Charged rho condensation in external magnetic field

Hao Liu

Institute of High Energy Physics,CAS

collaboration with Mei Huang and Lang Yu

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Outline

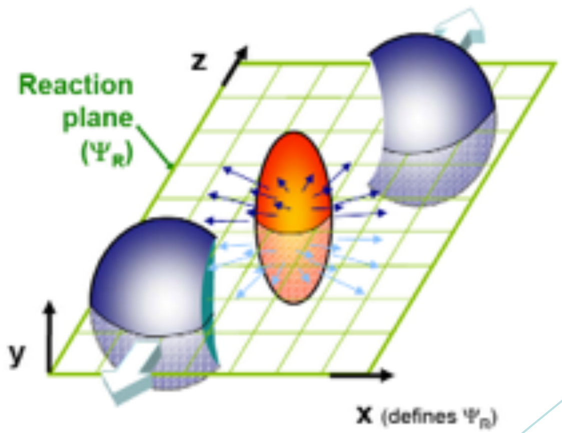
arxiv.1408.1318

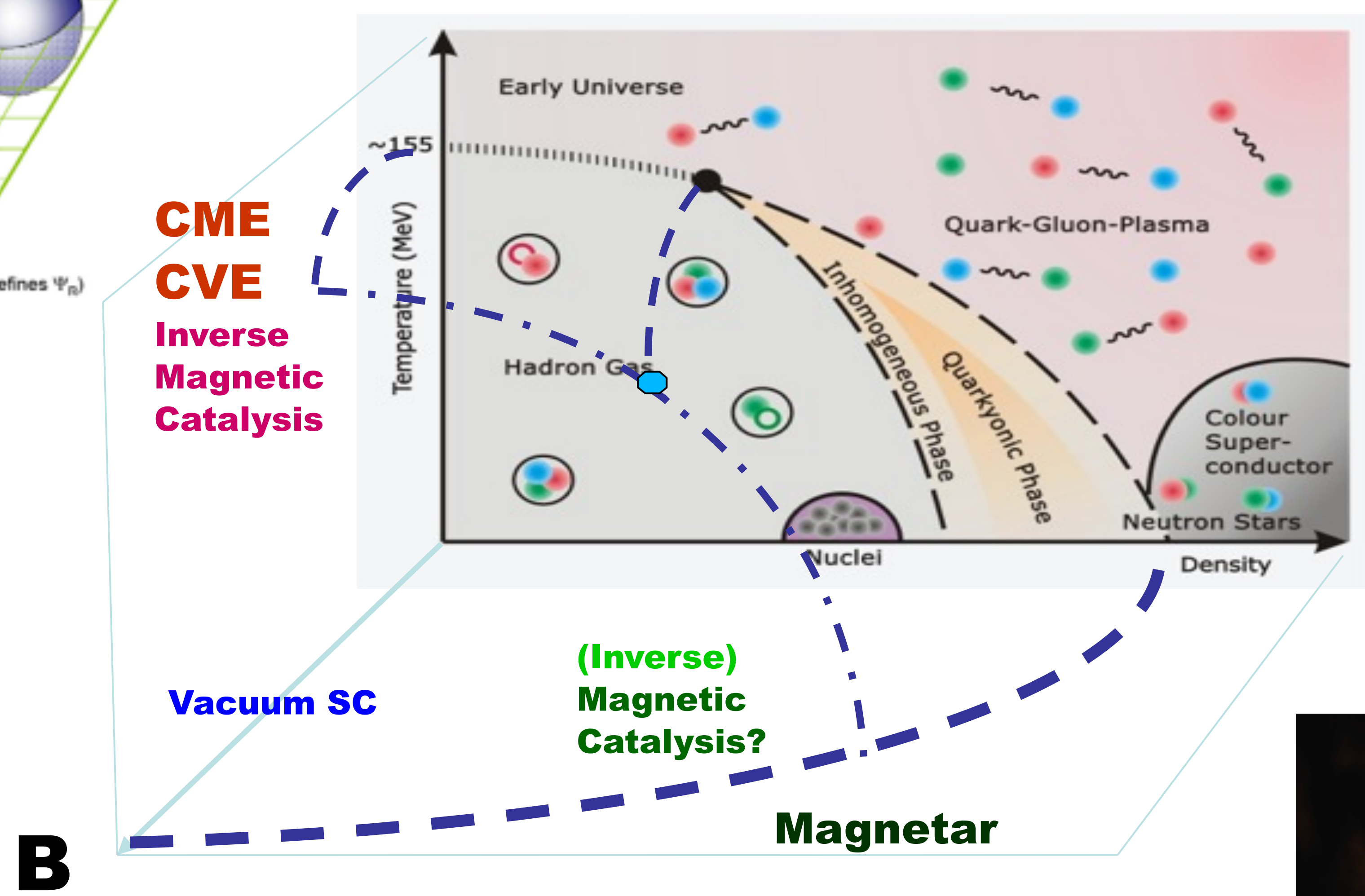
- Introduction
- NJL model and analysis result
- Numerical Result and Discussion
- Conclusions

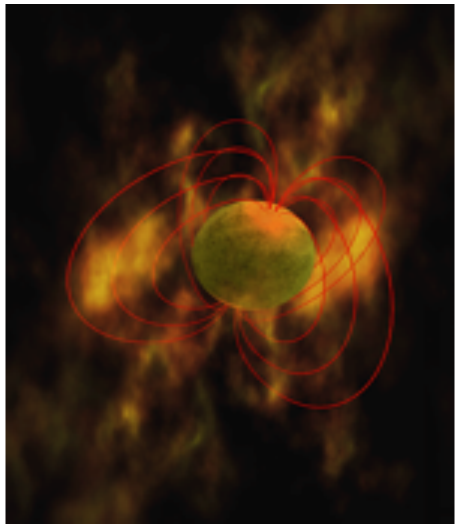
Strong Magnetic Fields in QCD

- $\mathbf{1MeV^2 = 1.44 \times 10^{13} Gauss, m_\pi^2 = 2.8 \times 10^{17} Gauss}$
- **Early Universe: up to** $\mathbf{10^{24}\ Gauss}$
- **Magnetars: about** $\mathbf{10^{14}\ Gauss}$
- **Heavy ion collisions:** $\mathbf{10^{18}\ to\ 10^{20}\ Gauss}$

QCD Phase Diagram under Strong Magnetic Field







Vacuum Superconductor

M. N. Chernodub, Phys. Rev. Lett. 106 (2011) 142003 [arXiv:1101.0117 [hep-ph]]

-Energy of relativistic particle in the external magnetic field B:

$$E^2_{n,s_z}(p_z) = p_z^2 + (2n - 2sgn(q)s_z + 1)eB + m^2$$

- p_z: the momentum along the external magnetic field
- n: nonnegative integer number
- s_z: projection of spin on the direction of magnetic field

-Masses of ρ mesons and π in magnetic field:

$$\mathrm{m}^2_{\pi^\pm}(\mathrm{B}) = \mathrm{m}^2_{\pi^\pm} + \mathrm{eB}$$ **becomes larger**

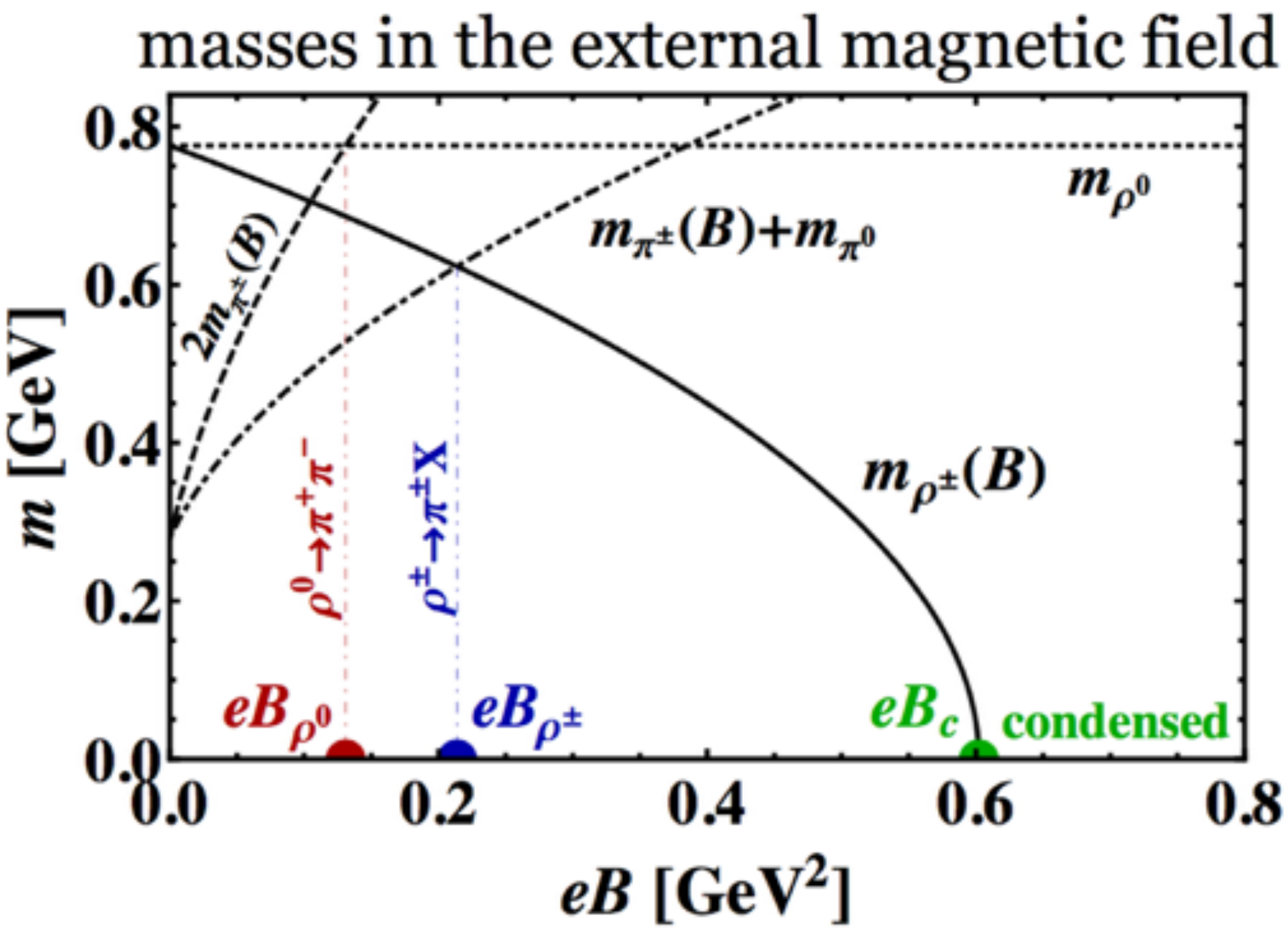
$$\mathrm{m}^2_{\rho^\pm}(\mathrm{B}) = \mathrm{m}^2_{\rho^\pm} - \mathrm{eB}$$ **becomes lighter**

where $\mathrm{m}_{\rho_\pm} = 768\mathrm{MeV}$, $\mathrm{m}_{\pi_\pm} = 140\ \mathrm{MeV}$

Vacuum Superconductor

The charged rho becomes massless and condensates at a critical magnetic field : $\mathrm{eB}_c = \mathrm{m}^2_{\rho^\pm}$

M. N. Chernodub, Phys. Rev. Lett. 106 (2011) 142003 [arXiv:1101.0117 [hep-ph]]



The pions become heavier while the charged vector mesons become lighter in the external magnetic field

The $\rho^\pm \to \pi^\pm \pi^0$ **decay stops at a critical eB**

Vacuum Superconductor?

- **A point particle model for the charged rho :**

$$\mathrm{eB}_c = \mathrm{m}^2_{\rho^\pm}$$

- **NJL Model (LLL):** $\mathrm{eB}_c > 1\ \mathrm{GeV}^2$

M. N. Chernodub, Phys. Rev. Lett. 106 (2011) 142003 [arXiv:1101.0117 [hep-ph]]

- **NJL Model:** $\mathrm{eB}_c = 0.978\ m_q^2$

M. Frasca, JHEP 1311, 099 (2013) [arXiv:1309.3966 [hep-ph]]

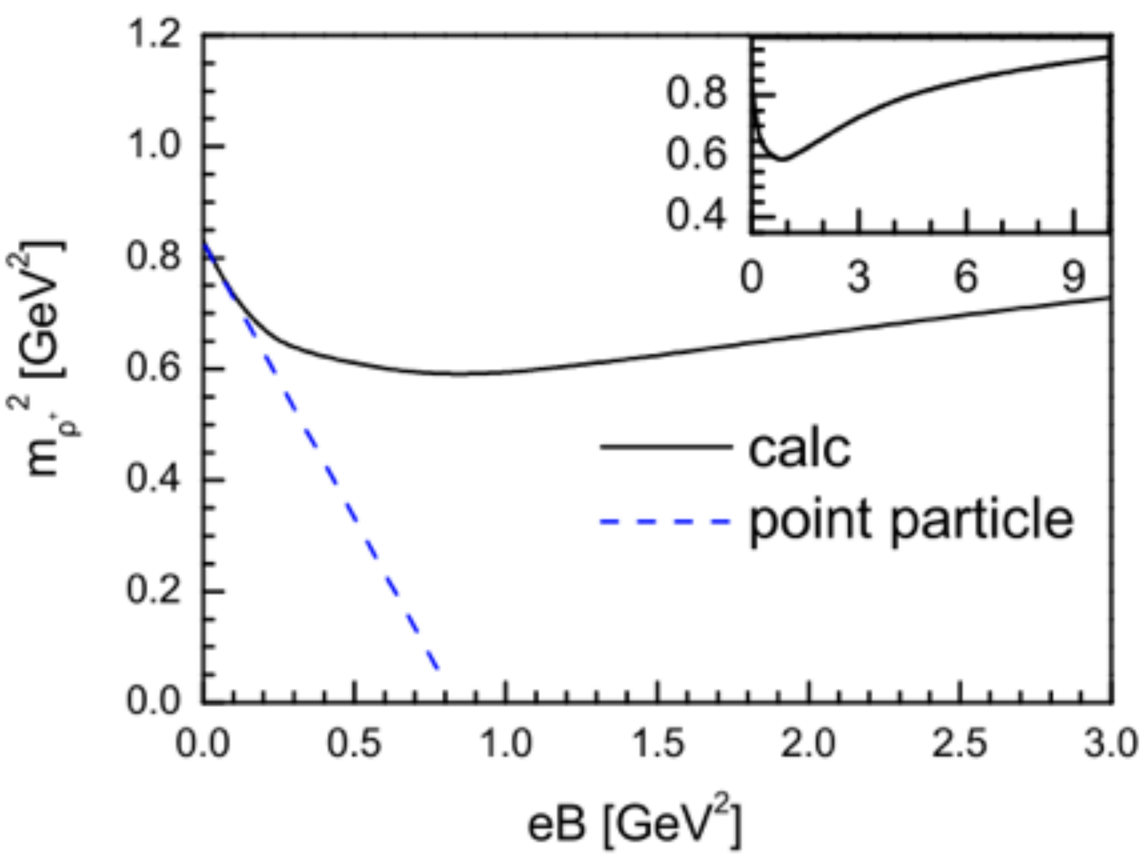
Vacuum Superconductor?

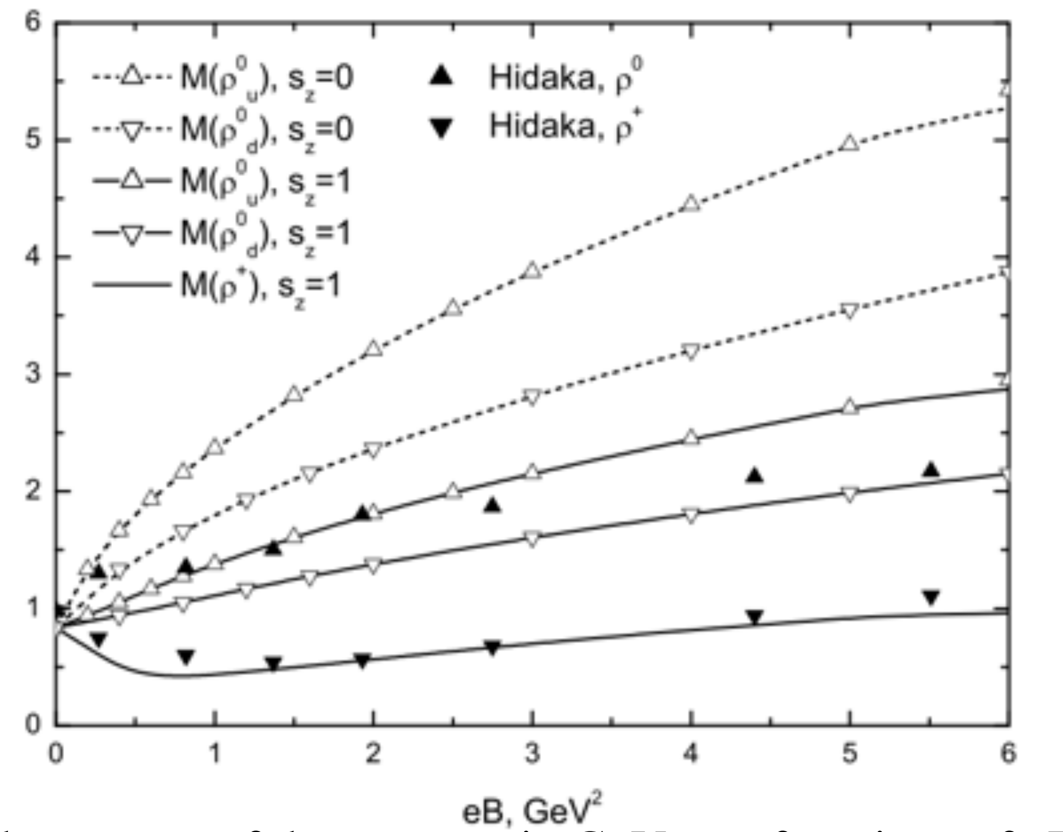
- DSE and BSE:

  Kunlun Wang PhD thesis

- Quark-antiquark Green Function and effective Hamiltonian (LLL)

  M. A. Andreichikov, B. O. Kerbikov, V. D. Orlovsky and Y. . A. Simonov, Phys. Rev. D 87, no. 9, 094029 (2013) [arXiv:1304.2533 [hep-ph]]





The masses of the systems in GeV as a functions of eB

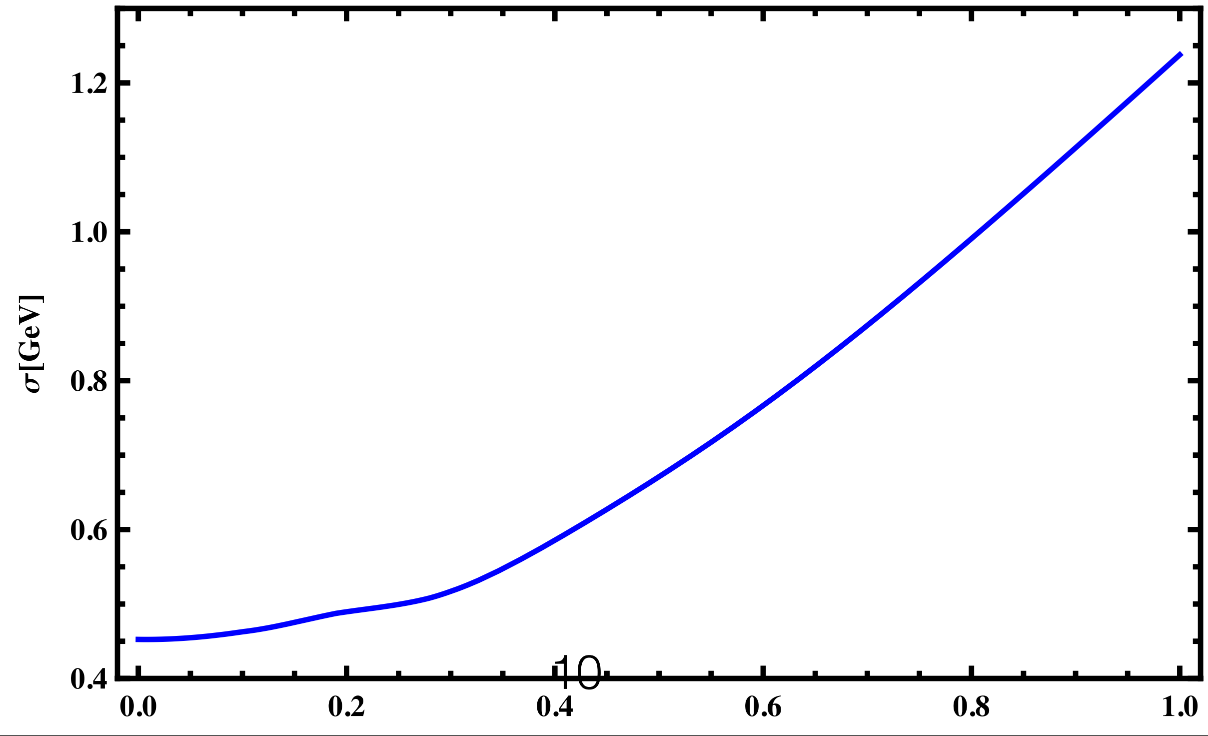
Our work

- We explore the character of rho meson in magnetic field by NJL Model
- We analyse the different results from different models

NJL Model and Analysis Result

$$\mathcal{L} = \bar{\Psi}(i\not{D} - m_0)\Psi + G_1\left[(\bar{\Psi}\Psi)^2 + (\bar{\Psi}i\gamma^5\tau\Psi)^2\right]$$
$$-G_2\left[(\bar{\Psi}\gamma^\mu\tau\Psi)^2 + (\bar{\Psi}\gamma^\mu\gamma^5\Psi)^2\right]$$

$\sigma = -2G_1 < \bar{\Psi}\Psi >$ and $m = m_0 + \sigma$

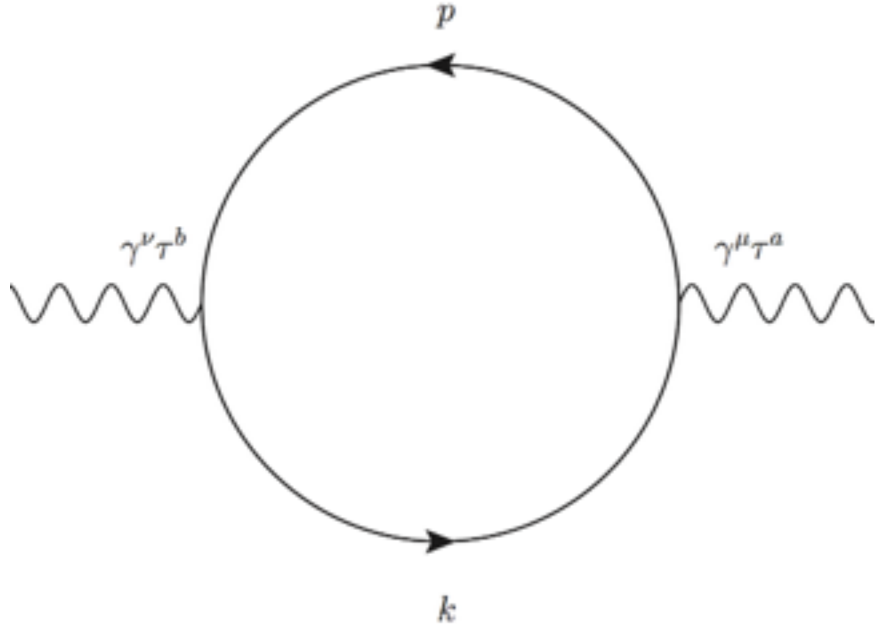


NJL Model and Analysis Result

$$\Pi^{\mu\nu}_{ab} = i \sum_{p,k=0}^{\infty} \int \mathcal{D}\tilde{p}\, \mathcal{D}\tilde{k} \int d^4x\, e^{-i(\tilde{p}-\tilde{k}-q)\cdot x} \Lambda^{\mu\nu}_{pk,ab}(\bar{p}, \bar{k}, x_1)$$

where

$$\Lambda^{\mu\nu}_{pk,ab}(\bar{p}, \bar{k}, x_1) = \mathrm{tr}_{sfc}\big[\gamma^\mu \tau^a P_p(x_1) D_Q^{-1}(\bar{p}) P_p(0) \gamma^\nu \tau^b \times K_k(0) D_Q^{-1}(\bar{k}) K_k(x_1)\big]$$



NJL Model and Analysis Result

Ritus fermion propagator:

$$S_Q(x,y) = i \sum_{p=0}^{\infty} \int \mathcal{D}\tilde{p}\; e^{-i\tilde{p}\cdot(x-y)} P_p(x_1) D_Q^{-1}(\bar{p})\; P_p(y_1)$$

V. I. Ritus, Annals Phys. 69, 555 (1972)

K. Fukushima, D. E. Kharzeev and H. J.Warringa, Nucl.Phys. A 836, 311 (2010) [arXiv:0912.2961 [hep-ph]

Sh. Fayazbakhsh and N. Sadooghi, Phys. Rev. D 88, no.6, 065030 (2013) [arXiv:1306.2098 [hep-ph]]

$$P_p(x_1) = \frac{1}{2}[f_p^{+s}(x_1) + \Pi_p f_p^{-s}(x_1)]\,, D_Q(\bar{p}) = \gamma \cdot \bar{p}_Q - m$$

$$f_p^{+s}(x_1) = \phi_p\left(x_1 - s_Q p_2 \ell_B^2\right) \qquad a_p = (2^p p! \sqrt{\pi}\ell_B)^{-1/2}$$

$$f_p^{-s}(x_1) = \phi_{p-1}\left(x_1 - s_Q p_2 \ell_B^2\right) \qquad \ell_B = |QeB|^{-1/2}$$

$$\phi_p(x) = a_p \exp\left(-\frac{x^2}{2\ell_B^2}\right) H_p\left(\frac{x}{\ell_B}\right)$$

NJL Model and Analysis Result

-In the rest frame of charged ρ meson:

$$\Pi^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \Pi^{11} & \Pi^{12} & 0 \\ 0 & \Pi^{21} & \Pi^{22} & 0 \\ 0 & 0 & 0 & \Pi^{33} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & ib & 0 \\ 0 & -ib & a & 0 \\ 0 & 0 & 0 & c \end{pmatrix}$$

-In the rest frame of neutral ρ meson:

$$\Pi^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \Pi^{11} & 0 & 0 \\ 0 & 0 & \Pi^{22} & 0 \\ 0 & 0 & 0 & \Pi^{33} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & d & 0 \\ 0 & 0 & 0 & e \end{pmatrix}$$

NJL Model and Analysis Result

$$\Pi^{\mu\nu}_{ab} = \left[\Pi_1 P^{\mu\nu}_{-} + \Pi_2 P^{\mu\nu}_{+} + \Pi_3 L^{\mu\nu} + \Pi_4 u^\mu u^\nu\right]\delta_{ab}$$

$$P^{\mu\nu}_{+} = -\epsilon^{\mu *}_1 \epsilon^\nu_1, P^{\mu\nu}_{-} = -\epsilon^{\mu *}_2 \epsilon^\nu_2, L^{\mu\nu} = -b^\mu b^\nu$$

$$b^\mu = (0,0,0,1), u^\mu = (1,0,0,0)$$

with $\epsilon^\mu_1, \epsilon^\mu_2$ are right-and left-handed helicities.

Π_1 **: the projection of spin is -1**

Π_2 **: the projection of spin is 1**

Π_3 **: the projection of spin is 0**

The RPA relation for vector meson: $1 + 2\mathrm{G}_2\Pi_i = 0$

NJL Model and Analysis Result

-For charged rho :

- **Condensation:** $1 + 2G_2\Pi_2 = 0$, $1 + 2G_2\Pi_1 = 0$

where $\Pi_1 = -(a+b), \Pi_2 = b - a$

Get the condensation for ρ^+

Get the condensation for ρ^-

-For neutral rho :

$$\Pi_1 = \Pi_2 = -d, \Pi_3 = -e$$

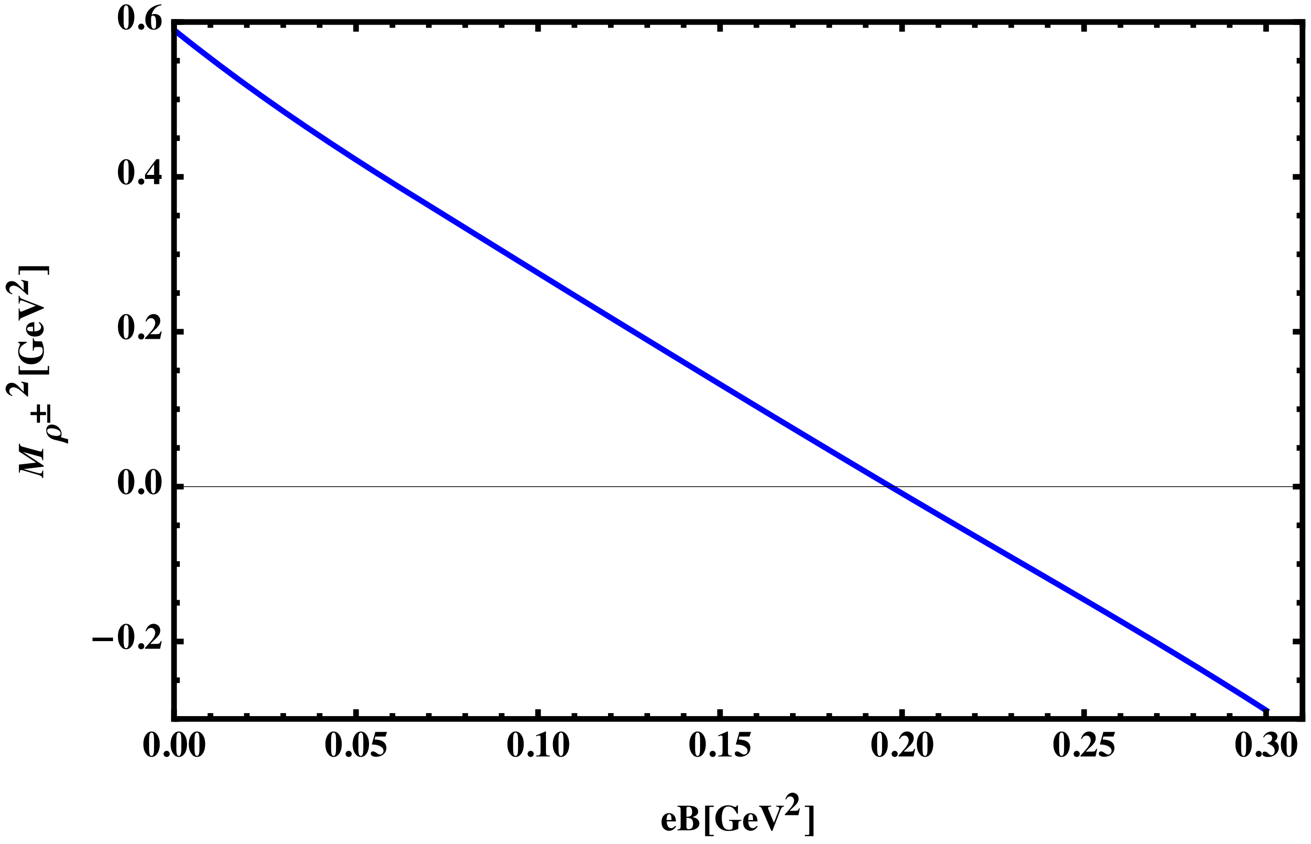
Numerical Results and Discussion

$$f_\pi = 95MeV, m_\pi = 140MeV, M_\rho = 768MeV$$

$$m = 458MeV, m_0 = 5MeV$$

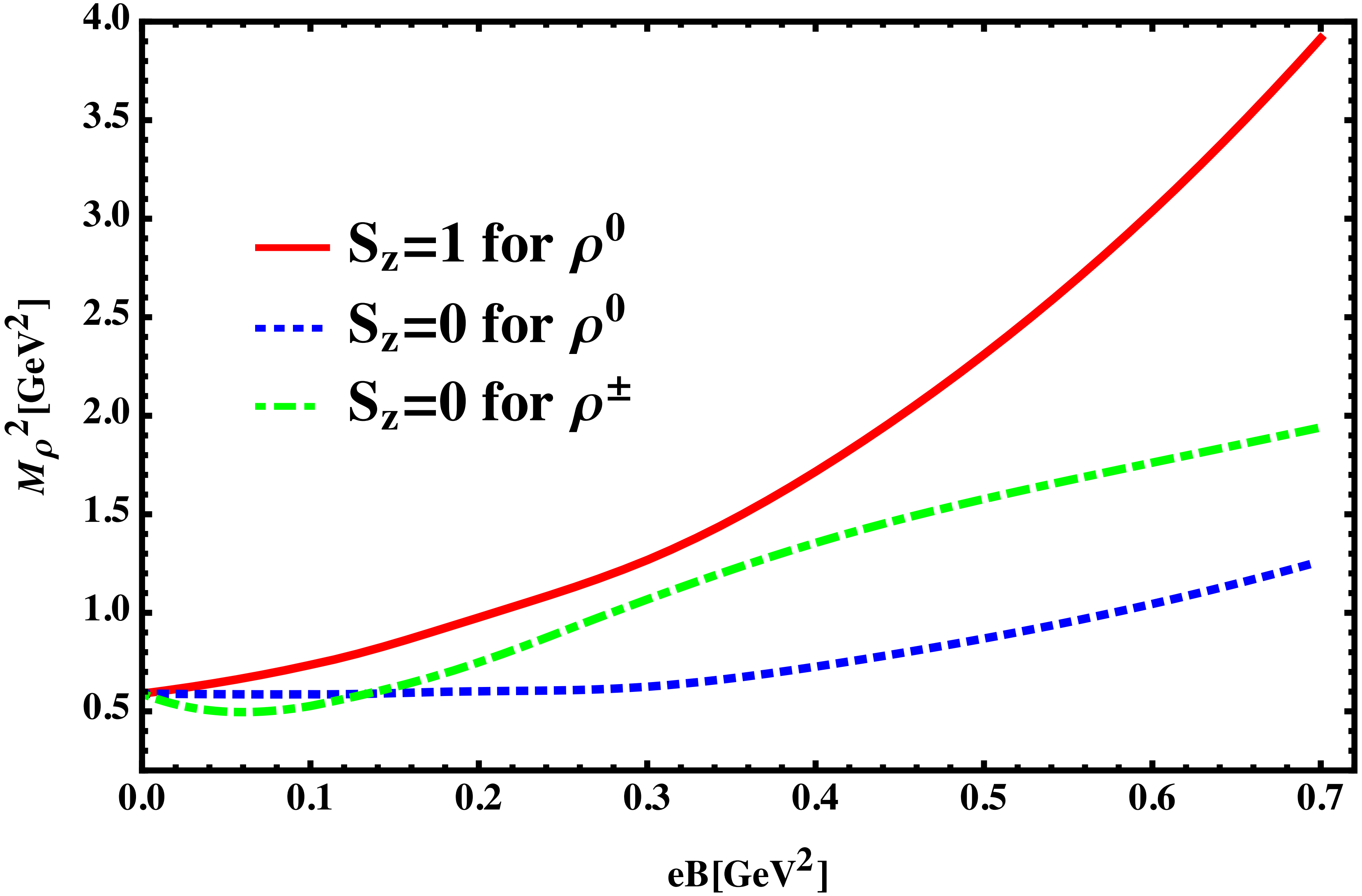
$$\Lambda = 582MeV, G_1\Lambda^2 = 2.388, G_2\Lambda^2 = 1.73$$

Numerical Results



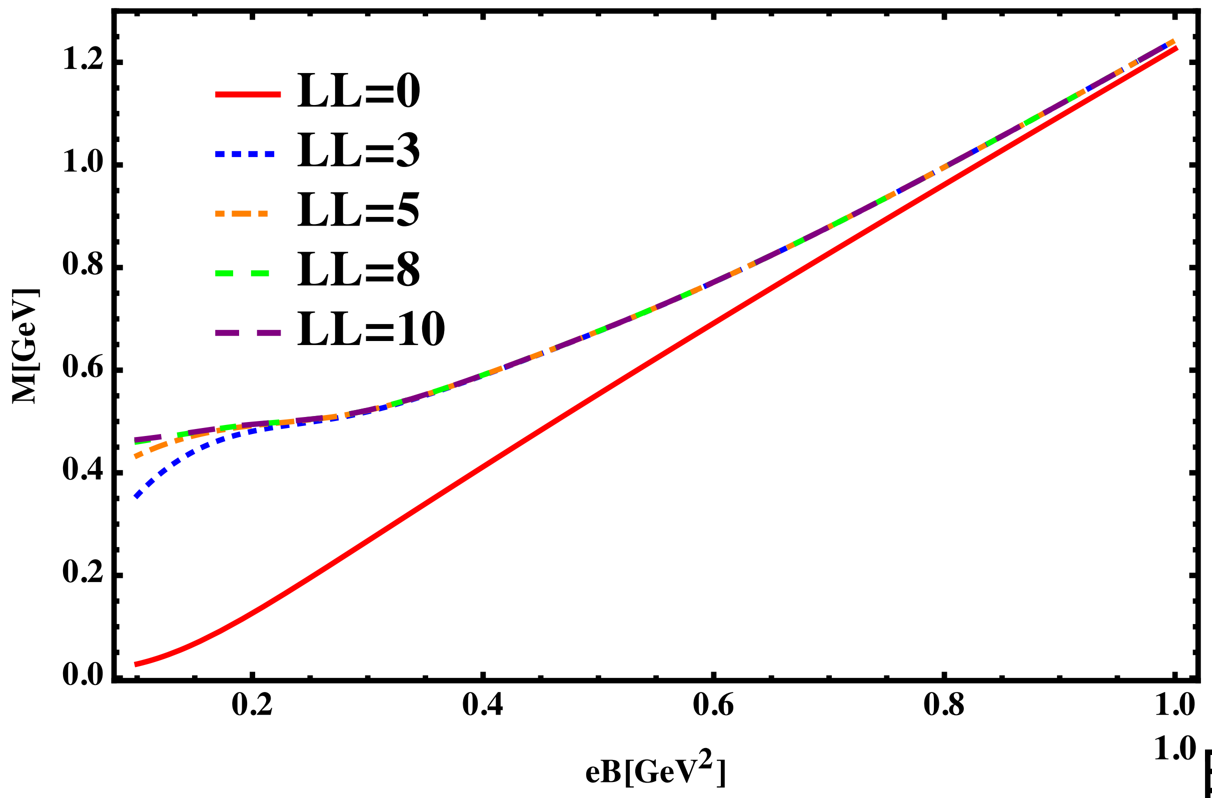
The masses of $\rho^{\pm}$ decrease and condensate at $\mathrm{eB}_c \approx 0.2\ \mathrm{GeV}^2$!

Neutral Particle



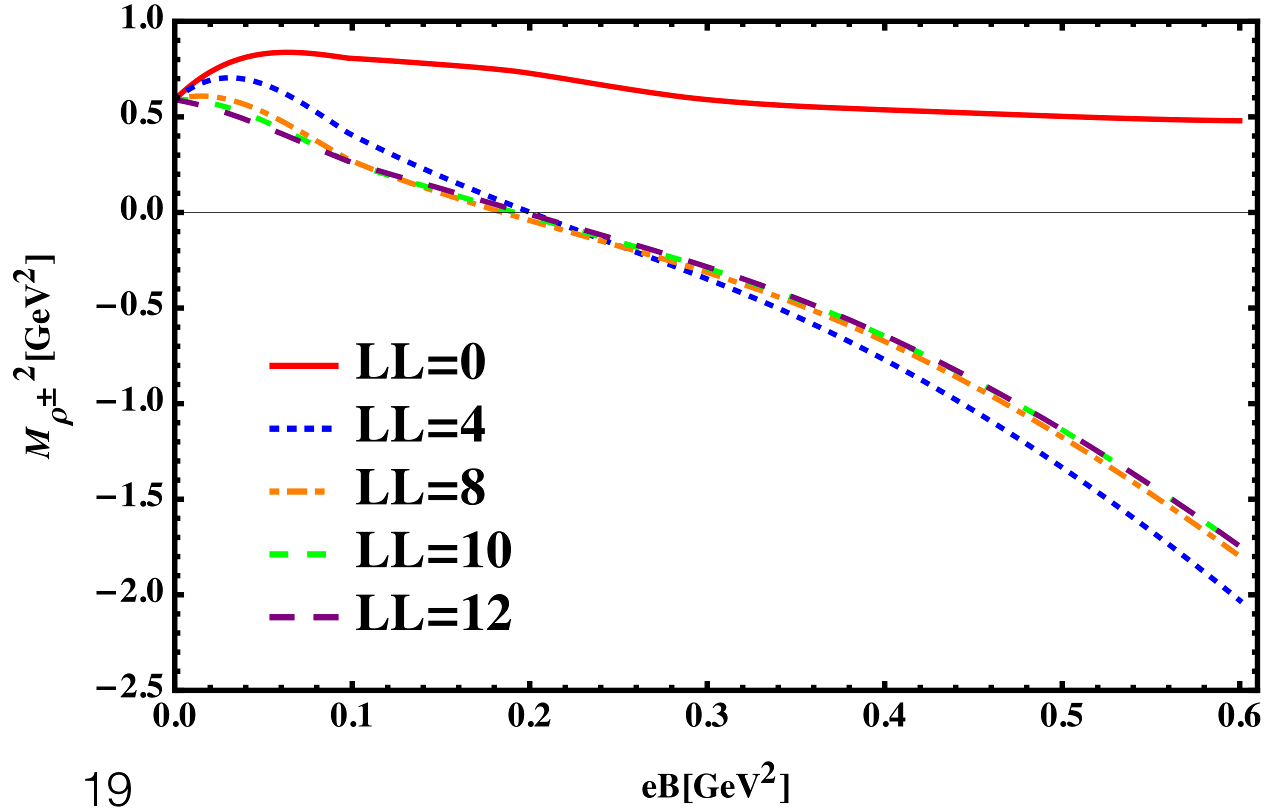
The magnetic field affects u and d quark differently and the projection of spin on the direction of magnetic field is more sensitive to the magnetic field.

Discussion



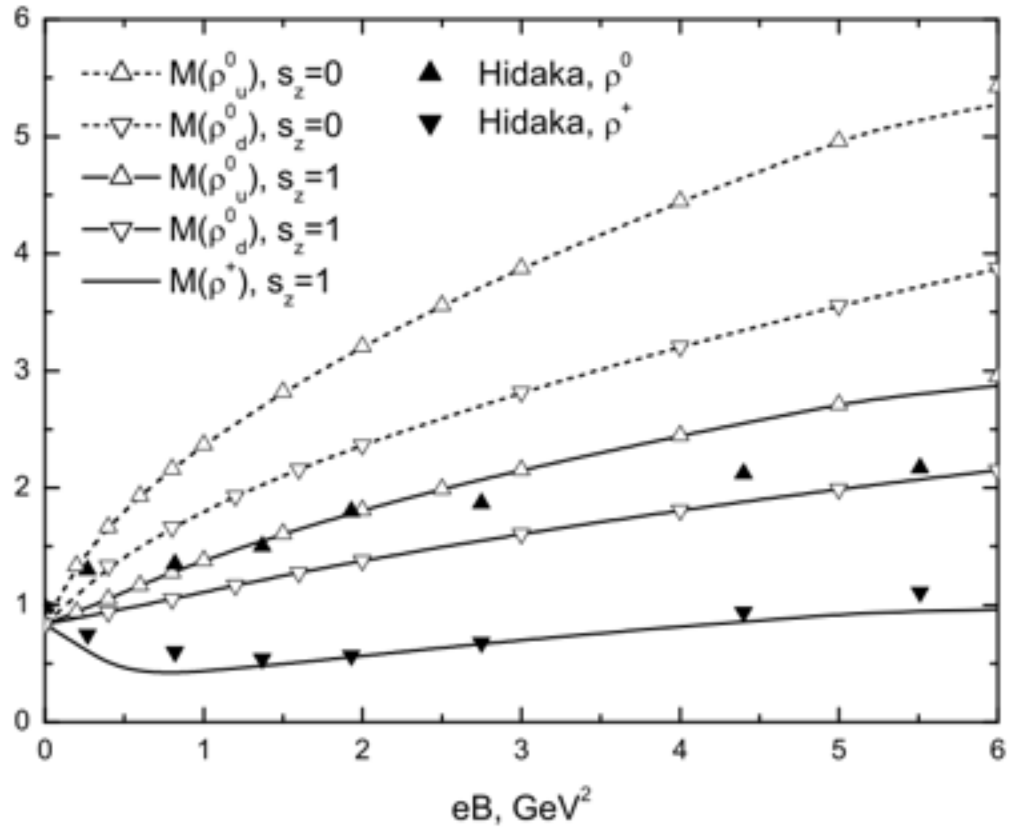
LLL is good for the quark constitute mass in strong magnetic field

LLL can not give charged rho condensation!



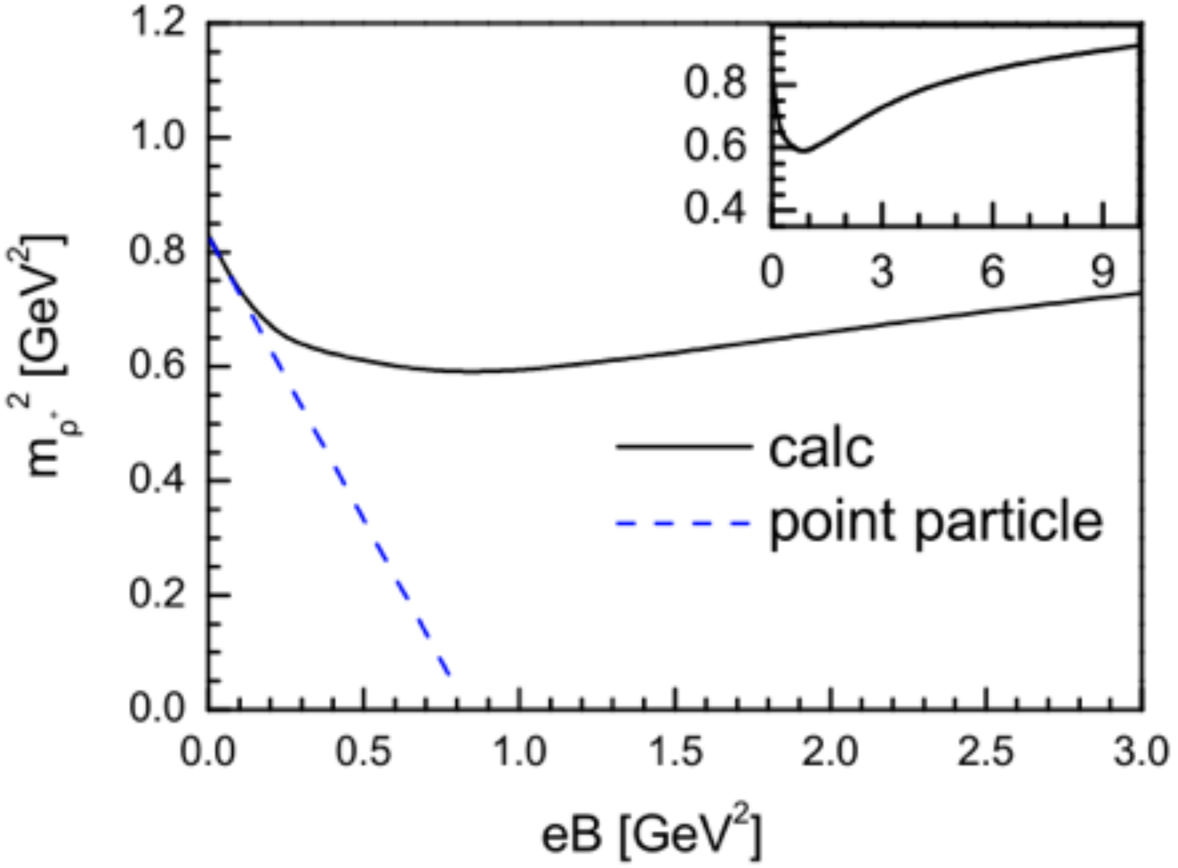
Discussion

LLL approximation



Simonov's group

LLL approximation?



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Conclusions

- **Charged rho meson condensates at** $\mathrm{eB}_c \approx 0.2\ \mathrm{GeV}^2 < 0.6\ \mathrm{GeV}^2$
- **The point particle model ignores the vacuum polarization**
- **LLL can not give the charged rho condensation**
- **LLL approximation is not good enough for the calculation of rho mesons' masses**
- **The external magnetic field shows different influence on the masses of rho mesons due to different electric charges and spin projections**

Thanks for your attention!