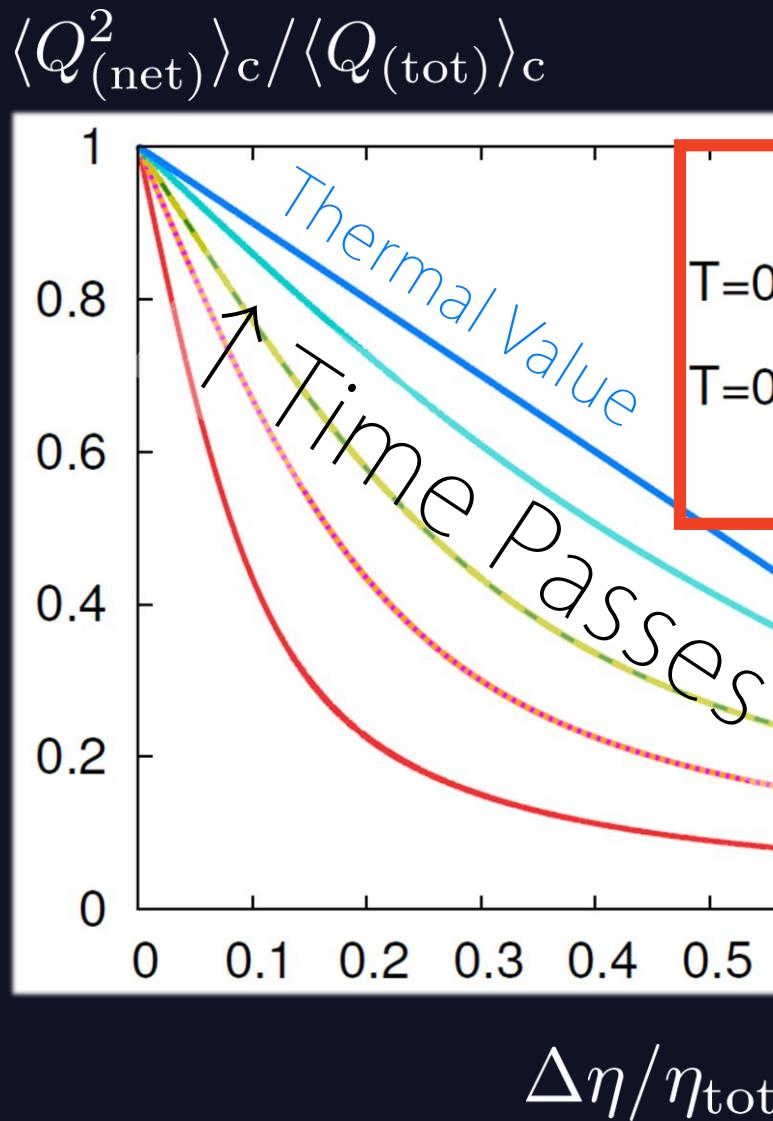
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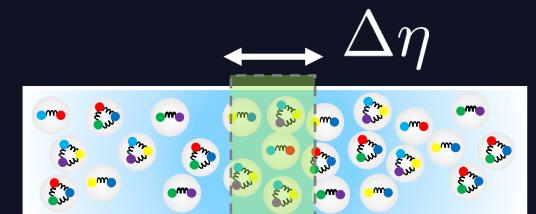


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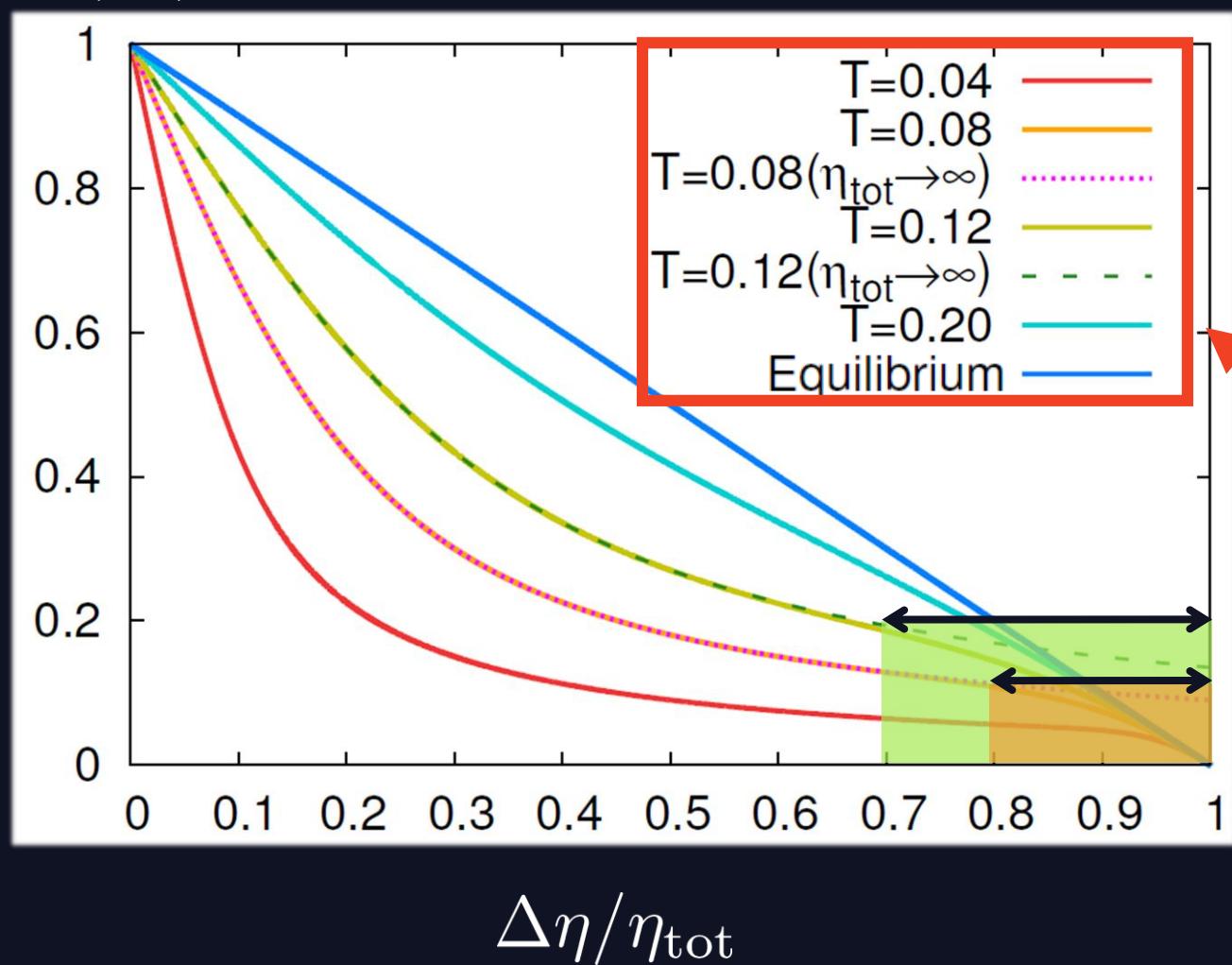
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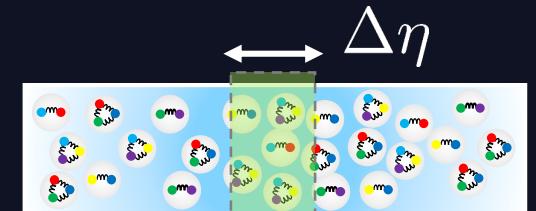


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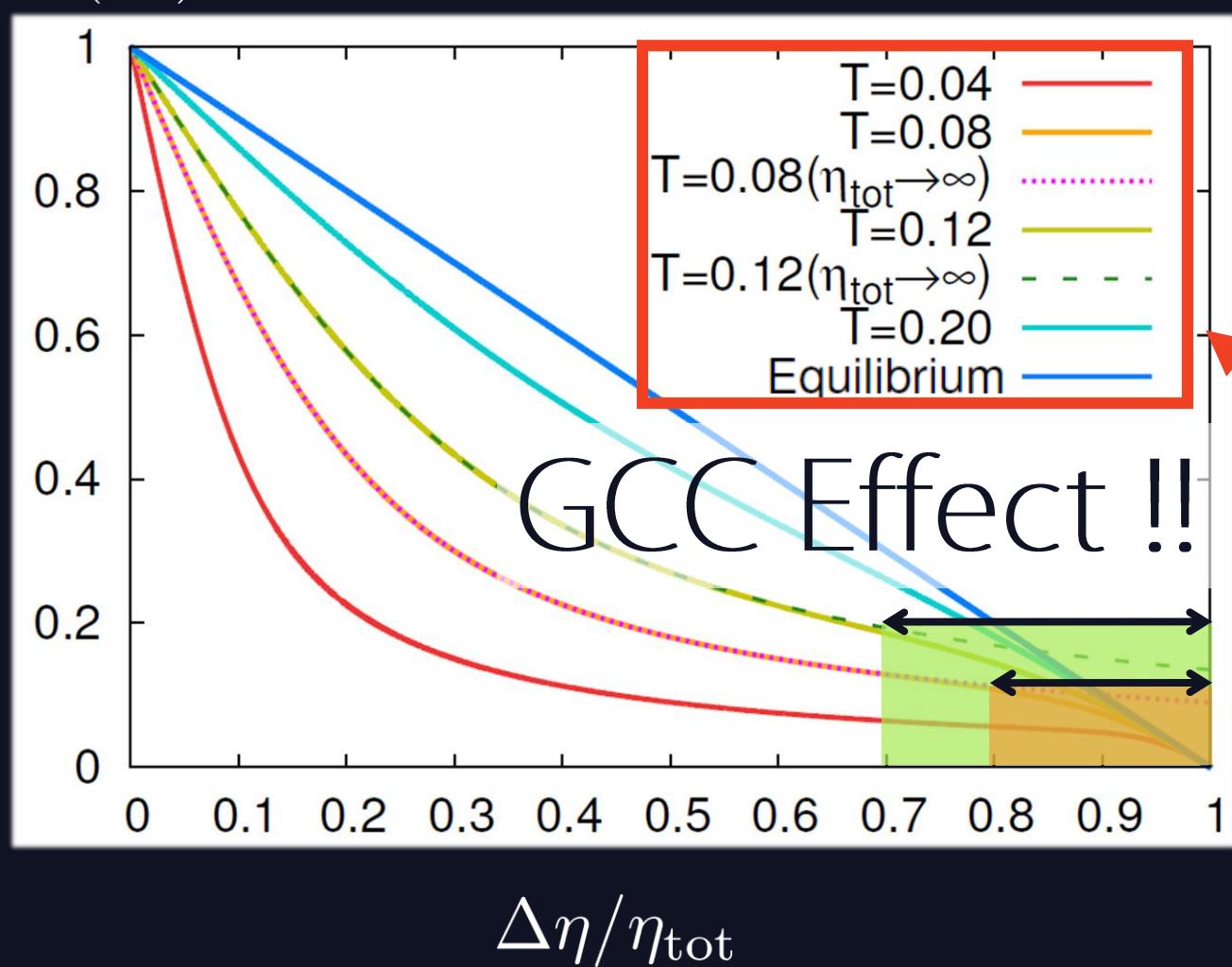
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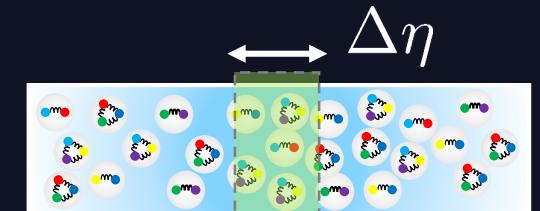


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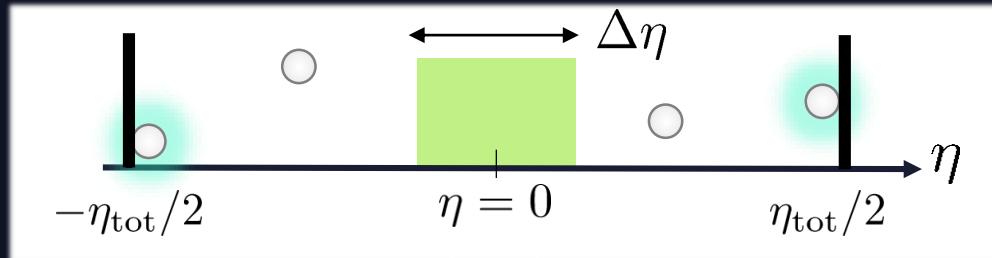
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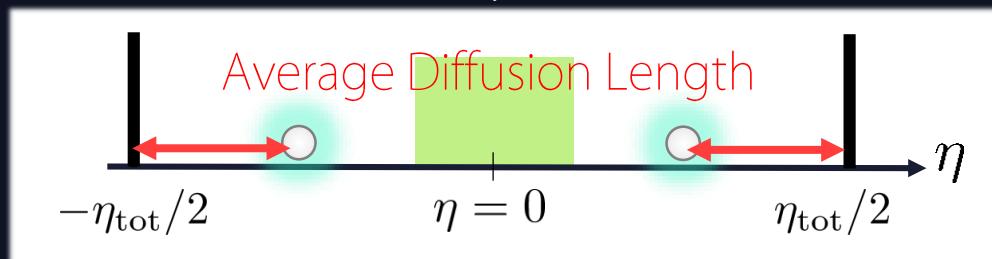
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Interpretation of Result for GCC Effect

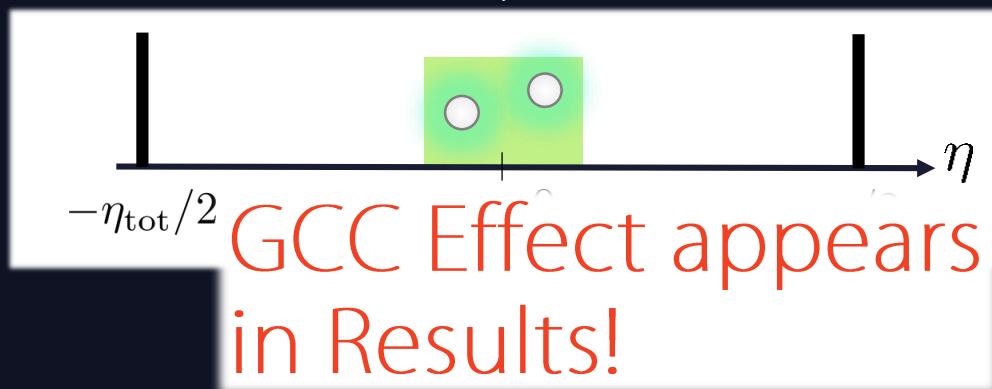
At Initial



Time Passes...

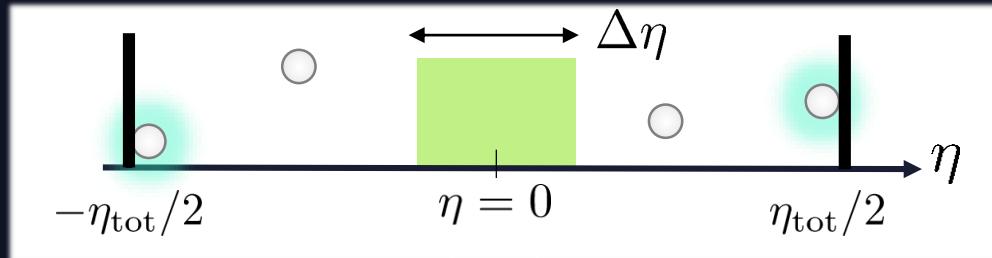


Time Passes...

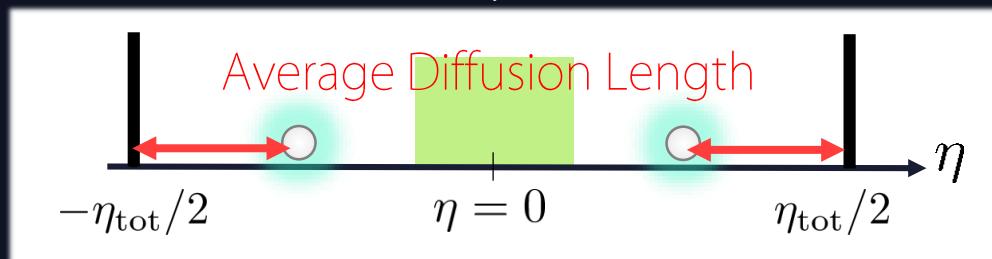


Interpretation of Result for GCC Effects

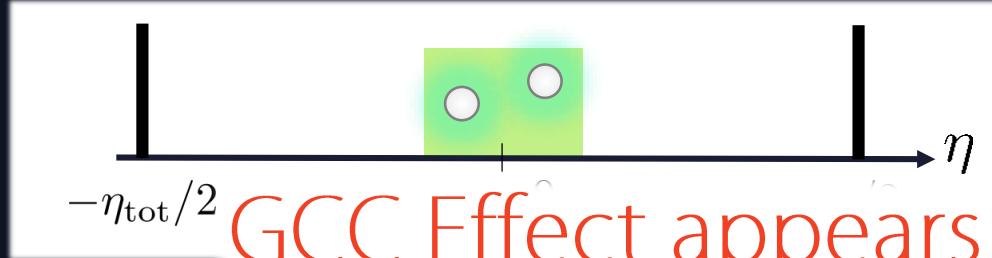
At Initial



Time Passes...



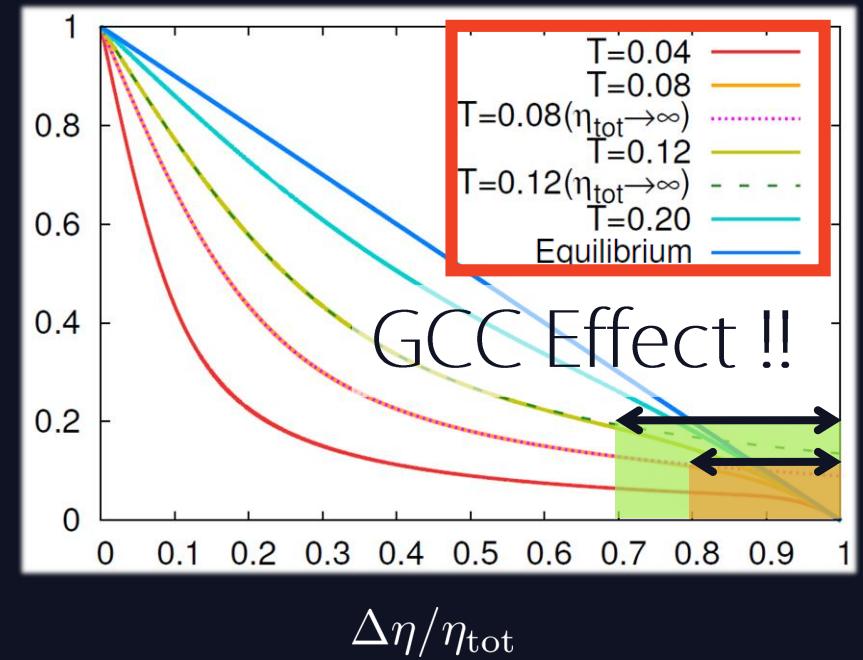
Time Passes...



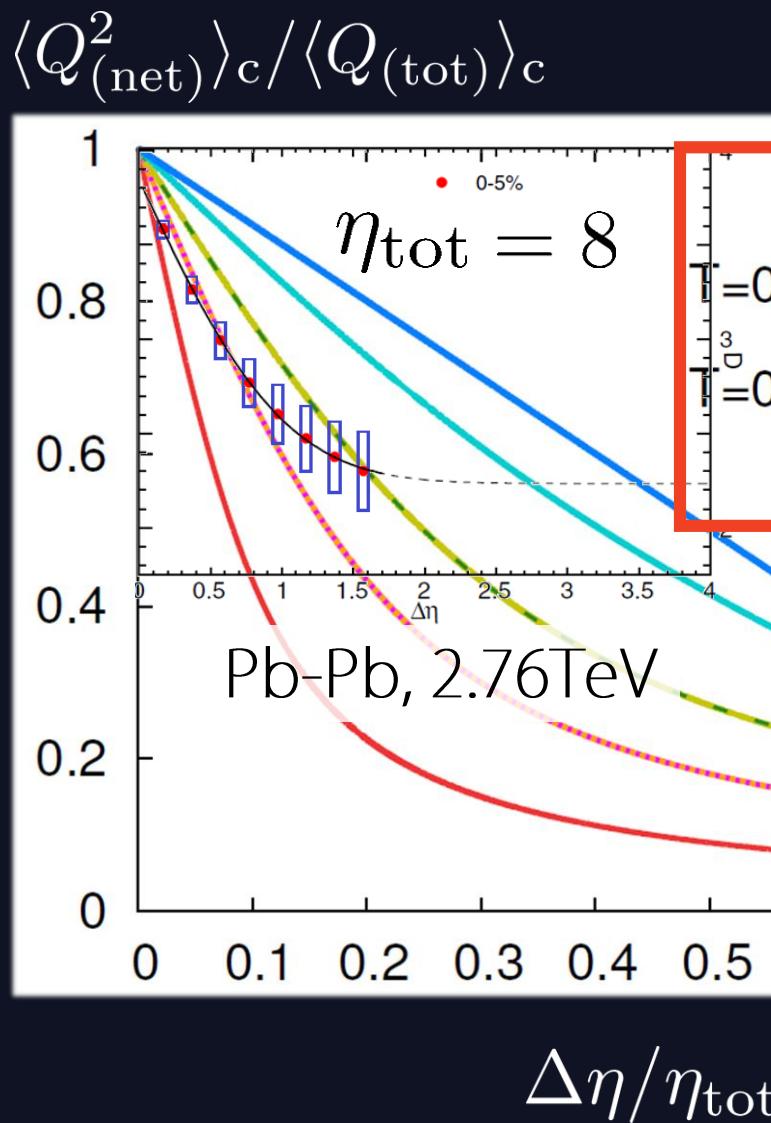
GCC Effect appears
in Results!

GCC Effect appears
ONLY near boundaries!!

$$\langle Q_{(\text{net})}^2 \rangle_c / \langle Q_{(\text{tot})} \rangle_c$$



Comparison with Experimental Result @ALICE

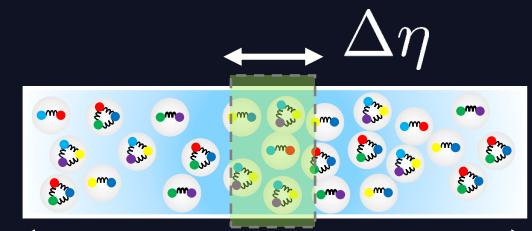


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: Average Diffusion Length

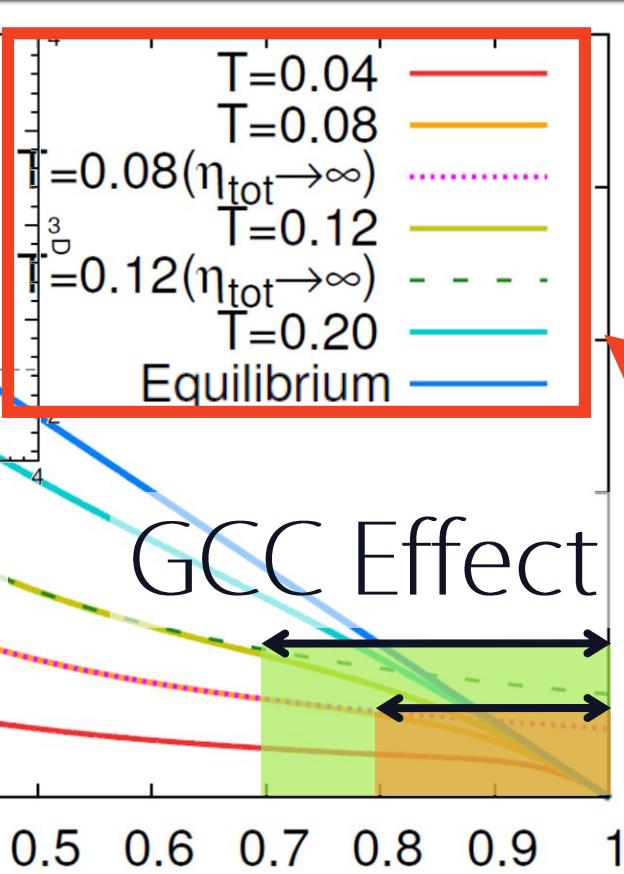
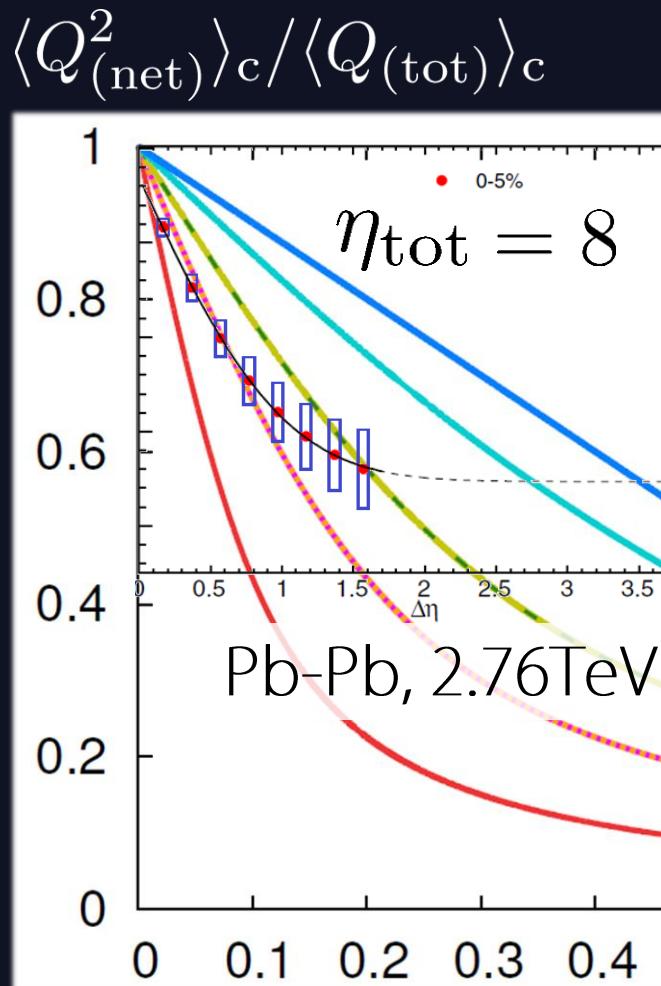
$D(\tau)$: Diffusion Coefficient

$$T = \frac{d(\tau)}{\eta_{\text{tot}}}$$



η_{tot} : Total Rapidity Length

Comparison with Experimental Result @ALICE

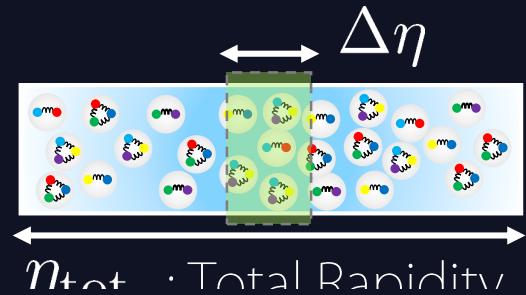


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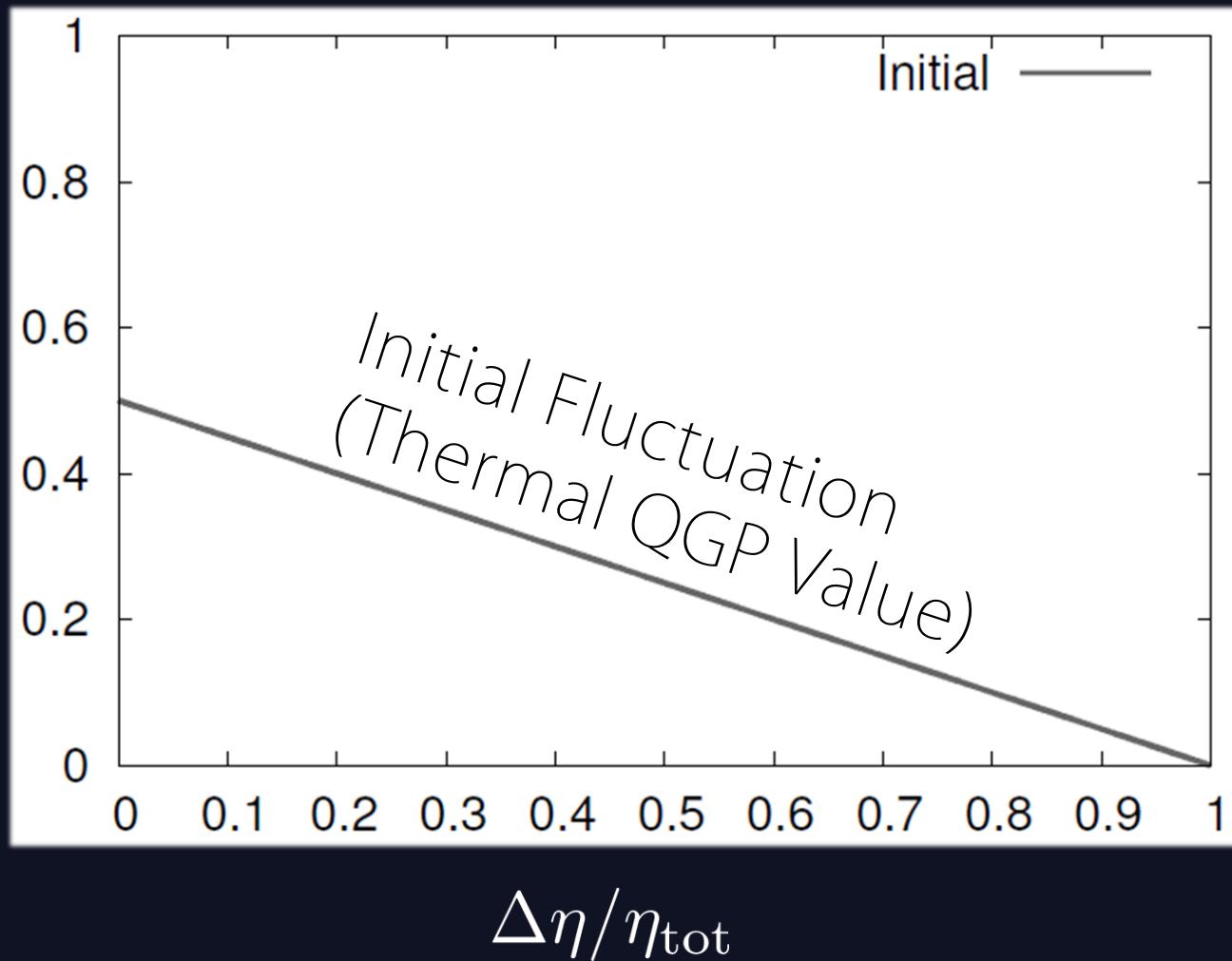
$$T = \frac{d(\tau)}{\eta_{\text{tot}}}$$



GCC Effect is almost negligible !

$\Delta\eta$ Dep. of Fluctuations (+ Initial Fluctuation)

$$\langle Q_{(\text{net})}^2 \rangle_c / \langle Q_{(\text{tot})} \rangle_c$$

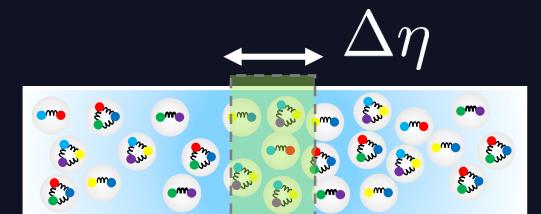


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: Average
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$D(\tau)$: Diffusion
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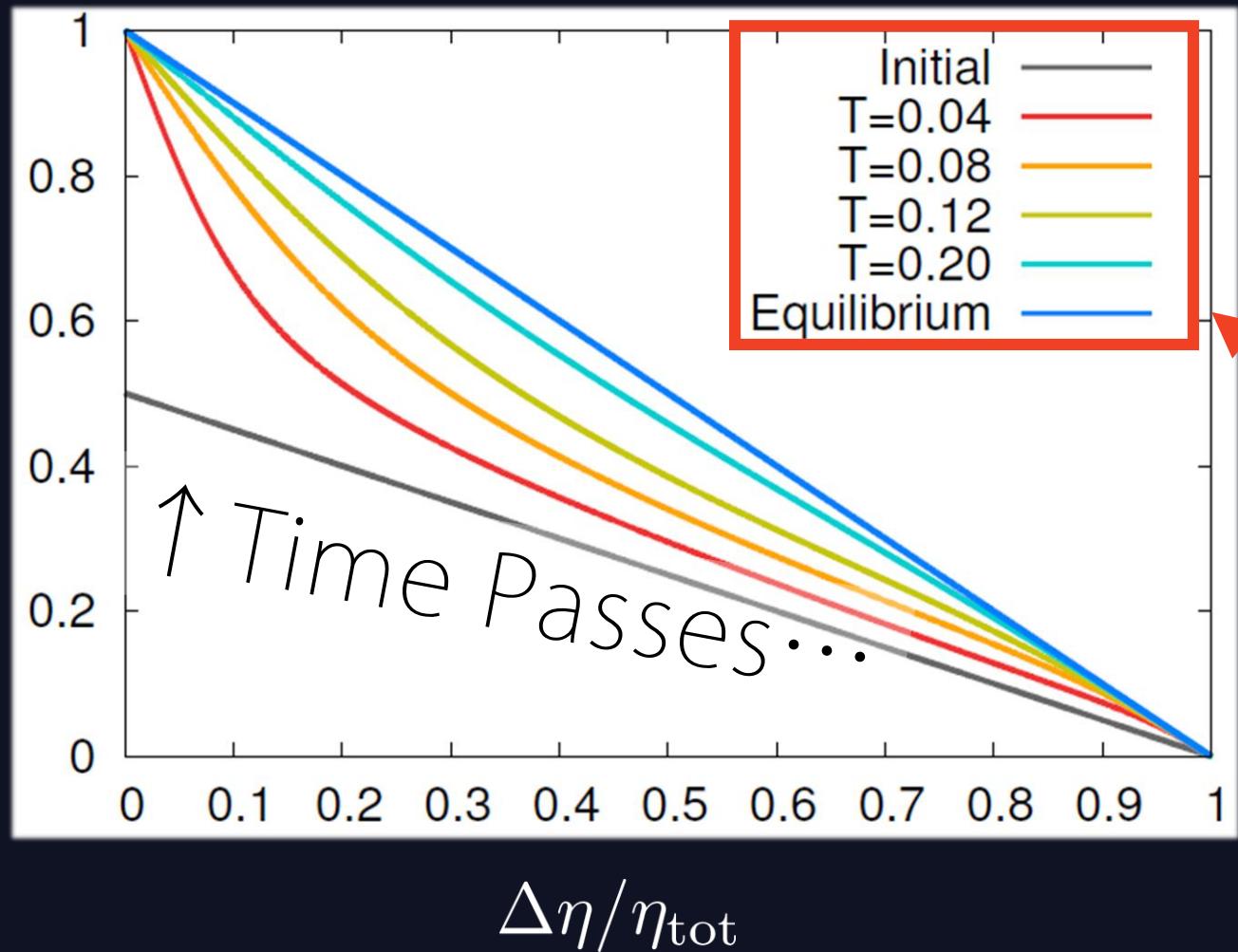
$$T = \frac{d(\tau)}{\eta_{\text{tot}}}$$



η_{tot} : Total Rapidity
Length

$\Delta\eta$ Dep. of Fluctuations (+ Initial Fluctuation)

$$\langle Q_{(\text{net})}^2 \rangle_c / \langle Q_{(\text{tot})} \rangle_c$$

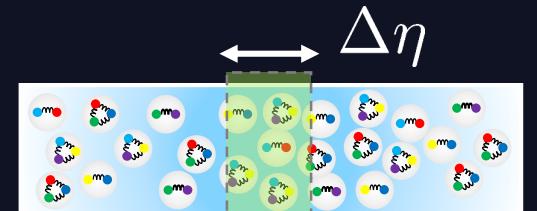


$$d(\tau) = \sqrt{2 \int_{\tau_0}^{\tau} D(\tau') d\tau'}$$

: Average Diffusion Length

$D(\tau)$: Diffusion Coefficient

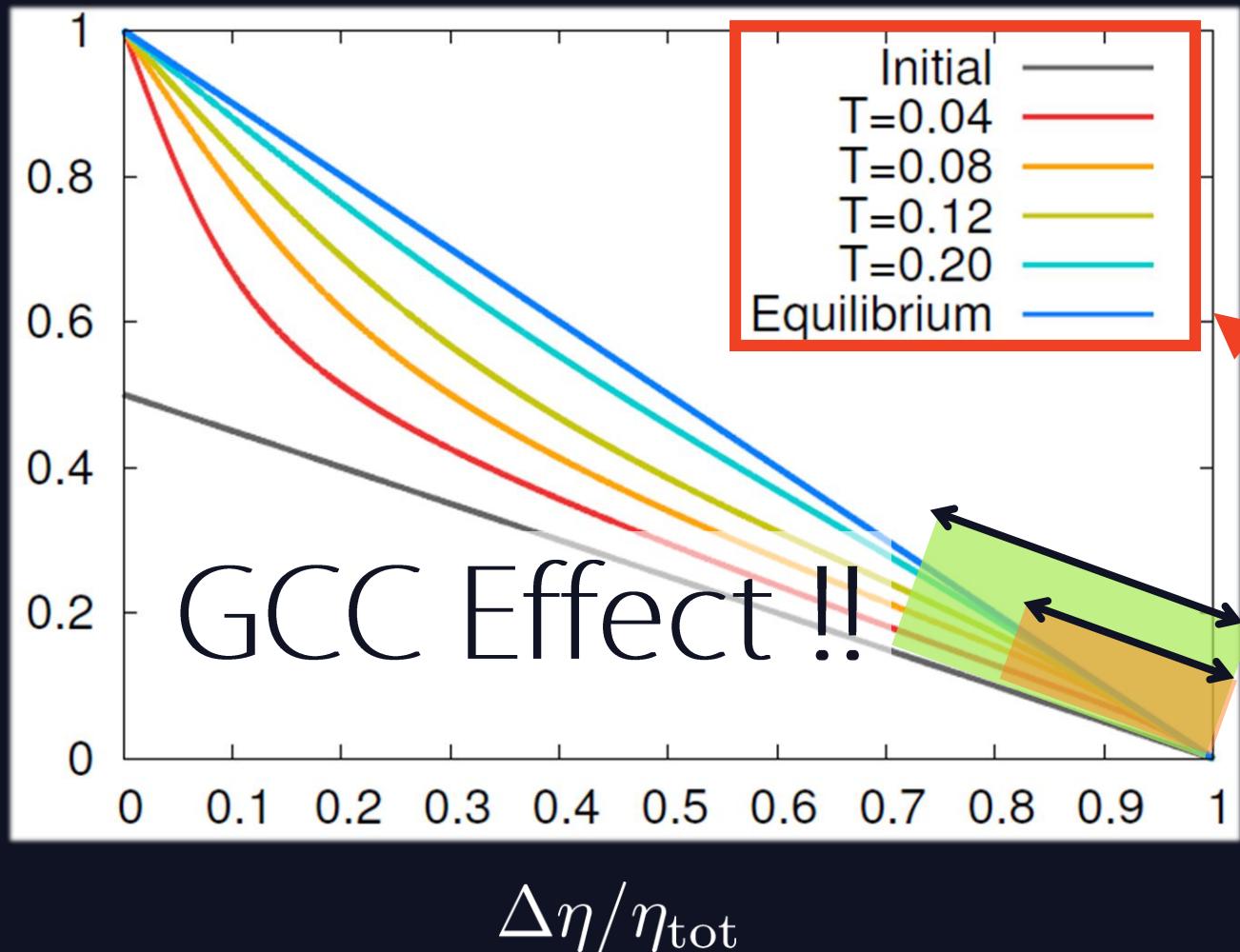
$$T = \frac{d(\tau)}{\eta_{\text{tot}}}$$



η_{tot} : Total Rapidity Length

$\Delta\eta$ Dep. of Fluctuations (+ Initial Fluctuation)

$$\langle Q_{(\text{net})}^2 \rangle_c / \langle Q_{(\text{tot})} \rangle_c$$

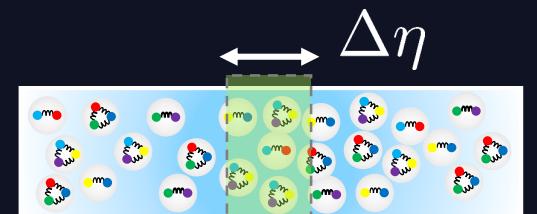


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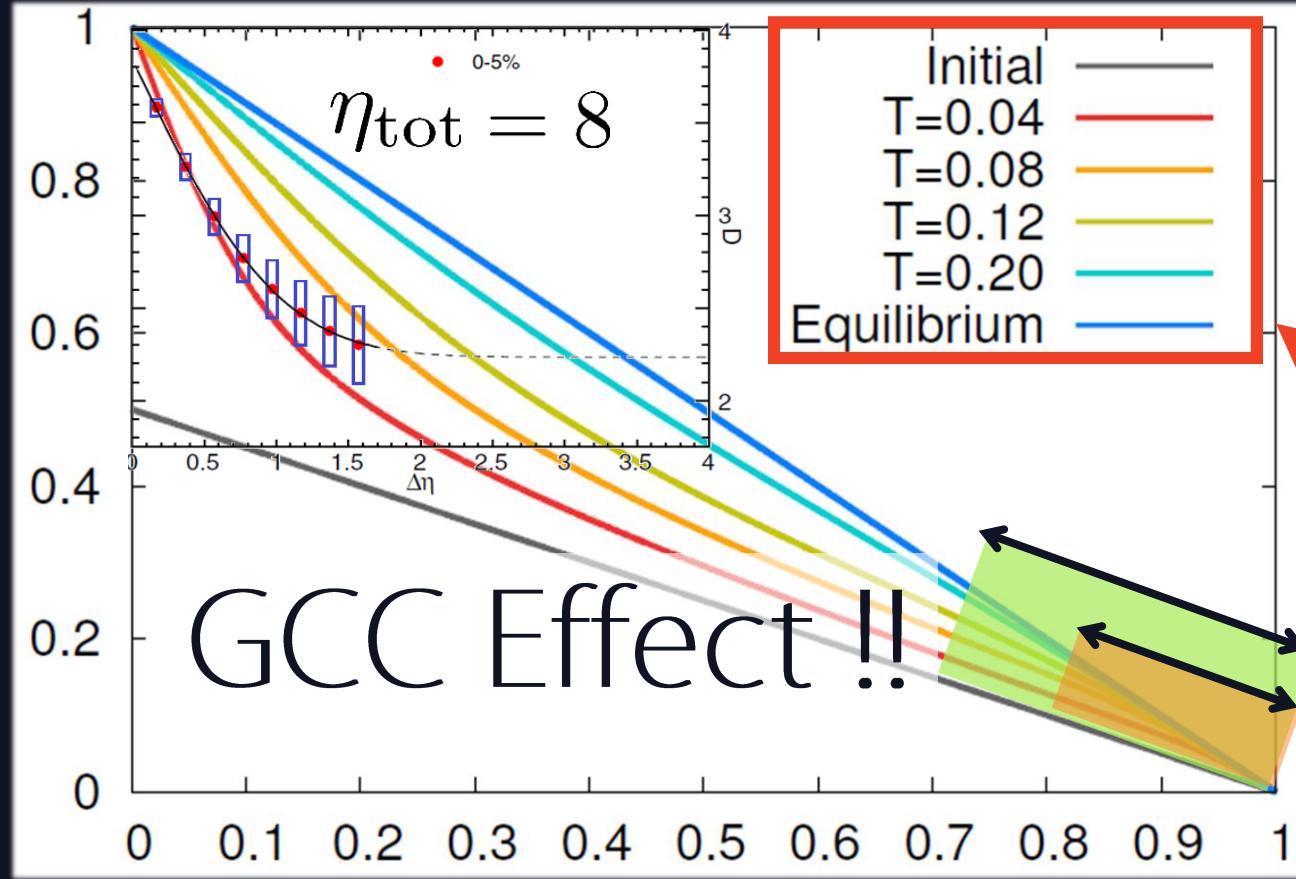
$$T = \frac{d(\tau)}{\eta_{\text{tot}}}$$



η_{tot} : Total Rapidity Length

Comparison with Experiment (+ Initial Fluctuation)

$$\langle Q_{(\text{net})}^2 \rangle_c / \langle Q_{(\text{tot})} \rangle_c$$

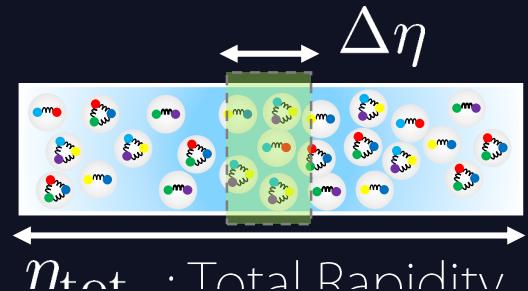


$$d(\tau) = \sqrt{2 \int_{\tau_0}^{\tau} D(\tau') d\tau'}$$

: Average Diffusion Length

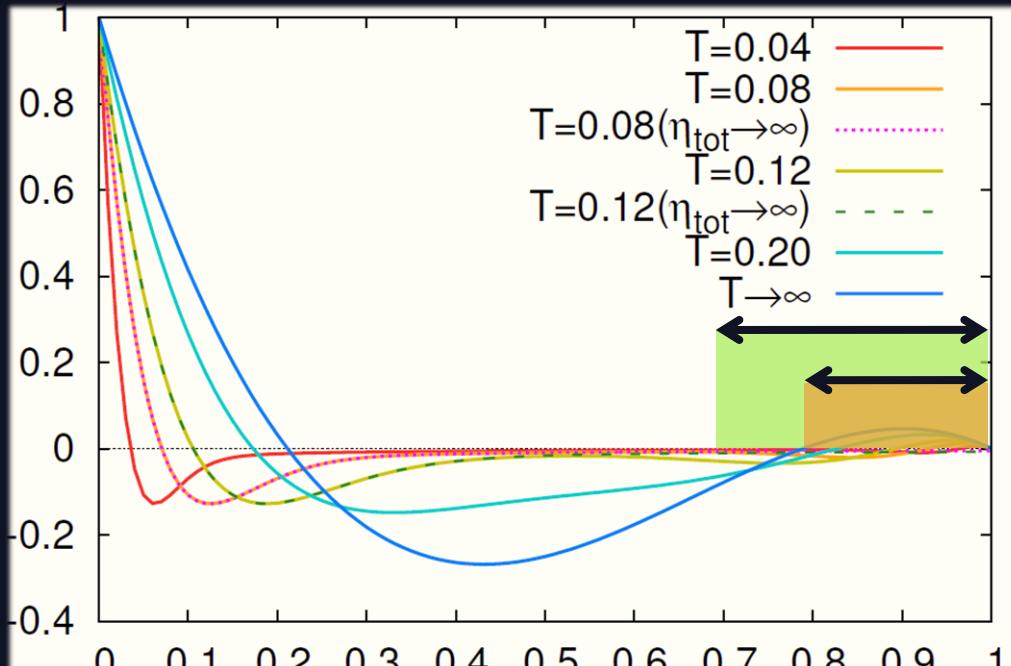
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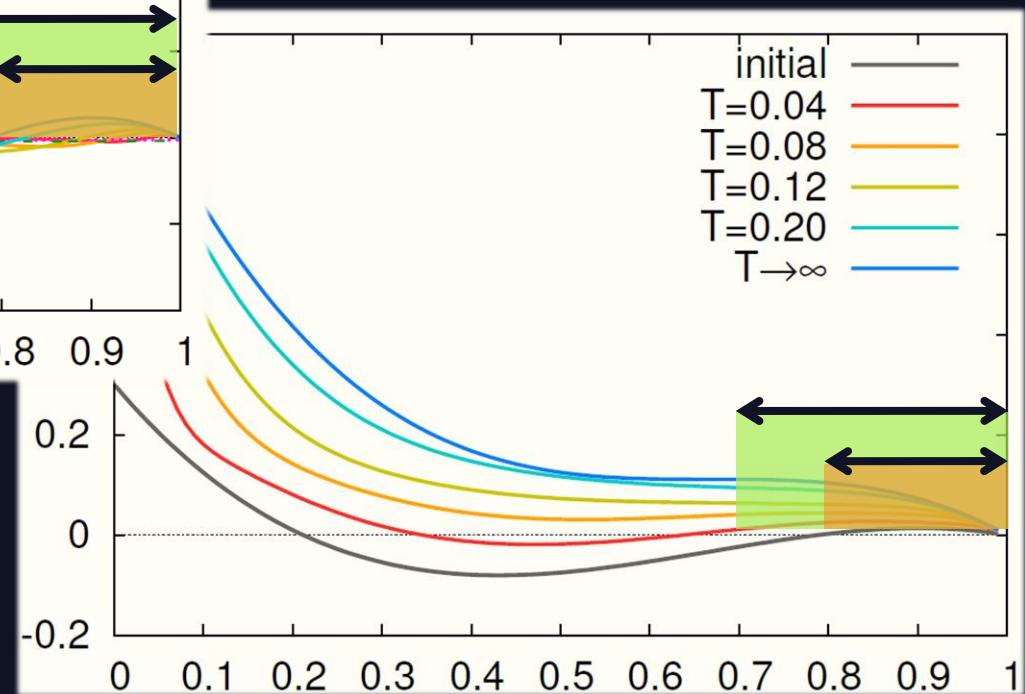


GCC Effect is almost negligible !

4th order Fluctuations



+ Initial Fluctuation



No Initial Fluctuation

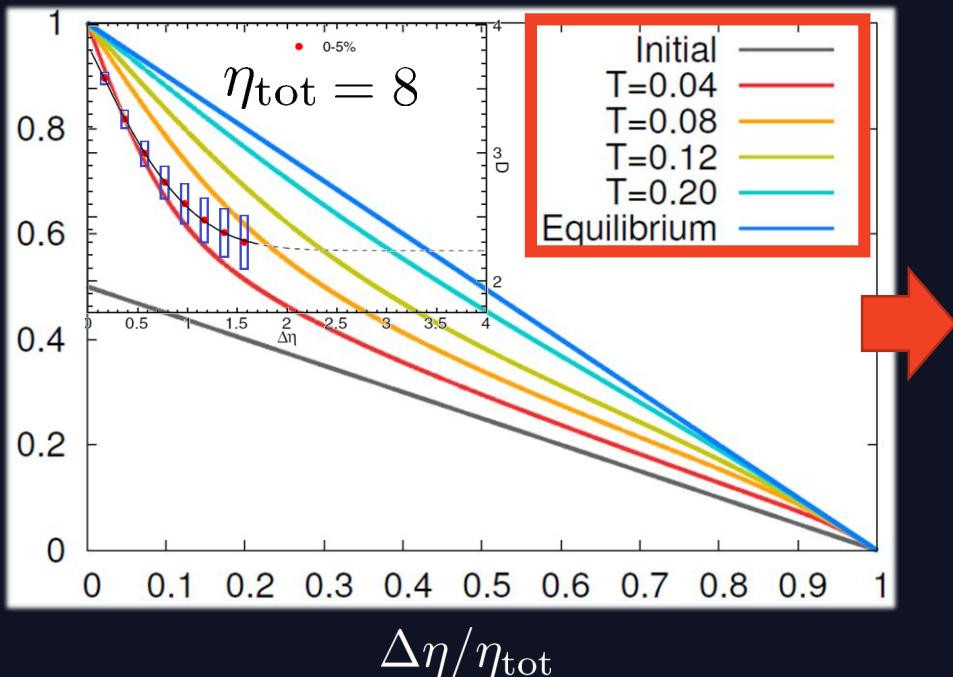
GCC Effect appears ONLY near Boundaries !

What information can we obtain?

Suppression of Charge Fluctuation observed @ALICE
→ Global Charge Conservation

Fluctuations are NOT Equilibrated.

$$\langle Q_{(\text{net})}^2 \rangle_c / \langle Q_{(\text{tot})} \rangle_c$$



We can know about

- * Fluctuation in QGP
- * Time Evolution
- * Diffusion Coefficient

etc...

Summary

- *We investigated the Rapidity Window Dependence of Fluctuations of Conserved Charges and GCC Effect.
 - *GCC Effect appears ONLY near boundaries.
 - *GCC Effects is almost negligible in Results at ALICE.

Fluctuations of Conserved Charges tell us much Information about the Hot Medium.

Fluctuations in QGP, Time Evolution of Hot medium, Diffusion Coefficient, Hadronization Mechanism, etc...

Back Up

Outline

1. Fluctuations of Conserved Charges

- How to measure?
- Why Fluctuation?

2. Problem

- Global Charge Conservation

3. Method

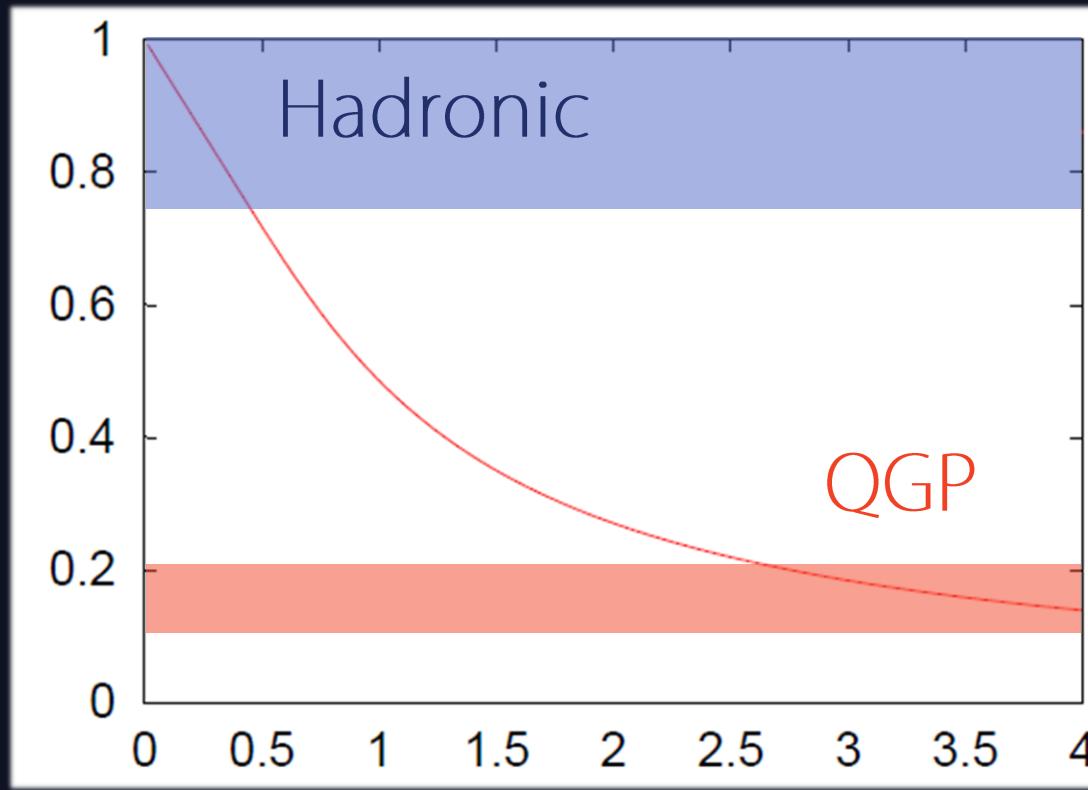
- Diffusion Model (1D Brownian Motion)

4. Results

Time Evolution of 2nd order Fluctuations

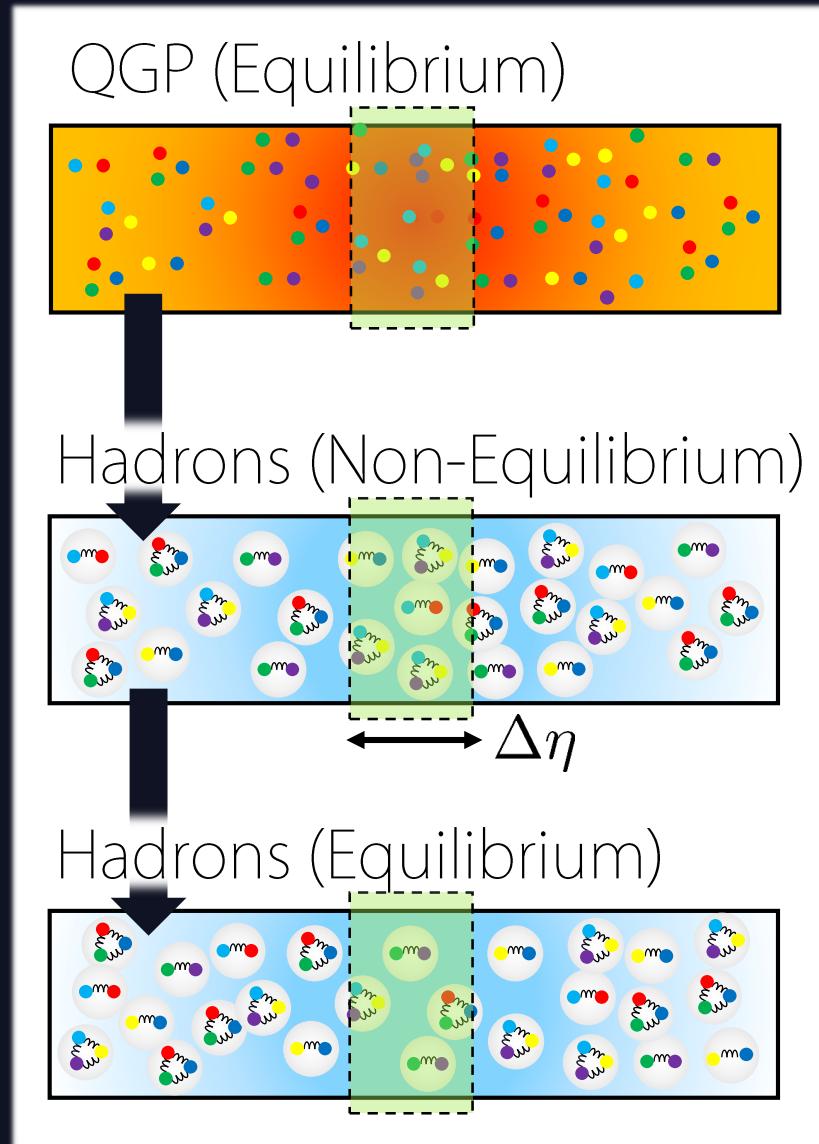
Fluctuating Hydro.
(Stochastic Diffusion Eq.)

$$\frac{\langle \delta N_Q^{(\text{net})2} \rangle_c}{\langle N_Q^+ + N_Q^- \rangle_c}$$

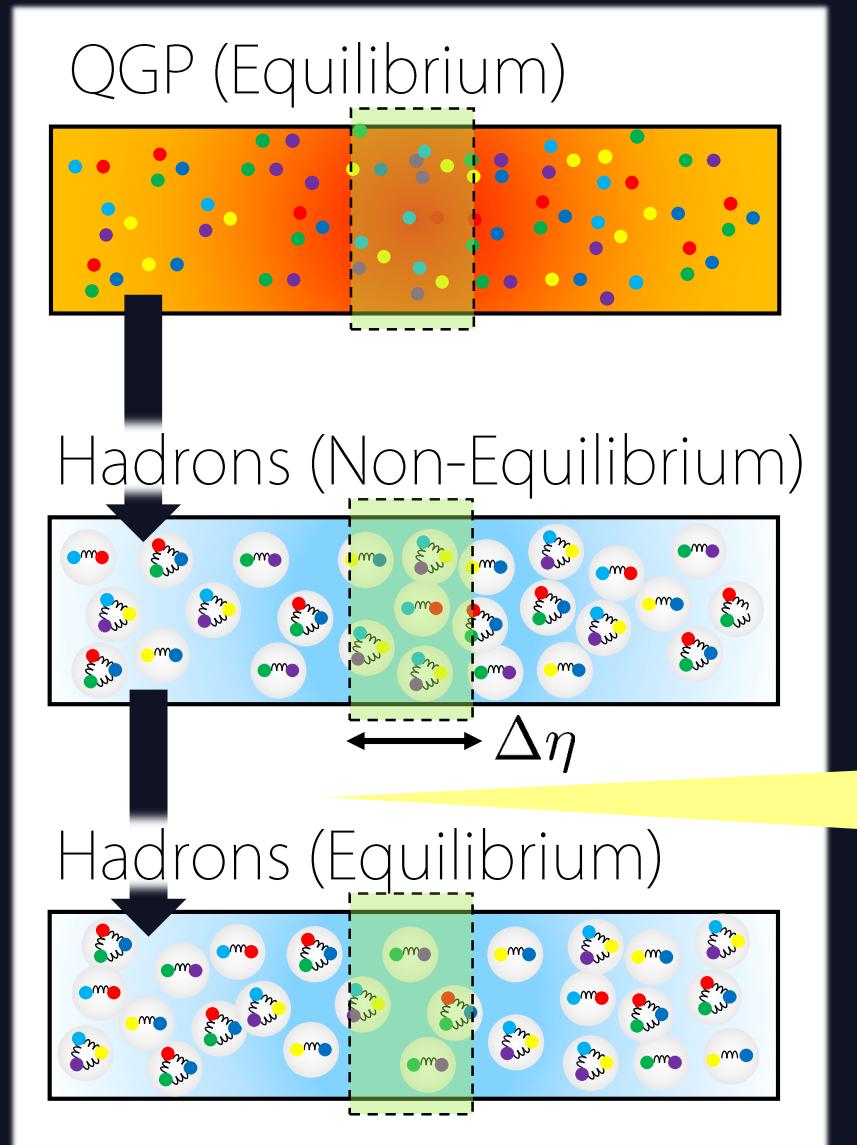


$\sim \Delta\eta$

Time Evolution of Fluctuations

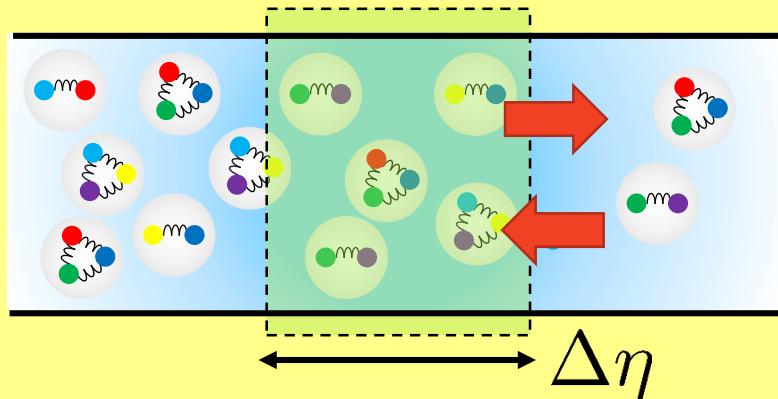


Time Evolution of Fluctuations



Relaxation can **only** proceed by **Diffusion of Charges !!**

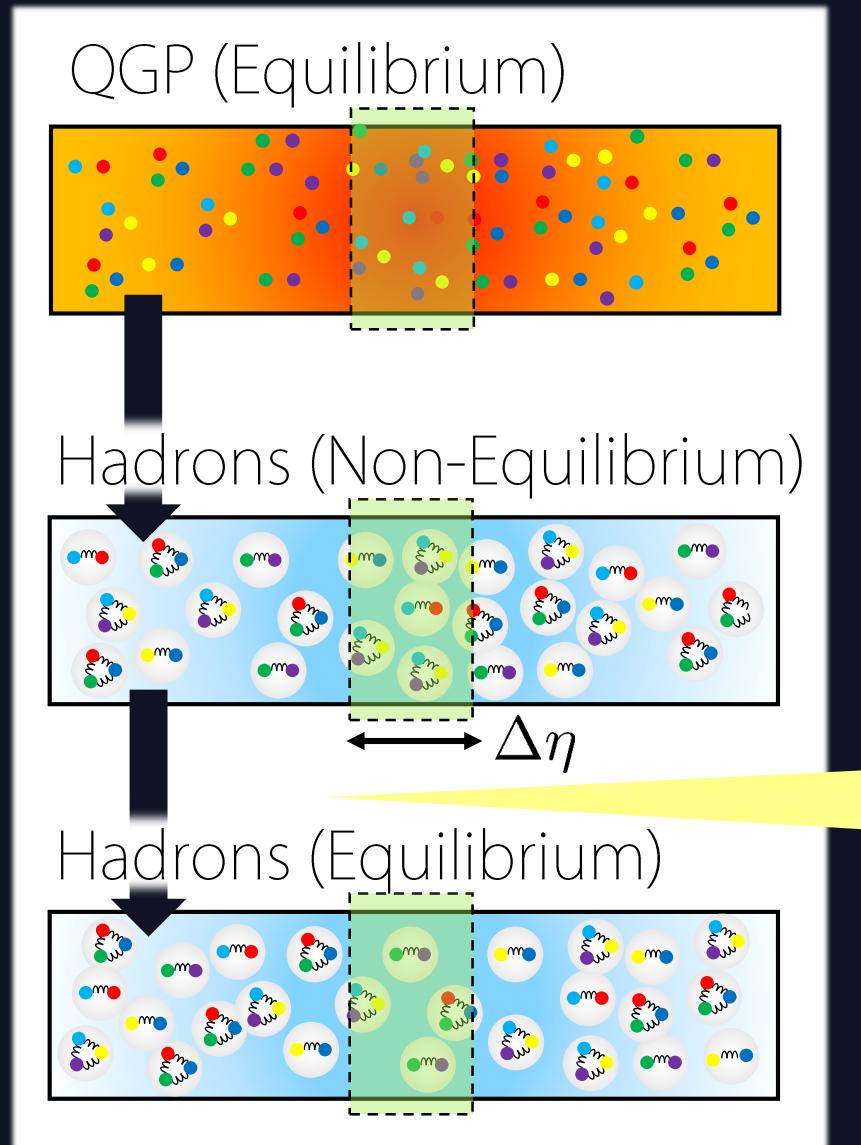
Shuryak, Stephanov (2001)



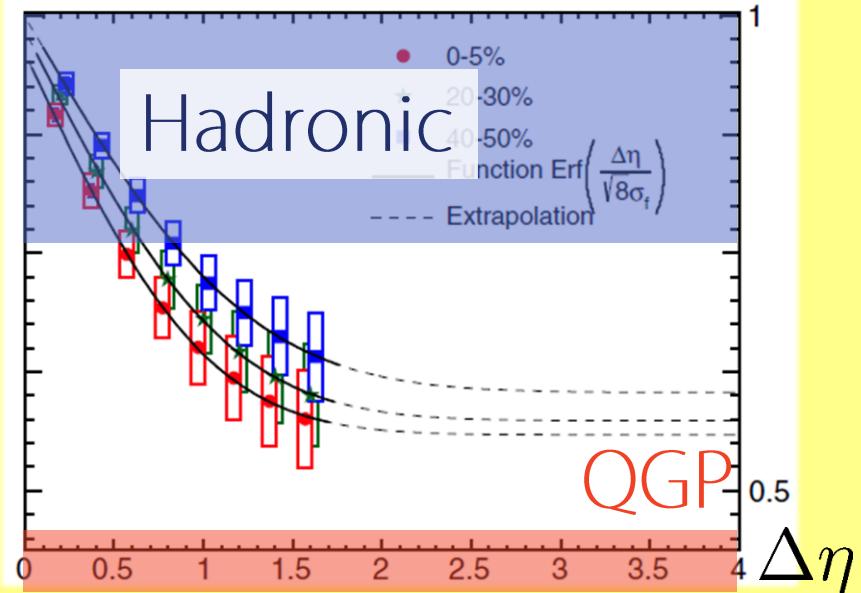
$\Delta\eta \rightarrow$ Larger

→ Relaxation Time → Longer
(More the QGP Value)

Time Evolution of Fluctuations



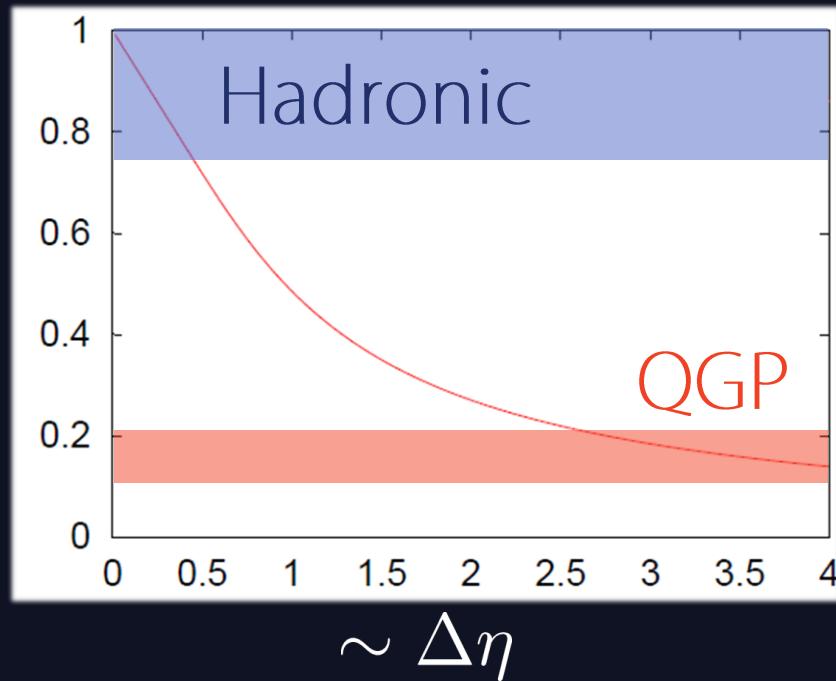
2nd order Fluctuation @ ALICE



$\Delta\eta \rightarrow$ Larger

→ Relaxation Time → Longer
(More the QGP Value)

Time Evolution of Fluctuations

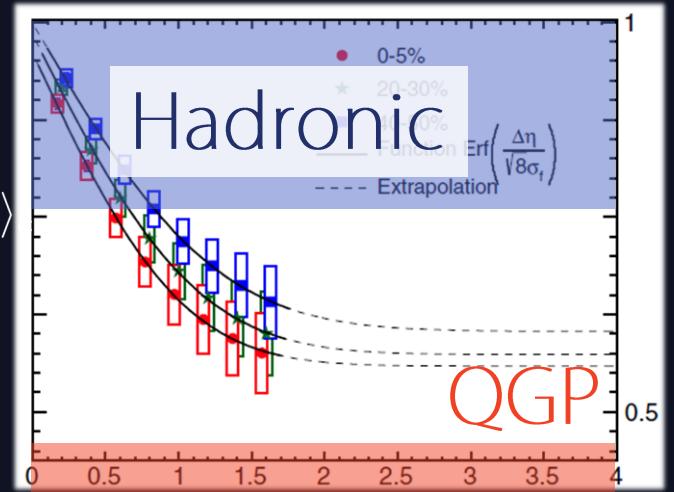


Fluctuating Hydro.
(Stochastic Diffusion Eq.)

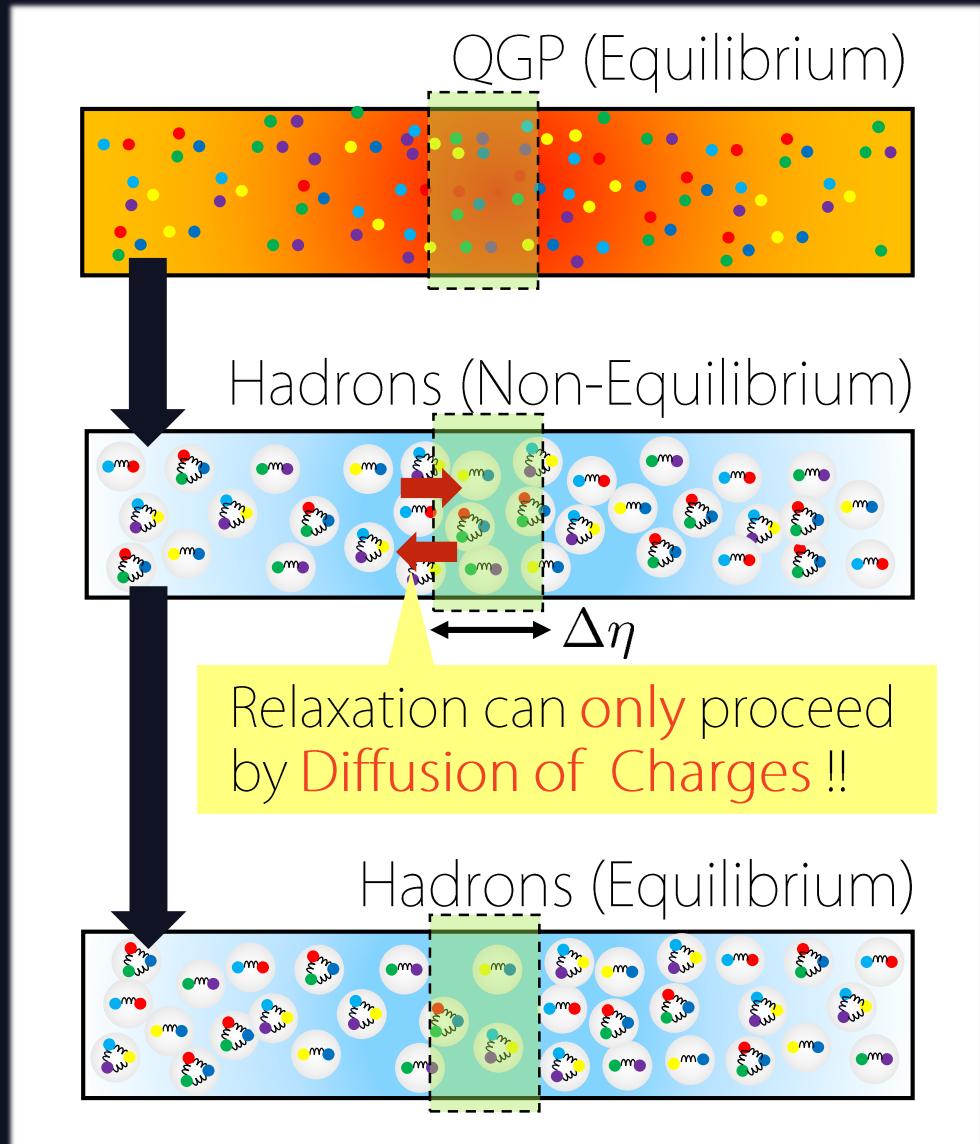
Shuryak, Stephanov (2001)

$$\langle Q_{(\text{net})}^2 \rangle_c / \langle Q_{(\text{tot})} \rangle$$

ALICE, PRL110, 152301 (2013) $\Delta\eta$

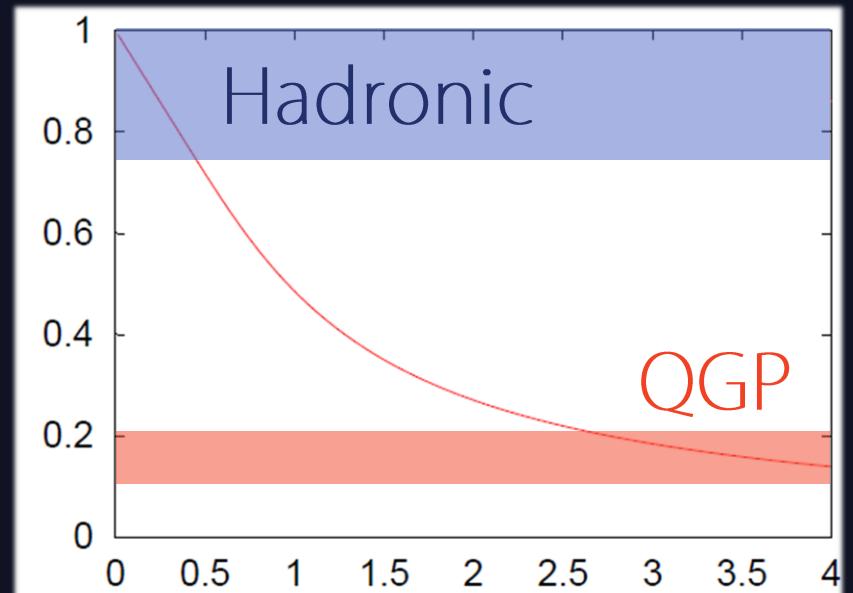


Time Evolution of Fluctuations



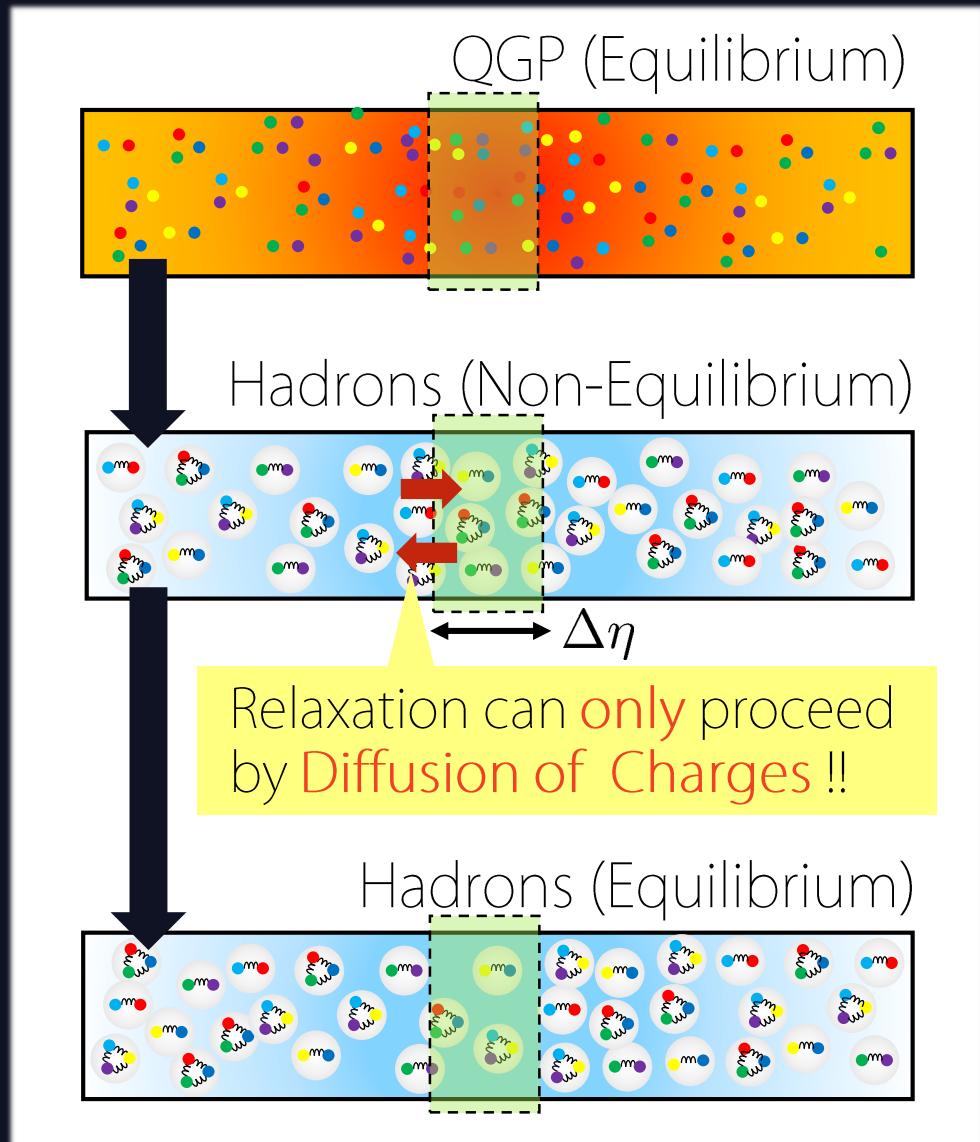
Fluctuating Hydro.
(Stochastic Diffusion Eq.)

2nd order Fluctuation

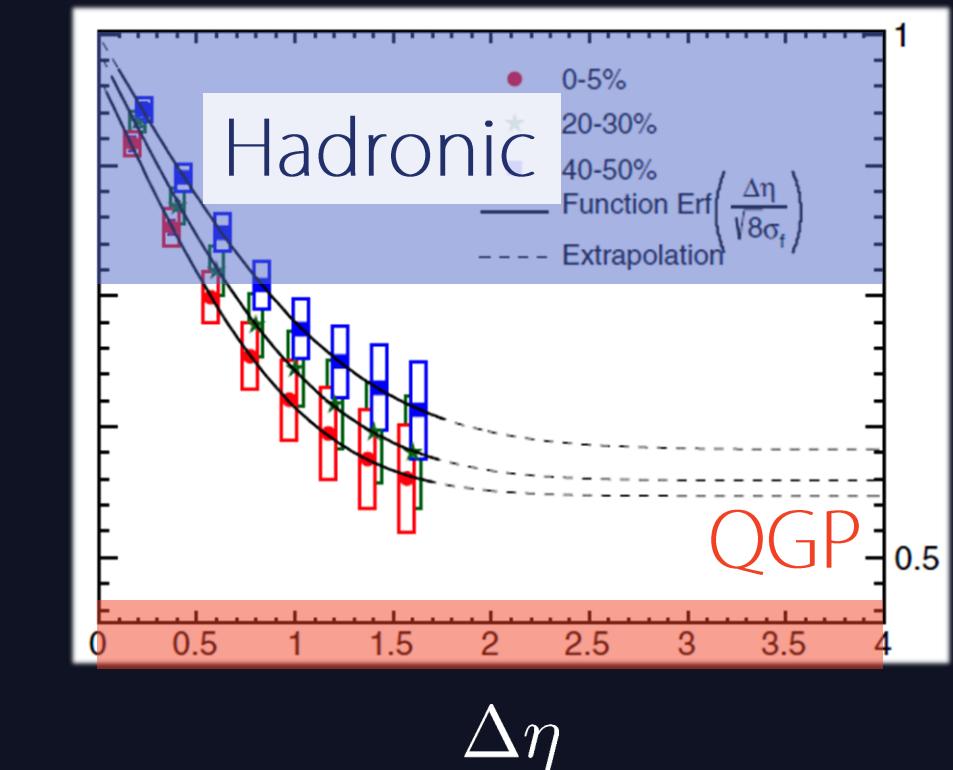


$\sim \Delta\eta$: Rap. Win.
Shuryak, Stephanov (2001)

Time Evolution of Fluctuations



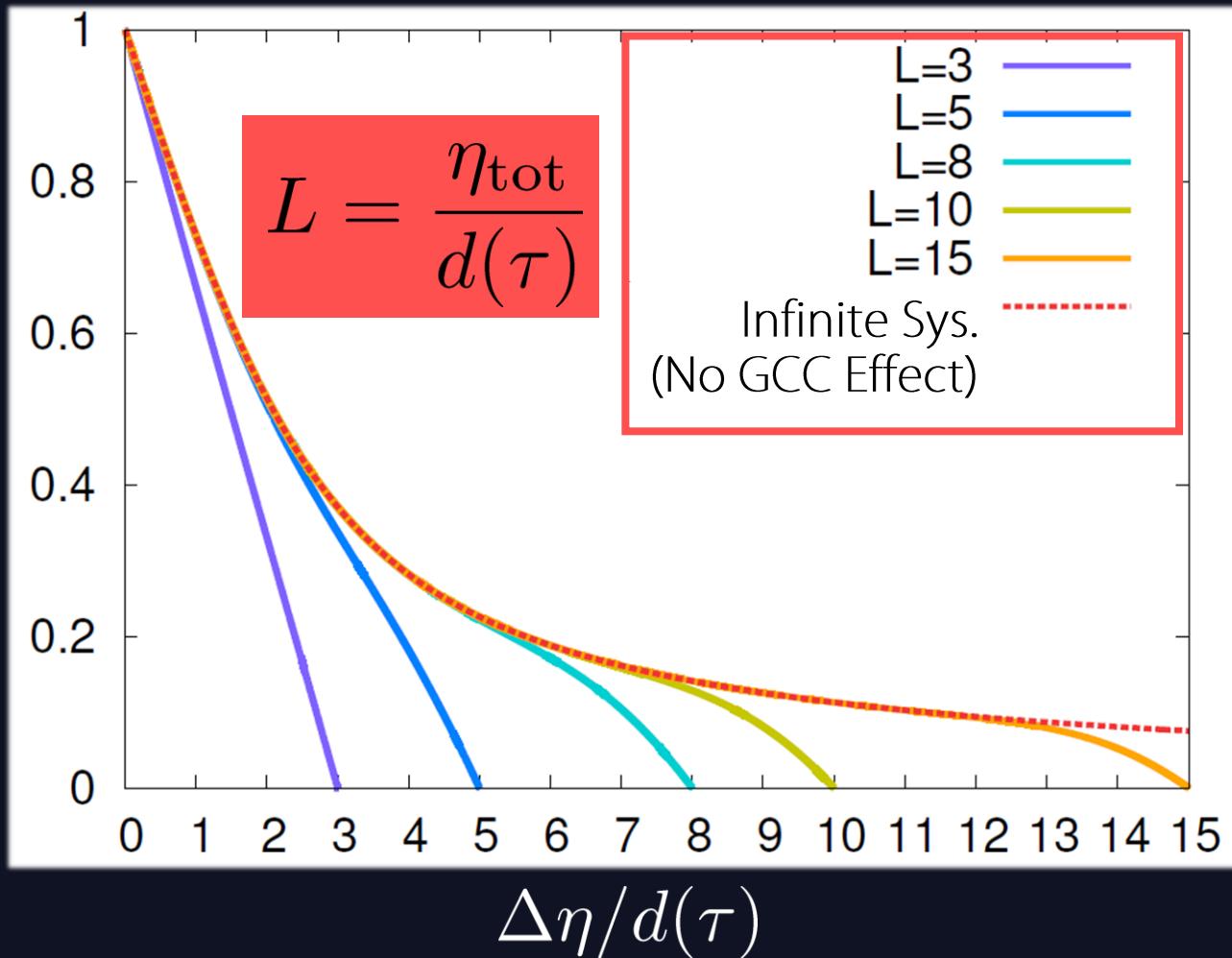
2nd order Fluctuation
ALICE, PRL110, 152301 (2013)



Shuryak, Stephanov (2001)

$\Delta\eta$ Dep. of Fluctuations (No Initial Fluctuation)

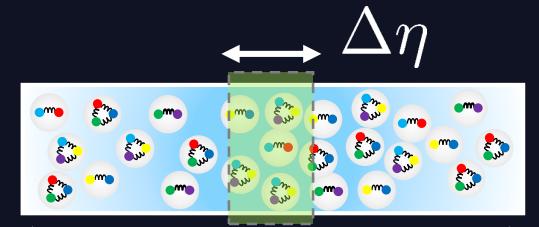
$$\langle Q_{(\text{net})}^2 \rangle_c / \langle Q_{(\text{tot})} \rangle_c$$



$$d(\tau) = \sqrt{2 \int_{\tau_0}^{\tau} D(\tau') d\tau'}$$

: Average
Diffusion Length

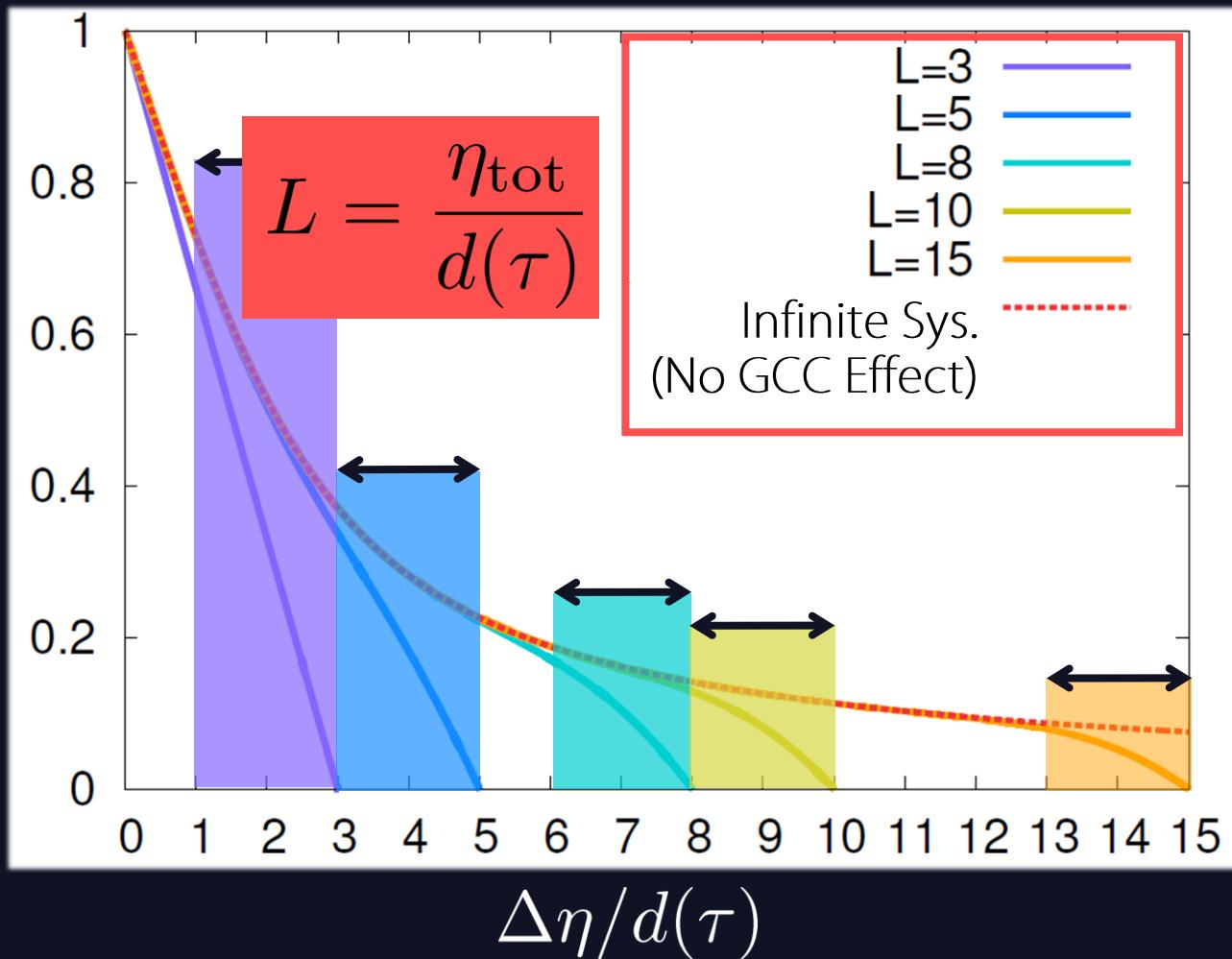
$D(\tau)$: Diffusion
Coefficient



η_{tot} : Total Rapidity Length

$\Delta\eta$ Dep. of Fluctuations (No Initial Fluctuation)

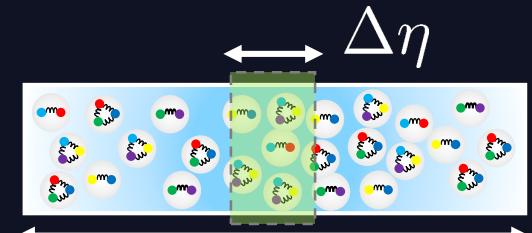
$$\langle Q_{(\text{net})}^2 \rangle_c / \langle Q_{(\text{tot})} \rangle_c$$



$$d(\tau) = \sqrt{2 \int_{\tau_0}^{\tau} D(\tau') d\tau'}$$

: Average Diffusion Length

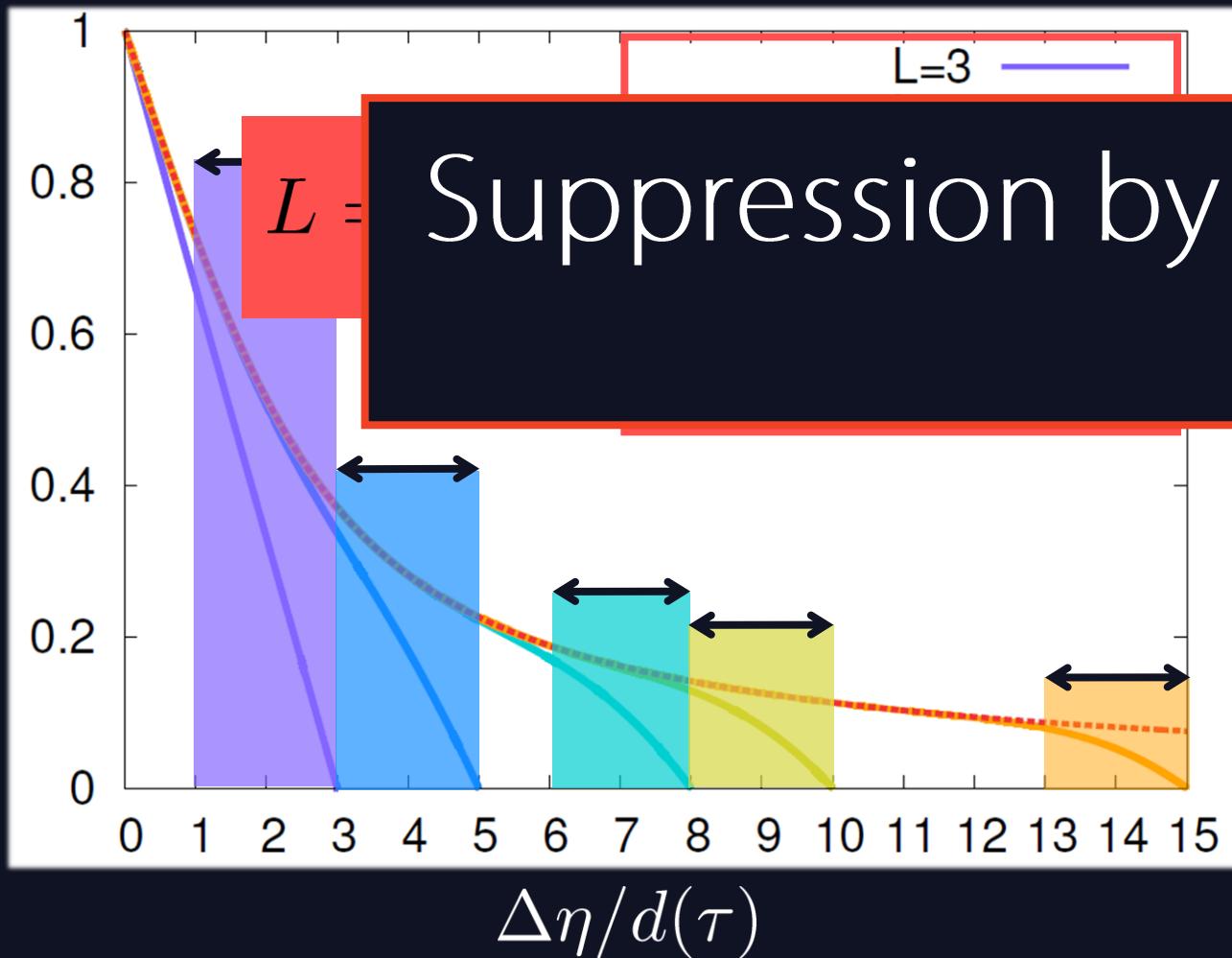
$D(\tau)$: Diffusion Coefficient



$\Delta\eta$: Rapidity Width
 η_{tot} : Total Rapidity Length

$\Delta\eta$ Dep. of Fluctuations (No Initial Fluctuation)

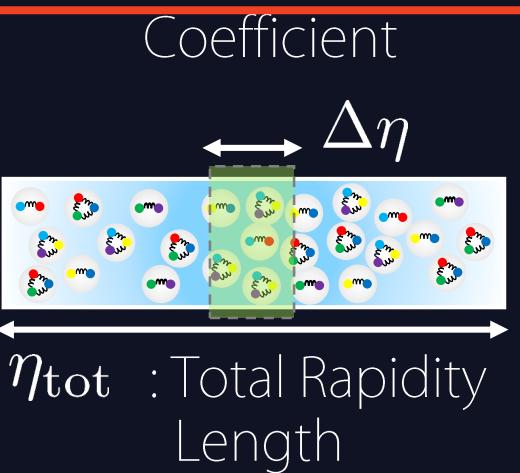
$$\langle Q_{(\text{net})}^2 \rangle_c / \langle Q_{(\text{tot})} \rangle_c$$



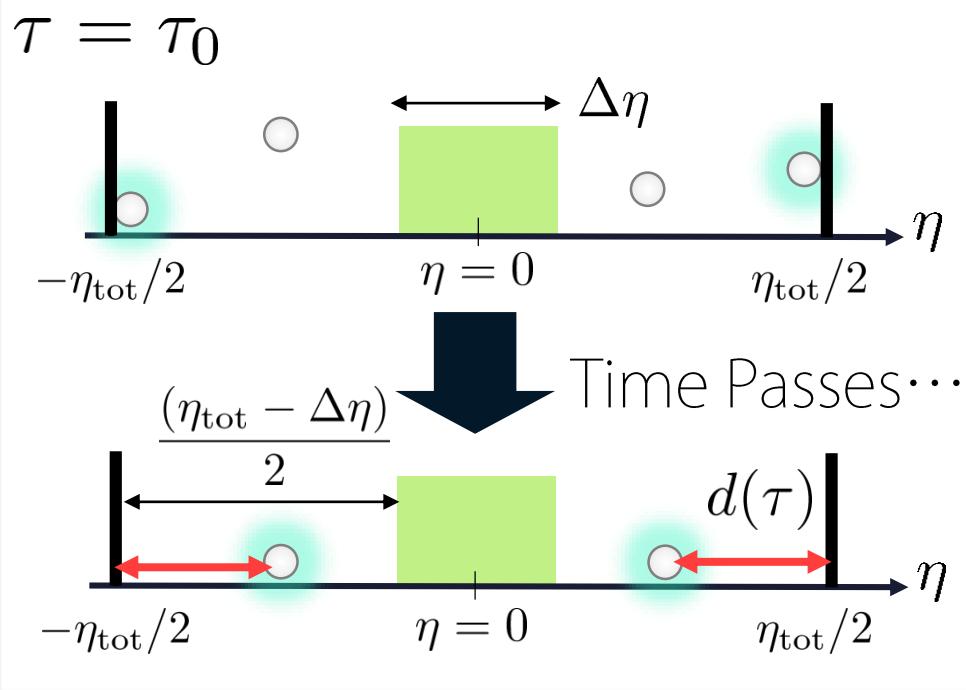
$$d(\tau) = \sqrt{2 \int_0^\tau D(\tau') d\tau'}$$

Suppression by GCC Effects

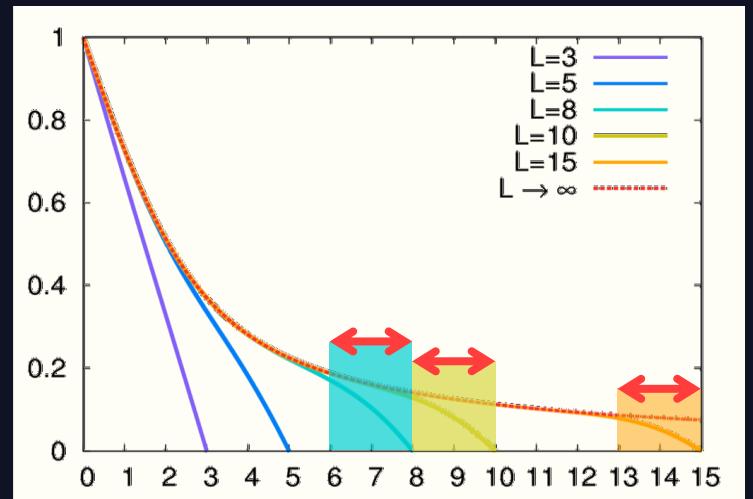
$$\Delta\eta/d \geq L - 2$$



Interpretation of Results for GCC Effect



$d(\tau)$: Averaged Diffusion Length
 $D(\tau)$: Diffusion Coefficient
 η_{tot} : Total Length of Matter

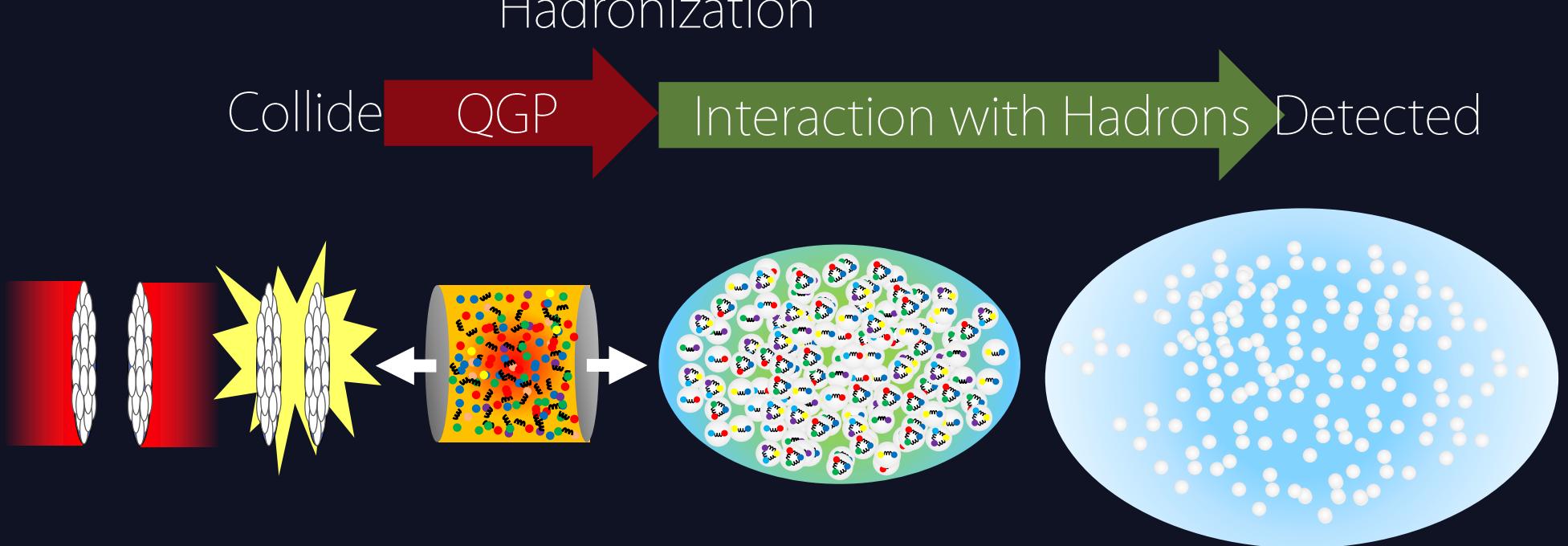


Condition for effects of the GCC

$$\Delta\eta/d \geq L - 2 \Leftrightarrow \frac{\eta_{\text{tot}} - \Delta\eta}{2} \leq d$$

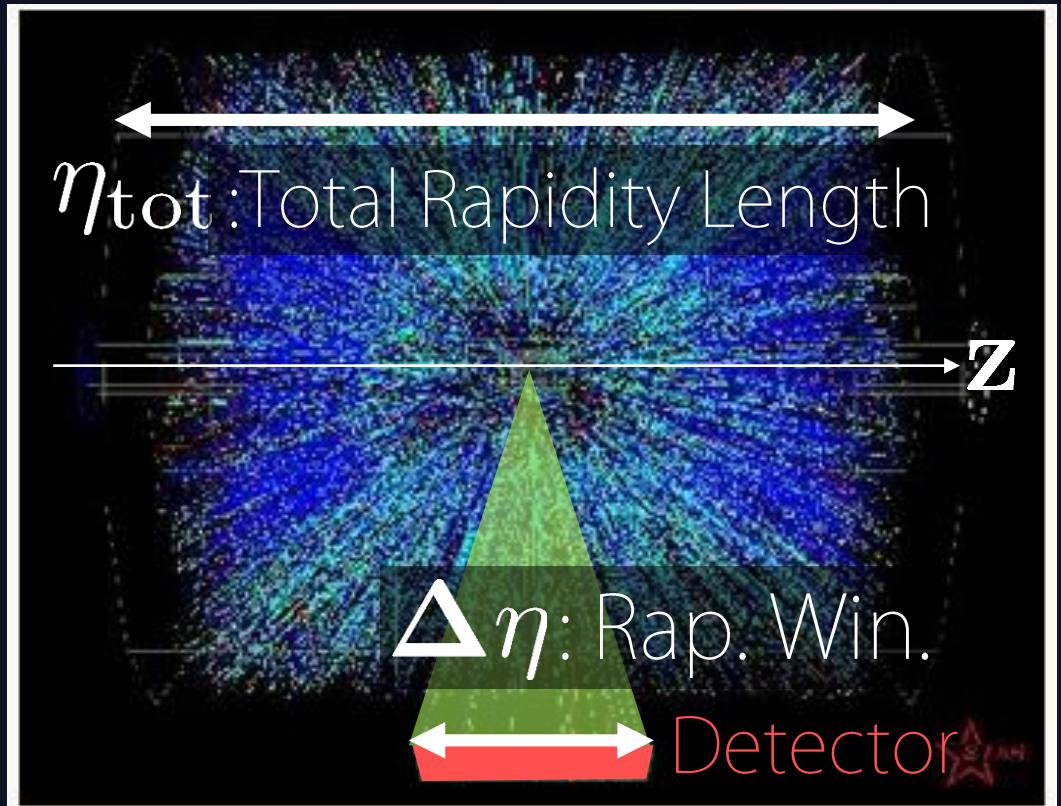
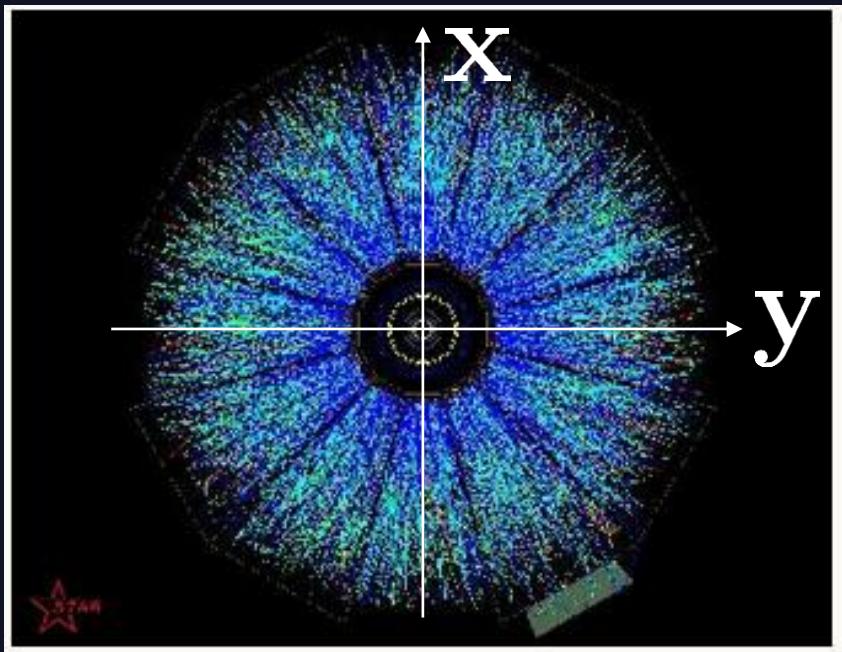
Effects of the GCC appear only near the boundaries.

Time Evolution of Heavy Ion Collisions



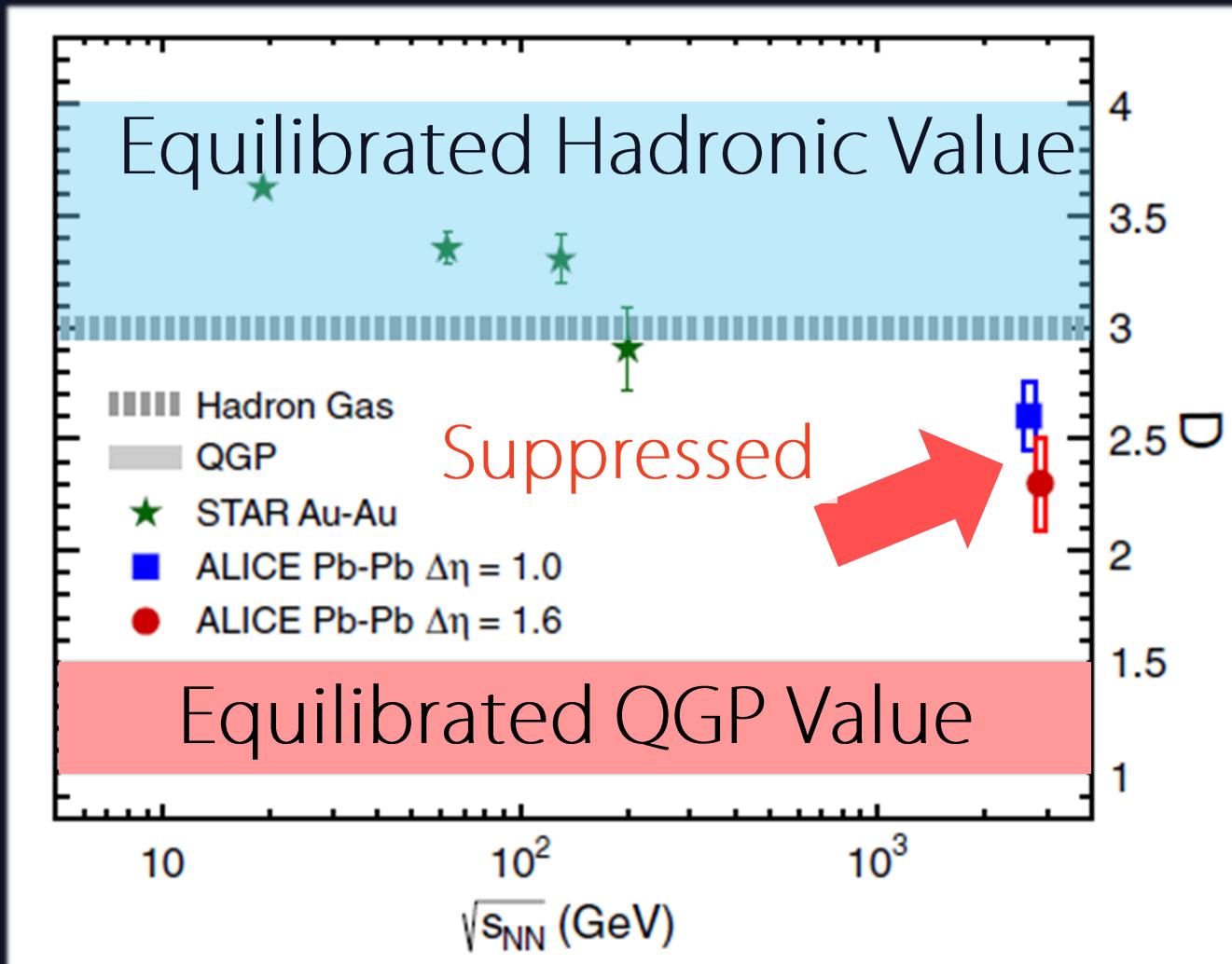
- Detected Particles : Hadrons
 - After Phase Transition, Interaction
- Lose Information About QGP

Event-by-Event Fluctuations



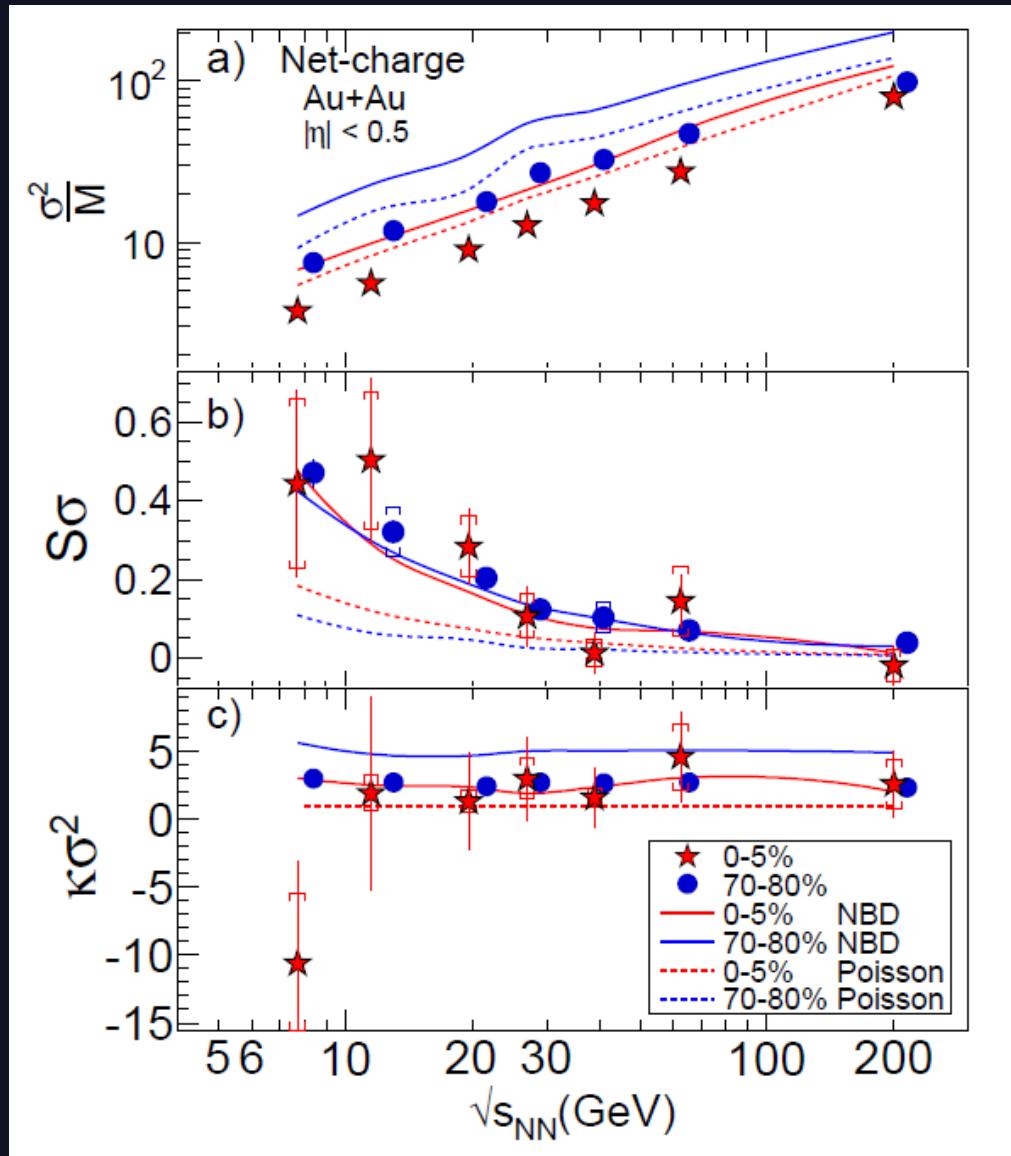
Net Electric Charge Fluctuation @ ALICE

ALICE, PRL110, 152301 (2013) ALICE, PRL110, 152301, 2013



$$D = \frac{4\langle(\delta N_Q^{(\text{net})})^2\rangle_c}{\langle N_Q^+ + N_Q^-\rangle_c}$$

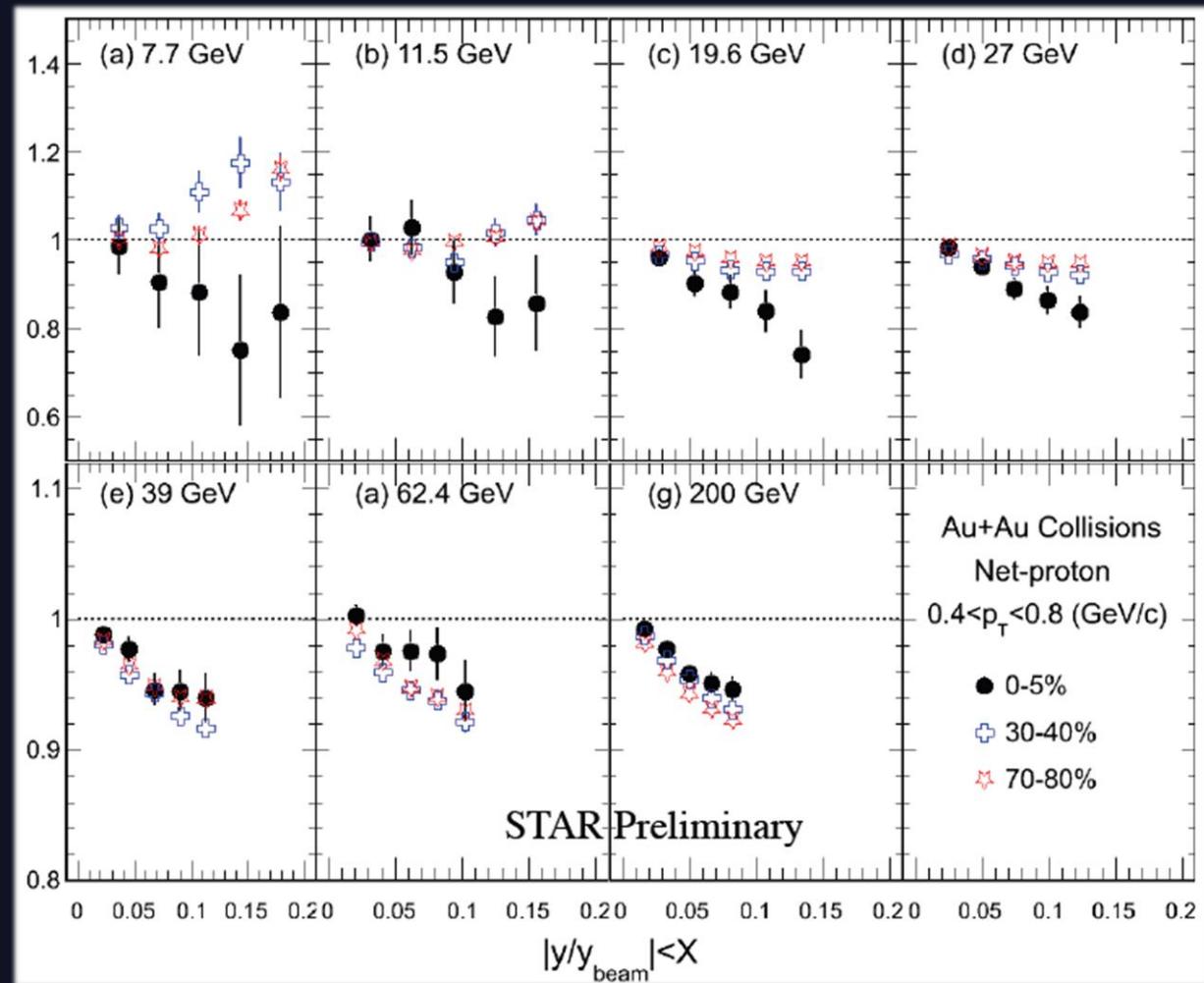
Net Proton Number Fluctuation @ BES



RHIC (2014)

Rapidity Window Dependence of Net Proton Number Fluctuation @ STAR

$$\frac{\langle \delta N_p^4 \rangle_c}{\langle \delta N_p^2 \rangle}$$



Binomial and Poisson Distribution

Binomial Distribution

$$B_p(n; N) = p^n (1 - p)^{N-n} {}_n C_N$$

 $p \rightarrow 0$

Poisson Distribution

$$P_\lambda(n) = \frac{\lambda^n e^{-\lambda}}{n!}$$

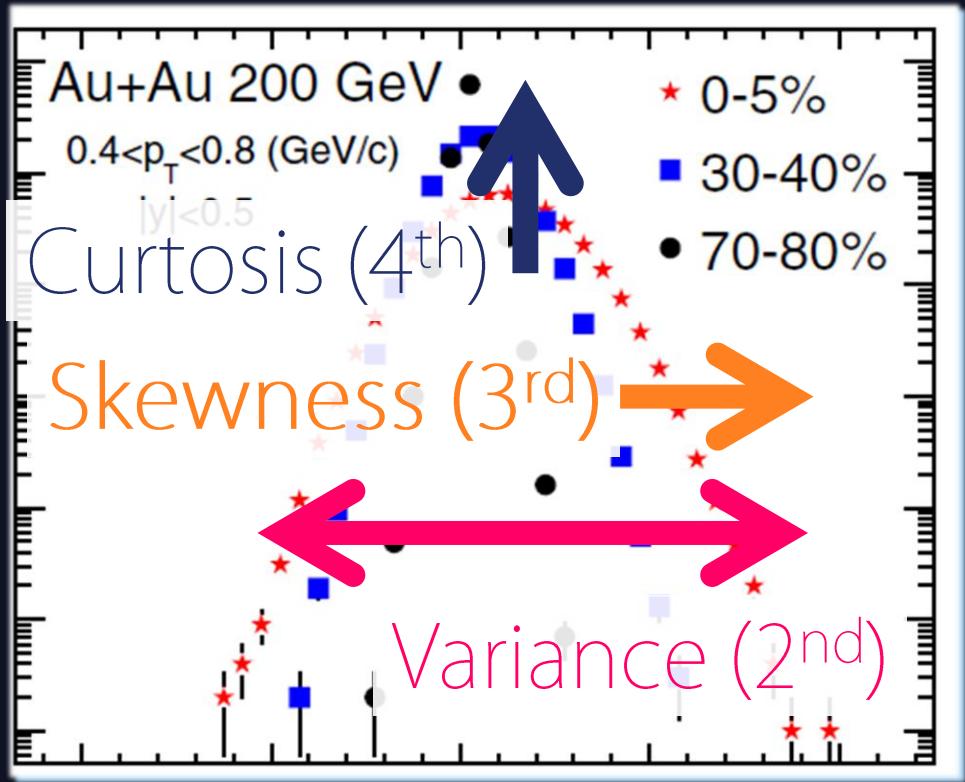
Cumulants $\langle \delta N^n \rangle_c$

Quanta characterizing
the Probability Distribution

$$\langle (\delta N)^2 \rangle_c = \langle (\delta N)^2 \rangle = \sigma^2$$

$$\langle (\delta N)^3 \rangle_c = \langle (\delta N)^3 \rangle = S\sigma^3$$

$$\langle (\delta N)^4 \rangle_c = \langle (\delta N)^4 \rangle - 3\langle (\delta N)^2 \rangle^2 = \kappa\sigma^2$$

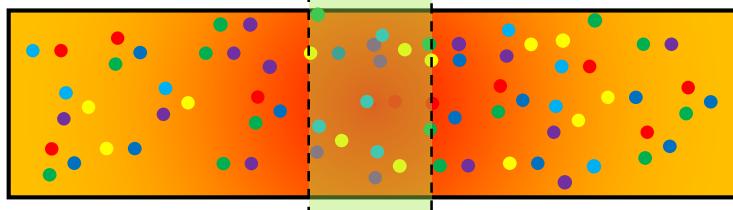


Baryon Number Fluctuation

Classical Free Gas in Equilibrated Infinite System

$$\rightarrow \text{Poisson} \quad \langle \delta N^n \rangle_c = \langle N \rangle$$

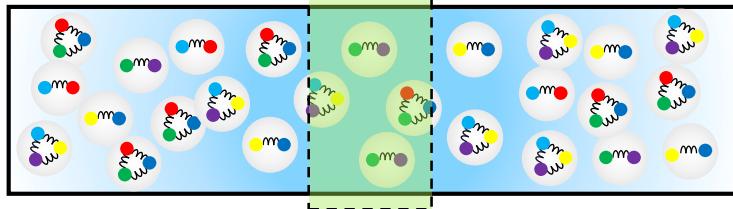
QGP (Equilibrium)



$$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$$

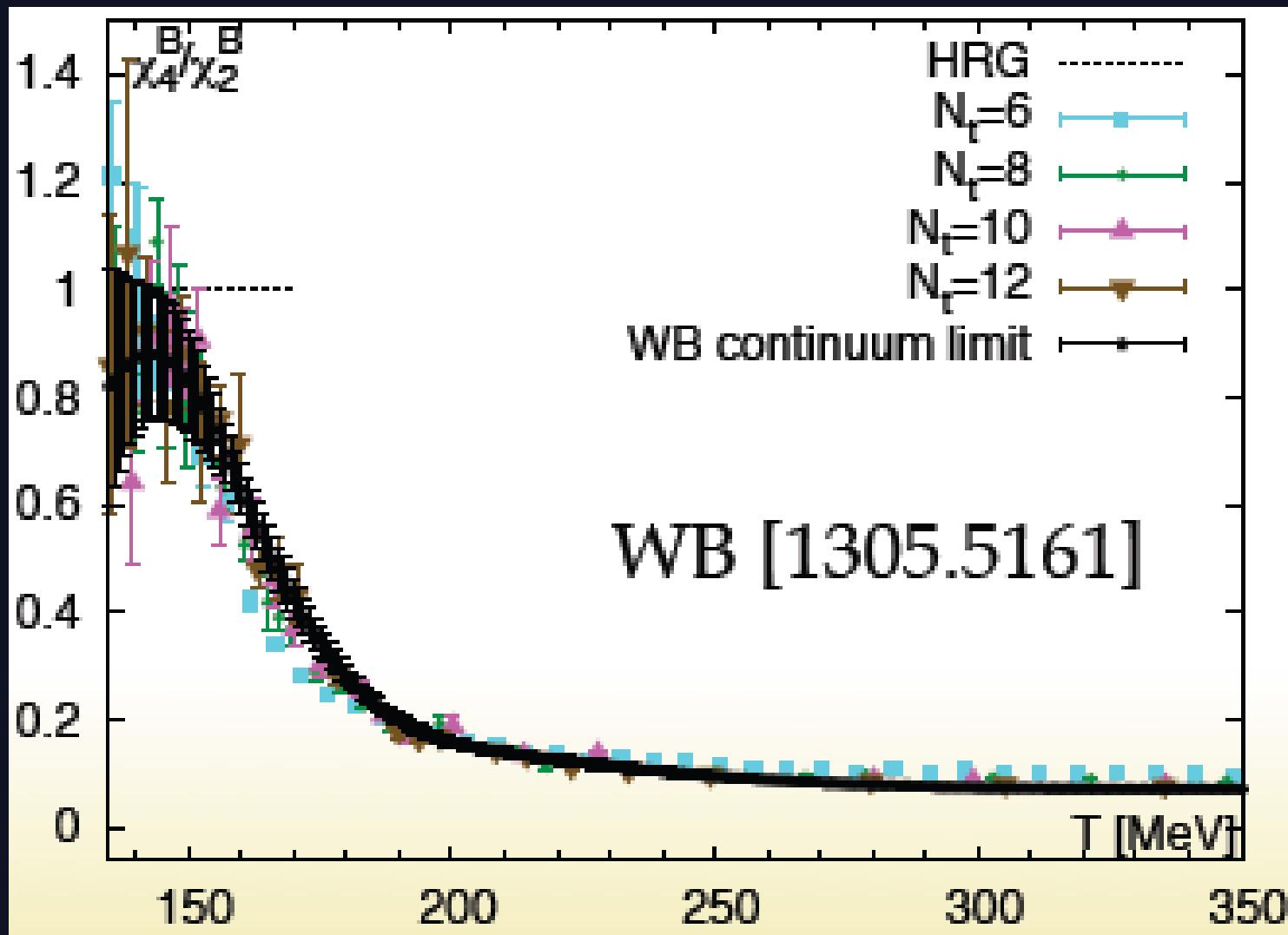
$$3N_B = N_q$$
$$\langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$$

Hadrons (Equilibrium)



$$\langle \delta N_B^n \rangle_c = \langle N_B \rangle$$

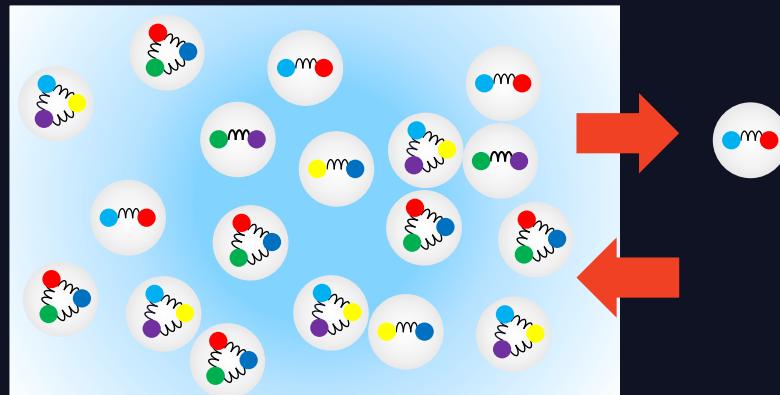
Baryon Number Fluctuation



Conserved VS Non-Conserved Charges

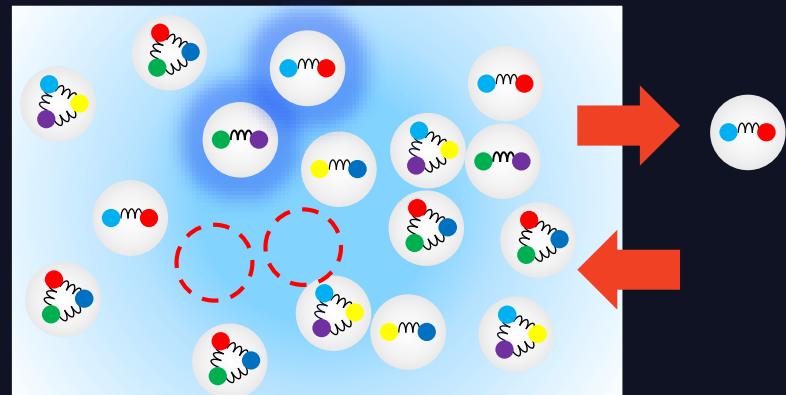
Relaxation time are larger than typical time scale(?)

Conserved Charges



ONLY by Diffusion of
Charges

Non-Conserved Charges



Can change everywhere

Information about QGP is easy to survive !!

Stochastic Diffusion Eq.

$$\partial_\tau n = D(\tau) \partial_\eta^2 n + \partial_\eta \xi$$

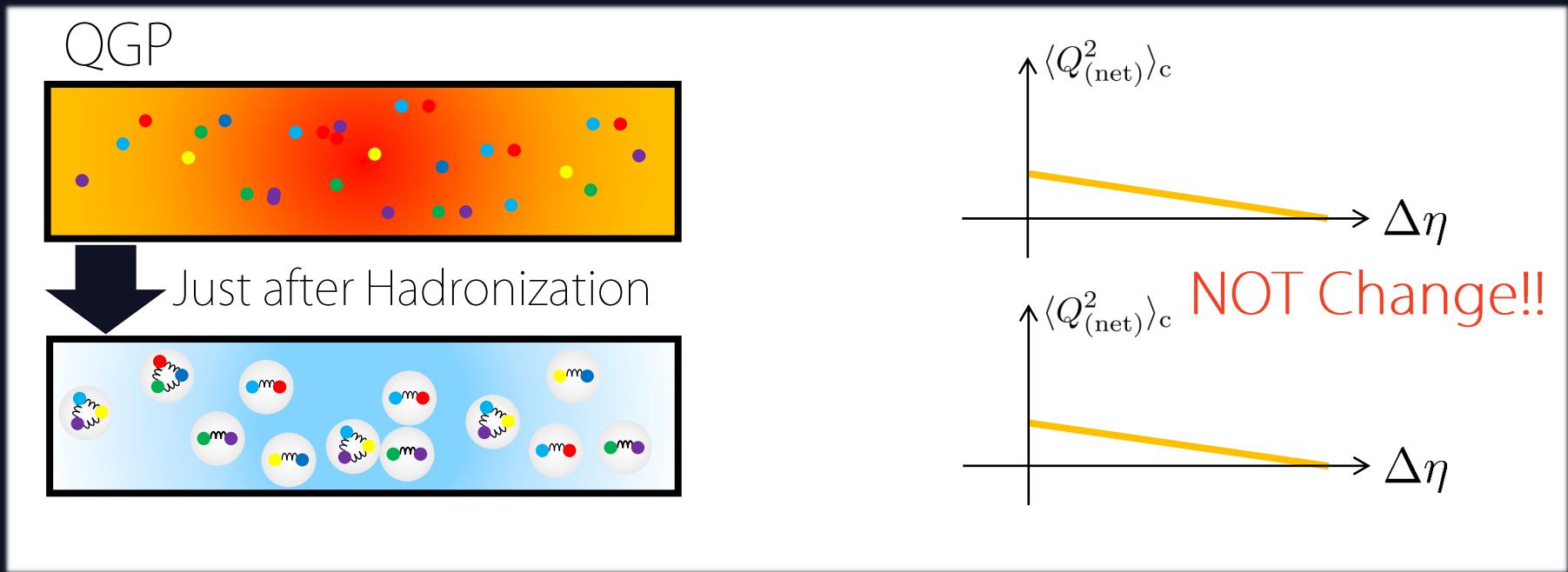


$$\gamma(\tau) a^2 = D(\tau)$$

Diffusion Master Eq.

$$\begin{aligned} \partial_t P(\mathbf{n}, \tau) = & \gamma(\tau) \sum_m [(n_m + 1) \{ P(\mathbf{n} + \mathbf{e}_m - \mathbf{e}_{m+1}, \tau) \\ & + P(\mathbf{n} + \mathbf{e}_m - \mathbf{e}_{m-1}, \tau) \} - 2n_m P(\mathbf{n}, \tau)] \end{aligned}$$

Initial Fluctuations



Assumption

- Fluctuation does not change just after hadronization
- Thermal Value in the final QGP State

Parameters

Initial Fluctuation

$$\langle Q_{(\text{net})}^2 \rangle_c \quad \langle Q_{(\text{net})}^4 \rangle_c \quad \langle Q_{(\text{net})}^2 Q_{(\text{tot})} \rangle_c$$

Small compared with Equilibrated Hadronic Value

$$\langle Q_{(\text{tot})}^2 \rangle_c$$

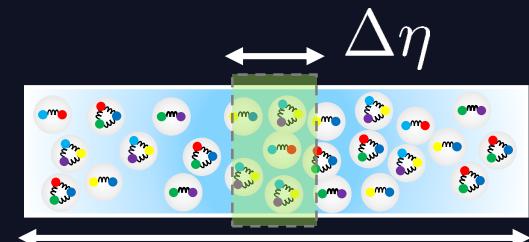
Large?

Free Parameter

$$T = \frac{d(\tau)}{\eta_{\text{tot}}}$$

$$d(\tau) = \sqrt{2 \int_{\tau_0}^{\tau} D(\tau') d\tau'} \quad : \text{Average Diffusion Length}$$

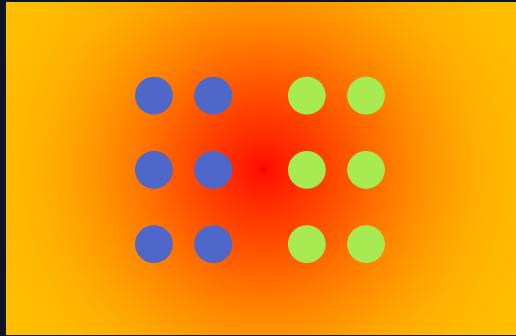
$D(\tau)$: Diffusion Coefficient



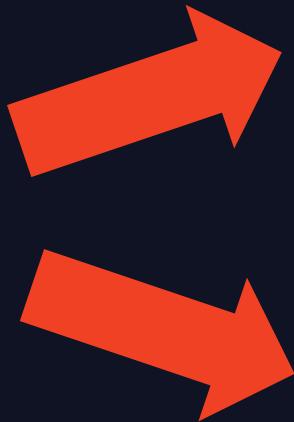
η_{tot} : Total Rapidity Length

$$\langle Q_{(\text{tot})}^2 \rangle_c$$

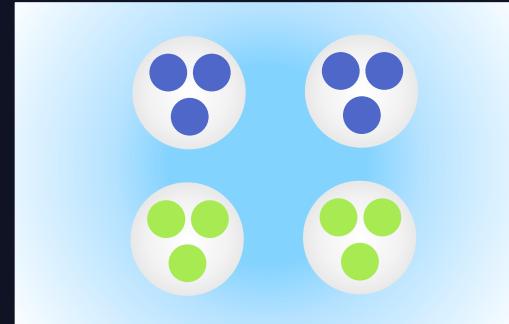
$$N_B^{(\text{net})} = 0$$



- {
- Quark
 - Anti-Quark



$$\left\{ \begin{array}{l} N_B^{(\text{net})} = 0 \\ N_B^{(\text{tot})} = 4 \end{array} \right.$$



$$\left\{ \begin{array}{l} N_B^{(\text{net})} = 0 \\ N_B^{(\text{tot})} = 0 \end{array} \right.$$

$\langle Q_{(\text{tot})}^2 \rangle_c$ strongly depends on Hadronization Mechanism.

$$D = (T \cdot \eta_{\text{tot}})^2 \frac{\tau_0 \cdot \tau_{\text{f.o.}}}{\tau_0 - \tau_{\text{f.o.}}}$$

$$\langle Q_{(\text{net})}(0)^2 \rangle_c = 0.5 \langle Q_{(\text{tot})}(0) \rangle_c$$

$$T = 0.04$$

$$\eta_{\text{tot}} = 8$$

| | | | | | | | | | |
|---------------------------|------|------|------|------|------|------|------|------|------|
| τ_0 [fm] | 8 | 8 | 8 | 10 | 10 | 10 | 12 | 12 | 12 |
| $\tau_{\text{f.o.}}$ [fm] | 20 | 25 | 30 | 20 | 25 | 30 | 20 | 25 | 30 |
| D [fm] | 1.37 | 1.20 | 1.12 | 2.05 | 1.71 | 1.54 | 3.07 | 2.36 | 2.05 |