

New method for precise determination of top quark mass at LHC

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Outline

1. Introduction
2. Weight function method
3. Top mass reconstruction
4. Summary and future works

1. Introduction

Motivation to measure the top quark mass

- ◆ One of the fundamental parameters of the SM
- ◆ Top mass is an important input parameter to

- EW precision tests of SM
- beyond SM
- SM vacuum stability

Current status of top mass measurements

(direct measurement)

Tevatron+LHC m_t combination [arXiv:1403.4427](https://arxiv.org/abs/1403.4427)

$$m_t = 173.34 \pm 0.76 \text{ GeV} \quad 0.4 \% \text{ precision !}$$

Current status of top mass measurements

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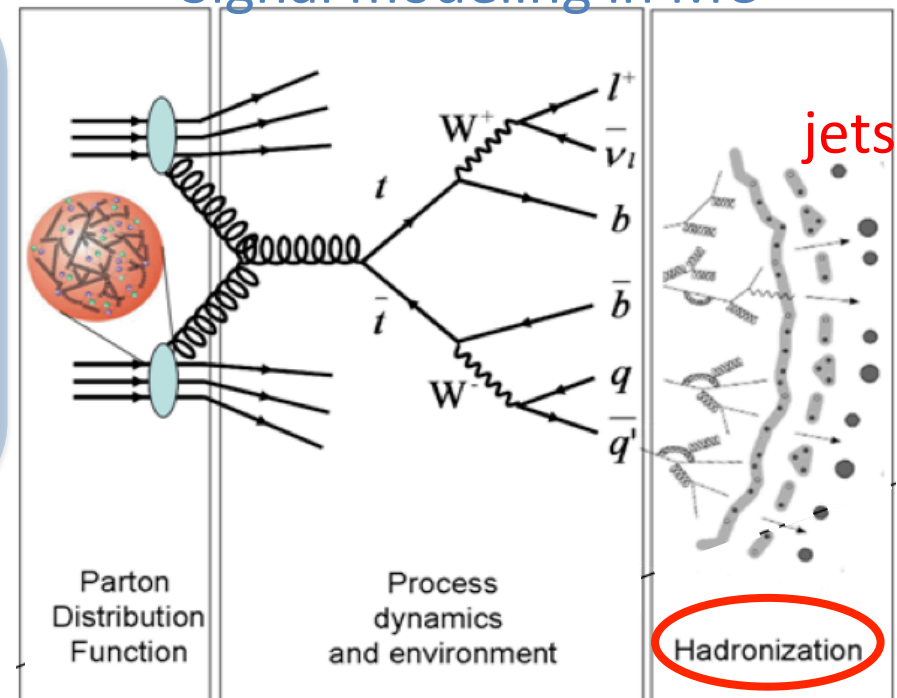
$m_t = 173.34 \pm 0.76$ GeV 0.4 % precision !

What kind of mass?

- ◆ Measurements use jet momenta and fit the data with MC
- ◆ Hadronization in MC
 - Cannot be treated within perturbative QCD
 - Assume model

➔ The measured mass is not well-defined in theory

Signal modeling in MC



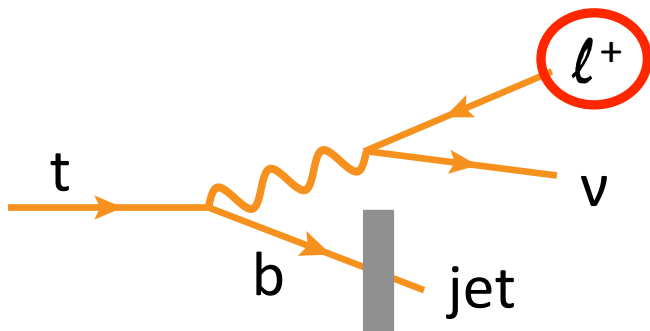
2. Weight function method

SK, Y.Shimizu, Y.Sumino, H.Yokoya, PLB 710, 658 (2012)

SK, Y.Shimizu, Y.Sumino, H.Yokoya, JHEP 08, 129 (2013)

A new method to measure the mass of parent particle.

- Only **lepton energy distribution** is needed
 - ➔ **Free from the ambiguity of hadronization models**
 - ➔ We can determine the $\overline{\text{MS}}$ mass of top quark
- Does not depend on the velocities of top quarks
 - ➔ Independent of production process of top quark



Weight functions and the weighted integrals

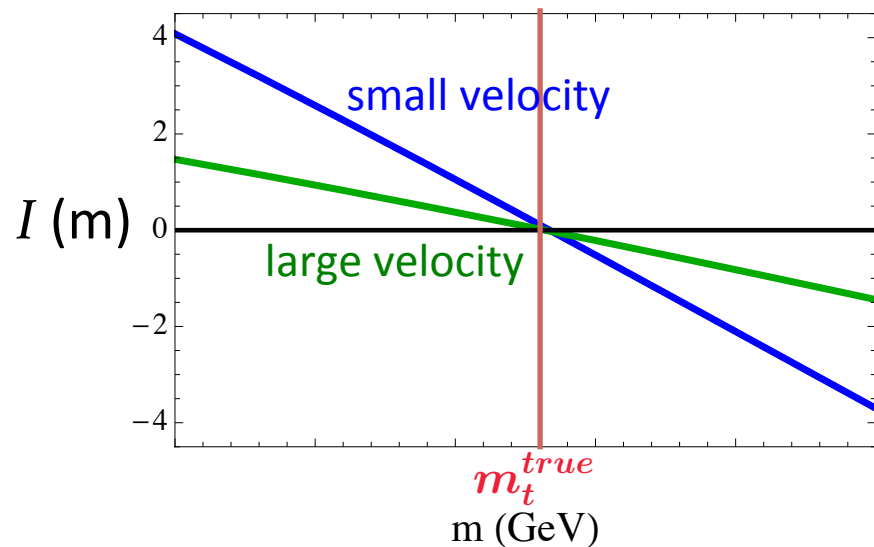
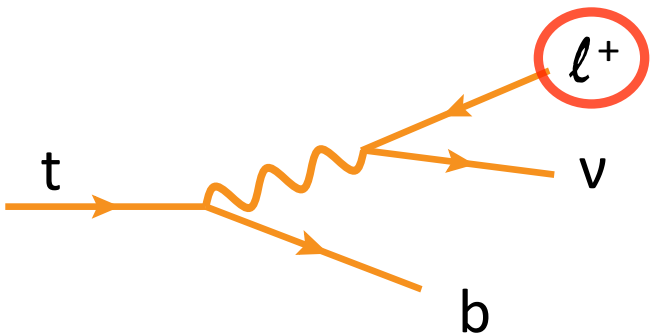
$$I(m) \equiv \int dE_l D(E_l) \boxed{W(E_l, m)}$$

↑ Lepton energy distribution in the lab. frame

Lepton energy distribution in the lab. frame

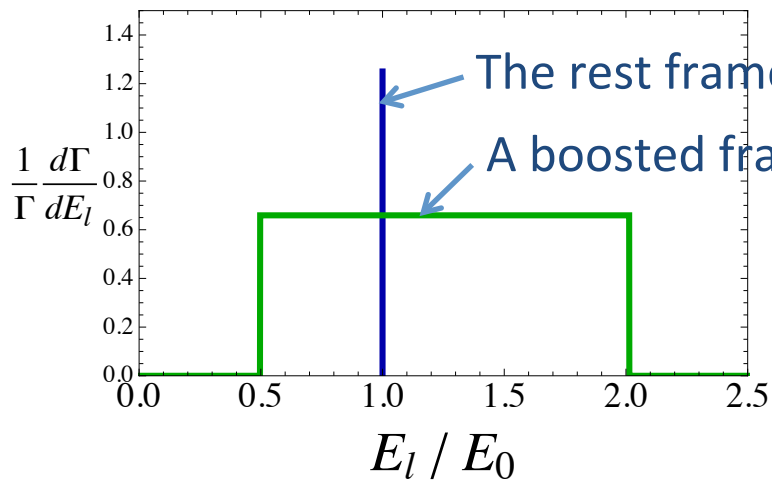
There exist an infinite number of weight functions which satisfy

$I(m = m_t^{\text{true}}) = 0$ for an arbitrary velocity distribution of top quarks

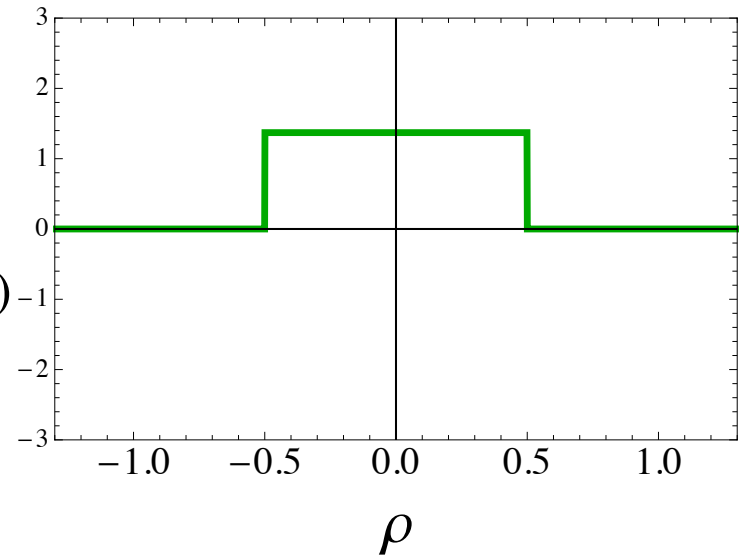


Explicit form of weight functions

For a two-body decay : $X \rightarrow \ell + Y$ (X is scalar or unpolarized)



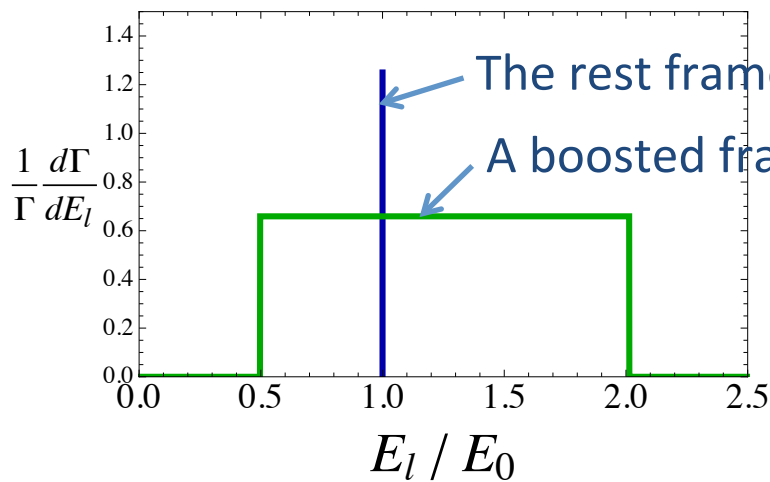
$$\rho = \ln (E_l / E_0)$$



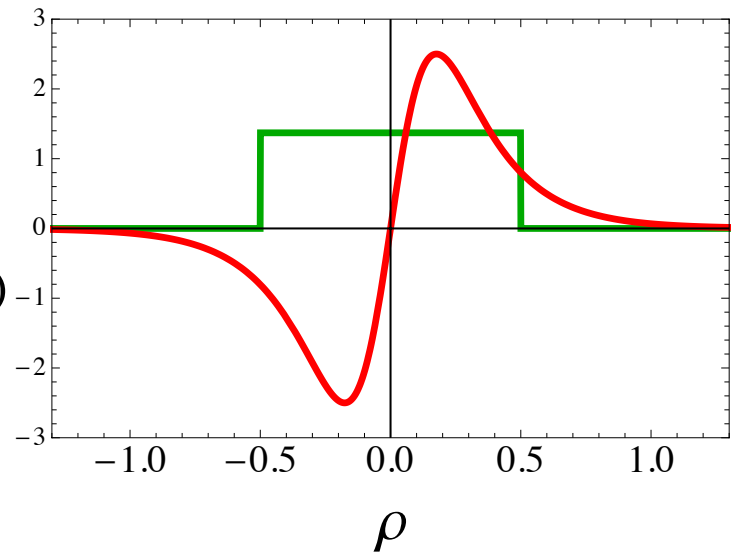
Lepton energy distribution

Explicit form of weight functions

For a two-body decay : $X \rightarrow \ell + Y$ (X is scalar or unpolarized)



$$\rho = \ln (E_l / E_0)$$



Lepton energy distribution

$$\int dE_l D(E_l) W(E_l, m_X^{true}) = 0 \iff \int d\rho (\text{even func. of } \rho) (\text{odd func. of } \rho) = 0$$

$$d\rho \propto e^{-\rho} dE_l$$

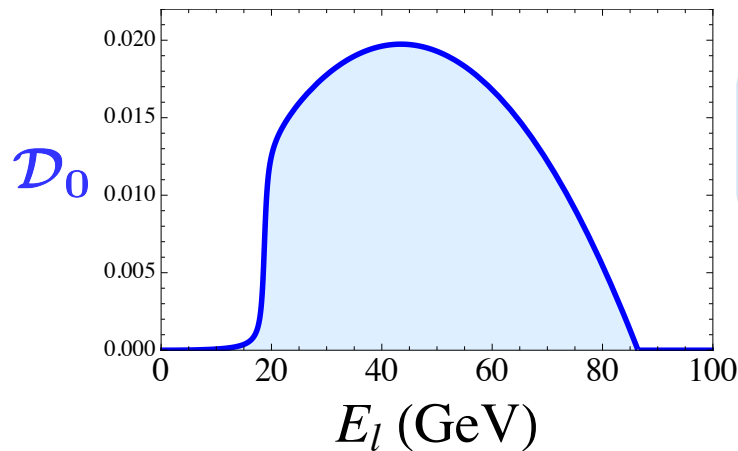


$$W(E_l, m_X^{true}) = e^{-\rho} (\text{odd func. of } \rho) \Big|_{e^\rho = E_l / E_0}$$

Explicit form of weight functions

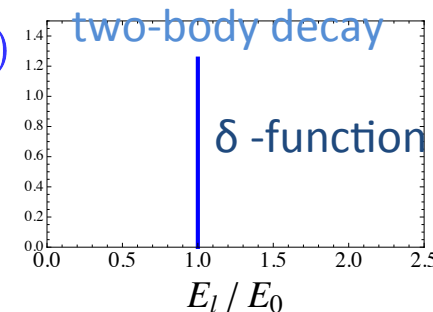
For a many-body decay : $X \rightarrow \ell + \text{anything}$ (X is scalar or unpolarized)

Lepton energy distribution in the rest frame of X



Can be expressed as a superposition of lepton distribution for a two-body decay

$$\mathcal{D}_0(E_l) = \int dE \mathcal{D}_0(E) \delta(E_l - E)$$



A weight function would be also a superposition of that for a two-body decay



$$W(E_l, m) = \int dE \mathcal{D}_0(E; m) \frac{1}{EE_l} (\text{odd func. of } \rho) \Big|_{e^\rho = E_l/E}$$

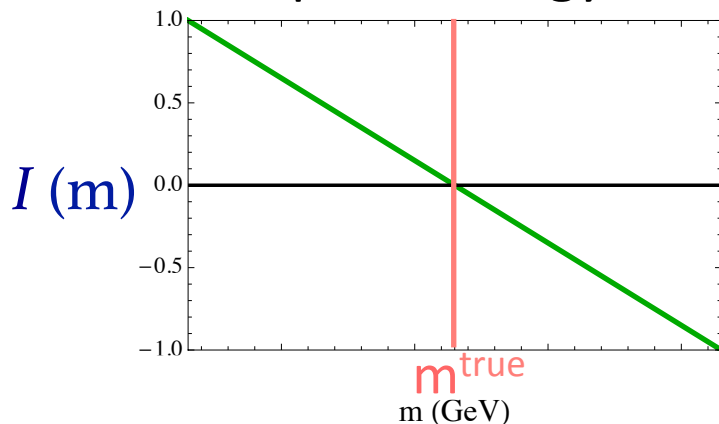
Summary of the weight function method

1. Construct weight functions for the process

$$W(E_l, m) = \int dE \mathcal{D}_0(E; m) \frac{1}{EE_l} (\text{odd func. of } \rho) \Big|_{e^\rho = E_l/E}$$

Lepton energy dist. in the rest frame of parent particle, which can be calculated in pert. QCD

2. Use the lepton energy distribution measured by experiment as $D(E_l)$



$$I(m) \equiv \int dE_l D(E_l) W(E_l, m)$$

3. Obtain the zero of $I(m)$ as m^{true}

$$I(m = m^{\text{true}}) = 0$$

There are deviations from the above ideal case. In this first study, we focus on deviations due to experimental aspects.

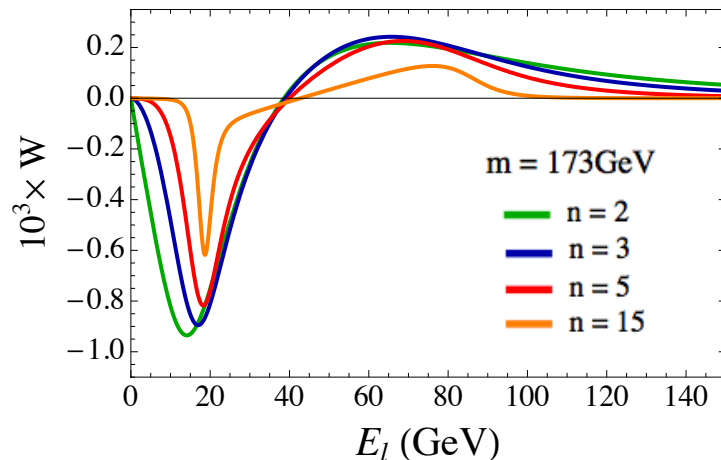
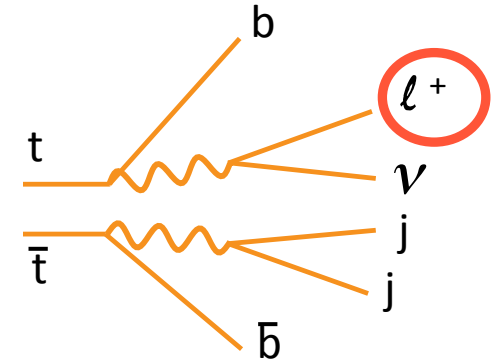
3. Top mass reconstruction (Simulation analysis: LO)

arXiv:1405.2395 [hep-ph]

Setup of the analysis

LHC $\sqrt{s} = 14$ TeV

- **Signal** $t\bar{t}$ events, Lepton(μ)+jets channel
- **Background** W +jets, other $t\bar{t}$ events
- **Weight functions used in this analysis**



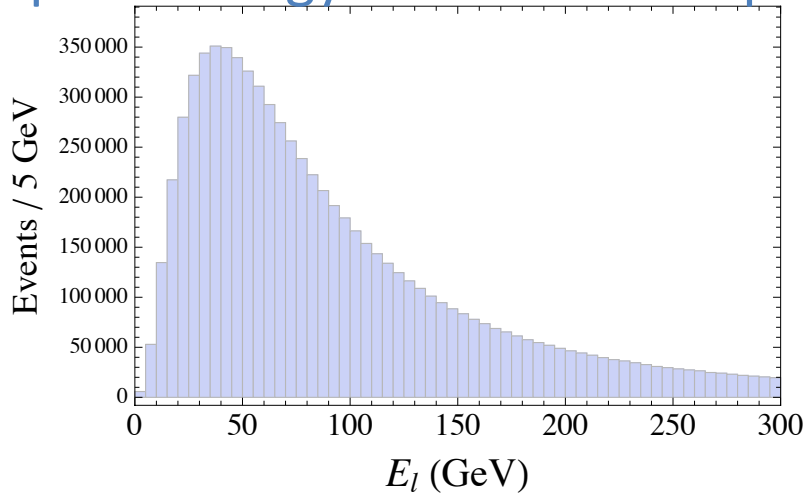
$$W(E_l, m) = \int dE \mathcal{D}_0(E; m) \frac{1}{EE_l} (\text{odd func. of } \rho) \Big|_{e^\rho = E_l/E}$$

$$(\text{odd func. of } \rho) = \frac{n \tanh(n\rho)}{\cosh(n\rho)}$$

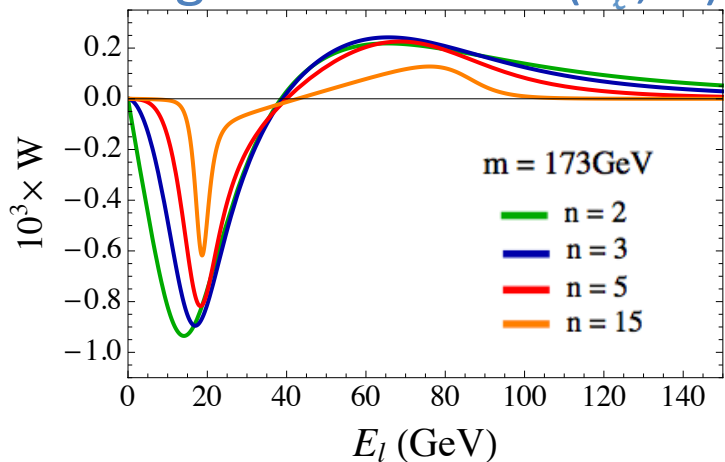
➡ Small weight at $E_l \sim 0$

Parton level analysis

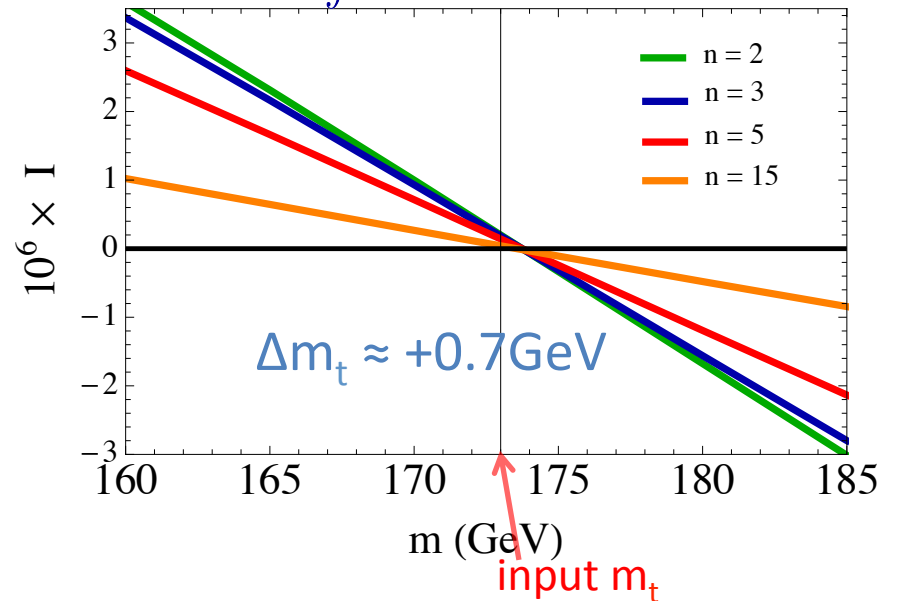
Lepton energy distribution at parton level (signal)



Weight function $W(E_l, m)$



$$I(m) \equiv \int dE_l D(E_l) W(E_l, m)$$



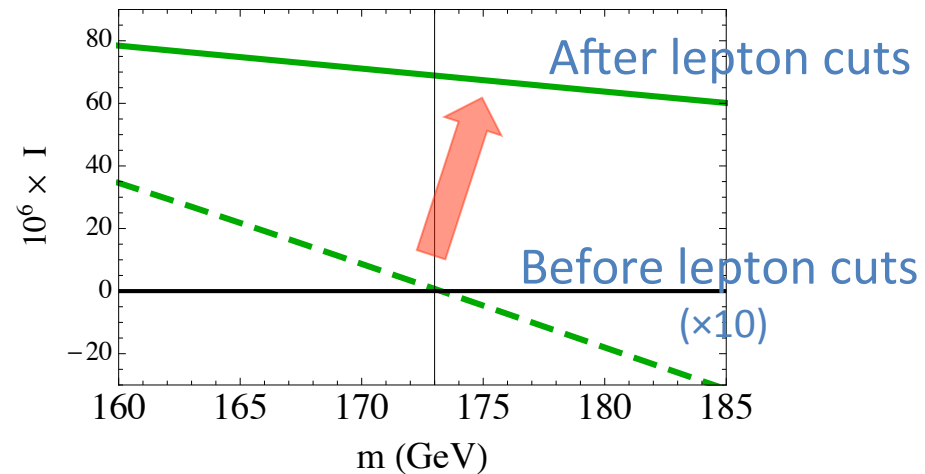
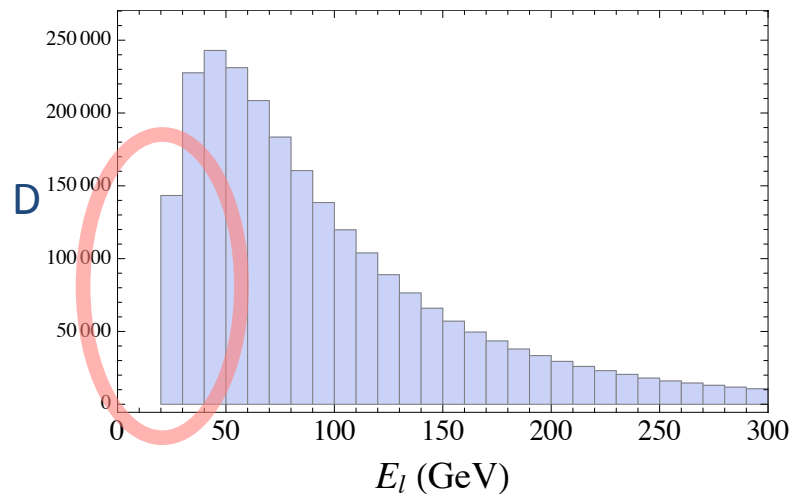
{ Effect of Γ_t : +0.34 GeV
MC stat. error : 0.4 GeV

➔ Consistent with expectation

In principle, our method works

Effect of lepton cuts

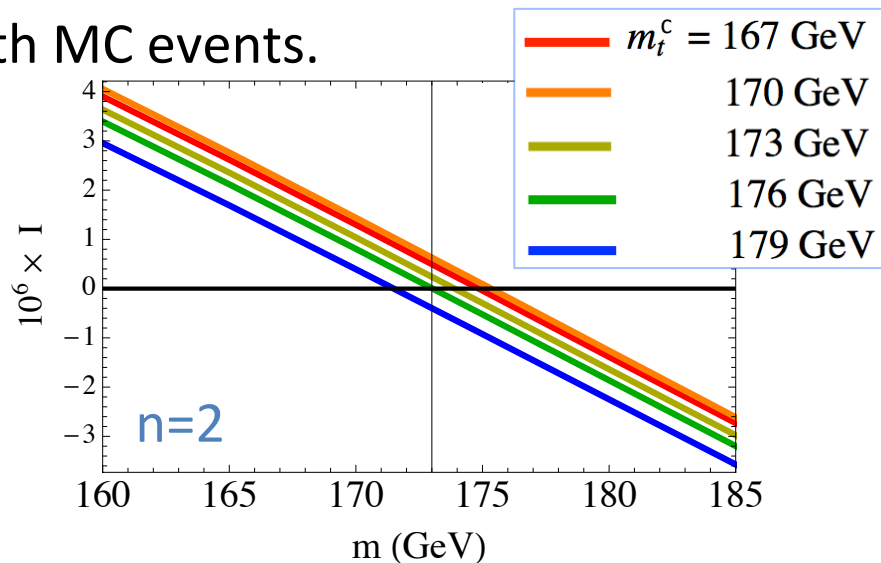
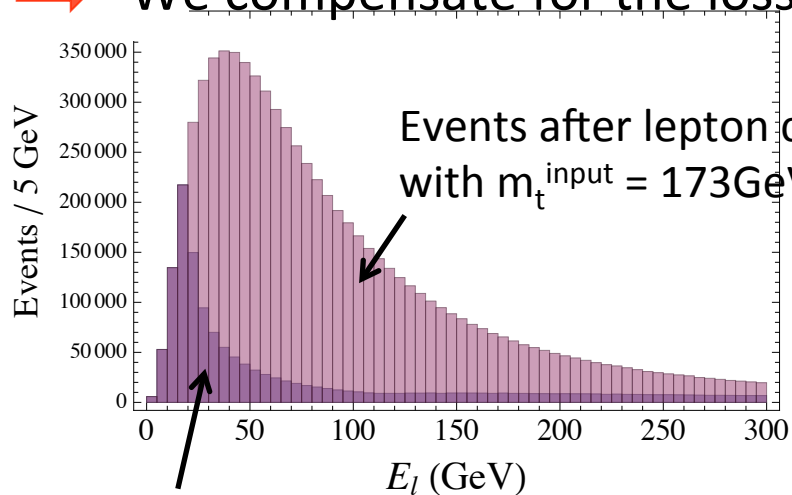
- Event selection cuts and backgrounds deform the lepton distribution.
The major effect is from the lepton cuts ($p_T(l) > 20$ GeV, $|\eta(l)| < 2.4$).



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➔ We compensate for the loss with MC events.

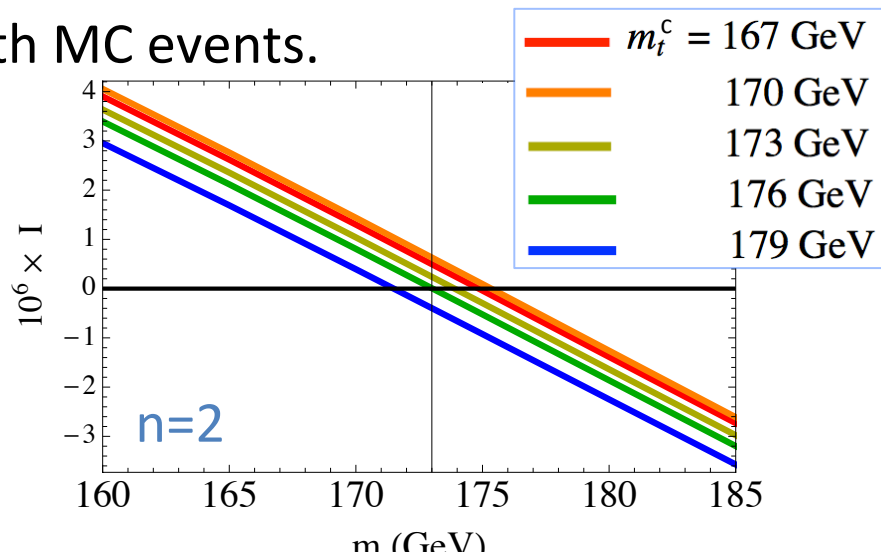
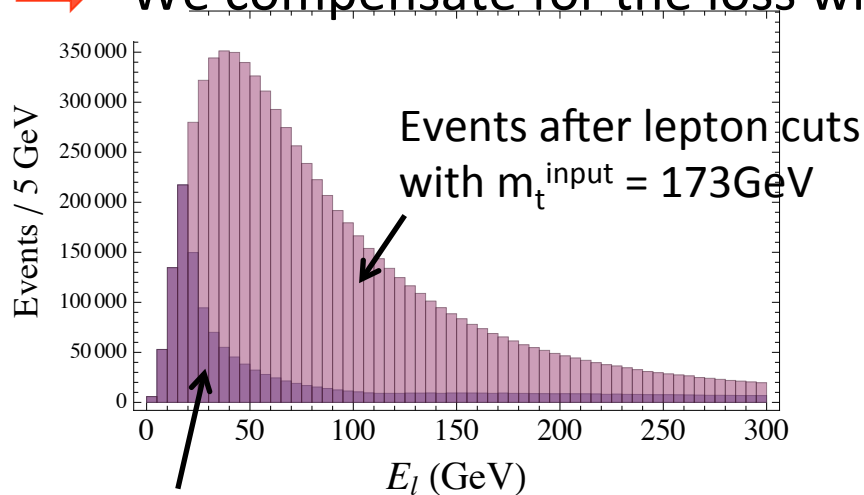


Compensated MC events with m_t^c

Effect of lepton cuts

- Event selection cuts and backgrounds deform the lepton distribution.
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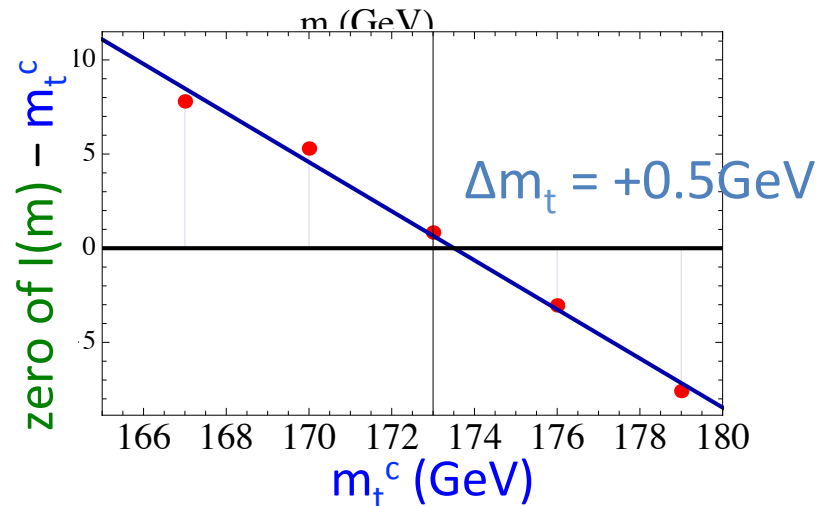
➔ We compensate for the loss with MC events.



Compensated MC events with m_t^c

$$m_t^c = m_t^{\text{input}} \Rightarrow \text{zero of } I(m) = m_t^c$$

$$m_t^c \neq m_t^{\text{input}} \Rightarrow \text{zero of } I(m) \neq m_t^c \text{ (guess)}$$

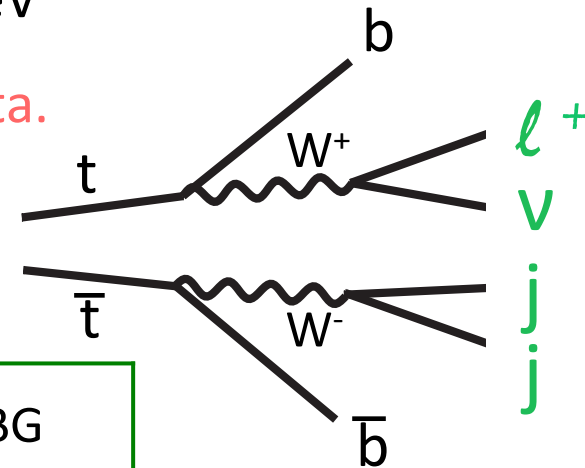


Event selection cuts

In addition to lepton cuts, we impose the following cuts to events:

- 1 muon with $p_T > 20\text{GeV}$, $|\eta| < 2.4$ (lepton cuts)
- At least 4 jets
- At least 1 b-tag with the b-tag efficiency 0.4 independent of p_T and η (in the region $p_T > 15\text{GeV}$ and $|\eta| < 2.5$)
- $p_T(j_1) > 55$, $p_T(j_2) > 25$, $p_T(j_3) > 15$, $p_T(j_4) > 8\text{GeV}$

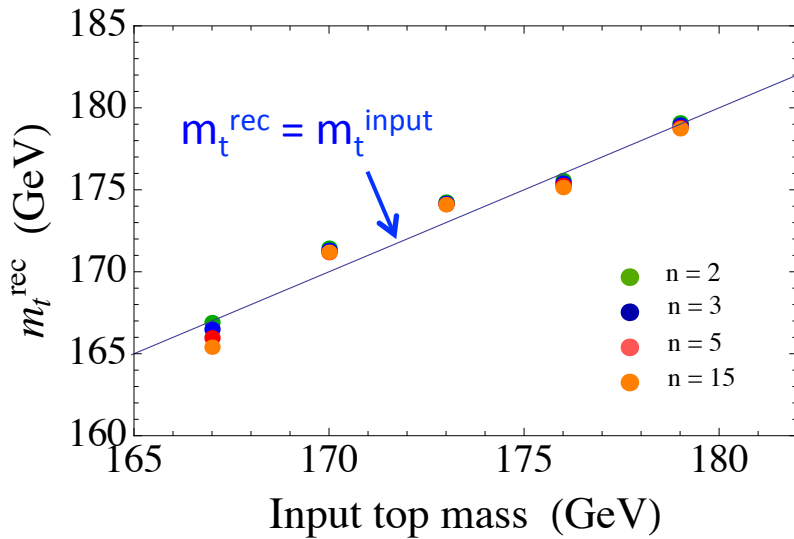
We do not use cuts concerning missing momenta.



Cross section after all cuts

Signal ($m_t=173\text{GeV}$)	W+jets BG	Other $t\bar{t}$ BG
22.4 pb	1.8 pb	5.7 pb

Sensitivity of mass determination



n=2

Input top mass(GeV)	167	170	173	176	179
m_t^{rec} (GeV)	166.9	171.4	174.2	175.6	179.1

- At 100 fb^{-1}
- Lepton(e, μ)+jets channel
- Assuming that the error of electron mode is the same as the muon mode

Uncertainties [GeV]

	Signal stat. error	μ_f scale	JES	BG stat. error
n = 2	0.4	+1.5/-1.6	+0.0/-0.1	0.4
3	0.4	+1.5/-1.5	+0.1/-0.3	0.4
5	0.5	+1.4/-1.4	+0.2/-0.4	0.5
15	0.5	+1.5/-1.3	+0.2/-0.6	0.6

Can be improved by including NLO

4. Summary and future works

- We proposed a new method to measure a theoretically well-defined top quark mass at LHC.
- We performed a simulation analysis of top mass reconstruction with lepton+jets channel at LO.
- The estimated stat. error is about 0.4GeV with 100fb⁻¹. Major systematic uncertainties are under good control.

Future works

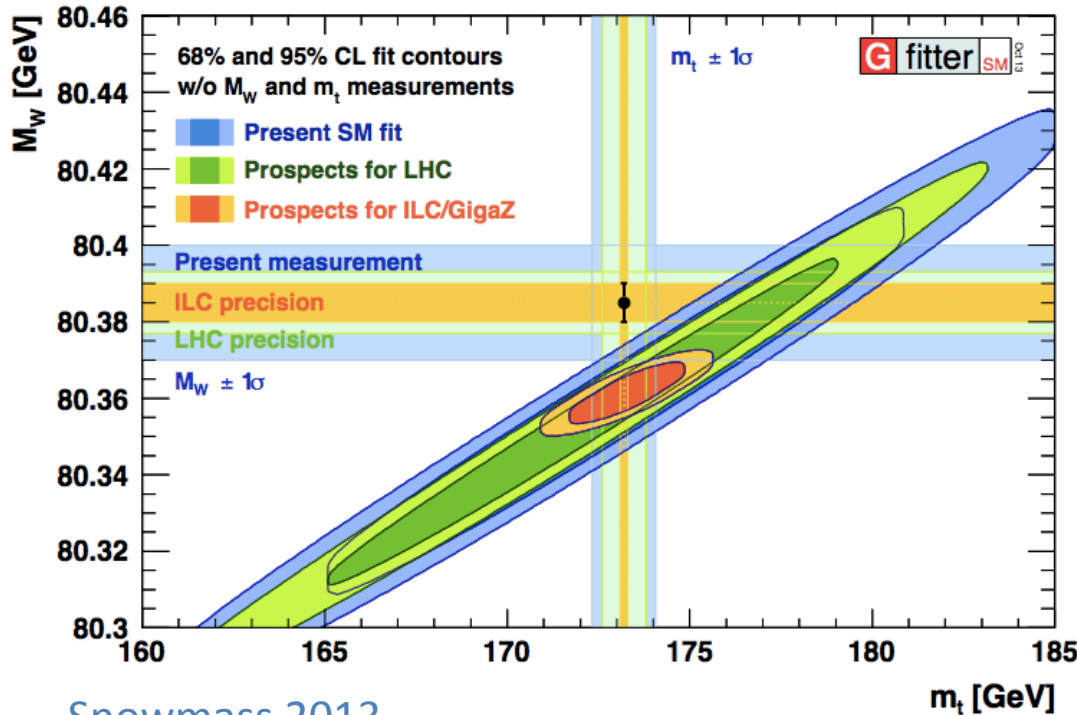
★ NLO, NNLO

Include NLO, NNLO corrections to the top decay process in weight functions. → $m_t^{\text{pole}}, m_t^{\overline{\text{MS}}}$

★ Effects of top off-shellness

backup

EW precision tests for SM

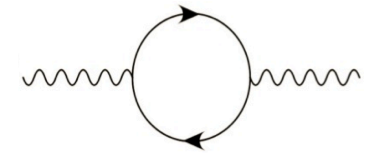


Snowmass 2013

Radiative corrections are often sensitive to the top quark mass

$$M_W^2 = \frac{\pi\alpha}{\sqrt{2}G_F \sin^2\theta_W} (1 + \Delta r)$$

Radiative corrections



$$\Delta r = \frac{\alpha}{\pi s^2} \left(-\frac{3}{16} \frac{m_t^2}{m_W^2} \frac{c^2}{s^2} + \frac{11}{48} \ln \frac{m_H^2}{m_Z^2} \right) + \dots$$

$s^2 = \sin^2\theta_W, c^2 = \cos^2\theta_W$

It is important to test for deviations from SM predictions in the EW sector

Beyond SM

For example, SUSY

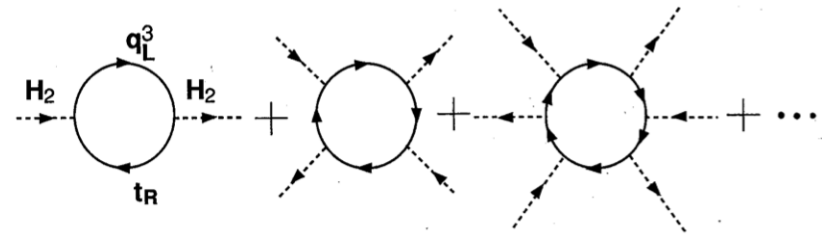
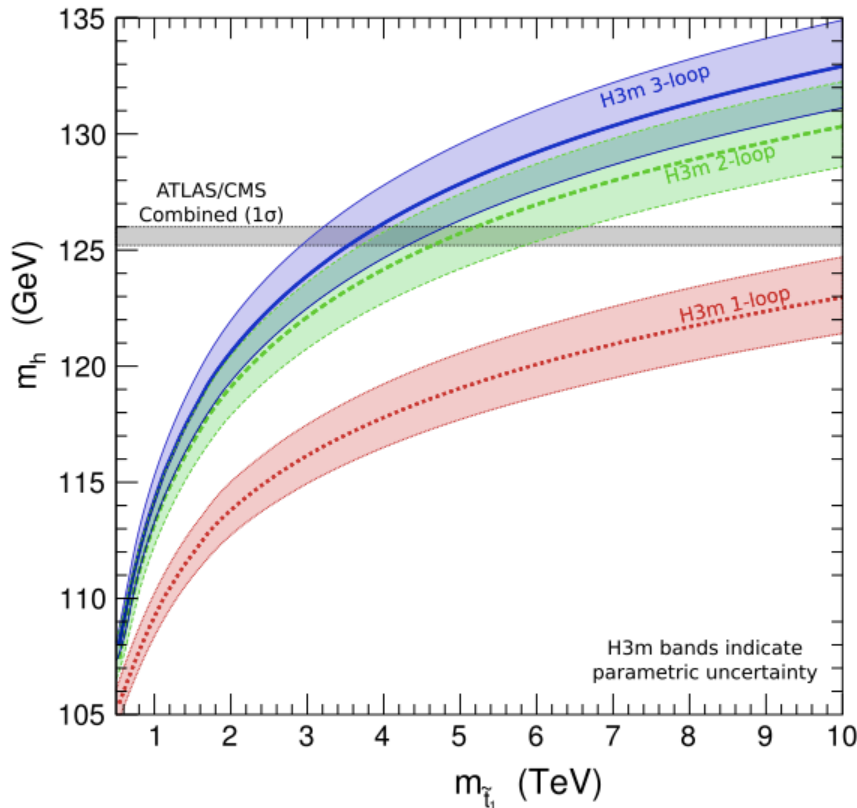


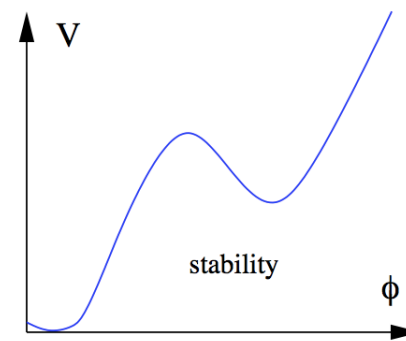
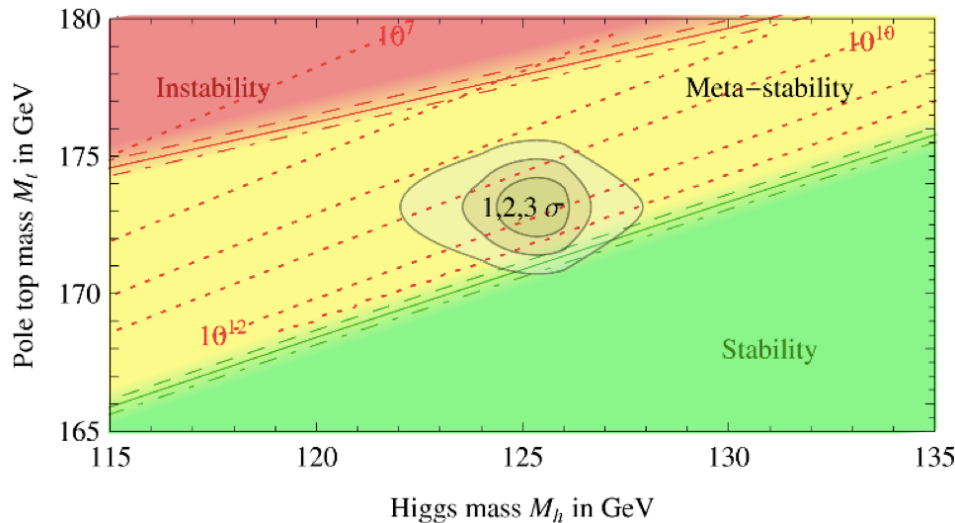
FIG. 1. The Higgs boson mass m_h from H3M at 1-, 2-, and 3-loops for nearly degenerate ($m_{\tilde{t}_L} = m_{\tilde{t}_R}$), unmixed ($X_t = 0$) top squarks, as a function of the physical mass $m_{\tilde{t}_1}$. The renormalization scale is fixed to $M_S = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$, we set $\tan \beta = 20$, $\mu = 200$ GeV, all other sfermion soft parameters equal to $m_{\tilde{t}_{L,R}} + 1$ TeV, and assume gaugino mass unification with $m_{\tilde{g}} = 2$ TeV. The thickness of the bands indicates the parametric uncertainty from the uncertainties in $m_t^{\text{pole}} = 173.3 \pm 1.8$ GeV and $\alpha_s(m_Z) = 0.1184 \pm 0.0007$; it is **dominated by the m_t^{pole} uncertainty**. The horizontal bar is the experimentally allowed range $m_h = 125.6 \pm 0.4$ GeV.

Feng et al., PRL 111,131802(2013)

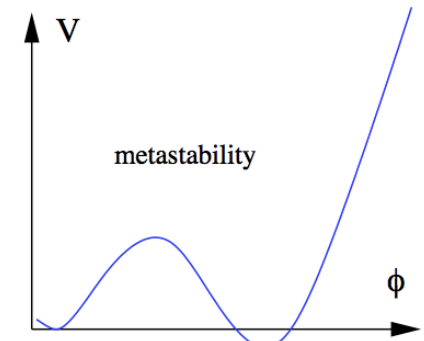
The predictions beyond the SM often depend strongly on the value of the top quark mass.

SM vacuum stability at the Planck scale

Is the SM vacuum stable or metastable ?



Stable



Meta-stable

Type of error	Estimate of the error	Impact on M_h
M_t	Experimental uncertainty in M_t	± 1.4 GeV
α_s	Experimental uncertainty in α_s	± 0.5 GeV
Experiment	Total combined in quadrature	± 1.5 GeV
λ	Scale variation in λ	± 0.7 GeV
y_t	$\mathcal{O}(\Lambda_{\text{QCD}})$ correction to M_t	± 0.6 GeV
y_t	QCD threshold at 4 loops	± 0.3 GeV
RGE	EW at 3 loops + QCD at 4 loops	± 0.2 GeV
Theory	Total combined in quadrature	± 1.0 GeV

Degrassi et al. '12

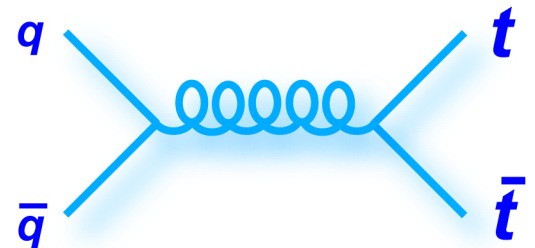
Precision measurements of m_t , α_s , m_H are needed.

Top quarks at Tevatron and LHC

Tevatron

- $p\bar{p}$ collider
- discovered the top quark in 1995
- shut down in 2011

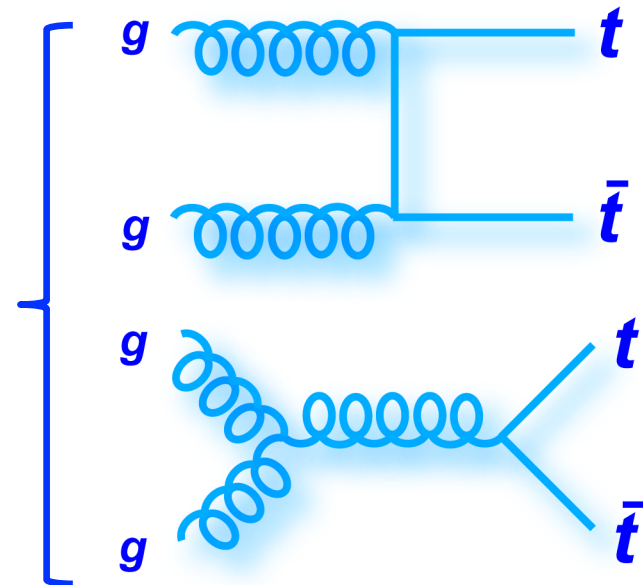
Main production process
: $t\bar{t}$ pair production



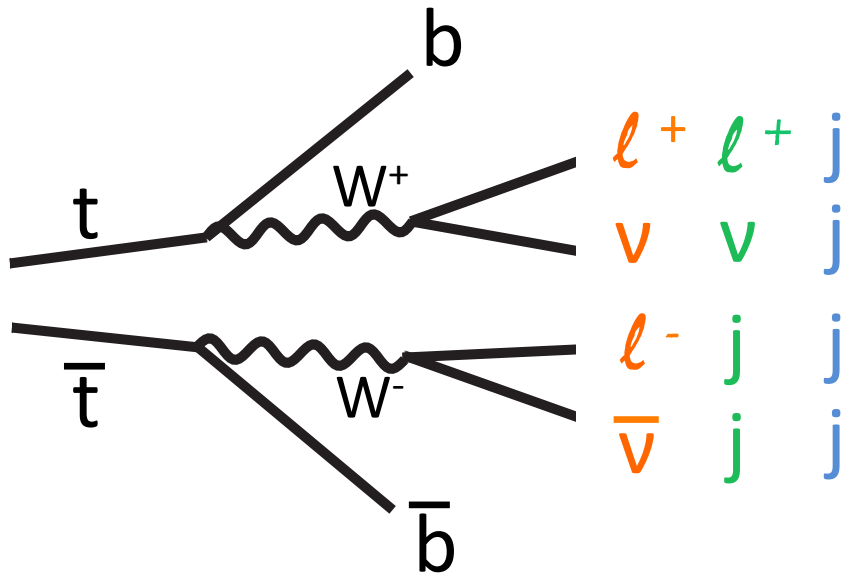
LHC

- pp collider
- ended data taking at $\sqrt{s} = 7, 8\text{ TeV}$
- restart in 2015 at $\sqrt{s} = 13/14\text{ TeV}$

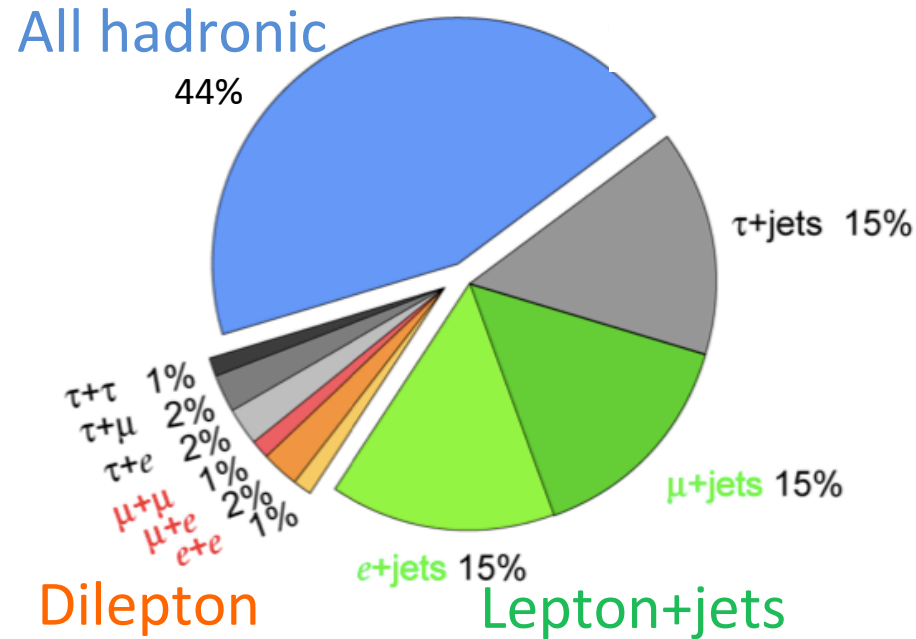
$\sigma(t\bar{t}) \sim 900\text{pb}$
($\sqrt{s} = 14\text{ TeV}$)



$t\bar{t}$ decay channel



$t\bar{t}$ branching ratio



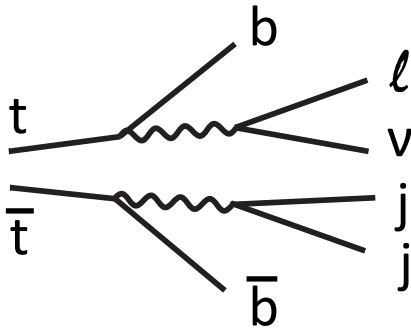
	Cross section	S / N
Dilepton	Small	Very good
Lepton+jets	Medium	Good
All hadronic	Large	Not good

Main method to measure the top mass

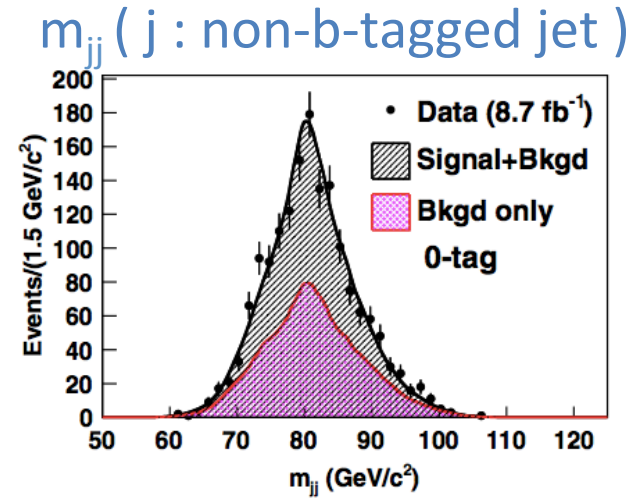
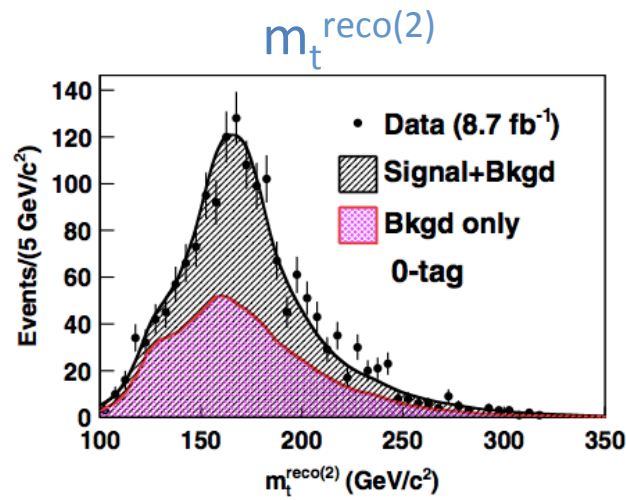
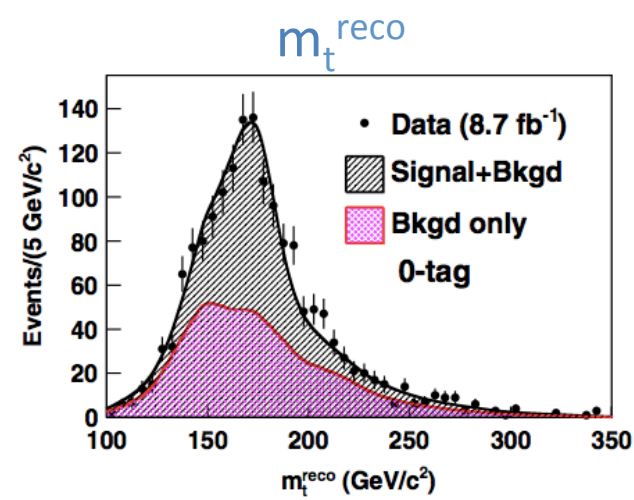
Template Method

: compare some distributions of data with templates from MC

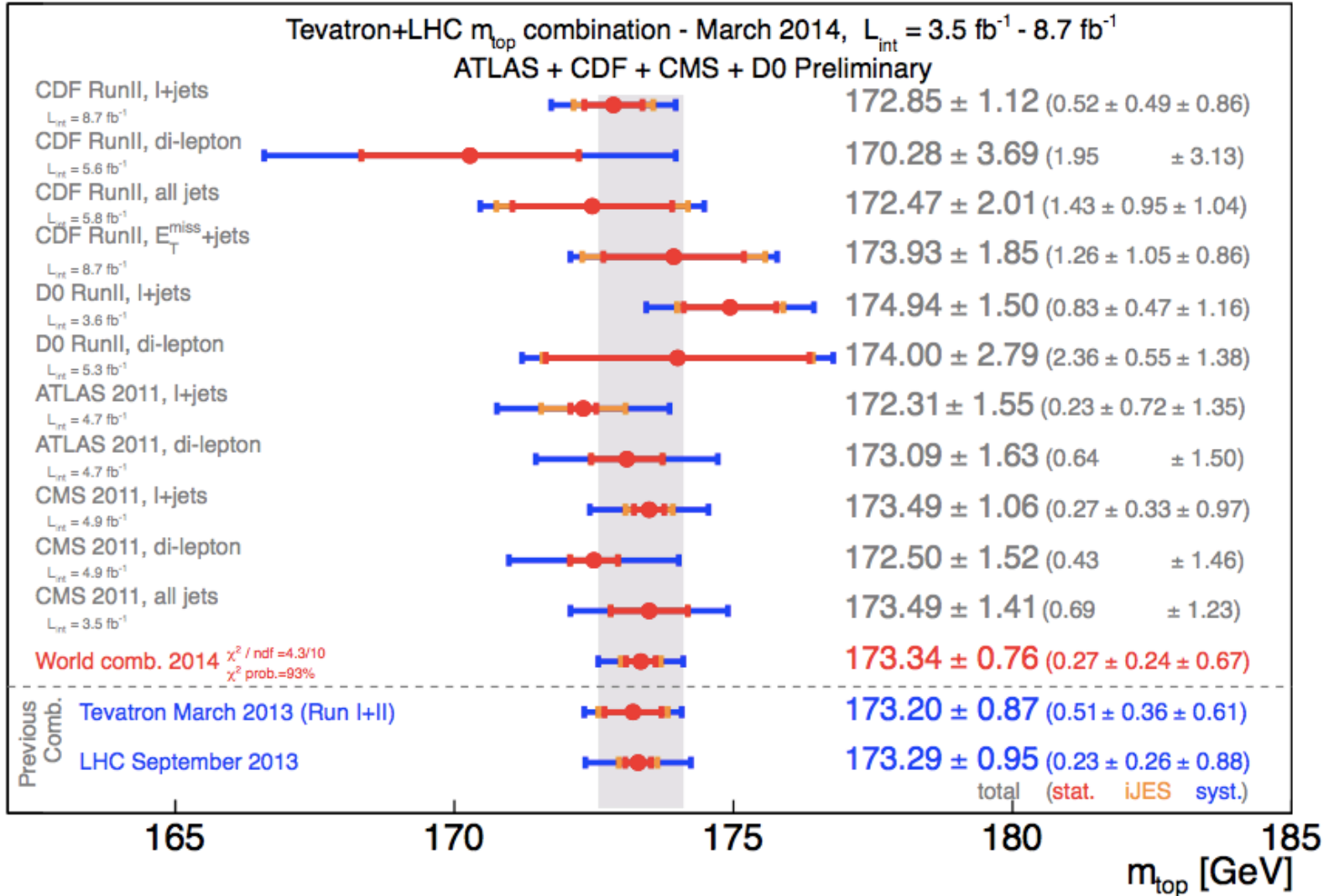
Example



1. Reconstruct invariant top mass (m_t^{reco} , $m_t^{\text{reco}(2)}$) and invariant W mass (m_{jj}) in each event
2. Create MC distributions at discrete values of m_t and correction factor of JES (**templates**), and interpolate
3. **Fit the MC distribution to data**



Tevatron and LHC m_t combination



Top quark mass?

- m_t^{Pythia} : Not a parameter defined in perturbative theory

- m_t^{pole} : Top quark has a color, so the physical on-shell quark cannot exist

➔ Far from a fundamental param.

$$\frac{1}{\not{p} - m_0 - \Sigma(p, m_0)} = \frac{c}{\not{p} - m}$$



$$m = m(\mu) \left(1 + \alpha_s(\mu) d^1 + \alpha_s^2(\mu) d^2 + \dots \right)$$

- $m_t^{\overline{\text{MS}}}$: Short-distance mass
Free from IR contamination

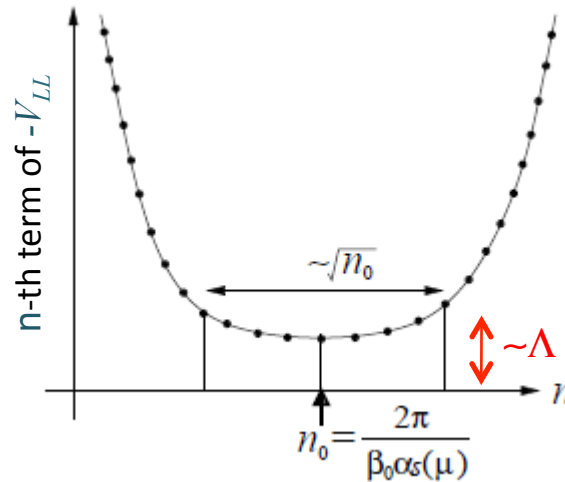
$$m_0 = m(\mu) \left(1 + \frac{\alpha_s}{\pi} \left[\frac{1}{\epsilon} \right] \right)$$

➔ Known as **a good parameter in pert. QCD**

Important to determine $m_t^{\overline{\text{MS}}}$ accurately

Pole mass and $\overline{\text{MS}}$ mass

From Sumino-san's slide

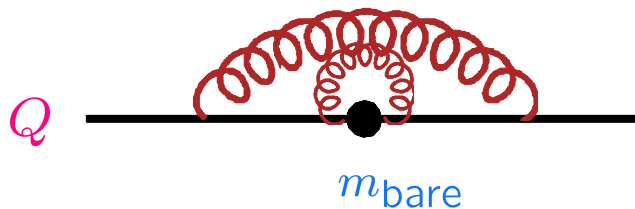


$$m_{\text{pole}} \simeq m_{\overline{\text{MS}}}(\mu) + \frac{1}{2} \int_{q < \mu} \frac{d^3 \vec{q}}{(2\pi)^3} C_F \frac{4\pi \alpha_{1L}(q)}{q^2}$$

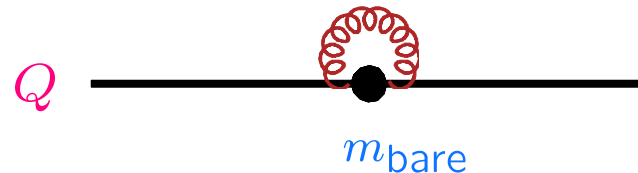
Pole mass m_{pole}

$\overline{\text{MS}}$ mass $\bar{m} \equiv m_{\overline{\text{MS}}}(m_{\overline{\text{MS}}})$

$$0 < \lambda_g < \infty$$



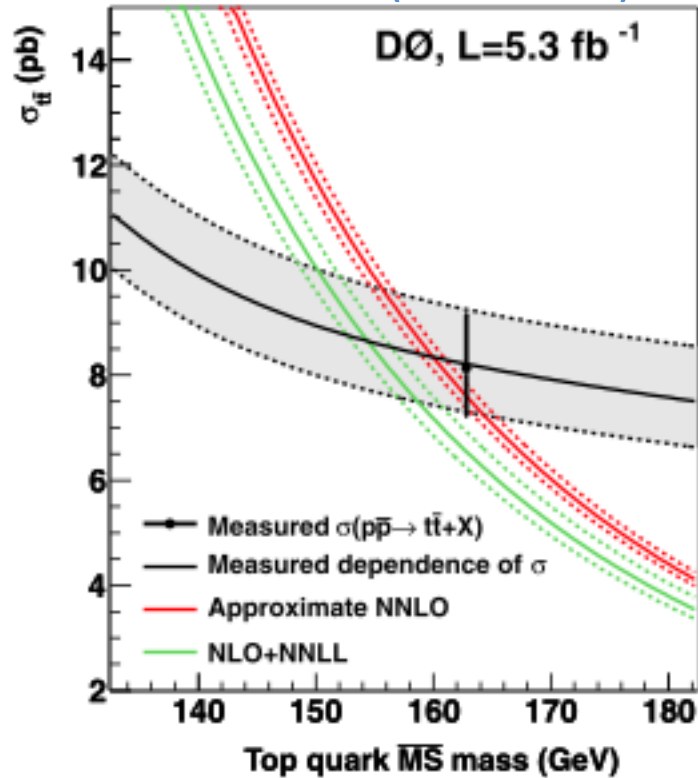
$$0 < \lambda_g < 1/\bar{m}$$



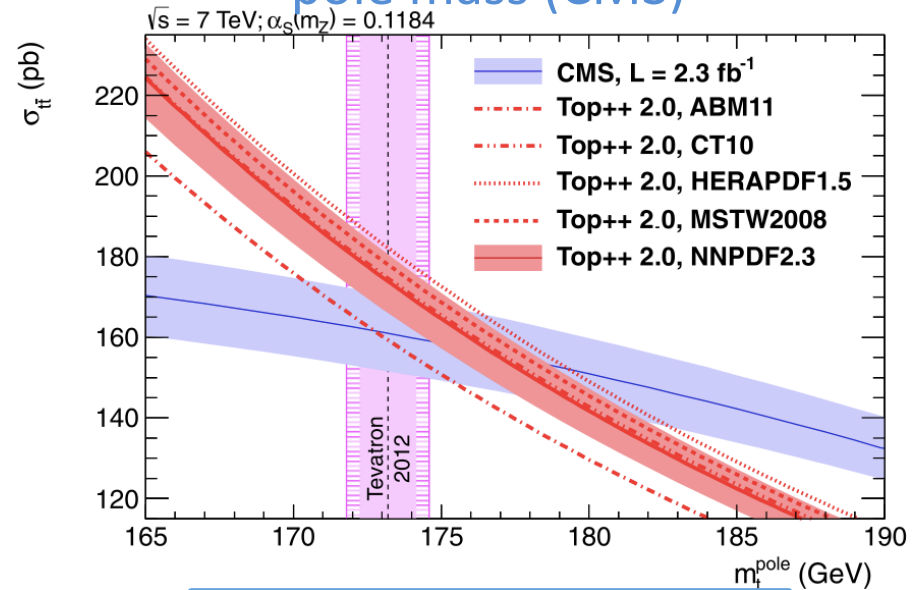
Measurement of \overline{MS} mass (and pole mass)

(from $t\bar{t}$ cross section)

\overline{MS} mass (Tevatron)



pole mass (CMS)



$$m_t^{pole} = 176.7^{+3.8}_{-3.4} \text{ GeV}$$

$$m_t^{\overline{MS}} = 160.0^{+5.1}_{-4.5} \text{ GeV}$$

$$m_t^{\overline{MS}} = 154.5^{+5.2}_{-4.5} \text{ GeV}$$

The measured \overline{MS} mass still has a large error

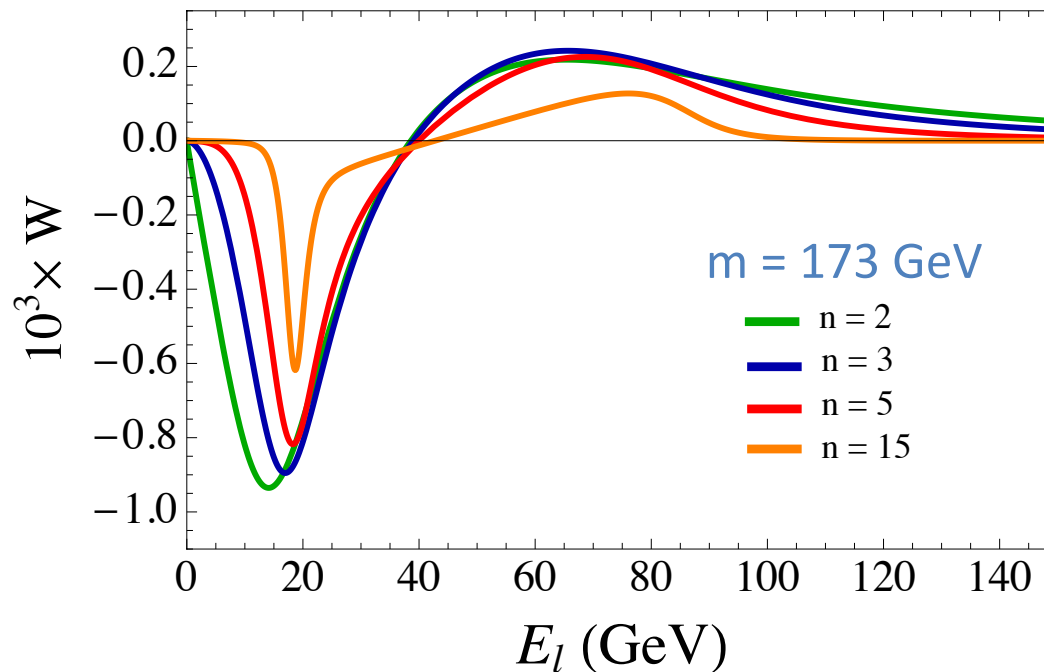
- The sensitivity of $\sigma_{t\bar{t}}$ to m_t is not so strong
- Theoretical uncertainties $\sim 1.5 - 2 \text{ GeV}$

Examples of weight functions

For a top quark decay : $t \rightarrow Wb \rightarrow \ell \nu b$

$$W(E_l, m) = \int dE \mathcal{D}_0(E; m) \frac{1}{EE_l} (\text{odd func. of } \rho) \Big|_{e^\rho = E_l/E}$$

for (odd func. of ρ) = $n \tanh(n\rho) / \cosh(n\rho)$



$$W(E_l, m) = \int dE \mathcal{D}_0(E; m) \frac{2nE_l^{n-1} E^{n-1} (E_l^{2n} - E^{2n})}{(E_l^{2n} + E^{2n})^2}$$

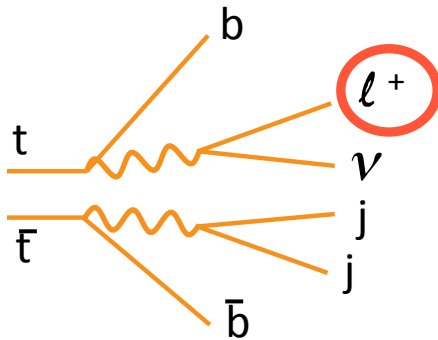
Polarization of top at LHC

Top quarks in $t\bar{t}$ production at the LHC

$$\frac{1}{\sigma} \frac{d\sigma}{d \cos \theta_{lt}} = \frac{1}{2} (1 + B \cos \theta_{lt}) \quad \text{longitudinal polarization}$$

$$\left\{ \begin{array}{l} \text{SM: } B \sim 0.003 \quad \text{W.Bernreuther, Z.-G.Si, PLB 725(2013)115} \\ \text{ATLAS: compatible with zero } (\Delta B \sim 0.08) \end{array} \right.$$

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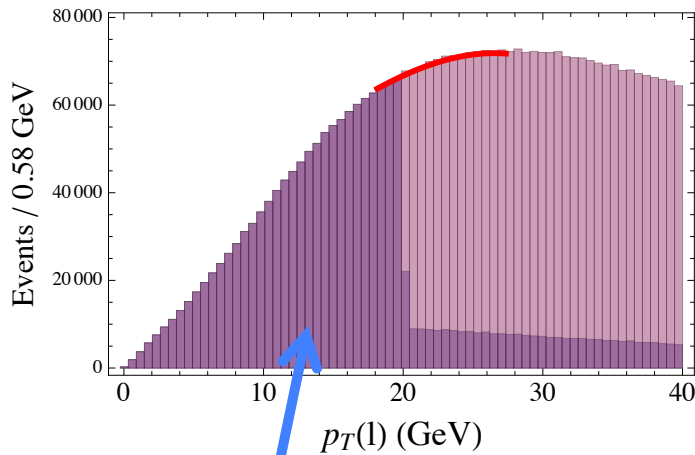
Effects of lepton isolation and photon emissions

These effects are well understood, it should be possible in principle to estimate them and restore the parton-level lepton distributions.

- ➔ We assumed that they can be estimated and restored completely for signal events in this analysis.
 - Estimate of lepton isolation
 - ◆ Isolation cone angle $\rightarrow 0$
 - ◆ Compensate for the loss caused by the lepton isolation
 - Effect of photon emissions
 - ◆ Can include it into weight function by calculating the lepton distribution with the effect

Compensating method

We **compensate** for the loss ($p_T(l) < 20\text{GeV}$ or $|\eta(l)| > 2.4$) using MC events.



Compensated MC events

To fix the normalization of the compensated events, perform a χ^2 fit so that $p_T(l)$ distribution are connected smoothly.

We do not use detailed knowledge on the global shape of the distribution

➔ As a result, we obtain a good feature that $l(m)$ does not depend strongly on the top quark mass of the compensated events.

We expect that we can improve the quality of the fit.

(fitting function, correction to the p_T distribution, raise lepton p_T cut)

Uncertainties from compensated events

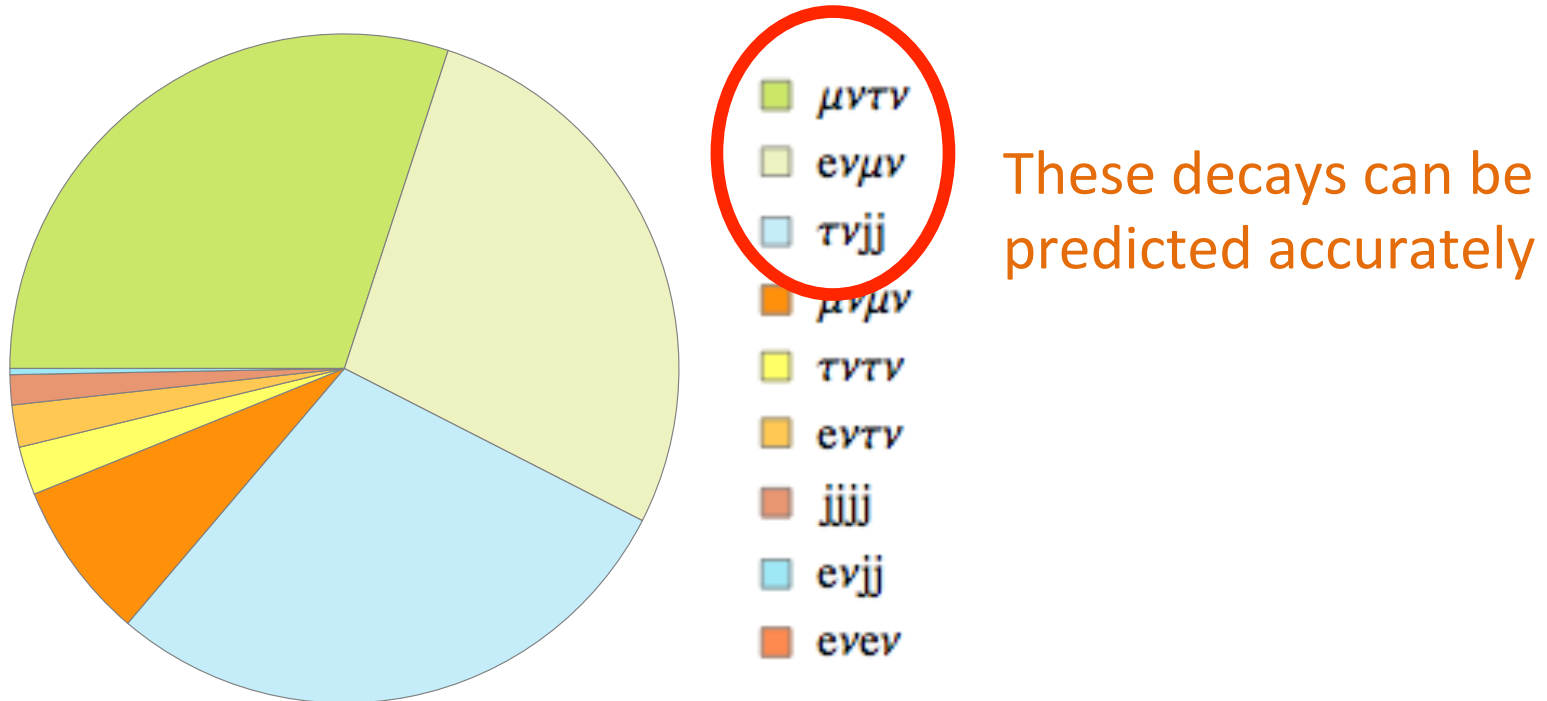
MC lepton energy and p_T distributions with $p_T(l) < 20\text{GeV}$ or $|\eta(l)| > 2.4$

Possible uncertainties

- μ_F scale uncertainties
 - ➔ We have estimated in this analysis
- PDF model
 - ➔ Future work?

We found that $I(m)$ does not depend strongly on the top mass m_t^c of the compensated events.

Other $\tau\bar{\tau}$ BG after cuts



- $\mu\nu\tau\nu$, $e\nu\mu\nu$: Can be included in the signal events in principle
- **Muon from tau decay** : Can be regarded as a signal process by including the contribution of the muon energy distribution into weight functions

Flat b-tagging efficiency

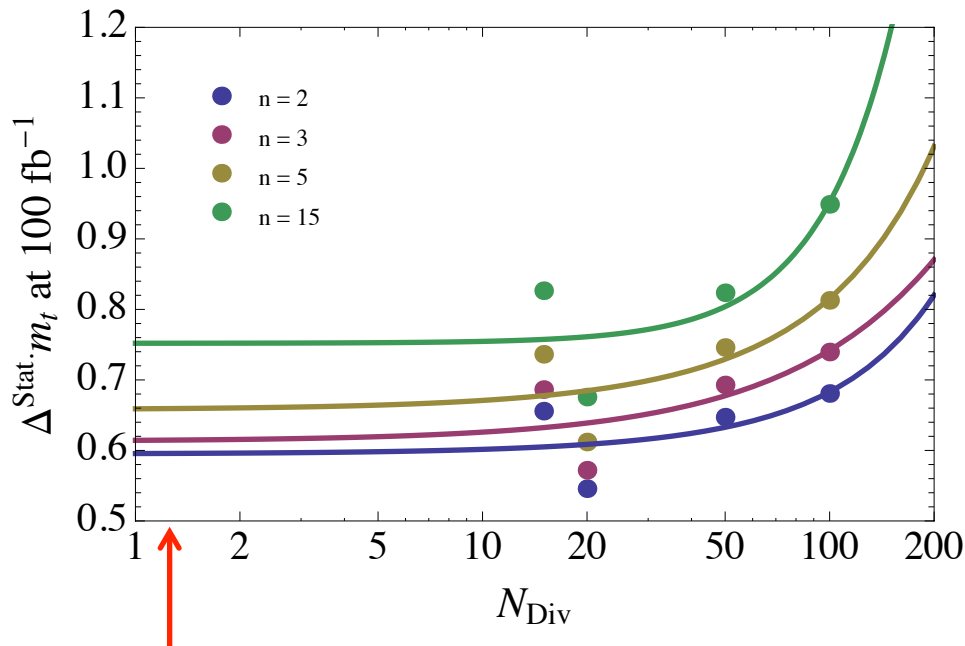
At least 1 b-tag with the b-tag efficiency 0.4 independent of p_T and η (in the region $p_T > 15\text{GeV}$ and $|\eta| < 2.5$)

- This flat efficiency would be attainable in experiments in principle.
- If a b-tagging efficiency $\varepsilon(p_T, \eta)$ can be estimated with a good accuracy, multiplying the lepton energy distribution by $\varepsilon(p_T, \eta)^{-1}$ can be an alternative way.

Signal statistical error

$m_t^{\text{input}} = 173\text{GeV}$

We divide the generated events into 15, 20, 50 and 100 subgroups of equal sizes and perform the top mass reconstruction.



Results of fits depend on the number of events

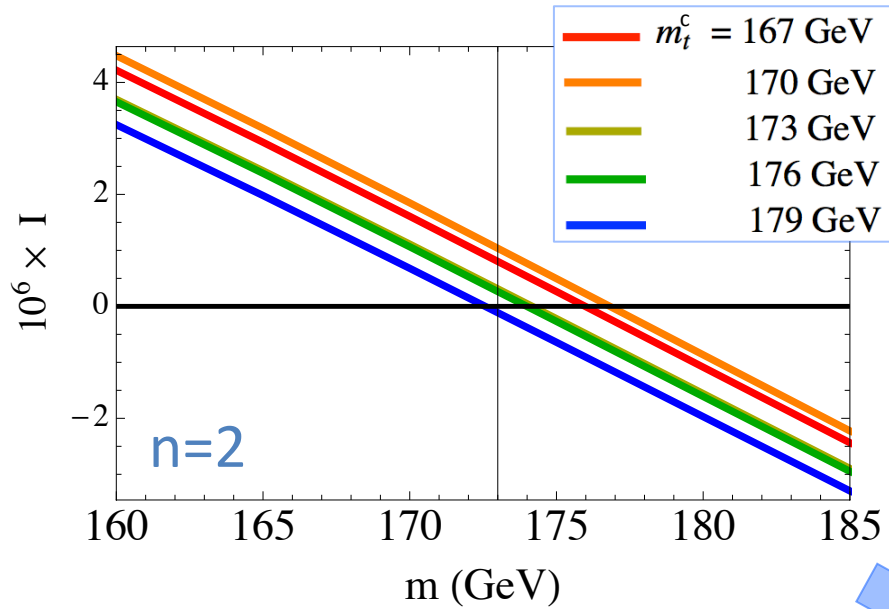
We extrapolate statistical errors at the number of events for 100fb^{-1}

N_{Div} corresponding to the number of events with $100 \text{ fb}^{-1} = 1.2$

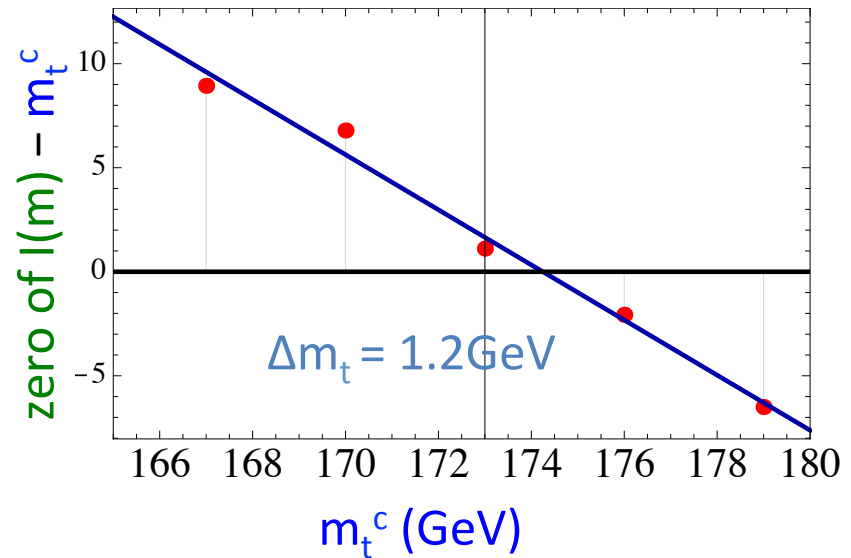
	n=2	n=3	n=5	n=15
only μ mode	0.60	0.61	0.66	0.75
semi-leptonic channel	0.42	0.43	0.47	0.53

Top mass reconstruction (after event selection cuts)

The weighted integrals with various m_t^c



$m_t^c = m_t^{\text{input}} \Rightarrow \text{zero of } I(m) = m_t^c$
 $m_t^c \neq m_t^{\text{input}} \Rightarrow \text{zero of } I(m) \neq m_t^c \text{ (guess)}$



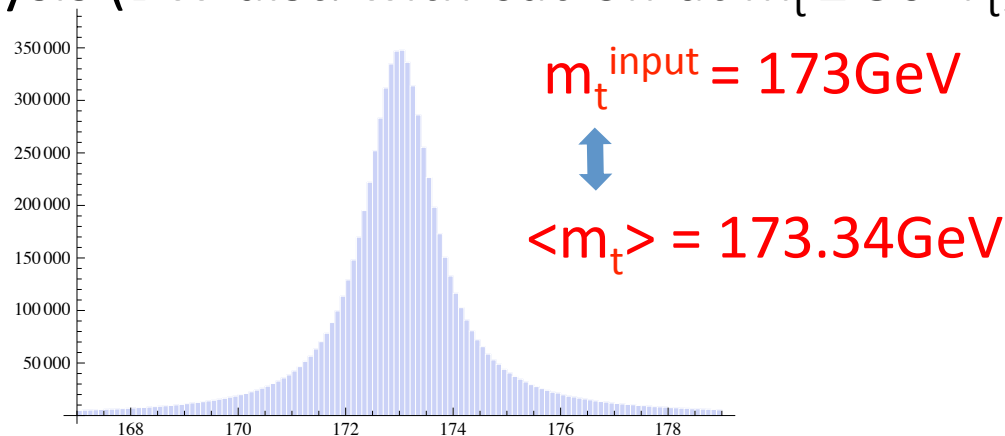
Effects of top width and MC stat. error

Effects of top width

The weight function method assumes that the top quark is on-shell

➔ We should include corrections for off-shellness of the top quark

In our analysis (BW dist. with cut-off at $m_t \pm 50 \cdot \Gamma_t$)



Top invariant mass distribution at parton level

MC statistical errors after all cuts [GeV] ($m_t^{\text{input}} = 173 \text{ GeV}$)

n = 2	3	5	15
0.5	0.5	0.5	0.7

Future work: NLO and NNLO

- Lepton energy distribution in the rest frame of the top quark at NLO, NNLO
 - ➔ Weight function at NLO, NNLO
- MC simulator with NLO top decay (in progress)
 - ➔ Compensated events at NLO
- MC simulator with NNLO top decay
 - We can obtain parton level lepton distributions using the lepton dist. in the rest frame of top quark at NNLO and velocity dist. of top quarks from MC.
 - ➔ Compensated events at NNLO
- MC simulator with NLO top production
 - ➔ Reduce uncertainties due to μ_F scale dependence

Future work: top off-shellness

Incorporate the effects of off-shellness of the top quark into weight function:

- Contributions from diagrams with top quark


Use the superposition of the lepton distribution in the top rest frame with the weight of top invariant mass distribution

- Contributions from diagrams without top quark (Irreducible background)

Define “the rest frame of the top quark” as the center-of-mass frame of b and W boson, and incorporate this contribution into the lepton dist.

We do not know whether these work. At least, we can estimate corrections caused by top off-shellness effects.

Future work: $\overline{\text{MS}}$ mass

- Determination of m_t^{pole}  $m_t^{\overline{\text{MS}}}$
3-loop relation

- Direct determination of $m_t^{\overline{\text{MS}}}$

Lepton energy dist. in the top rest frame with $m_t^{\overline{\text{MS}}}$

α_s expansion is not a good approximation
in a part of phase space

 But good for the weighted integral $I(m)$?

Also, Including effects of

- final state radiation,
- top off-shellness,

the convergence will improve.